APPENDIX C

EVALUATION OF AVERAGES OF ISOSPIN SCALAR AND NON-SCALAR OPERATORS

In this appendix we briefly describe the evaluation of averages of operators of scalar and non-scalar isospin rank of the form $\langle K^{\omega_T}\rangle$, $\langle K^{\omega_T} \times H\rangle$, $\langle K^{\omega_T} \times H^2\rangle$, $\langle (O^{\lambda} \times H \times O^{\lambda})^{0,\omega_T}\rangle$ etc. where $K^{\omega_T} \equiv K^{0,\omega_T} = (O^{\lambda} \times O^{\lambda})^{0,\omega_T}$ with O^{λ} a one-body operator of spin and isospin rank $\lambda \equiv \lambda_J, \lambda_T$ and H, the Hamiltonian. The procedure followed is to write the operator K^{ω_T} in normal form, then convert it to p-n form and perform unitary decomposition and then finally obtain the traces in fixed-T_z (z-component of the isospin $\vec{T}$) space $\left(\text{this is a subspace of the scalar space } \binom{N}{m} \supset \binom{N_n}{m_n} \times \binom{N_p}{m_p}\right)$. The last part is done following the well known[1),2)] formalism of expressing the traces as sums of the form $\sum_{\alpha} I_{\alpha} P_{\alpha}$, where I_{α} and P_{α} stand for the input coefficient and the propagator respectively. Traces evaluated in fixed-T_z spaces are then transformed to double-barred traces in fixed-T space. The double-barred average is then obtained by dividing the double-barred trace by the appropriate dimensionality. Having thus obtained the double-barred average of R^{ω_T} (R^{ω_T} standing for any one of the operators mentioned earlier) for each ω_T = 0,1,2 in fixed-T space, one can then perform the Racah sum[3)] to get the unpolarised sum rule quantities, given by

$$R(m,T \longrightarrow T') = (-1)^{T'-1-T} [T']^{1/2} \times$$

$$\sum_{\omega_T} (-1)^{\omega_T} U(T1T1:T'\omega_T) \langle mT|| R^{\omega_T} ||mT\rangle \qquad (C-1)$$

Here T' is the isospin of the final states which can be connected by the

R^{ω_T} operator acting on $|mT\rangle$ space.

A. Expressing $K^{\omega} = (O^{\lambda'} \times O^{\lambda})^{\omega}$ in the normal form as a (1+2)-body operator

If $O^{\lambda} = \sum_{a,b} \epsilon^{\lambda}(ab)\ [\lambda]^{-1/2}\ (A^{a} \times B^{b})^{\lambda}$ is a one-body operator, then $T^{\omega} = (O^{\lambda'} \times O^{\lambda})^{\omega}$ is a (1+2)-body operator. We give below the expression for T^{ω} in normal multipole (in (j,t)) form

$$T^{\omega} = \sum_{abc} [\lambda\lambda']^{-1/2}\ \epsilon^{\lambda}(ab)\ \epsilon^{\lambda'}(bd)\ U(\lambda b\omega d:a\lambda')\ U(abab:\lambda 0) \times [b]^{1/2}\ (A^{a} \times B^{d})^{\omega}$$

$$- \sum_{abcd:pq} [\lambda\lambda']^{-1/2}\ \epsilon^{\lambda}(ab)\ \epsilon^{\lambda'}(cd) \overline{\begin{Bmatrix} a & b & \lambda \\ c & d & \lambda' \\ p & q & \omega \end{Bmatrix}} \times \left[(A^{a} \times A^{c})^{p} \times (B^{b} \times B^{d})^{q}\right]^{\omega}$$

(C-2)

Here $\overline{\begin{Bmatrix} a & b & \lambda \\ c & d & \lambda' \\ p & q & \omega \end{Bmatrix}}$ is the normalised 9j coefficient [and is related to ordinary 9j coefficient by a multiplicative factor $[\lambda\lambda' pq]^{1/2}$]. For our purpose we require averages of operators in spaces like $|mT\rangle$ or its configuration analogue $|\tilde{m}T\rangle$, where the initial space J-value is averaged out. Therefore, for $\lambda = \lambda'$, after J-averaging we have $\omega_J = 0$ and an additional $\sqrt{3}$ factor in eq. (C-1)[4]. Now with $\lambda = \lambda' = 1,1$ we give the explicit expressions for the one- and two- body parts of T^{ω_T} for different values of ω_T.

