CHAPTER – I

GOAL PROGRAMMING

1.1 GENERAL

The traditional mathematical programming approach to the modeling of farm/firm decisions rests on certain basic assumptions about the situation being modeled and the decision maker (DM) himself. One fundamental assumption is that the decision maker seeks to optimize a well defined single objective. In reality this is not the case, as the DM is usually seeking an optimal compromise amongst several objectives, many of which can be in conflict on trying to achieve satisfactory levels of his goals. For instance, a subsistence farmer may be interested in securing adequate food supplies for the family, maximizing cash income, increasing leisure, avoiding risk, etc., but not necessarily in that order. Similarly a commercial farmer may wish to maximize gross margin, minimize his indebtedness, acquire more land, reduce fixed costs, etc.

Despite the recognition given to the existence of multiple objectives in farm/firm decision making, not much is done to develop and use methodologies that model the decision realistically. This is particularly intriguing when one notices the amount of effort that has been devoted to the development and use of multiple criteria decision making (MCDM) techniques in the related disciplines.
1.2 TRADITIONAL PARADIGM FOR DECISION MAKING

The traditional paradigm that has been used for the analysis of decision making presupposes the existence of three elements: a decision maker (an individual or a group recognized as a single entity); an array of feasible choices; and, a well defined criterion such as utility or profit. The given criterion can then be used to associate a number with each alternate so that the feasible set can be ranked or ordered to find the optimal value.

This paradigm has served us well so far. Notwithstanding the fact that this paradigm is logically sound, it does not faithfully reflect the real life decision situations. The DM is usually not interested in ordering the feasible set according to just a single criterion but seems to be striving to find an optimal compromise against several objectives.

As an illustration, consider the problems of decision making in some large corporation which is interested not just in maximizing profits but in optimizing other objectives such as sales or the growth of the firm. In other instances such firms seem to achieve some goals that have been fixed ex ante for some of their objectives rather than optimize any generally stated objective. Precise conceptual and mathematical distinctions between goals and objectives will be established below. In managing a natural resource like fisheries, a balance among a set of conflicting values prices, objectives such as cost, income for fisherman sustainability of fish population etc. must be established. In engineering field, for example, the problem may be to optimize conflicting objectives of cost, horse power and mileage, say, in the design of an engine. Likewise in capital budgeting decision the DM is interested not only in maximizing the
net present value of the portfolio of investments but also in achieving a certain rate of growth in the sales of the company, in maintaining a certain level of employment and so on.

Another drawback of the traditional paradigm lies in assuming that the constraints that define the feasible set are rigid and cannot be violated under any circumstances. In real life this is not always the case and in many cases it is possible to accept a certain amount of violation of at least some of the constraints. This is specifically true in such problems as the formulation of rations or in choosing fertilizer combinations where the technical detail is not precise enough to impose rigid constraints for instance, in representing crude protein requirements in a diet or nutrient quantities in a fertilizer mix.

It is possible to distinguish between economic and technological problems of choice depending upon whether or not multiple criteria are involved. Technological problems consist only of the process of search and measurement. These processes can be undertaken using simple tools or very sophisticated methods. In the technological problem strictly there is no decision making as one is only searching and measuring. The real decision making problem arises only when several criteria determine the choice of the optimum decision.

1.3 HISTORICAL ORIGIN

The analysis of problems involving MCDM had been perhaps the fastest growing area of Operational Research and Management Science (ORIMS) during the last 30 years. According to Vincke (1986) in 1975, 3.5% of the papers presented to the Congress organized by the Association of European Operational Research Societies (EURO
congress) were devoted to MCDM topics. That percentage has increased to 14% in 1985, that is in the EURO congress – these days one out of every seven papers is concerned with the field of MCDM. Recent bibliographies on MCDM include more than 2,000 references. Likewise surveys of some specific approaches to MCDM such as goal programming (GP) include more than 400 references.

This scientific revolution on MCDM started first with the two papers by Hoopman (1951) and Khun and Tucker (1951). The first paper has developed the concept of efficient or non-dominated vector, which plays a crucial role in MCDM; while in the second one the multi objective or vector maximization problem is formulated and the optimality conditions for the existence of non-dominated solutions are derived. Another crucial work in the development of the MCDM paradigm is that of Charnes and Cooper (1961). The undoubted success of the MCDM paradigm has led some researches in this area to suggest that it may be advisable to establish societies and Journals devoted to MCDM separately. The single objective decision making is just an old paradigm which has been superseded by the new MCDM one. The old can be considered a simplification, that is, a particular case of the MCDM paradigm.

1.4 BASIC CONCEPTS

It should be noted that some of the concepts introduced here may have the same dictionary meanings, for example, goals and objectives, and in the context of some problems, can be used interchangeably without creating confusion. However, in the context of the economic problems within a MCDM framework, certain conceptual differences have to be established. Further the theoretical concepts have a meaning and
usefulness only within the theoretical structure within which they have been created. Hence, the meaning and use of concepts in the analysis of decision making problems change according to the theoretical structure, single or multi objective frame work, within which the problem is being studied.

