CHAPTER IV

A SEMI-IMPICIT THREE DIMENSIONAL MULTINESTED GRID PRIMITIVE EQUATION NUMERICAL MODEL FOR SIMULATION OF TROPICAL STORM

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4.1. Introduction

In spite of the fact that necessary climatological and geographical conditions for the formation of tropical storm prevail over large areas of the earth during storm seasons, actual appearance of storms is a relatively rare phenomenon. According to statistics, only about 80 tropical storms with maximum sustained wind 40-50 kts are observed per year over the entire globe; of these, between one half and two third attain hurricane strength. This indicates that there must be a rare coincidence of circumstances before development of a storm. Very little is yet known about detailed mechanism of its incipient stage of development. The formation always occurs in connection with some kind of pre-existing disturbance associated with a deep cloud layer. All of these disturbances do not intensify into tropical storm. Only a small percentage of these systems starts intensifying. Numerous studies have been made to clarify the process of their formation; but no general mechanism has yet been accepted. The appearance of an upper level (500 - 200 mb) warm area during its formation is commonly observed.

Palmen (1948) pointed out that the extremely low sea level pressure in the centre of a fully developed tropical storm far exceeds the values that could be caused by hydrostatic effect of the temperature rise in the central cloud free area due to release of latent heat of condensation.

Gray (1977) states that direct cumulus cloud induced enthalpy change due to condensation processes is probably too small to explain increase needed for initial storm genesis and intensification. Condensation energy is almost wholly expended in suppor-
ting the cluster’s vertical motion. The magnitude of this energy appears not to be fundamental potential for transformation to a storm. The various types of CISK (Conditional Instability of Second Kind) parameterization are directly linked with the amount of frictional convergence in the boundary layer. These are not likely very relevant to pre-storm disturbance maintenance and intensification. Arnold (1977) and Gray (1977) have observed that the low level circulation centres of growing cloud clusters are often found outside the convective regions. They hypothesise that the upper tropospheric outflow from the cluster is blocked by the large scale flow and forced to subside. The subsidence warms the air and hydrostatically lowers the surface air pressure. This surface pressure fall is larger than that which could result from direct cumulus heating alone. Gray (1979) considers a dynamical forced subsidence to be a necessary requirement for the genesis. Yanai (1963) states that weak absolute vorticity at upper levels together with warming of middle tropospheric air is an almost necessary and sufficient condition for the development of a wave disturbance into typhoon.

Surface friction in presence of low-level vorticity produces upward motion in region of positive vorticity and sinking motion where relative vorticity is negative. The magnitude of frictional convergence is approximately proportional to the relative vorticity (Anthes, 1982). Charney and Eliassen (1974) state that friction performs a dual role. It acts to dissipate kinetic energy, but because of the frictional convergence in the moist surface boundary layer, it also acts to supply latent heat energy to the system. The dual role of surface friction as discussed by
them and numerical experiments performed by Ooyama (1969), Yamasaki (1968) and Sundqvist (1970) were attempts to simulate tropical cyclone through parameterization of the diabatic heating due to cumulus convection in such a way that convective heating is proportional to the cyclone scale horizontal mass convergence in the friction layer. Numerical experiments of tropical cyclones have also been made by Rosenthal (1969), Anthes et al. (1971), Mathur (1974), Kurihara and Tuleya (1974) and many other workers in the same line.

Yamasaki (1977) has demonstrated that surface friction is not important at the early stage of development of a tropical storm when its vorticity is not large. With a two-dimensional model, he has shown that (i) a disturbance may develop even when surface friction does not exist, and (ii) surface friction is one of the important factors to determine the horizontal scale of the disturbance. He also pointed out that it is uncertain whether a CISK mode exists in the actual atmosphere.

One of the main objectives of this study is to make an attempt to understand the role of surface friction as pointed out by Yamasaki (1977) based on the hypothesis of Arnold (1977), Gray (1977) and Yanai (1961) for the initial development of a tropical storm. For this purpose preliminary numerical experiments have been accomplished on a three-dimensional multi-nested grids varying coefficient of surface drag and imposing artificial heating through an analytical function considered as the effect of forced subsidence.

Filtered models do place a definite limit on the accuracy of forecast. The motion in the tropics, and particularly in tropical
storm is essentially ageostrophic; hence, they can only be represented by the primitive equation. In the primitive equation model, after elimination of acoustic wave propagation through incompressibility assumption, does include inertia-gravitational wave as the fastest moving mode as a solution. For this reason, the time steps in primitive equation forecast must be considerably shorter than those allowed for filtered model with equal grid length, as it has no gravity wave solution, and depends upon the maximum wind speed and grid length. Although, the use of an implicit method removes the dependance of the time step on grid length, it is difficult to apply it to a non-linear system of equations. Also there is undesirable damping of physical modes in many implicit schemes.

The numerical simulation of synoptic and mesoscale phenomena of the atmosphere, such as frontal wave, tropical storm etc. requires a grid with very small mesh size. Due to limitation of computing capacity and time, it is extremely difficult to fill the entire region with such a fine mesh and handle it. In order to overcome these difficulties a "nested grid" was first used by Hill (1968) for quasi-geostrophic model. The system consists of a limited fine mesh area in the region of interest embedded within a larger domain of coarse mesh. Multiple grids are also used stepwise so as to reduce the grid size less suddenly and are called the "telescopic grid".

One of the most crucial problem in nesting of grid is how to connect the solutions from various meshed grids. Matsuno (1966) and Browning et al. (1973) found that wave motions in two unequal meshes have different phase speeds due to different truncation
errors, and as a result numerical problem usually develops.

The applications of multinested grid in problems of tropical storms have been presented by several authors. Mathur (1974) was the first to use two way interactive grids for the simulation of a real hurricane. The results of his integration show that the movement and rate of intensification of a weak depression into a hurricane agree fairly well with observations for hurricane Isbell (1964). For simulation of hurricane, works of Madala and Piacses (1975) and Jones (1977) and for real data forecast, works of Ley and Elsberry (1976) and Ookochi (1978) are also informative. The results of these studies have provided with useful information and encouragement to the present study. In our limited area model, we have used triply-nested grids and semi-implicit scheme for time integration.

In semi-implicit scheme for integration, the time steps may be increased significantly. Robert et al. (1972) have extended this scheme to multilevel primitive equation model. Chen and Miyakoda (1974) pointed out that in semi-implicit scheme the speed of integration is about four times faster than the leapfrog method for non-nested grid problem, and is about eight times faster than Euler backward method of explicit scheme. For the nested grid calculation, this advantage is even greater.

But while adopting semi-implicit scheme coupled Helmholtz equations for one of the appropriate variables are to be solved in each layer at each time step offsetting some of the computer time savings achieved by larger time step. To retain the advantage of this technique, an efficient method is necessary to solve the Helmholtz equations. The commonly used iterative method such
as successive over relaxation (SOR) is fast for diagonally dominant equations; however, it converges slowly or may even diverge for the weakly or non-diagonally dominant equations. In SOR method the optimal value of the overrelaxation coefficient depends on the number of grid points, specific form of the coefficients of the equation and error distribution. A theoretical estimate may be made only for a simplified form of equation. As such, one must resort to a sort of trial and error method to determine it (Sturat and O'Neill, 1967). The coefficient matrix of the Helmholtz equations obtained in this study is not symmetric or diagonally dominant at each level. Moreover, the equations are coupled among levels. So to solve these equations, we first used Stabilized Error Vector Propagation method (SEVP:Madala, 1978), an efficient direct solver. Simultaneously, we hunted for other superior technique, and ultimately have developed an iterative method which we call "Simultaneous Multi-level Relaxation (SMR)"; as during iteration, improvements over the previous values on all levels for any grid point \((i,j)\) are made simultaneously. This method took lesser time than SEVP method.