I. The One-body part

(i). $$T^{\omega_T=0}(1) = \sum_{rs} [\lambda]^{-1/2}\, \bar{\epsilon}_{rs}\, [r]^{1/2}\, (A^r \times B^s)^{\omega_J=0,\omega_T=0} \qquad \text{(C-3)}$$

$$\text{where } \bar{\epsilon}_{rs} = \delta_{rs} \sum_{b} \left| \epsilon^{11}(rb) \right|^2 [r]^{-1} \qquad \text{(C-4)}$$

(ii) $$T^{\omega_T=1}(1) = \sum_{rs} [\lambda]^{-1/2}\, \bar{\epsilon}_{rs}\, [r]^{1/2}\, (A^r \times B^s)^{\omega_J=0,\omega_T=1} \qquad \text{(C-5)}$$

$$\text{where } \bar{\epsilon}_{rs} = \delta_{rs} \sum_{b} (2/3)^{1/2} \left| \epsilon^{11}(rb) \right|^2 [r]^{-1} \qquad \text{(C-6)}$$

(iii) $T^{\omega_T=2}(1) = 0$ because $U(1\ \frac{1}{2}\ 2\ \frac{1}{1} : \frac{1}{2}\ 1)$ which appears in the one-body part in equation (C-2) is equal to zero.

$\bar{\epsilon}_{rs}(\omega_T=0)$ for the GT operator in sd and fp shell is $\bar{\epsilon}_{rs} = 9/2\ \delta_{rs}$; $\bar{\epsilon}_{rs}(\omega_T=0)$ for the isovector M1 operator in sd shell is given by, $\bar{\epsilon}_{11}$ = 16.34141, $\bar{\epsilon}_{22}$ = 7.91705, $\bar{\epsilon}_{33}$ = 11.90250 ($\bar{\epsilon}_{rs}$= 0 for $r \neq s$); and similarly, $\bar{\epsilon}_{rs}(\omega_T=1) = 3(3/2)^{1/2}\ \delta_{rs}$ for the GT operator in sd and fp shell and $\bar{\epsilon}_{rs}$ with ω_T = 1 for isovector M1 operator in sd shell is given by, $\bar{\epsilon}_{11}$ = 13.34208, $\bar{\epsilon}_{22}$ = 6.46221, and $\bar{\epsilon}_{33}$ = 9.71298, (again $\bar{\epsilon}_{rs}$ = 0 for $r \neq s$).

II. The Two-body Part

(i). $$T^{\omega_T=0}(2) = -\frac{1}{4} \sum_{rstu:\Gamma \equiv J,T} (1+\delta_{rs})^{1/2}\, (1+\delta_{tu})^{1/2}$$

$$[\Gamma]^{1/2}\ W^{\Gamma}_{rstu}\ \left[(A^r \times A^s)^{\Gamma} \times (B^t \times B^u)^{\Gamma} \right]^0$$

$$= \sum_{\substack{r\leq s\\ t\leq u\\ \Gamma}} [\Gamma]^{1/2}\ W^{\Gamma}_{rstu} \left(Z^{\Gamma}(rs) \times \bar{Z}^{\Gamma}(tu) \right)^0 \qquad \text{(C-7)}$$

with $(1+\delta_{rs}) = \zeta^{-2}_{rs}$, $Z^{\Gamma}(rs) = -\zeta_{rs}(A^r \times A^s)^{\Gamma}$ and $\bar{Z}(tu) = +\zeta_{tu}(B^t \times B^u)^{\Gamma}$. W^{Γ}_{rstu} is the fully antisymmetrised two-body matrix element and is given by[5)]

$$W^{\Gamma}_{rstu} = 2\zeta_{rs}\ \zeta_{tu} \left[\beta^{\Gamma}_{rstu} - (-1)^{r+s-\Gamma}\ \beta^{\Gamma}_{srtu} \right] \qquad \text{(C-8)}$$

where

$$\beta^{\Gamma}_{rstu} = [\lambda]^{-1/2}\ \epsilon^{\lambda}(rt)\ \epsilon^{\lambda}(su)\ (-1)^{r+u-\Gamma}\ W(rtsu{:}\lambda\Gamma) \qquad \text{(C-9)}$$