1.4.1 Attributes, objectives and goals

Attributes can be defined as a DM’s values related to an objective reality. These values can be measured independently from a DM’s desires and in many cases can be expressed as a mathematical function $f(x)$ of the decisional variables. Examples of attributes include gross margin, seasonal cash exposure, indebtedness etc in a farm planning problem and intake of crude protein, metabolizable energy, etc in a ration formulation model.

Now the concept of an objective represents directions of improvement of one or more of them. The improvement can be interpreted in the sense either “more of the attribute, the better” or “less of the attribute, the better”. The first case means of maximization process and in the second situation, minimization is at work. Hence, objectives imply the maximization or the minimization of the function representing one or several attributes reflecting the values of the DM. Thus, maximizing value added, minimizing risk, minimizing cost, etc. are typical examples of objectives. In general, objectives take the form: $\max f(x)$ or $\min f(x)$. In short, objectives are not attributes even though they are derived from them. If a DM considers two attributes represented by the functions $f_1(x)$ and $f_2(x)$, then the objective maximization would be given by:

$$\text{Max: } w_1 f_1(x) + w_2 f_2(x)$$
where $w_1$ and $w_2$ represent the importance attached by the DM to each attribute.

To define the concept of a goal we need the definition of aspiration level or a target. A target is an acceptable level of achievement for any one of the attributes. On combining an attribute with a target we have a goal. For example if the DM wants a cropping pattern or a firms total output given a value added of at least 2 crores then we have a goal. The mathematical expression will be $b_1x_1+b_2 x_2 \geq 2$ crores. In some cases the DM may aim for an exact achievement of the target. In this case the goal is of the form $x_1+x_2$ = the target value. Hence in general goals take the form $f(x) \leq t$ or $f(x) = t$, where $t$ is the parameter representing the aspiration level or target value.

It should be pointed out that the model builder can consider two types of goals. The first type are the goals which represent the DM’s desires like when the gross margin or the value added of a particular farm/firm plan is required to reach a specific value. The second type of goals refer to the existence of limited resources, such as water for irrigation, labour during certain peak periods or to the fulfillment of any explicit or implicit constraints, such as technical constraints referring to the frequency and sequence of certain crops. In this sense, the goals do not represent the DM’s desires in the strict sense, but only as flexible constraints. Such goals are very practical in model building because they relax the complete rigidity of the traditional constraints, thus approximating the model to real life where not all constraints are rigid and inflexible as assumed in the traditional LP framework.

In short then, in a farm planning problem gross margin is an attribute; to maximize gross margin, an objective; and, to achieve a gross margin of at least a certain target, a
goal. Finally, a criterion is a general term comprising the three preceding concepts. That is criteria are the attributes, objectives or goals to be considered relevant for a certain decision making situation. Hence MCDM is a general framework or a paradigm involving several attributes, objectives and goals.

1.4.2 Distinction between goals and constraints

Any one may now wonder as what is the exact difference between goals and constraints. In fact, goals and constraints have the same mathematical structure and look exactly the same as both of them are inequalities. The difference between them lies in the meaning attached to the right-hand of the inequality. With goal the right-hand side is a target aspired by the DM, which may be achieved or not. With constraints, however, the right-hand side must be satisfied, otherwise an infeasible solution ensures.

Thus goals can be considered as soft constraints which could be violated without producing infeasible solutions. The amount of violation can be measured by introducing positive and negative deviational variables.

For example,

\[ b_1x_1 + b_2x_2 + n - p = \text{goal level, or target}. \]

The variables \( n \) and \( p \) account for deviations from the achievement of a goal from its target. For example if the target is Rs. 2,000,000 and in the solution \( n=\text{Rs.}500,000 \) it means that the goal has fallen short by Rs.500,000. Hence the amount of violation of a goal in the sense of an under-achievement is represented by the negative deviational variable \( n \). The positive deviational variable does the opposite, that is, they indicate the amount by which a goal has surpassed its target. For instance \( p=\text{Rs.}300,000 \) means that...
the goal has exceeded its target by Rs.300,000. Hence the amount of violation of goal in the sense of an over-achievement is represented by the positive deviational variable p. Thus a goal can be, in the mathematical form expressed as \( f(x) + n - p = t \).

The device of the deviational variables is very useful for two different reasons. First, it is a simple and interesting way to impart flexibility to constraints; that is, to convert rigid constraints into goals or soft constraints. Second, it is the first step to build a goal programming model, which is the most widely used approach within the general MCDM framework.

### 1.4.3 Pareto Optimality

The concept of Pareto optimality plays a vital role in the traditional farm/firm analysis and is also a cornerstone for all the different approaches within the MCDM paradigm, particularly for the multi objective programming. In fact, all the MCDM approaches look for efficient or Pareto optimal solution.

The efficient or Pareto optimal solutions are feasible solutions such that no other feasible solution can achieve the same or better performance for all the criteria under consideration and strictly better for at least one criterion. In other words, a Pareto optimal solution is a feasible solution for which an increase in the value of one criterion can only be achieved by degrading the value of at least one other criterion.

