ADDENDUM

I. Some details of the strength and expectation value density formalism.

In section 2.4 of chapter II the strength and expectation value density formalism was described very briefly. We give some details here. The strength density $S(E,E')$ for the transition $|mE\rangle \xrightarrow{O} |mE'\rangle$ induced by the number conserving excitation operator O (generalisation to number non-conserving case is immediate) which connects an initial m particle Hamiltonian eigenstate $|mE\rangle$ at energy E with eigenvalue density $I_N^m(E)$ ($\equiv I_m(E)$) to a final one $|mE'\rangle$ at energy E' with eigenvalue density $I_N^m(E')$ ($\equiv I_m(E')$) is given by equation (2.4-1) of this section. Here N is the total number of underlying single particle states over which the m valence particles are distributed to form the many body space $|mE\rangle$ and $|mE'\rangle$. The positive definite quantity $S(E,E')$ represents a bivariate distribution with central moments defined in equation(2.4-3). Associated with this bivariate distribution are two marginal densities defined as

$$S(E) = \int_{-\infty}^{+\infty} S(E,E')\, dE'$$

$$\text{and} \qquad S(E') = \int_{-\infty}^{+\infty} S(E,E')\, dE \tag{a.1}$$

As pointed out in this section the asymptotic form for the density $S(E,E')$ is derived in the framework of random matrix theory by replacing the Hamiltonian H and the excitation operator O by EGOE (Embeded Gaussian Orthogonal Ensemble) ensembles of k and k' body operators. The ensemble averaged bivariate cumulants (related to the central moments μ_{pq} in

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equation(2.4-3) of $S(E,E')$ are obtained in the dilute limit :

$m \longrightarrow \infty$, $N \longrightarrow \infty$, $\frac{m}{N} \longrightarrow 0$ and $m \gg k,k'$. By construction cumulants $K_{pq} = 0$ for $p+q$ = odd. For example the fourth order bivariate cumulants are given by,

$$K_{40} = K_{04} = \binom{m-k}{k}\binom{m}{k}^{-1} - 1 = -\frac{k^2}{m} + \frac{1}{2}\frac{k^2(k-1)^2}{m^2} + 0(1/m^3)$$

$$K_{31} = K_{13} = \zeta\left\{\binom{m-k}{k}\binom{m}{k}^{-1} - 1\right\} = -\frac{k^2}{m} + \frac{k^2[(k-1)^2 + 2kk']}{2m^2} + 0(1/m^3)$$

$$K_{22} = \zeta^2\left\{\binom{m-k-k'}{k}\binom{m-k'}{k}^{-1} - 1\right\} = -\frac{k^2}{m} + \frac{k\,[(k-1)^2 + 2k'(2k'-1)]}{2m^2} + 0(1/m^3)$$

with
$$\zeta = \binom{m-k'}{k}\binom{m}{k}^{-1} = 1 - \frac{kk'}{m} + 0(1/m^2) \qquad \text{(a.2)}$$

Similarly one can derive sixth and other higher order ensemble averaged cumulants. For a bivariate Gaussian, cumulants of order three or larger should vanish. From the fourth order cumulants of equation(a.2) together with the sixth order ones derived by French et al it can be shown that in the dilute limit K_{pq} with $p+q \geq 4$ vanishes i.e. the CLT(Central Limit Theorem) form for $S(E,E')$ is a bivariate Gaussian $S_G(E,E')$, which is an asymptotic result.

An important point to note here is that as $K_{pq} \longrightarrow 0$, the correlation coefficient $\zeta \longrightarrow 1$ at the same rate. This leads to the violation of a property that a bivariate Gaussian should retain its form under the linear transformation of its variables. For this reason one considers the density $S'(X,X')$ with X and X' are respectively the "center of mass" and "difference" co-ordinates, with the transformation from E, E' to X, X' given in the text of this section. The cumulants K'_{pq} of $S'(X,X')$ are expressible in terms of K_{pq} and it can be shown that