Symmetry of β^{Γ}_{rstu} with respect to (j,t)-orbits r,s,t,u provides symmetry relations for W^{Γ}_{rstu}. This is given by

$$W^{\Gamma}_{rstu} = -(-1)^{r+s-\Gamma}\ W^{\Gamma}_{srtu} = -(-1)^{t+u-\Gamma}\ W^{\Gamma}_{rsut} = (-1)^{r+s-t-u}\ W^{\Gamma}_{srut} \qquad \text{(C-10)}$$

(ii) $$T^{\omega_T=1}_{T}(2) = \sum_{rstu:J} \epsilon^{\lambda}(rs)\ \epsilon^{\lambda}(tu)\ \ [\lambda]^{-1/2}\ \ (-1)^{j_t+j_u+J}$$

$$W(j_r j_s j_t j_u{:}1J)\ [J]^{1/2}\ \frac{1}{\sqrt{6}}\ \left[\left((A^r \times A^t)^{J,1} \times (B^s \times B^u)^{J,0} \right)^{0,1} + \right.$$

$$\left[\left((A^r \times A^t)^{J,0} \times (B^s \times B^u)^{J,1}\right)^{0,1}\right] \qquad \text{(C-11)}$$

and one can see immediately that antisymmetrisation is impossible. Therefore, $W^{\Gamma}_{rstu} = 0$ for $\omega_T = 1$.

(ii). $T^{\omega_T=2}(2)$ part is similar to $T^{\omega_T=0}(2)$ part and can be expressed as

$$T^{\omega_T=2}(2) = -\frac{1}{4} \sum_{rstu:\Gamma \equiv J,1} (-\zeta^{-1}_{rs}\,\zeta^{-1}_{tu})\;[\Gamma]^{1/2}\; W^{\Gamma}_{rstu}$$

$$\left[(A^r \times A^s)^{\Gamma} \times (B^t \times B^u)^{\Gamma}\right]^{0,2} \delta_{T1}$$

(C-12)

where $W^{\Gamma}_{rstu} = W^{J,1}_{rstu}$ and its expression is similar to equation (C-8) and also obeys the same symmetry relations of equation (C-10), but for $\omega_T = 2$ case,

$$\beta^{\Gamma}_{rstu} = \beta^{J,1}_{rstu} = -\frac{1}{3}[\lambda]^{-1/2}\,\epsilon^{\lambda}(rt)\,\epsilon^{\lambda}(su)\,(-1)^{j_r+j_u-J}\;W(j_r j_t j_s j_u : 1J) \qquad \text{(C-13)}$$

It is therefore clear, comparing equations (C-9) and (C-13) (and with W(1/2 1/2 1/2 1/2 : 11) = 1/6) that $\beta^{\Gamma}_{rstu}(\omega_T{=}2) = -2\,\delta_{T1}\,\beta^{\Gamma}_{rstu}(\omega_T{=}0)$ and thus

$$\boxed{W^{\Gamma}_{rstu}(\omega_T{=}2) = -2\,\delta_{T1}\,W^{\Gamma}_{rstu}(\omega_T{=}0)}$$

(C-14)

B. Evaluation of averages of $M^{\omega} = \left[O^{\lambda'}, \left[H, O^{\lambda}\right]^{\omega}_{-}\right]^{\omega}_{-}$ and its product with H