To clarify this concept, consider a farm/firm planning problem with three feasible solutions, whose performance according to three different criteria is as follows:
<table>
<thead>
<tr>
<th>Gross Margin (in Rs.)</th>
<th>Human Labour (in hrs.)</th>
<th>Borrowed Money (in Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution 1 200,000</td>
<td>500</td>
<td>50,000</td>
</tr>
<tr>
<td>Solution 2 200,000</td>
<td>600</td>
<td>50,000</td>
</tr>
<tr>
<td>Solution 3 300,000</td>
<td>700</td>
<td>60,000</td>
</tr>
</tbody>
</table>

Assuming that the DM wants a gross margin as large as possible and human labour and borrowing as small as possible then the second solution is clearly non-efficient, since it offers the same gross margin and borrowed money as the first one. However it requires more Human labour. Thus, the second solution will not be chosen by the DM. But the third solution, however, is Pareto optimal. It has a larger requirement of borrowed money and Human labour, but it offers a bigger gross margin. To choose among the first and the third solution is an economic problem, where a real decision must be taken according to the preferences of the DM for each one of the three attributes considered.

All the MCDM techniques aim to obtain solutions which are efficient in the Paretoian sense as discussed. Even within the multi objective programming approach the first step to be taken consists of obtaining the set of feasible solutions which are efficient. That is, the feasible set is partitioned into two disjoint subsets, namely the subset of feasible and non-efficient solutions and the subset of feasible and efficient solutions. After that, the preferences of the DM are introduced to establish a compromise among the feasible and efficient solutions.
1.4.4 Trade-offs-amongst Decision making criteria

The concept of Pareto optimal solution leads to another crucial concept in MCDM: the value of trade-off between two criteria. The trade-off between two criteria means the amount of achievement of one criterion that must be sacrificed to gain a unitary increase in the other one. Thus if we have two efficient solutions \( x^1 \) and \( x^2 \), the trade-off value between the \( j^{th} \) and \( k^{th} \) criteria would be given by:

\[
T_{jk} = \frac{f_j(x^1) - f_j(x^2)}{f_k(x^1) - f_k(x^2)}
\]

where \( f_j(x) \) and \( f_k(x) \) represent the two objective functions being considered. Thus in the example considered above the trade-off value between gross margin and Human labour for solutions one and third is:

\[
T_{12} = \frac{300,000 - 200,000}{700 - 500} = 500
\]

The trade-off, \( T_{12} \), indicates that each hour of decrease of Human labour implies a decrease of Rs.500 of gross margin. i.e., the opportunity cost of one hour of seasonal labour is Rs.500 of gross margin. The trade-off, \( T_{13} \), between gross margin and borrowed money and the trade-off, \( T_{23} \), between human labour and borrowed money would be given by

\[
T_{13} = \frac{300,000 - 200,000}{60,000 - 50,000} = 10
\]

\[
T_{23} = \frac{700 - 500}{60,000 - 50,000} = 0.02
\]
ie., the opportunity cost of increasing the borrowed money by Re.1/= is Rs.10/= of gross margin or 0.02 hour of human labour.

The trade-offs values besides being a good index for measuring the opportunity cost of one criterion in terms of another under consideration also play a key role in the analysis of interactive techniques.

1.4.a Data Envelopment Analysis (DEA) – Review of Literature

Data Envelopment Analysis (DEA) is a non-parametric method in Operations Research and Econometrics for multivariate frontier estimation and ranking. It is a systematic methodology for empirical production analysis originated from Afriat (1972), Hanoch and Rothschild (1972), Diewert and Parkan (1983) Varian (1984), based on Samuelson’s (1948) revealed preference theory called Data Envelopment Analysys. This methodology was adapted for farm-specific efficiency analysis (Cooper et al., 2000), based on the ‘classics’ by Debreu (1951), Koopmans (1951) and Farrell (1957). The methodology has established itself as an important analytic research instrument and practical decision-support tool (Chavas and Cox (1990, 1995), Fare et. Al. (1994), Liebenstein and Schlom (1992), Lim and Shumway (1992), and Mathijis and Swinnen (2001)). In the DEA context, different units that are compared against each other are called “Decision Making Units” (DMU) because they can identify and vary their input and outputs (Bala, K. and Cook, W.D.2003). The non-parametric approach compares a population of observations against the “best practice” observation (Parsons, L.J., 1992). Unlike parametric approaches like regression analysis it optimizes on each individual
observation and does not require a single function that suits best to all observations (Charnes A. et al., 1994 and Scheel, H., 2000)

As pointed out in Cooper, Seiford and Tone (2000), DEA has also been used to supply new insights into activities (and entities) that have previously been evaluated by other methods. Since DEA in its present form was first introduced in 1978 (Charnes et. al. 1978), researches in a number of fields have quickly recognized that it is an excellent and easily used methodology for modeling operational processes for performance evaluations (Zhu (2002), Takamura and Tone (2003)).

Our focus in this section is on DEA models for emasureing the efficiency of a DMU relative to similar DMUs in order to estimate a ‘best practice’ frontier. The initial DEA model, as originally presented in Charnes, Cooper, and Rhodes (CCR) (1978), built on the earlier work of Farrell (1957).