Other important purposes of the present study are to examine the computational capabilities of the proposed multinested grid, advantage of the semi-implicit scheme, and also testing of the finite difference scheme for the non-linear terms.

4.2. Design of the model

The numerical model used in this study is a three-dimensional primitive equation model. It contains three layers in the vertical (Fig. 4.1). The lowest layer is the surface boundary layer.
The hop layer contains most of the upper troposphere which can be called the outflow layer. The middle layer is characterized by strong tangential velocities with no pronounced radial mass flux. A comparison of the present model with those of Mathur (1974) and Madala and Piacsek (1975) are shown in Table 4-II.

4.2.1 Basic equations

The basic equations on an f-plane \( f = 5 \times 10^{-5} \text{sec}^{-1} \) may be written in Cartesian co-ordinate system (c.f. Rosenthal, 1969) as:

\[
\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial u v}{\partial y} + \frac{\partial u w}{\partial z} - f v = -\rho \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial z} \tag{4.1}
\]

\[
\frac{\partial v}{\partial t} + \frac{\partial u v}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial w v}{\partial z} + f u = -\rho \frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial z} \tag{4.2}
\]

\[
\frac{\partial \phi}{\partial z} = -\frac{g}{\theta} \tag{4.3}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4.4}
\]

\[
\frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial z} = \frac{\phi_H}{\theta} + \frac{K \phi}{\theta} \phi \theta \tag{4.5}
\]

\[
\theta = C_p T \tag{4.6}
\]

Equations (4.1) and (4.2) are the equations of motion in the east and north directions respectively. Since, horizontal dimension of a tropical storm is at least two orders of magnitude larger than the vertical dimension, the equation of motion for the vertical component of velocity can be simplified into the hydrostatic equation given by (4.3). \( u, v \) and \( w \) are the eastward, northward and vertical velocity components respectively; \('g'\) is the acceleration of gravity. Equation (4.4) is a simplified form of the continuity equation that filters acoustic wave solution (Ogu-
ra and Charney, 1962; Ogura and Phillips, 1962); use of this form to simulate tropical storm can also be justified by scale analysis. Equation (4.5) is the first law of thermodynamics. First term "\( H \)" on the right side of this equation represents diabatic heating (heat added or removed per unit mass of air per unit time, vide Sect. 4.5.1 & 4.5.4). Vertical diffusion of heat is not included in the model because of relatively insufficient knowledge about the behaviour of the vertical flux of heat in a tropical storm. Equation (4.6) is the equation of state. \( \bar{e} = C_p \left( \frac{p}{p_0} \right) \frac{R}{C_p} \)

\[ C_p = 1004.64 \times 10^4 \text{erg g}^{-1} \text{deg}^{-1} \] is specific heat of air at constant pressure; \( R = 287.04 \times 10^4 \text{erg g}^{-1} \text{deg}^{-1} \) is specific gas constant of dry air; \( p \) is pressure, \( p_0 = 1000 \text{ mb} \); \( \bar{\rho} = \bar{\rho}(z) \) is the density of mean tropical atmosphere and function of vertical distance \( z \) only; \( T \) is temperature; \( \theta \) is potential temperature. The terrain effect is excluded for simplicity.

The horizontal and vertical momentum flux terms in the equation (4.1) and (4.2) are assumed to be of the following form:

\[
\begin{align*}
J_{xx} &= \vec{\rho}K_H \frac{\partial u}{\partial x}, \quad J_{yx} = \vec{\rho}K_H \frac{\partial u}{\partial y}, \quad J_{zx} = \vec{\rho}K_z \frac{\partial u}{\partial z} \\
J_{xy} &= \vec{\rho}K_H \frac{\partial v}{\partial x}, \quad J_{yy} = \vec{\rho}K_H \frac{\partial v}{\partial y}, \quad J_{zy} = \vec{\rho}K_z \frac{\partial v}{\partial z}
\end{align*}
\]

and at the surface, we assume

\[
\begin{align*}
J_{zx} &= \bar{\rho}(1)C_D u_1 \left( u_1^2 + v_1^2 \right)^{1/2} \\
J_{zy} &= \bar{\rho}(1)C_D v_1 \left( u_1^2 + v_1^2 \right)^{1/2}
\end{align*}
\]

where, \( K_H \) and \( K_z \) are respectively the horizontal and vertical eddy coefficients of viscosity; \( C_D \) is drag coefficient; \( \bar{\rho}(1) \) is the air density at the surface in the mean tropical atmosphere.
\( u, v \), are the horizontal components of velocity at the 0.5 km level in the model. Values of the coefficients used in the model are discussed in Sections 4.5.3 and 4.5.4.

The equivalent potential temperature of mean tropical boundary layer air \( \theta_{eb} \) is about 350K. Moist adiabatic ascent of this air in the form of convective clouds serves to transport sensible and latent heat upward. Below the level of convective cloud tops (150mb), the warmest virtual temperature lapse curve induced by the above process lies along the moist adiabat with equivalent potential temperature \( \theta_{eb} \). If \( \theta_{eb} \) remains constant, the lowest surface pressure obtained from the hydrostatic law is limited by the warmest virtual temperature lapse curve; it is about 1000mb. For the surface pressure to fall below 1000mb, the boundary layer air must pick up sensible and latent heat from the sea surface so that the moist adiabatic ascent takes place at a higher equivalent potential temperature (Mathur, 1974). Malkus and Riehl (1960) showed that \( \theta_{eb} = 365K \) is required in order to explain hydrostatically a central surface pressure of 962.6 mb in a hurricane.

The sensible heat transfer between the tropical storm and the ocean surface is included implicitly in the model by assuming that the temperatures at levels 1 and 2 of the model (Fig. 4.1) are held constant for all time. This eliminates the need for a temperature forecast at levels 1 and 2 and for an explicit formulation of the air sea exchange of sensible heat. Frank (1977) in a composite study of hurricanes showed that the boundary layer temperatures are relatively constant with radius despite decreasing pressure towards the centre. Hawkins and Rubsam (1968) verified the isothermal expansion by estimating the surface tempera-
tures of hurricane Hilda (1964). The present model includes the latent heat transfer by assuming that a parcel rises dry-adiabatically from the surface up to the lifting condensation level (L.C.L.) and saturated adiabatically thereafter. Since the temperatures at levels 1 and 2 of the model were held constant, the equivalent potential temperature of the air rising in the cumulus clouds depends only upon the pressure at L.C.L. (Cloud base). The L.C.L. of the surface air is assumed to be fixed for all time half way between levels 1 and 2 (500 m). The average specific humidity over the lowest 1 km is assumed to be the average saturation value for this layer. Above two assumptions eliminate the need for a water vapour convergence equation.

