CHAPTER V

STRENGTH DISTRIBUTION BY SPECTRAL DISTRIBUTION THEORY

5.1 THE METHOD

In this chapter we study the distribution of the transition strength as a function of the excitation energy of the final nucleus for GT and M1 excitations. Experimental studies[1),2),3)] of the spin-isospin properties of nuclei through (p,n), (n,p) and inelastic (e,e'), (p,p') reactions furnish valuable informations about the GT and M1 strength distributions. But the strength distribution extracted from these reaction studies are known to be quenched considerably and the quenching is thought to be caused mainly by two different mechanisms i) the mixing of ΔN^{-1} i.e. the Δ isobar-nucleon hole states with NN^{-1} i.e. nucleon particle-nucleon hole GT states causing a part of the strength to move higher up in energy far beyond the giant resonance region and ii) nuclear configuration[4)] mixing involving many orbits across major shells caused primarily by the tensor force. Though formally our method when extended to include many orbits belonging to different shells can be used to study the second mechanism, we as a first application in this chapter shall describe studies only within the sd shell orbits. So for comparisons, we shall not use experimental strength distributions extracted from reaction studies, but compare the theoretical predictions with the available shell model values. The transition strength $R(E,E')$ is defined as $R(E,E') = | \langle E' | O | E \rangle |^2$, where O is the excitation operator, E and E' are the energies of the initial and final states. We use the recently developed strength density formalism, as described in section 2.4 of chapter II, for studying the strength distribution and demonstrate its application to nuclei with ground state T equal to zero. In this method the quantity $R(E,E')I(E')$ is given by[5)]

$$R(E,E')I(E') = \langle O^{+}O\rangle \; S(E,E')/\rho(E) \qquad (5.1-1)$$

We shall approximate[5)] $S(E,E')$, the strength density, by its asymptotic bivariate Gaussian form $S_G(E,E')$, given by equation (2.4-4) of chapter II. To evaluate $S_G(E,E')$ we need to calculate the quantities $\langle O^{+}O\rangle$, ϵ_1, σ_1, ϵ_2, σ_2 and ζ. These six quantities in the scalar-T space are given in terms of the six averages

$$\langle (O^{\lambda}\times O^{\lambda})^{\omega}\rangle^{mT},\ \langle (O^{\lambda}\times O^{\lambda})^{\omega} H\rangle^{mT},\langle (O^{\lambda}\times O^{\lambda})^{\omega} H^{2}\rangle^{mT},\langle (O^{\lambda}\times H\times O^{\lambda})^{\omega}\rangle^{mT},$$

$$\langle (O^{\lambda}\times H^{2}\times O^{\lambda})^{\omega}\rangle^{mT},\ \text{and}\ \left\langle \left(O^{\lambda}\times\left(\frac{H-\epsilon_1}{\sigma_1}\right)\times O^{\lambda}\times\left(\frac{H-\epsilon_2}{\sigma_2}\right)\right\rangle^{mT}\right.$$

respectively. The method of evaluation of the first three have already been described in chapter IV in connection with the calculation of the strength sums. We now describe the evaluation of the other three quantities. We define[6)] the double commutator $M^{0} = [O^{\lambda},[H,O^{\lambda}]_-]_-^{\,0}$ where $[A,B]_-$ denotes the commutator of A and B and M^{0} is a (1+2)-body scalar operator. The average $\langle M^{0}\rangle^{mT=0}$ can then be expressed as

$$\langle M^{0}\rangle^{mT=0} = 2\left[\langle (O^{\lambda}\times H\times O^{\lambda})^{\omega}\rangle^{mT=0} - \langle (O^{\lambda}\times O^{\lambda})^{\omega} H\rangle^{mT=0}\right] \qquad (5.1-2)$$

Firstly one has to calculate the one and two -body matrix elements of M^{0} as described in Appendix C and then evaluate the average $\langle M^{0}\rangle^{mT=0}$ by spectral distribution methods. Then using equation (5.1-2)

$\langle (O^{\lambda} \times H \times O^{\lambda})^{\omega} \rangle^{mT=0}$ can be obtained in terms of $\langle (O^{\lambda} \times O^{\lambda})^{\omega} H \rangle^{mT=0}$.