Here we evaluate the averages of the operators $\langle M^{0}\rangle$ and $\langle M^{0} \times H\rangle$. Expression for the operator M^{0} in the standard (1+2)-body form is given by Halemane and French[5]. As O^{λ} is a one-body operator and H is a (1+2) -body operator, then M^{0} is also a (1+2) -body operator. Its one-body and two-body parts, $M^{0}(1)$ and $M^{0}(2)$, are determined by the one-body (H(1)) and two-body (H(2)) parts respectively of H. They are

$$M^{\omega}(1) = \left[O^{\lambda'}, \left[H(1), O^{\lambda}\right]^{\lambda}_{-}\right]^{\omega}_{-} = \sum_{abc} (\varepsilon_{aa} - \varepsilon_{bb})\ \epsilon^{\lambda}(ab)\ \epsilon^{\lambda'}(ac) \times$$

$$W(\lambda' a \omega b : c\lambda)\ (-1)^{\lambda'}\ \left[\left(A^{c} \times B^{b}\right)^{\omega} + \left(A^{b} \times B^{c}\right)^{\omega}\right] \quad \text{(C-15)}$$

Here ε_{aa}, ε_{bb} are the matrix elements of the one-body part of the Hamiltonian H. For the applications in this thesis we need explicit expressions for traces involving M^{ω_T} with $\omega_T = 0$ only. So we give here only those expressions. For $\lambda' = \lambda$ and $\omega_T = 0$, $M^{\omega_T=0}(1)$ i.e. the one body part of M^{ω_T} reduces to

$$M^{0}(1) = \left[O^{\lambda}, \left[H(1), O^{\lambda}\right]^{\lambda}_{-}\right]^{0}_{-} = \sum_{r,s} \eta_{rs}\ [r]^{1/2} \left(A^{r} \times B^{s}\right)^{0} \quad \text{(C-16)}$$

where

$$\eta_{rs} = 2\delta_{rs} \sum_{b} (\varepsilon_{bb} - \varepsilon_{rr})\ \epsilon^{\lambda}(rb)\ \epsilon^{\lambda}(br)\ (-1)^{r-b+\lambda}\ [\lambda]^{-1/2}\ [r]^{-1}$$

(C-17)

The three diagonal η_{rs} for the sd shell orbits $(1d_{5/2}1d_{3/2}2s_{1/2})$ using univ-sd interaction are given by (8.95101, −13.42651, 0.00000) and (18.90900, −28.36351, 0.00000) for GT and isovector M1 operators respectively. In the case of $\omega_T = 0$ the double commutator operator M^0 is converted to the standard (1+2) -body Hamiltonian form as given by

$$M^0 = \left[O^\lambda, \left[H, O^\lambda\right]_-^\lambda\right]_-^0 = \sum_{r,s} \eta_{rs} [r]^{1/2} \left(A^r \times B^s\right)^0$$

$$+ \sum_{\substack{rstu:\Gamma \\ r\leq s;\, t\leq u}} [\Gamma]^{1/2}\, \overline{\overline{W}}{}^\Gamma_{rstu} \left(Z^\Gamma(rs) \times \overline{Z}^\Gamma(tu)\right)^0 \qquad \text{(C-18)}$$

Here $\overline{\overline{W}}{}^\Gamma_{rstu}$ obeys same symmetry relations as in eq.(C-10) and

$$\overline{\overline{W}}{}^\Gamma_{rstu} = \zeta^2_{rs}\zeta^2_{tu} \left[X^\Gamma_{rstu} - (-1)^{r+s-\Gamma} X^\Gamma_{srtu} - (-1)^{t+u-\Gamma} X^\Gamma_{rsut} + (-1)^{r+s-t-u} X^\Gamma_{srut}\right] \qquad \text{(C-19)}$$

For the detailed expression of X^Γ_{rstu}, in terms of ϵ_{rs} and two-body matrix elements $\overline{W}{}^\Gamma_{rstu}$ of the Hamiltonian, we refer to Halemane and French[5]. One can now use the codes[6] for calculating averages of products of two isoscalar operators to evaluate $\langle M^{\omega_T=0}\rangle$ and $\langle M^{\omega_T=0} \times H\rangle$. The extension of this for $\omega_T \neq 0$ is also done.