The following boundary conditions are imposed on \( w \):

\[
w = 0; \text{ at } z = 0, \text{ and } z = Z_t
\]

where, \( Z_t \) denotes the upper boundary of the model atmosphere. When these conditions are imposed, external gravity waves are filtered out, because of the continuity equation (4.4)

We replace \( \theta \), by \( \alpha (z - \frac{1}{\theta}) \), as \( \alpha \) varies more linearly than \( \theta \) between levels. For computational accuracy, we divide \( \bar{\alpha} \) and \( \alpha \) into the basic (undisturbed) field which is a function of \( z \) only and perturbation part as below:

\[
\bar{\alpha} = \bar{\alpha}(z) + \phi(x,y,z,t) \quad (4.8b)
\]

\[
\alpha = \bar{\alpha}(z) + \alpha'(x,y,z,t) \quad (4.8b)
\]

4.2.2 Structure of the model

Mc Gregor and Leslie (1977) have shown that for semi-implicit schemes, the use of non-staggered grid with the usual time and space central finite difference approximations leads to a decou-
pling into four separate solutions on different elementary sub-
grids. A preferable procedure is to use a particular staggered
grid which has no solution separation. This results in better
treatment of geostrophic adjustment process and should predict
the structure of small scale features more accurately. So the
variables in this model are specified in staggered grids (similar
to Williams, 1969), as shown in Figs. 4.2 and 4.3. The horizontal
velocity components u and v are specified at levels 2, 4 and 6
(Fig. 4.1). The thermodynamic variables are specified at levels
1, 2, 4 and 6; the vertical velocity w is specified at level 1,
3, 5 and 7. The pressure variable $p$, and temperature variables $T$
and $\theta$ which are defined at the same points form the basic grid.
The velocity components u and v are defined at different points
interlacing with the basic grid. The velocity points, u lie on
the east-west $\xi$ grid line at mid way between its grid points;
similarly v points on north-south grid line. Finally, w points
lie on the vertical lines through $\zeta$ points. Standard values of
pressure, density, temperature, and potential temperature in the
mean tropical atmosphere at different levels of the model are
given in Table 4-I (Frank, 1977).

The continuity equation is applied at a $\zeta$ point and is valid
for the fluid unit surrounding that point. Through using an inter-
lacing grid system, the amount of averaging reduces to a mini-
mum; thus, improves accuracy. The continuity equation should
have a unique exact form which can only be achieved by such a
grid. This uniqueness is essential for deriving the Helmholtz
equations in semi-implicit scheme. To define $\zeta$, $\theta$ and $T$ at the
same point is desirable for consistency with equation of state.
4.2.3 Nested grids

In a tropical storm, the dependent variables vary rapidly in the radial direction near the centre and less rapidly away from it. Therefore, a finer resolution of 10-20 km is required for representing the motion on the scale of a tropical storm 'eye' and intensity forecast. But it is not practically feasible to increase the spatial resolution throughout the entire domain. As such, a nested grid arrangement is considered suitable for this study. One important limitation is that the ratio of reduction of mesh should be such that certain coincident points are to be maintained between meshes for all the variables. In the staggered grid used in this model, minimum mesh ratio for adjacent grids must be 3 : 1. Considering all these points we have used three nested meshes, each of uniform spacing. The innermost mesh with fine grid spacing of 18 km (here after called FG ) is centered near the centre of the vortex. This is surrounded by a medium grid mesh (MG) with grid length of 54 km, which is again surrounded by another coarse grid mesh (CG) of 162 km grid length. Each mesh consists of 32x32 point array of momentum points enclosing 31x31 point array of mass points (Fig. 4.4).

4.3.1 Semi-implicit scheme and computational procedure

The semi-implicit method developed by Kwizak and Robert (1971) treats implicitly those terms in the equations of motion that are primarily responsible for the propagation of gravity waves. The advection terms are treated in an explicit fashion. The stability criteria for this method can be made to depend mainly upon the maximum wind speed and grid length thus allowing longer time...
step. The time truncation errors associated with the permissible time step in semi-implicit method still remain an order of magnitude smaller than the errors associated with space truncation errors; and hence accuracy of the result is not lost. The way in which it is done is to evaluate certain terms implicitly as a mean over times \((t-\Delta t)\) and \((t+\Delta t)\), rather than at time \(t\). A coupled set of equations in time then results rather than decoupled set. The method used in the present model is similar to the one described by Robert et al. (1972). Here the basic part of pressure gradient force, divergence and vertical advection of potential temperature are evaluated implicitly as a mean over times \((t-\Delta t)\) and \((t+\Delta t)\), and calculating all other terms of the variables at time \(t\), (Madala and Piasek, 1975).

4.3.2 Spatial finite difference

To derive the finite-difference equations, the following sum and finite difference operators are used:

\[
\overline{\frac{\partial^2}{\partial s^2}} \equiv \frac{1}{2} \left[ \beta(s + \frac{\Delta s}{2}) + \beta(s - \frac{\Delta s}{2}) \right]
\]

\[
\overline{\frac{\partial}{\partial s}} \equiv \frac{1}{\Delta s} \left[ \beta(s + \Delta s) + \beta(s - \Delta s) \right]
\]

\[
\delta_s \beta \equiv \frac{1}{\Delta s} \left[ \beta(s + \frac{\Delta s}{2}) - \beta(s - \frac{\Delta s}{2}) \right]
\]

\[
\delta_s^2 \beta \equiv \delta_s \left( \delta_s \beta \right)
\]

where, \(\beta\) represents one of the dependent variables \(u, v, w, \xi\) or \(\alpha\), and \(s\) represents one of the independent variables \(x, y, z\) or \(t\), and \(\Delta s\) the grid interval.

Introducing the basic and perturbation parts of \(\xi\) and \(\alpha\) as in (4.8), the terms in the equations (4.1) to (4.5) are arranged
in such a way that all terms on the left hand side will be
treated implicitly using values at time \((t-\Delta t)\) and \((t+\Delta t)\); while
the terms appear on the right will be treated explicitly using
the values at time \(t\). Thus, the finite difference form of of the
above equations can be written in the following form:

\[
\frac{\delta_t \rho u}{\alpha} + \frac{\rho}{\alpha} \frac{\delta_x \rho \alpha^2}{\Delta t} = A_1
\]

\[
\frac{\delta_t \rho v}{\alpha} + \frac{\rho}{\alpha} \frac{\delta_y \rho \alpha^2}{\Delta t} = B_1
\]

\[
\frac{\delta_z \rho \alpha^2}{\Delta t} = - \frac{\sigma \alpha^2 \rho \alpha^2}{\Delta t^2}
\]

\[
\delta_x \left( \frac{\delta_t \rho u}{\alpha} \right) + \delta_y \left( \frac{\delta_t \rho v}{\alpha} \right) + \delta_z \left( \frac{\delta_t \rho w}{\alpha} \right) = 0
\]

\[
\frac{\delta_t \rho \alpha'}{\alpha} + \frac{\delta_z \left( \frac{\delta_t \rho \alpha'}{\alpha} \right)}{\Delta t} = C_1
\]

where,

\[
A_1 = - \rho \left( \frac{1}{\alpha} - \frac{1}{\alpha} \right) \delta_x \phi + \frac{\rho}{\alpha} \frac{\delta_y \rho \alpha^2}{\Delta t} - \delta_x \left( \frac{\delta_t \rho u}{\alpha} \right) - \delta_y \left( \frac{\delta_t \rho uv}{\alpha} \right)
\]

\[
B_1 = - \rho \left( \frac{1}{\alpha} - \frac{1}{\alpha} \right) \delta_y \phi - \frac{\rho}{\alpha} \frac{\delta_y \rho \alpha^2}{\Delta t} - \delta_x \left( \frac{\delta_t \rho u}{\alpha} \right) - \delta_y \left( \frac{\delta_t \rho uv}{\alpha} \right)
\]

\[
C_1 = \delta_x \left( \rho \omega' \right) - \delta_x \left( \frac{\delta_t \rho \omega}{\alpha} \right) - \delta_y \left( \frac{\delta_t \rho \omega}{\alpha} \right) + \frac{\rho \alpha^2 H}{\phi}
\]

+ \frac{\rho \alpha^2 K_H \theta H}{\phi} \left( \frac{1}{\alpha} \right)

The terms \(A_1\), \(B_1\) and \(C_1\) will be evaluated first explicitly,
and the implicit method will be applied to the remaining
calculations.