The evaluation of $\langle (O^{\lambda} \times H^2 \times O^{\lambda})^{\omega} \rangle^{mT}$ in terms of $\langle (O^{\lambda} \times O^{\lambda})^{\omega} H^2 \rangle^{mT}$

with $\omega_T = 0,1,2$ is described again in Appendix C. Finally for the

calculation of $\langle (O^{\lambda} \times \left(\frac{H-\epsilon_1}{\sigma_1}\right) \times O^{\lambda} \times \left(\frac{H-\epsilon_2}{\sigma_2}\right) \rangle^{mT}$, the new average required

is $\langle (O^{\lambda} \times H \times O^{\lambda} \times H) \rangle^{mT}$. For that we first note

$$\left[[O^{\lambda},[H,O^{\lambda}]_-]_-^{\omega} \times H\right]^{\omega} = \left(1+(-1)^{-\omega}\right) (O^{\lambda} \times H \times O^{\lambda} \times H)^{\omega}$$

$$-(O^{\lambda} \times O^{\lambda} \times H^2)^{\omega} - (-1)^{-\omega}(H \times (O^{\lambda} \times O^{\lambda})^{\omega} \times H)^{\omega} \qquad (5.1\text{-}3)$$

Taking the trace of both sides in (mT=0) space, the averages can be expressed

$$\langle M^0 H \rangle^{mT=0} = 2 \langle (O^{\lambda} \times H \times O^{\lambda} \times H)^0 \rangle^{mT=0} - \langle (O^{\lambda} \times O^{\lambda})^0 \times H^2 \rangle^{mT=0}$$

$$- \langle H \times (O^{\lambda} \times O^{\lambda})^0 \times H \rangle^{mT=0}$$

Then for $O^{\lambda} = O^{11}$ we write

$$\langle (O^{11} \times H \times O^{11} \times H)^{0,0} \rangle^{mT=0}$$

$$= \frac{1}{2} \langle M^0 H \rangle^{mT=0} + \langle (O^{11} \times O^{11})^{0,0} \times H^2 \rangle^{mT=0} \qquad (5.1\text{-}4)$$

This completes the calculation of the traces needed involving the products of upto three scalar (1+2)-body operators. We modify the available computer programmes[7] for our purpose to evaluate these traces.

5.2 RESULTS AND DISCUSSIONS

In table 5.2-T1 we display the values of the marginal centroids ϵ_1, ϵ_2, marginal widths σ_1, σ_2 and the strength correlation coefficient ζ for five self-conjugate sd shell nuclei for both GT and M1 excitations. Though we evaluate both σ_1 and σ_2 explicitly and use them for the strength distribution calculations, but table 5.2-T1 clearly shows an approximation which assumes $\sigma_2 = \sigma_1$ is also fairly good for both the excitation operators. We also observe that though the values of ϵ_1 and ϵ_2 increase fast with the number of valence particles (holes) the widths σ_1 and σ_2 show a much slower variation. We note here that ζ calculated using EGOE ensembles[5] for both H (k-body) and the excitation operator (k'-body) in finite spaces is given by $(1-kk'/m) + O(1/m^2)$. So the ensemble averaged value of ζ with k=2 and $k'=1$ is close to the exact values given in table (5.2-T1).

Table 5.2-T1

The marginal strength centroids and strength widths along with the strength correlation coefficient (defined in equations (2.4-5a) and (2.4-5b) respectively) for five self-conjugate sd shell nuclei, with GT and isovector M1 excitation operators.

operator	Nucleus	ϵ_1	ϵ_2	σ_1	σ_2	ζ
GT	^{20}Ne	-15.24	-14.60	7.38	6.82	0.74886
	^{24}Mg	-44.94	-44.33	10.76	10.47	0.86205
	^{28}Si	-87.36	-86.78	11.84	11.45	0.88888
	^{32}S	-141.26	-140.71	10.78	10.49	0.84966
	^{36}Ar	-205.77	-205.23	7.76	7.27	0.71324
$M1^{\Delta T=1}$	^{20}Ne	-16.03	-15.23	7.20	6.67	0.77152
	^{24}Mg	-45.55	-44.79	10.70	10.39	0.87562
	^{28}Si	-87.67	-86.94	11.82	11.58	0.88831
	^{32}S	-141.16	-140.46	10.77	10.52	0.86463
	^{36}Ar	-205.14	-204.47	7.73	7.30	0.75250