The dual problems revised by Cooper and Rhodes (1978) readily extended the above ideas to multiple outputs and multiple inputs in ways that could locate inefficiencie in each input and each output for every DMU. Something more was nevertheless desired in the way of summary measures. At this point, Cooper invited Charnes to join him and Rhodes in what promised to be a very productive line of research. Utilizing the earlier work of Charnes and Cooper (1962), which had established the field of “fractional programming”, Charnes was able to put the dual linear programming problems devised by Cooper and Rhodes in to equivalent ratio form and provided a basis for unifying what had been done in DEA.
In many farms, the outcomes of the production process are affected in a nontrivial way by external risk factors that are beyond the control of the firm. In agriculture, uncontrollable climatic and pest factors can substantially affect production. The parametric approach to production analysis has made some steps towards including production uncertainty in the analysis (example Chambers and Quiggin (1998, 2000), Pope and Just (1996, 1998) and Moschini, 2001). However, as far as we know, the nonparametric approach currently does not account for production uncertainty. An important difficulty is how to model uncertainty without imposing overly restrictive structure and compromising the nonparametric orientation. Still, as discussed in Post and Spronk (2000, 2001), it is possible to include production uncertainty in the nonparametric approach e.g. by combining multi-factor risk models with stochastic dominance conditions.

However, the problem of model selection is relatively unimportant in large samples, as minimal assumptions can be used in large samples with relatively little sample error. Paradoxically, this is reflected in the fact that many tests rely on the asymptotic convergence of a model that imposes the hypothesized production property (e.g. constant returns-to-scale) and a relaxed model that does not impose the property. Unfortunately, relatively little is known about the statistical performance of the existing test in small samples. Moreover, the existing evidence suggests that these test are sensitive to finite sample error.

1.4.b Example
In this we shall explain the working of the Goal programming with the help of a hypothetical farm planning situation:

Table 1.1 represents the planning data for the hypothetical farm:

It is assumed that there are two types of crops A and B each needs five years for bearing. It can be coconut and Aracnut. The capital amount available with the farmer for the first year Rs.30,000/= while for the second, third and fourth years it is limited to Rs.14,000/= per annum. Annual availability of human labour for 8,000 hrs, the labour available for harvesting is 4,000 hrs per annum. A maximum of 2,000 own tractor hours are available during every year for tillage and the crops are harvested during different periods. The details of planning data are in Table 1.1.

**Table 1.1**

**Planning data for hypothetical farm planning**

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Crop A-ha(x₁)</th>
<th>Crop B-ha(x₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Present value of investment in trees (Rs/ha)</td>
<td>12500</td>
<td>10,000</td>
</tr>
<tr>
<td>Resources requirements Working capital (Rs/ha)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year one</td>
<td>1,100</td>
<td>800</td>
</tr>
<tr>
<td>Year two</td>
<td>400</td>
<td>350</td>
</tr>
<tr>
<td>Year three</td>
<td>600</td>
<td>500</td>
</tr>
<tr>
<td>Year four</td>
<td>650</td>
<td>400</td>
</tr>
<tr>
<td>Annual labour – Pruning (man hrs/ha)</td>
<td>240</td>
<td>360</td>
</tr>
<tr>
<td>– Harvesting</td>
<td>800</td>
<td>900</td>
</tr>
<tr>
<td>Machinery for tillage (hours/ha)</td>
<td>70</td>
<td>70</td>
</tr>
</tbody>
</table>

The objectives that the farmer is likely to have are:

a. Maximize the net present value (NPV) of investment in plantation.
b. Minimize borrowing for working capital over the next four years.
c. Minimize hiring casual labour for pruning and harvesting and
d. Minimize the use of contracted tractor hours.

This clearly is a situation where maximizing NPV can be in conflict with minimizing the dependence on borrowed money, hired labour and contracted use of tractors.

One could solve this problem as an ordinary LP model, by first isolating one of the objectives, such as NPV, and then maximize it. The other objective can be considered as constraint defining the availability of several resources. In the constraint for each resource the possibility of transferring the surplus in one year to the next has been included and thus the problem now is,

Max $Z=f(x_1, x_2)=12500x_1+10,000x_2$

Subject to

\[
\begin{align*}
1100x_1 + 800x_2 & \leq 30,000 \\
1500x_1 + 1150x_2 & \leq 44,000 \\
2100x_1 + 1650x_2 & \leq 58,000 \\
2750x_1 + 2050x_2 & \leq 72,000 \\
240x_1 + 360x_2 & \leq 8,000 
\end{align*}
\]
\begin{align*}
800x_1 & \leq 4,000 \\
900x_2 & \leq 4,000 \\
70x_1+70x_2 & \leq 2,000 \\
\text{And } x_1, x_2 & \geq 0 \\
\end{align*}

The solution for the above is \( x_1 = 105 \text{ ha}, \ x_2 = 8.88 \text{ ha} \) with an NPV of Rs.1,06,900/=. The harvesting labour is completely used, other resources are not.

This, though a solution by LP will be non-acceptable since it yields low NPV and leaves considerable amounts of various resources unused. Again if borrowing is minimized by considering other objectives as restrictions, an infeasible solution is obtained. Hence LPP cannot give satisfactory solution when there is more than one goal. Now let us formulate the GP for the same with the introduction of the two deviational variables.