The non-linearity in the equations of motion introduce compu­
tational instability from the finite difference approximation to
the non-linear terms of the equations. Since, this instability results from the space truncation error of non-linear terms, it can not be eliminated either by reducing time step or by using implicit or semi-implicit method. One method is to suppress this instability by introducing artificial viscosity terms in finite difference equations. Grammeltvedt et al (1969) showed that this can be controlled by devising a finite difference scheme for non linear terms which satisfies certain linear and quadratic integral constraints. The finite difference form for these terms applied in this model conserve mass, momentum and total energy over a closed domain. In addition to these, difference equations approximately conserve mean square vorticity for non-divergent barotropic motion which is discussed in Appendix-4.A.

4.4.1 Computational procedure

The time averaged set of prediction equations for \( u, v \) and \( \alpha' \) from (4.9), (4.10) and (4.13) can be written as:

\[
\frac{-2}{\rho} \frac{\partial}{\partial t} \frac{2}{a} \Delta t \delta x \varphi^{21} = A
\]

\[
\frac{-2}{\rho} \frac{\partial}{\partial t} \frac{2}{a} \Delta t \delta y \varphi^{21} = B
\]

\[
\frac{-2}{\rho} \frac{\partial}{\partial t} \frac{2}{a} \Delta t \delta z \varphi^{21} = C
\]

where, \( A = A_1 \Delta t + \overline{\rho u(t-\Delta t)} \), \( B = B_1 \Delta t + \overline{\rho v(t-\Delta t)} \), \( C = -C_1 \Delta t + \overline{\rho \alpha'(t-\Delta t)} \)

In order to determine \( \varphi^{21} \), equations (4.14) and (4.15) are differentiated with respect to \( x \) and \( y \) respectively and added.
Then eliminating the divergence part via continuity equation (4.12), we get:

\[ \frac{-\rho \Delta t}{\alpha} \left( \delta_x^2 + \delta_y^2 \right) \psi^t - \delta_z \psi^t = \psi A + \psi B \]  

(4.17)

Applying hydrostatic equation (4.11) to the three layers of the model and energy equation (4.16) at the levels 4 and 6, \( \overline{\rho c}^t \) can be eliminated to get \( \overline{\rho w}_1^t \) and \( \overline{\rho w}_2^t \) for the levels 3 and 5 respectively as:

\[ \overline{\rho w}_1^t = FP1 \]  

(4.18)

\[ \overline{\rho w}_2^t = FP2 \]

where, FP1 and FP2 are functions of \( \phi^t \), grid lengths, time step, basic part of the variables and dependent variables at time \( t-\Delta t \) and \( t \).

Thereafter, applying equation (4.17) at the levels 2, 4 and 6 and eliminating \( \psi^t \), the following set of Elliptical (Helmholtz) equations in \( \phi^t \) can be obtained.

\[ \nabla^2_{H} \phi_1^t + a_{11} \phi_1^t + a_{12} \phi_2^t + a_{13} \phi_3^t = F_1 \]  

(4.19)

\[ \nabla^2_{H} \phi_2^t + a_{21} \phi_1^t + a_{22} \phi_2^t + a_{23} \phi_3^t = F_2 \]  

(4.20)

\[ \nabla^2_{H} \phi_3^t + a_{31} \phi_1^t + a_{32} \phi_2^t + a_{33} \phi_3^t = F_3 \]  

(4.21)

where, \( \nabla^2_{H} \) represents horizontal Laplacian operator; the coefficients \( a_{ij} \ldots \ldots a_{33} \) are functions of the basic parts of thermodynamic variables, grid lengths and time step \( \Delta t \); \( F_1, F_2, \) and \( F_3 \) are forcing functions computed explicitly at time \( t \).

Having solved for \( \phi_1^t, \phi_2^t, \) and \( \phi_3^t \) by any direct or
iterative method, the values of $\phi$, $\mathbf{u}$, $v$, $w$, and $\alpha'$ at time step $(t+\Delta t)$, can be calculated from the values of these variables at time $(t-\Delta t)$, $\bar{\rho}u$, $\bar{\rho}v$, $\bar{\rho}w$ and $\bar{\rho}\alpha'$ using implicit relation:

$$\Gamma(t+\Delta t) = \frac{1}{2} \Gamma^{-2\Delta t} \Gamma(t-\Delta t)$$

(4.22)

where, $\Gamma$ is a general variable.

The CFL stability condition of the type of grid used in the study is given by:

$$\Delta t \leq \Delta x \left[ 2 \left( c^2 + V^2 \right)^{1/2} \right]^{-1}$$

where, $\Delta t$ and $\Delta x$ are the time and space increment; $V$ is the maximum wind speed; and $'c'$ is the phase speed of the fastest moving explicitly treated mode. It was considered prudent to use a conservative time steps, so initially, a time step of 360 seconds was used for all grids. Subsequently, it was reduced according to the stability criteria.

4.4.2 Solution of the system of Helmholtz equations

In principle, it is possible to solve a system of Helmholtz equations by simultaneous relaxation method. Sela and Scolnik (1972) mentioned that this method is neither efficient nor elegant. They pointed out that by decoupling the efficiency becomes double. But their method of decoupling can not be adopted in this problem as the coefficient matrix is not symmetric.

As the system of Helmholtz equations (19)-(21) are coupled in different levels, to apply the direct method, such as error vector propagation method (EVP), each of the equations is required to be decoupled to an equation in a single dependant variable.
at each time step. The EVP method can not be applied in an area with large number of grid points. This has been overcome in the stabilized error vector propagation method (SEVP), a modification of EVP method developed by Madala (1978) by dividing the integration region into blocks each of which is stable for EVP method. In the present problem this has been accomplished by dividing integration region of each mesh of 31x31 grid points into three overlapping blocks. In this, or other direct method several subsidiary calculations are involved. Moreover, in the present problem the coefficient matrix is also not symmetric, so the transformation has been achieved by using latent vectors of the transpose of the coefficient matrix. After solving the new equations for new variables independently of each other, were transformed back to the original set of variables by algebraic method. All these took sizeable computer time. It is not so easy for coding.

In the circumstances, we quest for other suitable method and have developed an iterative method (SMR) for such type of equations. It has been found that total computer time required for a particular period of integration by this method is nearly two third of the time that required by SEVP method. In our method there is no need for decoupling the equations, easy to code and no extra memory is required as in direct method (Appendix-4.B).

(a) Divergence correction during integration.

In deriving the Helmholtz equations (19) - (21), continuity equation (Divergence=0) has been used. If, these equations could be solved exactly there was no problem. In reality, however, a degree of round-off error is inevitable even with trigonometric method. This, in turn creates an artificial divergence which can
lead to computational instability. By inserting round-off divergence of step \((t-\Delta t)\) as a correction term in the forcing function for the solution for step \((t+\Delta t)\) (William, 1969), it was found that divergence remained bounded.

4.4.3 Boundary conditions and matching of the solutions

This is a numerical model with a limited part of the atmosphere. One of the difficulties in this connection is how to formulate the lateral boundary conditions on the outermost grid without imposing any physical constraint. To minimize the effect of the boundary conditions on the results, a square region with sides of 5184 km was selected for integration.

During the course of integration, all variables were held constant on the outermost grid boundaries. The spurious spatial oscillations were suppressed by applying a nine-point smoother (Shuman, 1957) with smoothing element equal to 0.1, within four grids from the boundaries.

The most crucial problem in nesting of grids is how to connect the solutions of different grids. Various interface boundary conditions have been employed to permit mass, momentum and energy to flow between grids. But, no condition can be perfect, because of the differences in resolvable waves with different grid lengths (Elsberry, 1979). In general, there are two approaches viz., one-way interaction and two-way interaction. In one-way interaction the boundary conditions for fine mesh grid are specified from independent solutions of coarse grid area and there is no feedback from the fine grid to coarse grid solution. This results in unbounded growth unless controlled very carefully. In
two-way interaction the solutions for the fine mesh grid is obtained by using boundary conditions from coarse mesh grid and solution thus obtained in the fine mesh grid are substituted on common grid points of coarse mesh grid. Thus the two mesh areas interact dynamically with each other. The scale interaction is more valid than reaction obtained with externally specified condition only. We have followed the second approach.