We now describe the strength distribution in terms of the quantity $R(E,E')I(E')$ evaluated by using the form of equation (5.1-1). Fig.5.2-F1 shows this quantity for the isovector M1 excitation for ^{28}Si as a function of the final state excitation energy i.e. setting the energy of the lowest T=1 state as zero and compares it with the shell model strength distribution, taken from Anantaraman et al[8) . For E we use the ground state energy of ^{28}Si (i.e. m=12,T=0). The first T=1 state in ^{28}Si is seen to be at an excitation energy of 8.19 MeV by SDM whereas by shell model it appears at an excitation energy of 10.81 MeV. The shell model strength distribution is narrow with the centroid at 12.3 MeV and the width of the corresponding smoothed distribution being 1.4 MeV. A continuous version of the shell model strength is generated by taking the envelope curve after summing all strengths with a 0.8 MeV (i.e. width = 0.4 MeV) Gaussian broadening function. The total strength for the shell model calculation i.e. the area below the curve corresponding to the smoothed shell model strength distribution is 8.8 μ_n^2. In the SDM calculation, the area below the strength curve for the energy above that of the 1st T = 1 state is 2.14 μ_n^2 , whereas the total sum rule strength $M_0(E)$ by SDM is 8.8 μ_N^2. This is because the calculation for the SDM strength function using the asymptotic form of $S(E,E')$ pushes a substantial part of the total strength in the unphysical region below the T = 1 state. In figure 5.2-F1 we also display a modified strength function where we renormalise the strength by a multiplicative factor to make the strength sum in this region equal to 8.8 μ_n^2 . This shows a better agreement with the shell model. But one actually needs to improve the form for the strength distribution predicted by SDM to eliminate the undesirable feature of the strength below T = 1 ground state. But the fact that the simple form of equation (5.1-1) brings the strength down to the correct energy domain is quite remarkable and this is achieved by the high value of ζ (=0.89) for isovector M1 operator. For the sake of comparison we observe that the final state density has its centroid at an excitation energy of 56.7 MeV and a form for the strength distribution making an expansion of $R(E,E')$ and keeping the first four terms of the expansion in equation (2.2-3) gives a peak for the isovector M1 excitation of ^{28}Si at very high excitation energy of 35.7 MeV. If for the T = 0 ground state binding energy of ^{28}Si one uses the SDM value instead of the shell model one, the peak appears at an excitation energy of 45.0 MeV . It is already known that the expansion of the strength distribution in terms of the strength density $S(E,E')$ has good convergence properties for large values of the strength correlation coefficient ζ close to 1. This is indeed the case for both GT and M1 excitations in sd shell. But in these cases the double polynomial expansion for strength shows a very slow convergence[9].

In fig.5.2-F2 we show the GT excitation strength distribution evaluated using the form of equation (5.1-1) for the nucleus ^{28}Si. As the ground state of the final space (m = 12 , T = 1 space) lies 8.19 MeV higher than the ^{28}Si ground state, one can observe only the upper part of the distribution. This type of excitation strength function is useful for (n,p) studies where the strength goes from $T_i \longrightarrow T_f = T_i + 1$ only where T_i stands for the initial T value. Our calculation should be extended to nuclei with non-zero initial isospin so that the strength function for transitions $T_i \longrightarrow T_i-1$, T_i, T_i+1 can all be observed. This will then correspond to strength functions observed in (p,n) reactions and comparisons with strength forms extracted from experiment will be easier to make.