The goal equations are:

\begin{align*}
12,500x_1 + 10,000x_2 + n_1- p_1 & = 4,000,000 \text{ ie } (g_1) \\
1,100x_1 + 800x_2 & + n_2- p_2 = 30,000 \text{ ie } (g_2) \\
1,500x_1 + 1,150x_2 & + n_3- p_3 = 44,000 \text{ ie } (g_3) \\
2,100x_1 + 1,650x_2 & + n_4- p_4 = 58,000 \text{ ie } (g_4) \quad \ldots \quad (2) \\
2,750x_1 + 2,050x_2 & + n_5- p_5 = 72,000 \text{ ie } (g_5) \\
240x_1 + 360x_2 & + n_6- p_6 = 8,000 \text{ ie } (g_6) \\
800x_1 & + n_7- p_7 = 4,000 \text{ ie } (g_7) \\
900x_2 & + n_8- p_8 = 4,000 \text{ ie } (g_8) \\
\end{align*}
70x₁ + 70x₂ + n₉ - p₉ = 2,000 \text{ ie (}g₉\text{)}

An artificially high target of Rs.4,00,000 for NPV which is not possible to be achieved, given the resources assumed in our example, has been set to point out that what the DM really wants it to maximize NPV.

The deviational variables account for deviations from achievement of a goal from its aspiration level. For instance if n₁=Rs.1,00,000/=, it means that g₁ has fallen short by Rs.1,00,000/=. In other words, the actual attainment of g₁ is Rs.3,00,000/=. So under achievement of a goal is represented by a negative deviational variable.

The positive deviational variable does the opposite, that is, they indicate the amount by which a goal has surpassed its aspiration level. For instance P₉=200 means that goal g₉ has surpassed its target by 200 hrs, that is, the number of tractor hours required is 2200. ie the positive deviational variables represent over – achievement of goals.

A goal cannot be both under-achieved and over-achieved. Hence in a solution at least one of the deviational variables for each goal is zero. When a goal (gᵢ) matches its aspiration level exactly then both nᵢ=pᵢ=0. If a certain goal’s achievement must be greater than or equal to its target then it’s negative deviational variable must take the smallest possible value. Hence, in this case nᵢ’s are minimized. If a certain goal must be less than or equal to its target then the positive deviational variable takes the smallest possible value. So in this situation, pᵢ’s are minimized. Finally, if a certain goal must be exactly equal to its target, then both positive and negative deviational variables must take the smallest possible value. Here we must minimize pᵢ+nᵢ.
The general purpose of GP is to minimize the deviations between achievement of the goals and their targets. This minimization can be undertaken in different ways. Among them the most widely used in practice is to attach absolute or pre-emptive weights to the deviational variables and attach relative or non-pre-emptive weights to the deviational variables. We shall now discuss these two approaches.

Note:-
In the above the first year surplus of cash (if available) is, $30,000 - 1100x_1 - 800x_2$, the second year cash available will be 14,000 scheduled plus the above surplus. Hence the actual inequality securing a financial equilibrium during the second year will be:

$$400x_1 + 350x_2 \leq 14,000 + 30,000 - 1,100x_1 - 800x_2$$

i.e., $1,500x_1 + 1150x_2 \leq 44,000$.

The cash flow constraint of the 2nd and 3rd years is obtained in a similar way.

1.5 **LEXICOGRAPHIC – GOAL PROGRAMMING (LGP)**

This approach was first introduced by Charnes and Cooper (1961) and developed by Ijiri (1965), Lee (1922) and Iqnizio (1976). It assumes that a DM can explicitly define all the goals that are relevant to a planning situation. Further, it assumes that he can attach priority to these goals, in a pre-emptive fashion, i.e. the fulfillment of the goals in a specific priority, $Q_i$, is immeasurably preferable to the fulfillment of any other set of goals situated in lower priority, $Q_j$. In LGP higher priority goals are satisfied first – it is only then that lower priorities are considered; hence the name Lexicographic order.

To illustrate the structure of LGP, in the hypothetical farm example considered, assume that the DM’s priority $Q_1$ in made up of goals $g_2$, $g_3, g_4$ and $g_5$. That is, for the DM the first goal that must be satisfied in an absolute and pre-emptive way is the one which assumes the equilibrium between out flows and the financial resources available
permitting the transfer of the funds from the periods of surplus to the ones with a deficit. Thus, the first component to minimize in the lexicographic process will be given by: \(p_2 + p_3 + p_4 + p_5\). The next priority in the order of importance, \(Q_2\) is made up of goal \(g_9\) which refers to the mechanization of tillage practices. Thus, the second component is given by the positive deviational variable, \(p_9\). Priority \(Q_3\) is made up of goal \(g_1\), referring to the maximization of NPV, thus giving the third component, that is \(n_1\). Finally, the last priority, \(Q_4\), is made up of goals \(g_6, g_7, \) and \(g_8\) referring to the minimization of hired causal labour for pruning and harvesting. Thus, the last component of the minimization process in given by \(p_6 + p_7 + p_8\). The whole lexicographic minimization problem is then:

\[
\text{Min } a = \{(p_2 + p_3 + p_4 + p_5), (p_9), (n_1), (p_6 + p_7 + p_8)\} \ldots (3)
\]

This vector, called the achievement function, replaces the objective function in the LP model. Each component of this vector represents the deviation variables (positive or negative) that must be minimized in order to make sure that the goals ranked in this priority come closer to the established achievement levels.