C. Conversion to p-n language

Here we recast equations (C-3), (C-5), (C-7) and (C-12) in p-n language. The basic components of this conversion are[4]

$$A^J(M_T=1/2) = a^+_{J,1/2,M_J,1/2} = A^J_n$$

$$A^J(M_T=-1/2) = a^+_{J,1/2,M_J,-1/2} = A^J_p$$

$$B^J(M_T=1/2) = (-1)^{J+1/2+M_J+1/2}\, a_{J,1/2,-M_J,-1/2} = -B^J_p$$

$$B^J(M_T=-1/2) = (-1)^{J+1/2+M_J-1/2}\, a_{J,1/2,-M_J,1/2} = B^J_n \quad \text{(C-20)}$$

Then we can write (for isospin part only),

$$\left(A^{j_r,1/2} \times B^{j_s,1/2}\right)^{0,0}_{0} = \sum_{\mu} C^{1/2\ \ 1/2\ \ 0}_{\mu\ \ -\mu\ \ 0} \left(A^{j_r,1/2}_{\mu} \times B^{j_s,1/2}_{-\mu}\right)^{0,0}_{0}$$

$$= \frac{1}{\sqrt{2}} \left[\left(A_n^{j_r} \times B_n^{j_s}\right)^0 + \left(A_p^{j_r} \times B_p^{j_s}\right)^0 \right]$$

where we put appropriate Clebsch-Gordan coefficient involved in the sum. Therefore the one- and two- body parts of T^{ω_T} in p-n language are

I. The one-body part

$$\text{(i).}\ T^{\omega_T=0}(1) = \sum_{rs} [\lambda]^{-1/2}\, \bar{\epsilon}_{rs} \left[\hat{n}^r_n + \hat{n}^r_p \right] \quad \text{(C-21)}$$

where $\hat{n}^r_{n(p)}$ is the number operator for neutron(proton) and $\bar{\epsilon}_{rs}$ are as defined earlier in section A of this appendix. Similarly

$$\text{(ii).}\ T^{\omega_T=1}(1) = \sum_{rs} [\lambda]^{-1/2}\, \bar{\epsilon}_{rs} \left[\hat{n}^r_n - \hat{n}^r_p \right] \quad \text{(C-22)}$$

II. The Two-body Part

(i). To obtain two-body part of H(2), $T^{\omega_T=0}(2)$ and $T^{\omega_T=2}(2)$ in p-n language, we first note that

$$\left[(A^r \times A^s)^{J,1} \times (B^t \times B^u)^{J,1}\right]^{0,0}_{0}$$

$$= \sum_{\mu} C^{1\ \ 1\ \ 0}_{\mu\ \ -\mu\ \ 0} \left[\left(A^{j_r,1/2} \times A^{j_s,1/2}\right)^{J,1}_{\mu} \times \left(B^{j_r,1/2} \times B^{j_s,1/2}\right)^{J,1}_{-\mu}\right]^{0,0}_{0}$$

(C-23)

and

$$\left[(A^{r}\times A^{s})^{J,1}\times(B^{t}\times B^{u})^{J,1}\right]^{0,2}_{0}$$

$$=\sum_{\delta} C^{\,1\ \ 1\ \ 2}_{\,\delta\ -\delta\ 0}\left[\left(A^{j_r,1/2}\times A^{j_s,1/2}\right)^{J,1}_{\delta}\times\left(B^{j_r,1/2}\times B^{j_s,1/2}\right)^{J,1}_{-\delta}\right]^{0,2}_{0}$$

(C-24)

where $C^{\,1\ 1\ 0}_{\,\mu\ -\mu\ 0}$ and $C^{\,1\ 1\ 2}_{\,\delta\ -\delta\ 0}$ are the Clebsch-Gordan(CG) coefficients for the isospin part only. Now putting the values of the CG coefficients in eq.(C-23) and (C-24) and again expanding operators like