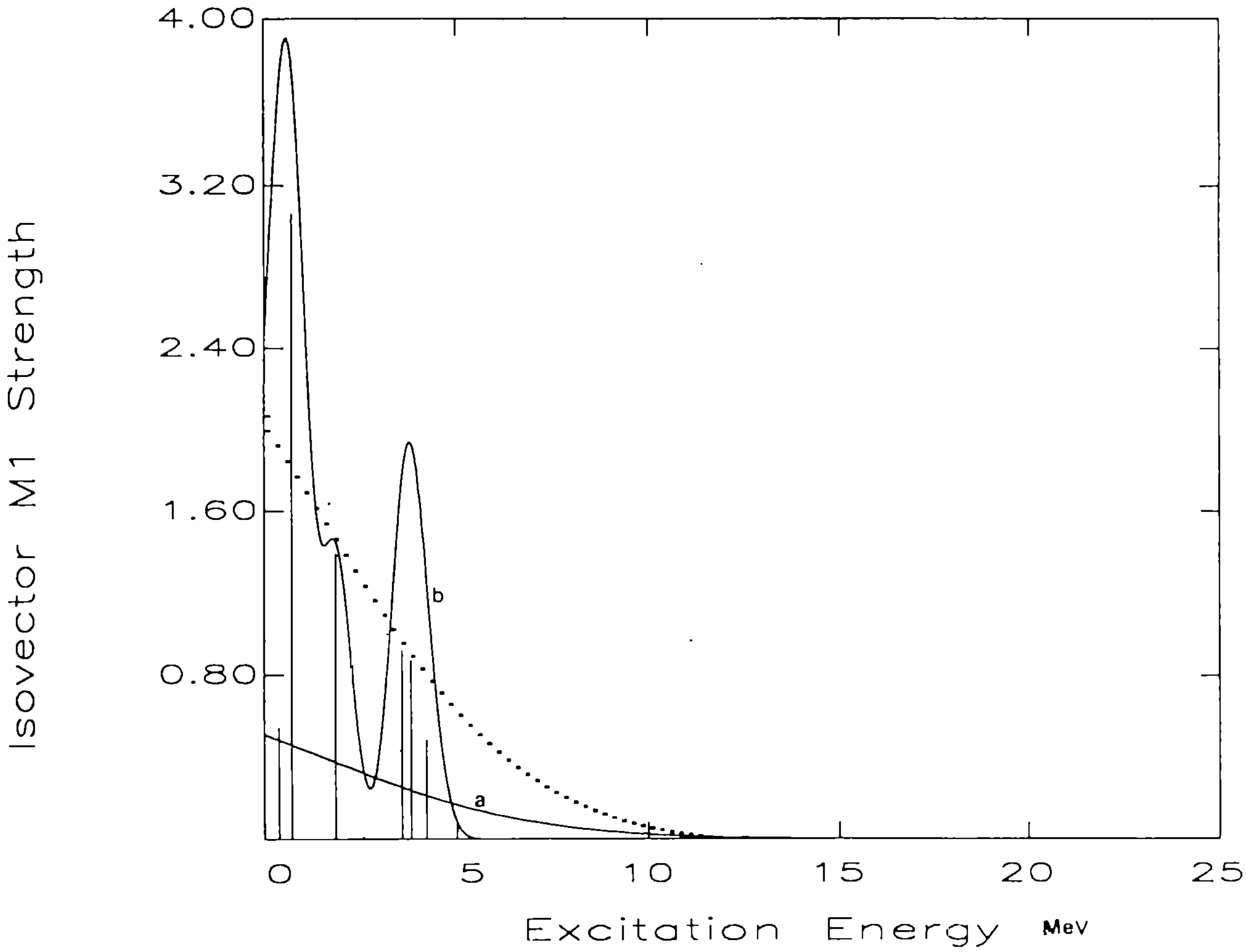

Figure 5.2-F1 : The isovector M1 strength distribution per unit energy expressed as $I(E') \times R(E_{gs},E')$ for ^{28}Si calculated by S-strength method is shown as a function (curve a) of the final state excitation energy E'. We set the zero of energy at the energy of the lowest T = 1 state. A continuous version of the shell model strength (curve b) generated by taking the envelope curve after summing all discrete ($B(M1^{\Delta T=1}$ values, given as spikes) strengths with a 0.8 MeV Gaussian broadening function is also shown. The dotted curve is the modified SDM strength function after renormalisation.