The computation proceeds from coarser grid to finer grid. First the forcing functions at all grid points are calculated. Then the diagnostic equations (4.19)-(4.21) for $\bar{\phi}^2$ are solved on the CG network. Boundary conditions of $\bar{\phi}^2$ for the MG network are then obtained from CG solutions by four point cubic Lagrangian interpolation along the grid length followed by eighth order linear filter (Francis, 1975) to remove two grid length irregularities. Initial guesses for MG mesh solutions are also obtained from CG data by similar interpolations from two orthogonal directions of the grids. This helps in reducing the number of iterations both for iterative and direct methods (in SEVP direct method, iterations are necessary to match the solutions of different blocks). Same process is repeated for the FG network. Having obtained the solutions for the three grid networks, at the common grid points between the FG and MG networks, the values of $\bar{\phi}^2$ on the MG are replaced with the corresponding values obtained on the FG. Same process of back substitution is adopted between MG and CG networks. Values of $\bar{\phi}^2$ at the second row of MG and FG networks are smoothed by interpolation so that the second derivative normal to the grid boundary is equal to that derivative at third row.

Using the values of $\bar{\phi}^2$ thus obtained, the values of $u$, $v$ and
\( \alpha' \) can be calculated from the prognostic equations (4.14)-(4.16); \( w \), from diagnostic equation (4.18) and \( \phi \) from implicit relation (4.22). Replacements of coarser grid values of all these variables by the finer grid data are done in the same way as in the preceding paragraph. The boundary values for \( u, v, w \) and \( \alpha' \) of the finer grid network are obtained by four point cubic Lagrangian interpolations of the coarser grid data. As on the finer grid along the eastern and western boundaries of \( u \), and northern and southern boundaries of \( v \), there is no coarser grid data, the interpolation is bi-cubic i.e. parallel and perpendicular to the interface. The values at the corner points are obtained as averages of values interpolated from two directions.

4.5 Numerical experiments

4.5.1 Diabatic heating due to forced subsidence

The amount and distribution of heat source due to \( \alpha_{\text{forced}} \) subsidence is not well known in disturbance region. For this preliminary experiment, our approach is to test the effect of it with an analytical expression and see what can give better agreement between the model and observations. The horizontal heat distribution due to forced subsidence in the model atmosphere is formulated in a similar way as proposed by Harrison(1973). In actual tropical storm, one expects to find maximum heating at the centre of the storm. So the horizontal heating function is given by:

\[
Q = Q_0 \exp \left[ -\frac{(X-X_0)^2}{\rho_0^2} \right] \exp \left[ -\frac{(Y-Y_0)^2}{\rho_0^2} \right]
\]

(4.23)

where \( (X_0,Y_0) \) is the horizontal locations of the heating maximum.
which is assumed to coincide with the centre of the storm; \( r_0 \),
is the distance of the maximum tangential velocity from \((x_0, y_0)\);
\( Q \) is the amount of heat available in calorie per square cm per
sec to increase the air temperature of layers 2 and 3, \( Q_0 \) being
its maximum value.

Since the model predict the temperature in layer 2 and 3 only,
we assume that heat \( Q \) is distributed between these two layers in
such a way that the amount of heat available to each layer is
proportional to the difference of pseudo-adiabat (cloud tempera­
ture, \( T_c \)) and the temperature\( (T) \) of the storm scale circulation.

The amount of heat \( \langle H_2 \rangle \) available per unit mass in layer 2 is

\[
H_2 = \frac{Q(T_{c2} - T_2)}{\Delta}, \quad \text{if } T_{c2} > T_2
\]

\[
= 0, \quad \text{otherwise}
\]

and amount of heat \( \langle H_3 \rangle \) available in layer 3 is

\[
H_3 = \frac{Q(T_{c3} - T_3)}{\Delta}, \quad \text{if } T_{c3} > T_3
\]

\[
= 0, \quad \text{otherwise}
\]

where, \( T_{c2} \) and \( T_{c3} \) are cloud temperatures at level 4 and 6 re­
respectively, and \( \Delta = \rho(4)\Delta Z_2(T_{c2} - T_2) + \rho(6)\Delta Z_3(T_{c3} - T_3) \). This
formula is similar to one used by Rosenthal (1969).

4.5.2. Initial fields

For this preliminary test, the time integration of the nume­
rical model was started from a weak circularly symmetric, cyclo­
nic vortex in gradient balance. The tangential wind, \( V_\theta \) is of
the following function of radius, \( r \).

\[
V_\theta = V_{\max} \left( \frac{r}{r_0} \right) \exp \left( -\frac{1}{2} \left[ 1 - (r/r_0)^2 \right] \right)
\quad (4.24)
\]
where, \( V_{\text{max}} = 9.8 \text{ m/sec} \) is the maximum wind which is located at \( r = r_0 = 162 \text{ km} \). Since, the winds are constant in the vertical the vortex is non-divergent and there is no radial or vertical motion. The choice of the initial vortex is of course to some degree arbitrary, as it is difficult to judge what should be the most realistic state. Sundqvist (1970) used the same form.

The gradient wind, after eliminating \( \theta \) with the help of equation of state, can be written as:

\[
\frac{V^2}{r} + fV = \frac{g}{\rho} \frac{\partial \ln \theta}{\partial r} \tag{4.25}
\]

This equation is solved for the distribution of \( \theta \), by assuming that the temperature \( T \) is constant on any horizontal level and equal to the temperature of the mean tropical atmosphere at that level. The value of \( \alpha \), and hence, \( \theta \) at any level is calculated from the hydrostatic equation.

4.5.3. Surface friction

In the concept of CISK mechanism to tropical cyclone theory, surface friction has been recognized as an essential factor for development of tropical storm. Yamasaki (1968), Ooyama (1969), Sundqvist (1970), and many others made numerical experiments on this concept. In their models, surface friction was indispensable to maintain the convective activity and disturbance should not develop when the drag coefficient concerning friction is reduced to zero. Linear analysis shows that the growth rates of tropical storm is directly proportional to the drag coefficient (Ooyama, 1969). On the other hand, increased surface drag leads to an increased dissipation of kinetic energy. In Yamasaki’s (1977) two
dimensional model, it is found that a disturbance may develop even if the surface friction is not taken into consideration.

Representation of the surface friction at the ground in terms of the large scale variables is a complex problem. In the conventional flux expressions, we need empirical determination of the drag coefficient $C_D$. Investigation in this context have not yielded a conclusive relationship between $C_D$ and wind. Some studies indicate a linear increase of $C_D$ with increasing wind (Miller, 1962); while according to Wu (1969), $C_D$ becomes constant for sufficiently strong wind ($>15$ m/sec). Consequently, the numerical values of this coefficient include some uncertainty too.

### 4.5.4 Three cases of the experiment

The effect of the surface friction can be investigated by changing the value of $C_D$. In view of the discussions in the preceding section, following three numerical experiments have been performed to see how the surface friction and its variation influence on the development of tropical storms in the early stage imposing artificial heating considered as the effect of forced subsidence.