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$$K'_{40} = -\frac{k^2}{m} + O(1/m^2)$$

$$K'_{31} = K'_{13} = 0$$

$$K'_{22} = -\frac{k(2k\ -1)}{4m} + O(1/m^2)$$

$$K'_{04} = (k-3/2)/k' + O(1/m) \qquad \text{(a.3)}$$

It is now obvious that X and X' become independent as $m \longrightarrow \infty$ i.e. $S'(X,X') \xrightarrow{m \longrightarrow \infty} S'_1(X)\ S'_2(X')$. At the same time the marginal density $S'_1(X) \longrightarrow S'_{1G}(X)$ (a Gaussian density). But $S'_2(X')$, the marginal density in the difference variable X', to approach a Gaussian requires k' to be large as can be seen from equation(a.3). But it is argued that as the variance $2(1-\zeta)$of the marginal density $S'_2(X')$ approach zero as $\zeta \longrightarrow 1$, one can safely approximate $S'_2(X')$ by a Gaussian even with small k' for large values of ζ. One also notes from the last one of equation(a.3) that for a (1+2)-body Hamiltonian taking $k_{effective} = 3/2$, $K'_{04} \longrightarrow 0$ even with small k' and therefore $S'_2(X') \longrightarrow S'_{2G}(X')$. Thus in general with (1+2)-body Hamiltonian and with usual excitation operators one approximates

$$S'(X,X') \longrightarrow S'_{1G}(X)\ S'_{2G}(X') \text{ and } S(E,E') \longrightarrow S_G(E,E').$$

For a detailed and excellent discussion of this theory section 2.4 refers to the work of French et al.

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II. A few important features of the calculations presented in chapter III.

Chapter III of this thesis is based on the work published in Phys. Rev. C36 (1987)2700. In this chapter calculation has been done in obtaining ground state energy, spectra and spherical orbit occupancies. The underlying shell model space used in these calculations is the configuration-isospin $(\tilde{m},T)$ space. Configuration partitioning of the space gives, as expected, an accuracy much more than achieved by scalar (m) or fixed (m,T) space. Configuration fixed (J,T) space is again an improvement over this. However all the results given in this chapter are fixed $(\tilde{m},T)$ results. Projecting fixed angular momentum J from the configuration-isospin space $(\tilde{m},T)$ and performing calculations in the $(\tilde{m},J,T)$ space for various nuclear properties is a future plan.

We describe below briefly the utility of this study done in chapter III.

It is well known that an important source of uncertainty in predictions using the spectral distribution methods(SDM) originates from the choice of the interaction Hamiltonian(H). Except for the Wildenthal's interaction all other sd shell interactions are not suitable globally i.e. for calculations throughout the sd shell. Wildenthal's interaction as mentioned in this chapter, is extremely successful in predicting various ground state and excitation properties throughout the sd shell. Therefore calculations were done with this interaction using the spectral distribution theory which extended the usefulness of this interaction to methods beyond shell model as well as provided a stringent test of the theory itself.

Cornish-Fisher expansion for the state density $\rho(E)$ has been used incorporating approximate γ_1(skewness) and γ_2(excess) corrections for the

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of the distribution. The effect of this is to replace the empirical attempted earlier for the prediction of the ground state properties spectra for nuclei in the sd shell using different interaction ltonians for the lower- and upper- sd shell nuclei separately. ılations in this chapter were done in a parameter free form throughout sd shell using Wildenthal's Hamiltonian and one concluded by comparing ults of spectral distribution theory for binding energy, spectra and ıerical orbit occupancies with earlier calculations using shell model that or future applications of SDM this new interaction is the appropriate one.

III. Usefulness of the beta decay rates of A>60 fp shell nuclei for the silicon-burning process in the presupernova evolution.

The beta decay rates calculated by using the simple model described in chapter VI is ıable enough for incorporation into the presupernova evolution codes the silicon-burning stage. In most previous calculations of the silicon burning stage the beta decays of these nuclei were either neglected or very approximately taken into account. But it is argued recently that at the temperatures and densities prevailing at this stage the abundances of these nuclei, particularly, some isotopes of Co and Cu which contribute substantially to the beta decay, can not be neglected. The impact of the incorporation of these rates into PreSN codes would be a change in the value of electron fraction Y_e at the begining of the gravitational collapse. Though some estimates are available but the precise amount of this change is difficult to predict without making a evolution calculation incorporating the decay rates because of the following reasons. These beta decays will give rise to a cooling through the loss of neutrinos produced in the process resulting in a reduction of the electron pressure

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and therefore of the size of the core that can support itself. At the lower temperatures the abundances of odd-odd and odd-A nuclei are also lower, the condition which is favourable for electron capture. So the question of the importance of the beta decays in the Si-burning phases can be answered only by studying the whole process self-consistently by incorporating the beta decay rates of the relevant nuclei into the stellar evolution code.

This simple model of this chapter of the beta decay rates can be improved by incorporating, i). experimentally observed strength (i.e. logft's) of the discrete low-lying states wherever available, ii). strength of specific forward as well as back resonances assuming simple shell model configurations for the mother and daughter nuclei. Work on this is already being carried out ["Beta Decay Rates of fp Shell Nuclei with A>60 In Massive Stars At Presupernova Stage" - K Kar, A Ray and S Sarkar - Submitted for publication].

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