APPENDIX B

EDGEWORTH, GRAM–CHARLIER AND CORNISH–FISHER EXPANSIONS

In the Gram–Charlier representation[1),2)] the density $\eta(\hat{x})$ [$\hat{x} = (x-x_c)/\sigma$, is called the standardized variable] is expanded in terms of polynomials, $P_\mu(\hat{x})$, defined by an asymptotic density $\eta_0(x)$,

$$\eta(\hat{x}) = \eta_0(\hat{x}) \left[1 + \sum_{\mu \geq 3} a_\mu P_\mu(\hat{x}) \right] \qquad \text{(B-1)}$$

where the a_μ are the polynomial moments expressible in terms of the shape parameters of $\eta(\hat{x})$. Using for $\eta_0(\hat{x})$ the asymptotic density, the Gaussian density

$$\eta_G(\hat{x}) = (2\pi)^{-1/2} \exp(-\hat{x}/2\ ) \qquad \text{(B-2)}$$

and retaining the terms upto order four in (B–1), a truncated GC series, with γ_1, γ_2 corrections, is obtained

$$\eta_{GC}(\hat{x}) = \eta_G(\hat{x}) \left[1 + \gamma_1/6\ (\hat{x}^3 - 3\hat{x}) + \gamma_2/24\ (\hat{x}^4 - 6\hat{x}^2 + 3\ ) \right] \qquad \text{(B-3)}$$

The EW series for $\eta(\hat{x})$ can be obtained from the asymptotic density $\eta_0(\hat{x})$ by

$$\eta(\hat{x}) = \exp \left[\sum_{r \geq 3} (-1)^r (\ K_r / \ r!) \frac{\partial^r}{\partial \hat{x}^r} \right] \eta_0(\hat{x}) \qquad \text{(B-4)}$$

where ϵ_r and $\epsilon_r + K_r$ are the cumulants for $\eta_0(\hat{x})$ and $\eta(\hat{x})$ and taking again $\eta_G(\hat{x})$ for $\eta_0(\hat{x})$, the truncated EW series which takes γ_1, γ_2 corrections up to order m^{-1} into account is given by,

$$\eta_{EW}(\hat{x}) = \eta_G(\hat{x}) \left[1 + \gamma_1/6\ (\hat{x}^3 - 3\hat{x}) + \left\{ \gamma_2/24\ (\hat{x}^4 - 6\hat{x}^2 + 3) + \gamma_1^2/72\ (\hat{x}^6 - 15\hat{x}^4 + 45\hat{x}^2 - 15) \right\} \right] \qquad \text{(B-5)}$$

It is important to note here that the truncated expansion forms in equations (B-3) and (B-5) are not valid in general for all x; that is, the expansions are not positive definite for all values of x, with fixed values of γ_1, γ_2. These truncated expansions are valid only in certain domain of γ_1, γ_2 values.

Alternatively, instead of expanding $\eta(\hat{x})$ around $\eta_G(\hat{x})$, one may look for a transformation $y = f(\hat{x})$, with y having a known asymptotic density $\eta_0(y)$; then,

$$\eta(\hat{x}) = \eta_0(y) \; \left|\frac{\partial}{\partial x} f(x)\right| \qquad \text{(B-6)}$$

where the quantity $\left|\frac{\partial}{\partial x} f(x)\right|$ represents the transformation Jacobian. The only condition for this approach to be valid for all x is that $x = f^{-1}(y)$ be a single-valued function of y; that is, the value of $\frac{\partial}{\partial x} f(x)$ always has the same sign. In the Cornish-Fisher method one writes the transformation $y = f(\hat{x})$ as an asymptotic series using polynomials in $\hat{x}$ as $y = \sum_{\nu} c_{\nu} \; g_{\nu}(\hat{x})$. The polynomials $g_{\nu}(\hat{x})$ and the expansion co-efficients c_{ν} are derived using the EW expansion. Taking y to be a Gaussion random variable, the Cornish-Fisher expansion for y to order m^{-1} is given by

$$y = \hat{x} - \gamma_1/6 \; (\hat{x}^2 - 1) + [\; -\gamma_2/24 \; (\; \hat{x}^3 - 3\hat{x} \;) + \gamma_1{}^2/36 \; (\; 4\hat{x}^{\,3} - 7\hat{x}) \;] \qquad \text{(B-7)}$$

The inverse transformation to the same order (1/m) is

$$\hat{x} = y + [\gamma_1/6(y^2-1) \;] + [\; \gamma_2/24(y^3-3y) - \gamma_1^2/36 \; (2y^3 - 5y)] \qquad \text{(B-8)}$$

Using equations (B-7) and (B-8), the CF representation (truncated) of $\eta(\hat{x})$ is

$$\eta_{CF}(\hat{x}) = (2\pi)^{-1/2} \left| \left[1-\gamma_1/3\ \hat{x} + \gamma_1^2/36\ (12\hat{x}^2-7)-\gamma_2/8\ (\hat{x}^2-1) \right] \right|$$

$$\exp \left\{ -1/2\ [\hat{x}-\gamma_1/6\ (\hat{x}^2-1)-\gamma_2/24\ (\hat{x}^3-3\hat{x}) + \gamma_1^2/36\ (4\hat{x}^3-7\hat{x})]^2 \right\} \quad \text{(B-9)}$$

REFERENCES FOR APPENDIX B

1). Kendall M G and Stuart A, Advanced Theory of Statistics Vol-1, (Hafner, New YorK, 1969); Johnson N L and Katz S, Continuous Univariate Distribution Vol-1, (Houghton-Mifftin, Boston, 1970).

2). Kota V K B, Potbhare V and Shenoy P, Phys. Rev. C34 (1986) 2330