**Case A**

Horizontal coefficient of eddy viscosity, $K_H$ is $10^4$ m$^2$ sec$^{-1}$; vertical coefficient, $K_Z$ is $1$ m$^2$ sec$^{-1}$ at level 2 and decreased linearly to zero at the top of the model. $C_D$ is kept zero up to 20 hours, then linearly increased to 0.0025 at 40 hour and there after kept constant. The maximum diabatic heat, $Q_0$ is linearly increased from zero at the initial time to $68.8 \times 10^{-3}$ cal cm$^{-2}$ sec$^{-1}$ at $t=40$ hour; thereafter it is kept constant for the remaining
time of integration. Equivalent amount in Harrison's (1973) model is much higher than this value.

Case B

All of $K_H$, $K_Z$ and $C_D$ are zero during entire period of integration. $Q_0$ is linearly increased from zero at the initial time to $18.34 \times 10^{-8}$ cal cm$^{-2}$ sec$^{-1}$ at 6 hour; thereafter it is kept constant for the remaining period of integration. This value is much smaller than that in case A.

Case C

$K_H$, $K_Z$ and $Q_0$ are same as in Case A, except $C_D$ is linearly increased from zero at the initial time to 0.0025 during a period of 40 hours, and thereafter kept constant.

4.6 Discussion of the numerical results

The time integration of the model was carried out for 60 hours of real time for the case B, and 48 hours for A and C. The time variations of central sea level pressure and the maximum winds at 0.5 km level in three cases are shown in Fig. 4.5. Though, there are small fluctuations on all curves throughout the time of integration, it is more on the velocity curves at the initial stage. Short period oscillations are not properly reflected as hourly data are plotted. Yamasaki (1977) states that such oscillations are probably due to convective motion and gravity wave. Though, the rate of diabatic heating in frictionless case is much smaller than other two cases, the rate of fall of central sea level pressure and increase of surface wind and their magnitudes are highest in this case. In C, the fall of central pressure or
increase of surface wind up to 18 hours is very small. These are expected, since in B there is no dissipation of kinetic energy and in C, frictional term was included from the starting of integration. These curves indicate that the vortex is developing in all cases at a steady rate at the termination of integration. In B, the central pressure has fallen from 1007 mb to 997.75 mb in 60 hours; while in C, from same value to 1000.3 mb in 48 hours. Starting from a weak low (19.8 kts), the vortex has intensified to a minimum storm (35 kts) at 30 hour in B, and at 47 hour in A; but in C, it could not attain this strength.

Radial variations of mean (around the centre) tangential velocity at 0.5 km level in three cases at 48 hour are shown in Fig. 4.6 (bottom). In B, the radius of the maximum tangential velocity is 100 km; while it is only 36 km in A and C. Such small values at this stage of development may be due to small radius (162 km) of the maximum velocity of the initial vortex. Radial gradient of diabatic heating may also have influence on this decrease. In B, the belt of high wind speed covers larger area than other two cases. In A and C, the speeds in the outer region are almost same; but around the radius of maximum wind the profile of A has well defined peak typical to real data; but in C, it is flat and its value is less than that in A, due probably to early imposition of \( C_D \) in C. All these features may be attributed to the variation of \( C_D \). Radial variation of mean inflow velocity for A (Fig. 4.6 top) is smooth without any pronounced peak. Considering the early stage, this appears to be realistic. In C, it is of the same type; but in B, radial flow is very feeble.

Stream lines for the lower, middle (direction only) and upper
(including isotach) levels in A at 48 hour are shown in Figs.4.7 to 4.10. The stream lines clearly show that the flow is nearly symmetric at all levels. These are characterized by cyclonic inflow at the surface (Fig. 4.7) and outflow at the upper level (Fig. 4.9), cyclonic inside a radius of about 100 km and anticyclonic outside. But, middle level wind directions (Fig. 4.8) indicate practically no inflow or outflow. The centre of the flow at each level lies on the centre of the vortex on the surface. This is expected as the vortex was not allowed to interact with other system. The isotach analysis on the surface indicates that the wind speed is also nearly symmetric about the centre. The inner pressure profiles are nearly circular, and on the surface they are closely packed. Outer profiles indicate that matching of the solutions between grids of different lengths did not produce unwanted effect. There is a noticeable asymmetry of the isotach field in the upper level (Fig. 4.10) with higher wind speed to the northwest and southeast sectors.

The vertical velocity \( w_4 \) at the top of the boundary layer at 48 hour in A is shown in Fig.4.11. It increased slowly with time and region of high speed gradually moved toward the centre. At 48 hour it attained a speed of 13-15 cm/sec in an annular region between 36 and 50 km from the centre. One interesting feature of \( w_4 \) is that band like regions of upward motion are separated by descending motion. In the upper level, \( w_2 \) is mainly ascending around the centre (maximum 32 cm/sec) with weak descending motion at a large distance.

Warm core in the middle level is noticeable at this stage. The maximum positive temperature anomaly of 6°C is located at
the centre of the vortex. The anomaly profiles are nearly circular and concentric. The gradient is highest between 36 and 54 km and not at the centre. The maximum warming at the upper level is not situated at the centre. It is negative in a very small area in the centre and increases sharply with radius to more than 4°C, then decreases slowly outwards. This implies that warming in upper layer is not yet sufficient to compensate adiabatic cooling due to vertical motion in the centre.

In the frictionless case(B), at 24 hour alternate regions of ascending and descending motion are observed at the top of the boundary layer. In the central region of about 90 km in radius, the upward velocity is very small and in almost all parts of the outer region are covered with sinking motion. At the upper level the pattern is similar to that at lower level. This suggests that such patterns may be independent of friction and probably linked to the dynamics and thermodynamics of the flow itself. The potential temperature anomaly profiles at 24 hour are circular and symmetric. In the middle level, the anomaly is positive with its maximum at the centre(4°C). While in the upper level it is negative(-1°C) at the centre and weak positive at large distance.

Sea level pressure anomalies, streamlines at lower and upper levels and $w_4$ for this case at 60 hour are shown in Figs.4.13 to 4.16. The pressure anomaly pattern in the inner domain represents circular contours comparable to observation. The streamlines in the boundary layer are symmetric. The angle of inflow is very small(1-3 degree). This could be expected as the surface friction and diffusion terms are excluded in this experiment. The streamlines and inflow angle in the middle level (not shown) are simi-
lar to those in the boundary layer. In the upper level these are not symmetric. Large area in the centre is dominated by cyclonic outflow which is surrounded by a trough and two weak ridges. Anthes (1972) concluded that the asymmetry of the outflow layer results from dynamic instability.

Due to increase in the potential temperature of the middle and upper layer, the pressure gradient force increased steadily during the later part of integration. This increased pressure gradient force accelerated the radial influx of mass in the lower layer. The major part of the radial momentum thus created was converted into the tangential momentum by Coriolis force, thereby increasing the tangential momentum to restore partly the gradient wind balance. In frictional cases, there is a loss of tangential momentum to the ocean surface and gradient wind balance of the vortex was disturbed leaving to an unbalanced pressure gradient force directed to the centre. As a result, rings of air started to move toward the centre in the boundary layer and a part of which was also converted to tangential momentum. Thus the radius of the maximum tangential velocity in the frictional cases (A and C) decreased. In frictionless case, this reduction is very less in comparison with frictional cases. With friction, the rate of increase of surface wind depends upon the excess of tangential momentum generated by heating over the loss due to surface friction and turbulent diffusions. Hence, increase of tangential velocity is highest in B and lowest in C.

The cyclonic flow in the upper troposphere slowly disappeared and anticyclonic outflow formed and intensified with time. The air converging in the boundary layer rose and diverged out.
in the upper troposphere due to decrease of pressure gradient force. The absolute angular momentum of a parcel of air moving horizontally in the upper troposphere is nearly conserved, so the relative angular momentum of diverging air decreases with radius. Thus the initial cyclonic circulation in the upper level weakened slowly and anticyclonic outflow formed away from the axis. The vortex remained stationary and nearly symmetric during the period of integration in all cases, since it was not allowed to interact with other system of the atmosphere.