$\left(A^{j_r,1/2}\times A^{j_s,1/2}\right)^{J,1}_{\delta}$ with proper CG coefficients we finally get,

$$H(2)\left(\text{ or } T^{\omega_T=0}(2)\right)=-\frac{1}{4}\sum_{rstu:J}(1+\delta_{rs})^{1/2}\,(1+\delta_{tu})^{1/2}\,[J]^{1/2}$$

$$\Bigg[W^{J,1}_{rstu}\left[\left(A^{r}_{n}\times A^{s}_{n}\right)^{J}\times\left(B^{t}_{n}\times B^{u}_{n}\right)^{J}\right]^{0}$$

$$+W^{J,1}_{rstu}\left[\left(A^{r}_{p}\times A^{s}_{p}\right)^{J}\times\left(B^{t}_{p}\times B^{u}_{p}\right)^{J}\right]^{0}$$

$$+\frac{1}{2}\left(W^{J,1}_{rstu}+W^{J,0}_{rstu}\right)\left[\left(A^{r}_{n}\times A^{s}_{p}\right)^{J}\times\left(B^{t}_{n}\times B^{u}_{p}\right)^{J}\right]^{0}$$

$$+\frac{1}{2}\left(W^{J,1}_{rstu}+W^{J,0}_{rstu}\right)\left[\left(A^{r}_{p}\times A^{s}_{n}\right)^{J}\times\left(B^{t}_{p}\times B^{u}_{n}\right)^{J}\right]^{0}$$

$$+\frac{1}{2}\left(W^{J,1}_{rstu}-W^{J,0}_{rstu}\right)\left[\left(A^{r}_{n}\times A^{s}_{p}\right)^{J}\times\left(B^{t}_{p}\times B^{u}_{n}\right)^{J}\right]^{0}$$

$$+\frac{1}{2}\left(W^{J,1}_{rstu}-W^{J,0}_{rstu}\right)\left[\left(A^{r}_{p}\times A^{s}_{n}\right)^{J}\times\left(B^{t}_{n}\times B^{u}_{p}\right)^{J}\right]^{0}\Bigg]$$

(C-25)

and

$$T^{\omega_T=2}(2) = -\frac{1}{4}\sum_{rstu:J} (1+\delta_{rs})^{1/2} (1+\delta_{tu})^{1/2} [J]^{1/2} W^{J,1}_{rstu} \frac{1}{\sqrt{2}}$$

$$\Bigg[\left[\left(A^r_n \times A^s_n\right)^J \times \left(B^t_n \times B^u_n\right)^J \right]^0 + \left[\left(A^r_p \times A^s_p\right)^J \times \left(B^t_p \times B^u_p\right)^J \right]^0$$
$$- \left[\left(A^r_n \times A^s_p\right)^J \times \left(B^t_n \times B^u_p\right)^J \right]^0$$
$$- \left[\left(A^r_p \times A^s_n\right)^J \times \left(B^t_p \times B^u_n\right)^J \right]^0$$
$$- \left[\left(A^r_n \times A^s_p\right)^J \times \left(B^t_p \times B^u_n\right)^J \right]^0$$
$$- \left[\left(A^r_p \times A^s_n\right)^J \times \left(B^t_n \times B^u_p\right)^J \right]^0 \Bigg]$$

(C–26)

One may note that whereas the structure of $H(2)\left(T^{\omega_T=0}(2)\right) \longrightarrow$

$W^J_{rstu}(nn)$ [nnnn] + $W^J_{rstu}(pp)$ [pppp] +

$W^J_{rstu}(pn)$ [npnp + pnpn − nppn −pnnp],

the $T^{\omega_T=2}(2)$ part is given by

$$T^{\omega_T=2}(2) \longrightarrow \frac{1}{\sqrt{2}} W^{J,1}_{rstu} \text{ [nnnn + pppp − npnp − pnpn + nppn + pnnp]}$$

Where we have used[3] $W^J_{rstu}(pn) = \frac{1}{2\zeta_{rs}\zeta_{tu}}\left[W^{J,1}_{rstu} + W^{J,0}_{rstu}\right]$ and also the commutation relations of the p–n field. The double commutator operator M^{ω_T}

of section B has the same structure as T^{ω_T}, and can therefore be rewritten in p-n language in a similar fashion.