In general, the achievement function will be given by:

\[
\text{Minimise } a = [h_1(n,p), h_2(n,p), \ldots, h_1(n,p)] \ldots (4)
\]

or, alternatively,

\[
\text{Minimise } a = [a_1, a_2, \ldots, a_1]
\]

where \(a_1 = h_1(n,p)\) is a function of the deviational variables.

Here minimization in (4) implies that first find the smallest value of the first component \(a_1\), then the smallest value compatible with the value of \(a_1\) of the second component \(a_2\) and so on. Many papers on LGP write the achievement function as,
Minimise $z = Q_1 h_1(n,p) + Q_2 h_2(n,p) + \ldots + Q_1 h_1(n,p)$ ................................ (5)

where $Q_1$ denotes the first priority with an infinitely larger weight than priority $Q_2$ and so on. To express this as above is misleading as pointed out by Zeleny (1982) and Ignizio (1985). This is so because, as a summation (5) becomes a scalar and hence (4) is most appropriate.

By combining the achievement function (3) with (2) we obtain the following LGP model for our hypothetical farm example.

Min $a = \{(p_2+p_3+p_4+p_5), (p_9), (n_1), (p_6+p_7+p_8)\}$

Subject to

$$Q_3 \begin{cases}
12,500x_1 + 10,000x_2 + n_1 \cdot p_1 = 400,000 \text{ ie. } g_1 \\
1,100x_1 + 800x_2 + n_2 \cdot p_2 = 30,000 \text{ ie. } g_2 \\
1,500x_1 + 1,150x_2 + n_3 \cdot p_3 = 44,000 \text{ ie. } g_3 \\
2,100x_1 + 1,650x_2 + n_4 \cdot p_4 = 58,000 \text{ ie. } g_4 \\
2,750x_1 + 2,050x_2 + n_5 \cdot p_5 = 72,000 \text{ ie. } g_5 \\
240x_1 + 360x_2 + n_6 \cdot p_6 = 8,000 \text{ ie. } g_6
\end{cases}$$

$$Q_4 \begin{cases}
800x_1 + n_7 \cdot p_7 = 4,000 \text{ ie. } g_7 \\
900x_2 + n_8 \cdot p_8 = 4,000 \text{ ie. } g_8
\end{cases}$$

$$Q_2 \begin{cases}
70x_1 + 70x_2 + n_9 \cdot p_9 = 2,000 \text{ ie. } g_9
\end{cases}$$

$x_i \geq 0, n_j \geq 0, p_j \geq 0 \text{ i = } 1,2$
This LGP can be solved by many algorithms and the solution through one of which is
\[ x_1=38.36, \quad x_2=18.76 \]

While the optimum values of the deviational variables are
\[
\begin{align*}
n_1 &= Rs.66,500, \quad p_1=0, \\
n_2 &= Rs.1,398, \quad p_2=0 \\
n_3 &= Rs.4,442, \quad p_3=0 \\
n_4 &= Rs.2,244, \quad p_4=0 \\
n_5 &= n_6=0, \quad p_5=p_6=0 \\
n_7 &= 0, \quad p_7=11,344 \text{ hrs} \\
n_8 &= 0, \quad p_8=4,442 \text{ hrs} \\
n_9 &= 0, \quad p_9=0
\end{align*}
\]

This solution permits complete achievements of the goals which make up the first two priorities.

1.6 SEQUENTIAL LINEAR METHOD (SLM) or LGP

This is a simple method of solving the LGP. This requires only the conventional simplex method for solution. The SLM solves a sequence of LP problems.

The first LP problem of the sequence minimizes the first component of the achievement vector, subject to the constraints (inequalities) corresponding to the priority Q₁. The second LP minimizes the second component of the achievement vector, subject to the constraints corresponding to priorities Q₁ and Q₂, as well as the values of the deviation variables in priority Q₁ which were found in the preceding solution. This
sequential procedure continues until the last linear program is solved or until in one of the problems of the sequence there are no alternative optimum solutions.

On applying this approach to our problem we obtain the following sequence of LP problems:

**Problem 1-the first priority level**

Minimise $a_j=p_2+p_3+p_4+p_5$

subject to

\[
\begin{align*}
1,100x_1 + 800x_2 + n_2-p_2 &= 30,000 \\
1,500x_1 + 1150x_2 + n_3-p_3 &= 44,000 \\
2,100x_1 + 1650x_2 + n_4-p_4 &= 58,000 \\
2,750x_1 + 2050x_2 + n_5-p_5 &= 72,000 
\end{align*}
\]

There are alternative optimum solutions for decision variables and $p_2 = p_3 = p_4 = p_5 = 0$

(The existence of alternative optima can be checked easily from the final simplex tableau. If in that tableau for at least one non basic variable its reduced cost is zero, then alternative optimum solutions exist)

Infact, the alternative optimum solutions corresponds to the region of feasible solution in the graphical method. Actually the first solution and the graphical method do exactly the same job.