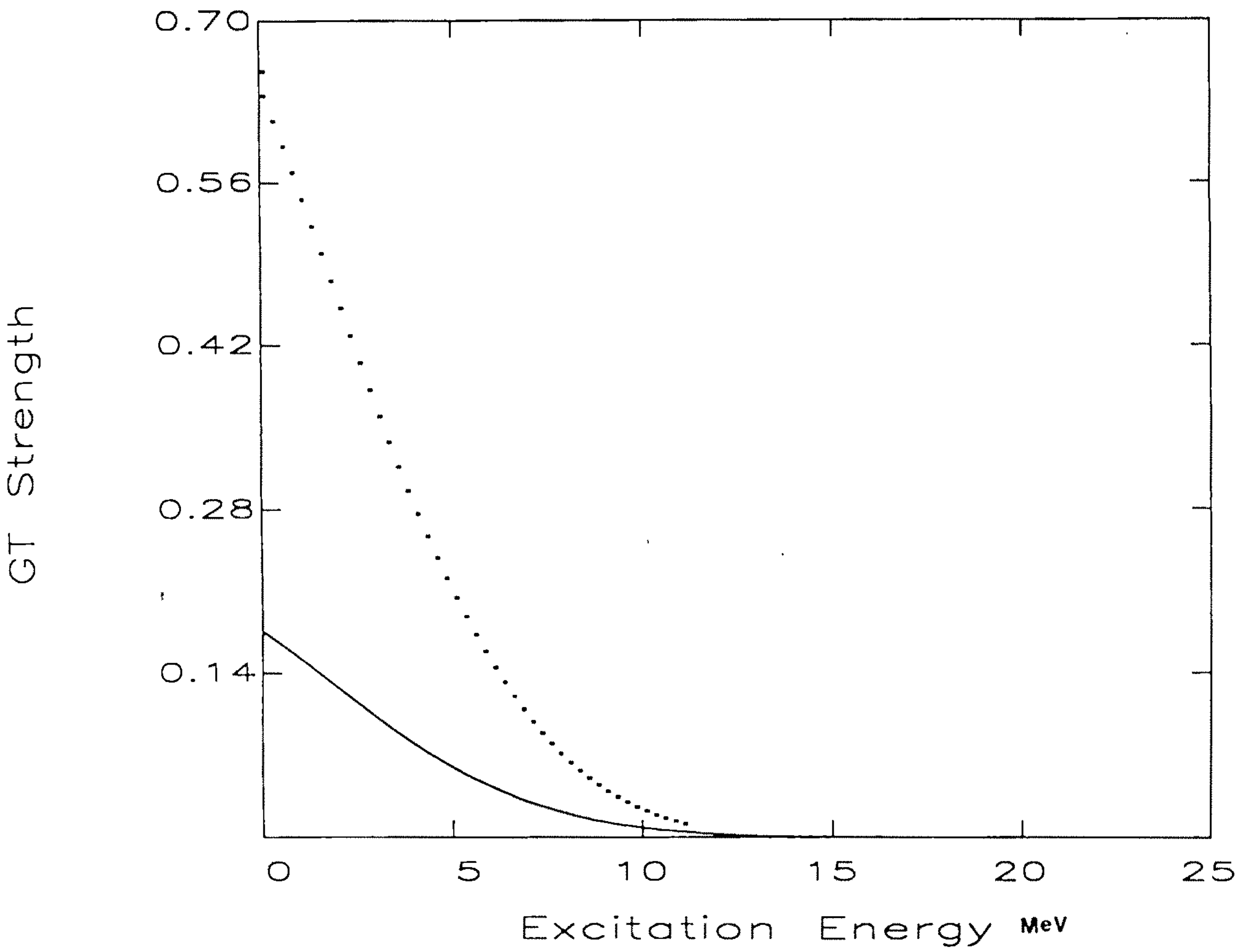

Figure 5.2-F2 : Gamow-Teller strength distribution per unit energy for ^{28}Si calculated using the S-strength form for the quantity $I(E') \times R(E_{gs}, E')$. The zero of the energy is set at the energy of the lowest T = 1 state. The dotted curve represents the renormalised SDM strength function.

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