4.7 Summary and remarks

From the results of three cases tested in this preliminary study, it is clear that the staggered grid scheme, finite difference method, matching of solutions between meshes of different grid lengths and the technique of integration have yielded smooth and encouraging results. Therefore, we may conclude that the present scheme shows satisfactory performance from the computational point of view. Using the new iterative method of solution for the system of Helmholtz equations developed in this study, the semi-implicit method has become more advantageous in speed for the nested grid model. In this sense the purpose of the present experiment has been achieved.

The results of the numerical experiment indicate that a disturbance may develop without surface friction. It may be inferred that surface friction may not play an essential role in the early developing stage of a storm when the vortex is weak, as the frictional convergence at this stage may not be sufficient for its growth. That is, the CISK mechanism may not be very relevant.
to pre-storm disturbance maintenance and intensification. On the other hand, initiation of initial development to the storm's intensity may be attributed to the warming caused by forced subsidence in the central region of the disturbance as postulated by several authors. The structure of the disturbance obtained in frictionless case at 60 hour is different from the real storm in the sense that it has very less convergence in the boundary layer. The radius of the maximum tangential velocity is also much larger than the frictional case. This suggests that surface friction comes into play when the strength of the vortex is increased and it is one of the important factors to determine the scale of the disturbance.

An interesting related application of this work, for example, would be experiments with tropical storm modification in which the behaviour of the storm system could be examined when heat is added at the upper level.

There are several deficiencies in the model. The heating due to forced subsidence has been incorporated through an analytic function. This is to be parameterized in terms of large scale flow. The vertical resolution of the model is poor. Heat transfer between ocean surface and the vortex has been formulated implicitly. Eddy coefficients of viscosity have been treated in a simple way. This model can be further extended and refined to study the life-cycle of a tropical storm taking into consideration of the variation of Coriolis parameter, environmental flow with movable nested grids, more sophisticated parameterization of physical processes, small grid lengths etc., if high speed computer is available.
Appendix - 4.A

Finite-difference of non-linear terms

The following finite-difference and average operators are defined on a uniform horizontal mesh of uniform grid length $\Delta s$, for a dependent variable $\beta$, as discrete values of independent variables $x=i\Delta s$, $y=j\Delta s$ ($\Delta s=\Delta x=\Delta y$):

\[
\beta_x \equiv \left( \beta_{i+\frac{1}{2}, j} - \beta_{i-\frac{1}{2}, j} \right)/\Delta s
\]

\[
\beta_x^N \equiv \left( \beta_{i+\frac{1}{2}, j} + \beta_{i-\frac{1}{2}, j} \right)/2
\]

\[
\beta_y \equiv \left( \beta_{i, j+\frac{1}{2}} - \beta_{i, j-\frac{1}{2}} \right)/\Delta s
\]

\[
\beta_y^N \equiv \left( \beta_{i, j+\frac{1}{2}} + \beta_{i, j-\frac{1}{2}} \right)/2
\]

In addition to these operators along the coordinate directions, we define similar operators along the diagonals of a grid square(Ookochi,1972):

\[
\delta_a \beta \equiv \left( \beta_{i+\frac{1}{2}, j+\frac{1}{2}} - \beta_{i-\frac{1}{2}, j-\frac{1}{2}} \right)/\Delta s
\]

\[
\delta_a^N \beta \equiv \left( \beta_{i+\frac{1}{2}, j+\frac{1}{2}} + \beta_{i-\frac{1}{2}, j-\frac{1}{2}} \right)/2
\]

\[
\delta_b \beta \equiv \left( \beta_{i+\frac{1}{2}, j-\frac{1}{2}} - \beta_{i-\frac{1}{2}, j+\frac{1}{2}} \right)/\Delta s
\]

\[
\delta_b^N \beta \equiv \left( \beta_{i+\frac{1}{2}, j-\frac{1}{2}} + \beta_{i-\frac{1}{2}, j+\frac{1}{2}} \right)/2
\]

The factor 2 that naturally occurs in the denominator of difference operators has been omitted here and combined with 2 that occurs when the components are resolved and combined along the co-ordinate axes.

In terms of the above operators, the finite difference equations for non-linear terms in first and second equations of motion and energy equation can be written in the following form:

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\[
\frac{\partial}{\partial x} (\rho u_x) + \frac{\partial}{\partial y} (\rho u_y) + \frac{\partial}{\partial z} (\rho u_z) = \frac{2}{3} \left[ \rho \delta_x (u_x^x u_x^x) + \rho \delta_y (u_y^y u_x^x) \right] \\
+ \frac{1}{6} \left[ \rho \delta_a (u_y^y u_y^y) + \rho \delta_b (u_y^y u_y^y) \right] \\
+ \rho \delta_z (u_z^z) \\
(4.A-3)
\]
\[
\frac{\partial}{\partial x} (\rho v_x) + \frac{\partial}{\partial y} (\rho v_y) + \frac{\partial}{\partial z} (\rho v_z) = \frac{2}{3} \left[ \rho \delta_x (v_x^x v_x^x) + \rho \delta_y (v_y^y v_x^x) \right] \\
+ \frac{1}{6} \left[ \rho \delta_a (v_y^y v_y^y) - \rho \delta_b (v_y^y v_y^y) \right] \\
+ \rho \delta_z (v_z^z) \\
(4.A-4)
\]
\[
\frac{\partial}{\partial x} (\rho w) + \frac{\partial}{\partial y} (\rho w) = \frac{2}{3} \left[ \rho \delta_x (w_x^a w_x^a) + \rho \delta_y (w_y^a w_x^a) \right] \\
+ \frac{1}{6} \left[ \rho \delta_a (w_y^a w_y^a) + \rho \delta_b (w_y^a w_y^a) \right] \\
+ \rho \delta_z (w_z) \\
(4.A-5)
\]

Appendix - 4.B

Iterative method for solution of a system of Helmholtz equations.

In iterative method, an initial guess of the solution is made and then progressively improved until an acceptable level of accuracy is reached.

To illustrate our Simultaneous Multi-level Relaxation (SMR) method, let us consider a rectangular region covered by a grid of equally spaced points. The values of the variable, \( \phi \) for which the solutions to the system of Helmholtz equations are sought, are specified on the boundaries. The equations in this study are of the following form:
\[ \nabla^2_{H} \phi_{i,j,1} + a_{11} \phi_{i,j,1} + a_{12} \phi_{i,j,2} + a_{13} \phi_{i,j,3} = F_{i,j,1} \] (4.B-1)

\[ \nabla^2_{H} \phi_{i,j,2} + a_{21} \phi_{i,j,1} + a_{22} \phi_{i,j,2} + a_{23} \phi_{i,j,3} = F_{i,j,2} \] (4.B-2)

\[ \nabla^2_{H} \phi_{i,j,3} + a_{31} \phi_{i,j,1} + a_{32} \phi_{i,j,2} + a_{33} \phi_{i,j,3} = F_{i,j,3} \] (4.B-3)

where \( \nabla^2_{H} \) is the 5-point Laplace operator; \( a_{11}, a_{12}, \ldots, a_{33} \) are coefficients and \( F_{i,j,1}, F_{i,j,2}, \) and \( F_{i,j,3} \) are forcing functions.

In Sequential Relaxation (SR) or Sussive Over Relaxation (SOR) method, the correction to estimate (or guess) value using residual is applied to each equation separately and independently of other equations. In our method (S M R) the residuals on a particular grid point \( 'i, j' \) on all levels are calculated first as in S . R method, and corrections to previous "guess values" at that particular point on all levels are done simultaneously taking into account of each and every terms of all equations, rather than each level separately, since the change in one level affects other levels also.