**Problem 2-second priority level**

In this iteration the second component of the achievement is minimized subject to the goals corresponding to priorities $Q_1$ and $Q_2$ and substituting $p_2, p_3, p_4$ and $p_5$ by the
optimum values obtained in the first iteration (ie \( p_2 = p_3 = p_4 = p_5 = 0 \)) In this way the LP problem is formulated as

Minimise \( a_2 = p_9 \)

subject to

\[
\begin{align*}
1,100x_1 + 800x_2 + n_2 &= 30,000 \\
1,500x_1 + 1,150x_2 + n_3 &= 44,000 \\
2,100x_1 + 1,650x_2 + n_4 &= 58,000 \\
2,750x_1 + 2,050x_2 + n_5 &= 72,000 \\
70x_1 + 70x_2 + n_9 - p_9 &= 2,000
\end{align*}
\]

Again there are alternative optimum solutions for decision variables and \( p_9 = 0 \) in the graphical method.

**Problem 3-the third priority level**

Following the logic of the SLM, in this iteration the third component is minimized, subject to the goals constraints corresponding to the second problem, making \( p_9 = 0 \) and augmenting that structure with the goal constraint which makes up priority \( Q_3 \). In this way the following LP is formulated:

Minimise \( a_3 = n_1 \)

subject to

\[
\begin{align*}
1,100x_1 + 800x_2 + n_2 &= 30,000 \\
1,500x_1 + 1,150x_2 + n_3 &= 44,000 \\
2,100x_1 + 1,650x_2 + n_4 &= 58,000
\end{align*}
\]
\[2,750x_1 + 2,050x_2 + n_5 = 72,000\]
\[70x_1 + 70x_2 + n_9 = 2,000\]
\[12,500x_1 + 10,000x_2 + n_1 - p_1 = 4,00,000\]

The optimum is \(x_1 = 38.36\) ha, \(x_2 = 18.76\) ha, \(n_1 = \text{Rs.66,500}\)

With this result we can go to the next priority level establishing the last problem, but as multiple optima do not exist for problem 3, the present solution is optimal with respect to all the priorities. All we do is to substitute the optimum values of the decision variables into goals \(g_6\), \(g_7\) and \(g_8\) in order to obtain the deviational variables.

### 1.7 WEIGHTED GOAL PROGRAMMING

This method considers all the goals simultaneously in a composite objective function which minimizes the sum of all deviations among the goals and their aspiration levels. The deviations are weighted according to the relative importance of each goal to the DM.

Since weights are arbitrary, different weights will give different solutions. If all the weights are equal to 1, then it is the solution in LGP. However use of fuzzy in allocating the weights and simulation exercise is made, on the intervals of weights for each, will bring better optimal solution.

### 1.8 DATA

In Tirupur, there are more than 6,000 units. After conducting a pilot study, data were collected from 150 firms, selected by simple random sampling method from the list of all firms arranged in the ascending order of their installed capacity. The required information is obtained from the records of the selected firms and further supplemented
by personal enquiry with the administrative heads of the units. Secondary data were also collected from the publications of the Association of Tiruppur Knitwear Producers for time series analysis of growth and instability in production and export.

1.9 ANALYSIS

Primary data are collected from the sample hosiery units for the year 2009-2010 for the details of units in operation, their installed capacity, employment of labour, use of fixed and variable capital, turnover, profit earned, value realized through exports, taxes and duties paid and complete details of production. The details on production include the type of products, turned out, quantity and value of production variety wise, prices received in domestic market and exports variety wise and the cost of production. The primary data were used to study turnover rates, capacity utilization and factor productivity.

The sample firms are in ascending order of their installed capacity and classified into three size groups A,B and C. The group A is comprised of top 50 firms in the list and represents the small units. The next 50 members of the firms (sl.no. 51 to 100 in the list) constituted the group B, of medium size and the remaining 50 firms formed group C – the large firms. It should be seen that the size groups A,B and C were called small, medium and large on the basis of classification adopted for the study and there was no such official classification. The groups were tested for the statistical difference among them in the mean size of installed capacity with the help of ‘t’ statistics for mean difference.
The major thrust of this study is to help the hosiery firms in identifying the product mix that would satisfy their organizational goals. All the firms produce more than three varieties of products and their goals were not unique. Hence, the decisions were to be made in multi-product and multi-goal context, with constraints in resource supply and market conditions. Necessarily, it is a normative analysis.

1.10 ECONOMICS OF HOSIERY PRODUCTIONS

Economics of hosiery production was studied at two levels; (i) With secondary data for hosiery industry of Tiruppur and (ii) Primary data for sample hosiery units. Attempt is to estimate the production function and to derive marginal productivities of capital and labour.

1.11 PRODUCTION FUNCTION ANALYSIS

An aggregate function of Cobb-Douglas was estimated for the time series data on Production, Capital use, and labour use in hosiery industry in Tiruppur. The functional form is,

\[ V_t = A K_t^\alpha L_t^\beta e^{u_t} \]  

(7)

where \( V_t \) is the aggregate production in Rs. Crores in 1985-'86 prices of hosiery industry of Tiruppur in year \( t=1, \ldots,25, \ t=1 \) being 1985.