C. Evaluation of operators of the form $\langle xT || \left(O^{\lambda} \times H^2 \times O^{\lambda}\right) || xT\rangle$

We have already described the method of evaluation of double-barred averages of the operators of the form $\langle xT' || \left(H^2 \times (O^{\lambda} \times O^{\lambda})^{\omega_T}\right)^{\omega_T} || xT'\rangle$. Now we find that each ω_T component of the double-barred average of the form $\langle xT || (O^{\lambda} \times H^2 \times O^{\lambda})^{\omega_T} || xT\rangle$ can be evaluated by connecting it to the double-barred averages $\langle zT'' || \left(H^2 \times (O^{\lambda} \times O^{\lambda})^{\omega_T}\right)^{\omega_T} || zT''\rangle$ in appropriate space $|zT''\rangle$. The relation between these two sets of averages is given by the general expression,

$$\langle x\Gamma || \left[(O^{\lambda} \times H^2 \times O^{\lambda})^{\omega}\right]^{\omega} || x\Gamma\rangle = \frac{[\Gamma]^{1/2}}{d(x\Gamma)} (-1)^{-\omega} \sum_{y\Gamma'} [\Gamma']^{-1/2} \; d(y\Gamma')$$

$$U(\Gamma\lambda\Gamma\lambda;\Gamma\omega') \sum_{\omega'} (-1)^{-\omega'} \; U(\Gamma'\lambda\Gamma'\lambda;\Gamma\omega) \; \langle y\Gamma' || \left(H^2 \times (O^{\lambda} \times O^{\lambda})^{\omega'}\right)^{\omega'} || y\Gamma'\rangle$$

(C-27)

This is derived using equation (3.22) and (3.23) of ref.3. In our calculations we have used eq.(C-27) to get

$$\langle mT=0 || \left((O^{\lambda} \times H^2 \times O^{\lambda})^{0,0}\right)^{0,0} || mT=0\rangle \text{ using}$$

$$\langle mT=1 || \left(H^2 \times (O^{\lambda} \times O^{\lambda})^{0,\omega_T}\right)^{0,\omega_T} || mT=1\rangle \text{ for } \omega_T = 0,1,2.$$

D. Conversion of traces from proton-neutron to isospin formalism

We first evaluate the traces of the operator O^{ω_T} for each ω_T (=0,1,2) in spaces with fixed T_z. As we need finally the traces in fixed -T space, we are to convert the fixed-T_z traces to fixed-T traces. For this we observe that the trace with a particular T_z consists of contributions from all $T \geq |T_z|$ allowed by the space, i.e. with $T \leq m/2$ for $m \leq N/2$ and $T \leq (N-m)/2$ for $m > N/2$, where m is the number of valence particles and N is the total number of single particle states. For isoscalar operators $O^{\omega_T=0}$ the method is direct, and to get the trace in spaces with fixed isospin T one simply subtracts the trace for T_z = T+1 from the trace with T_z = T i.e.

$$\langle\langle O^{\omega_T=0}\rangle\rangle^{T,T_z=T} = \langle\langle O^{\omega_T=0}\rangle\rangle^{T_z=T} - \langle\langle O^{\omega_T=0}\rangle\rangle^{T_z=T+1} \tag{C-28}$$

and in this case the double-barred trace is

$$\langle\langle || O^{\omega_T=0} || \rangle\rangle^{T} = \sqrt{(2T+1)}\, \langle\langle O^{\omega_T=0}\rangle\rangle^{T,T_z=T} \tag{C-29}$$

For non-isoscalar operators O^{ω_T} one has to generalise this procedure involving appropriate Clebsch-Gordan (CG) coefficients. One can show[7] that in a space consisting of m_n-neutrons and m_p-protons, the trace of an operator O^{ω_T} with isospin rank ω_T the general expression is

$$\langle\langle O^{\omega_T}\rangle\rangle^{m_n=m-k,\ m_p=k} = \sum_{i=0}^{k} (m-2i+1)^{-1/2}\, C^{m/2-i\ \ \omega_T\ \ m/2-i}_{m/2-k\ \ 0\ \ m/2-k} \times \langle\langle || O^{\omega_T} || \rangle\rangle^{m,T=m/2-i} \tag{C-30}$$

Here $C^{m/2-i\ \omega_T\ m/2-i}_{m/2-k\ 0\ \ m/2-k}$ represents the CG coefficient. One can then evaluate double-barred traces $\langle\langle||O^{\omega_T}||\rangle\rangle^{mT}$ by inverting the set of equations represented by the eq.(C-30) starting from $T = T_{max}$.

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