Let \( \phi_{1,j,k}^m \) represents the \( m\)th estimate of \( \phi_{1,j,k} \). Then the residuals \( R_{1,j,k}^m \) \((k=1,3)\), for the \( m\)th estimate is defined as:

\[
R_{1,j,1}^m = \phi_{i-1,j,1}^m + \phi_{i+1,j,1}^m + \phi_{i,j-1,1}^m + \phi_{i,j+1,1}^m - 4 \phi_{i,j,1}^m + a_{11} \phi_{i,j,1}^m + a_{12} \phi_{i,j,2}^m + a_{13} \phi_{i,j,3}^m - F_{i,j,1}
\] (4.B-4)

Similarly, \( R_{1,j,2}^m \) and \( R_{1,j,3}^m \) are calculated.

If, \( R_{1,j,1}^m, R_{1,j,2}^m \) and \( R_{1,j,3}^m \) just happen to vanish at all points, then \( \phi_{i,j,1}^m, \phi_{i,j,2}^m, \) and \( \phi_{i,j,3}^m \) would be the true solutions of the equations (4.B-1) to (4.B-3).

Let us now suppose that our guesses at the particular point
'i, j' on all three levels are changed by the amounts $\delta \phi_{i,j,1}$, $\delta \phi_{i,j,2}$, and $\delta \phi_{i,j,3}$ respectively, without altering the guess at any of the surrounding points. Since, the forcing functions are fixed, the equation (4.6-4) implies that the resulting changes in $\phi$'s will make residuals of all equations to zero, if the following relations are satisfied.

\[
(-4+a_{11})\delta \phi_{i,j,1} + a_{12}\delta \phi_{i,j,2} + a_{13}\delta \phi_{i,j,3} = - R_{i,j,1} \quad (4.6-5)
\]

\[
a_{21}\delta \phi_{i,j,1} + (-4+a_{22})\delta \phi_{i,j,2} + a_{23}\delta \phi_{i,j,3} = - R_{i,j,2} \quad (4.6-6)
\]

\[
a_{31}\delta \phi_{i,j,1} + a_{32}\delta \phi_{i,j,2} + (-4+a_{33})\delta \phi_{i,j,3} = - R_{i,j,3} \quad (4.6-7)
\]

Above three algebraic equations can be easily solved for $\delta \phi_{i,j,1}$, $\delta \phi_{i,j,2}$, and $\delta \phi_{i,j,3}$ and (m+1)th guess can be obtained from the following relations:

\[
\phi_{m+1}^{i,j,1} = \phi_m^{i,j,1} + \delta \phi_{i,j,1} \quad (4.6-8)
\]

\[
\phi_{m+1}^{i,j,2} = \phi_m^{i,j,2} + \delta \phi_{i,j,2}
\]

\[
\phi_{m+1}^{i,j,3} = \phi_m^{i,j,3} + \delta \phi_{i,j,3}
\]

This procedure is applied to all internal grid points and iterations are repeated and the solutions progressively improved until the maximum of the absolute value of residual becomes less than an optimum pre-assigned value.
### Table 4.1

Approximate values of pressure, temperature, potential temperature and density at various levels of the model in the mean tropical atmosphere (after Frank 1977)

<table>
<thead>
<tr>
<th>Level</th>
<th>Height in KM</th>
<th>Pressure in MB</th>
<th>Temperature $T$ in °K</th>
<th>Potential Temperature $\theta$ in °K</th>
<th>Density $\rho \times 10^3$ gm cm$^{-3}$</th>
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<td>299.700</td>
<td>298.850</td>
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### TABLE 4. II

A comparison of the present model with the models developed by Mathur (1974) and Madala and Piacsek (1975).

<table>
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<tbody>
<tr>
<td>Grid network and grid length</td>
<td>3-Nested grids</td>
<td>2-Nested grids</td>
<td>3-Nested grids</td>
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<tr>
<td>No of grid points on a grid</td>
<td>31x31 (32x32)</td>
<td>25x25</td>
<td>25x25 in 2 grids</td>
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<td>Domain size</td>
<td>5022 km x 5022 km</td>
<td>1760 km x 1760 km</td>
<td>9360 km x 2520 km</td>
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<td>Finite difference scheme for non-linear terms</td>
<td>Conservation of mass, momentum, energy and potential vorticity</td>
<td>Conservation of momentum, and energy.</td>
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<td>Grid matching</td>
<td>Two-way</td>
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<td>Time integration scheme</td>
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<td>Explicit</td>
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<td>Solution of Helmholtz Equations</td>
<td>Iterative method developed in this work</td>
<td>Not applicable</td>
<td>Successive Over Relaxation</td>
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<td>Initial condition</td>
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<td>Real data</td>
<td>Weak vortex</td>
</tr>
<tr>
<td>Sensible &amp; Latent Heat transfer between storm and ocean surface</td>
<td>Implicit</td>
<td>Explicit</td>
<td>Implicit</td>
</tr>
<tr>
<td>Diabatic heating</td>
<td>Forced subsidence (artificially)</td>
<td>Latent Heat of Condensation</td>
<td>Latent Heat of Condensation</td>
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<tr>
<td>Coriolis parameter</td>
<td>Constant</td>
<td>Variable</td>
<td>Const. &amp; variable</td>
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<tr>
<td>HEIGHT (KM)</td>
<td>LEVEL</td>
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<td>$\bar{p}(5)$, $\omega_2$</td>
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<td>$\bar{p}(1)$, $\Phi_0, \theta_0, \omega_0 = 0$</td>
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</tbody>
</table>

**FIG. 4.1 THREE LAYERS OF THE MODEL**
FIG. 4.2 GRID NETWORK IN X-Y PLANE
FIG. 4.3 GRID NETWORK IN X-Z PLANE
FIG. 4.5 TIME VARIATION OF
(a) Central Sea Level Pressure Anomaly(mb)
(b) Maximum Surface Wind(knots)

FIG. 4.5 TIME VARIATION OF
(a) Central Sea Level Pressure Anomaly(mb)
(b) Maximum Surface Wind(knots)
FIG. 4.6
(a) Mean Tangential Velocity (A, B, C) in Kts.
(b) Radial Wind (A) in Kts.
FIG. 4.7 STREAMLINES(A) OF 0.5 KM LEVEL AT 48 HOUR, 18 KM GRID
FIG. 4.8 WIND DIRECTIONS (A) OF MIDDLE LEVEL AT 48 HOUR, 18 KM GRID
FIG. 4.9 STREAMLINES (A) OF UPPER LEVEL AT 48 HOUR, 18 KM GRID
FIG. 4.10 ISOTACH (A) OF UPPER LEVEL AT 48 HOURS, 15 KM GRID, UNITS: KNOTS
FIG. 4.11 VERTICAL MOTION(A) AT 1.0KM(W1) 
AT 48 HOUR, 18KM GRID, UNITS: CM/SEC
FIG. 4.12 POTENTIAL TEMPERATURE ANOMALY (A) AT
UPPER LEVEL, AT 48 HOUR, UNIT: °C
FIG. 4.13 SEA LEVEL PRESSURE ANOMALY (B) AT 60 HOUR, 18 KM GRID, UNIT: MB
FIG. 4.14 STREAMLINES(B) ON 0.5KM LEVEL AT 60 HOUR, 18 KM GRID
FIG. 4.15 STREAMLINES (B) OF UPPER LEVEL AT 60 HOUR, 18 KM GRID
FIG. 4.16 VERTICAL MOTION (B) AT 1.0KM (W1) AT 60 HOUR, 18KM GRID, UNITS: CM/SEC