\( K_t \) = Capital use in Rs.Crores in year \( t \).

\( L_t \) = Labour use in year \( t \) (in 1000 persons)

\( A, \alpha, \beta \) are parameters to be estimated
\( u_t = \) random error term.

A is the measure of mean effect of technology \( \alpha \) and \( \beta \) are production elasticity of capital and labour respectively. The estimated form of the production function is linear in logarithms as

\[
y = a + \alpha x_{t1} + \beta x_{t2} + u_t \\
\text{.................................} (8)
\]

Table 1.2 below gives the details of the estimated parameters.

<table>
<thead>
<tr>
<th>Variables (in log)</th>
<th>Co-efficients</th>
<th>Standard Error</th>
<th>‘t’</th>
<th>Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production (Y)</td>
<td>A</td>
<td>6.5884</td>
<td>3.1726</td>
<td>2.0559</td>
</tr>
<tr>
<td>Capital (X₁)</td>
<td>( \alpha )</td>
<td>0.7346</td>
<td>0.3108</td>
<td>2.3636</td>
</tr>
<tr>
<td>Labour (X₂)</td>
<td>( \beta )</td>
<td>0.9258</td>
<td>0.2657</td>
<td>3.4844</td>
</tr>
<tr>
<td>( R^2 = 0.7115)</td>
<td>( F=16.6832)</td>
<td>( n=25)</td>
<td></td>
<td>D.W.1.29</td>
</tr>
</tbody>
</table>

As seen from Table 1.2 the value of \( R^2 = 0.715 \). This implies that the estimated equation could explain 71 percent of the variations in industry wide aggregate production of hosiery in Tiruppur. The co-efficient of both capital and labour have the expected positive sign and are statistically significant.

The co-efficient of the explanatory variables are the production elasticities. Their values are less than unity, but larger than zero, indicating that the use of resources is in the rational zone of production. The production elasticities of capital is 0.7346 while it is 0.9258 for labour. The sum being 1.6604 implies that there is increasing returns to scale.
The above inference on the resource use was further verified with the help of primary data collected from the 150 hosiery units. The units were post stratified into three size groups on the basis of installed capacity. The three size groups and their installed capacities are shown in Table 1.3.

Table 1.3
Size groups of hosiery units in Tiruppur

<table>
<thead>
<tr>
<th>Size Groups</th>
<th>Number of Units</th>
<th>Installed capacity (in Lakhs)</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
</tr>
<tr>
<td>Small</td>
<td>50</td>
<td>15</td>
<td>120</td>
</tr>
<tr>
<td>Medium</td>
<td>50</td>
<td>121</td>
<td>240</td>
</tr>
<tr>
<td>Large</td>
<td>50</td>
<td>241</td>
<td>360</td>
</tr>
<tr>
<td>Total</td>
<td>150</td>
<td>15</td>
<td>360</td>
</tr>
</tbody>
</table>

There are few firms with very small installed capacity of less than Rs.15 lakhs and larger than Rs.360 lakhs, but they were negligibly small in number and were hence treated as outliers and excluded from the sample. The hosiery units having installed capacity nearly equal to the mean value of each size group was selected as the representative units for normative analysis (to be discussed later). Before that the primary data were used to estimate the production functions separately for each group to study the resource use efficiency in them and for its comparison with industry wise performance.

1.12 PRODUCTION FUNCTION GROUPWISE
a) Small Enterprises

The mathematical form of the production function estimated for the small enterprises as,

\[ Y = 3.2581^* + 0.4172 \quad X_1^{**} + 0.5527 \quad X_2^{**} \]

\[ (1.4275) \quad (0.1364) \quad (0.2698) \]

\[ R^2 = 0.6638^{**} \quad n=50 \]

Here \( Y = \log \text{production} \)

\( X_1 = \log \text{(Capital)} \)

\( X_2 = \log \text{(no. of persons employed)} \)

The estimated equation has a good fit with statistically significant value of \( R^2 \) and the expected signs for their coefficients. The R-square indicates that 66 percent of the variations in the production could be explained by Labour and capital alone in this industry. The sum of the coefficients for \( X_1 \) and \( X_2 \) being equal to 0.9696 implies only constant returns to scale for this group of hosiery units.

b) Medium Enterprises

The production function equation fitted for this type of enterprises is

\[ Y = 5.1258^{NS} + 0.6334 \quad X_1^* + 0.8261 \quad X_2^{NS} \]

\[ (4.0861) \quad (0.2711) \quad (0.7658) \]

\[ R^2 = 0.8729^{**} \quad n=50 \]

The variables are as defined for the small size groups. The estimated equation has good fit with high R-square value of 0.8729. This implies that 87.29 percent of the
variations in the output are explained by these two explanatory variables. Since the sum of the b values in 1.4595, there is an increasing returns to scale.

(c) **Large Enterprises**

The production function of the same form is estimated for the sample of 50 large sized hosiery units. The fitted equation is,

\[ Y = 5.5607^{NS} + 0.7162X_1^* + 0.9052X_2^{**} \]

\[ \begin{align*} 
(3.9897) & \quad (0.23869) \quad (0.3018) \\
\end{align*} \]

\[ R^2 = 0.9604^{**} \quad n=50 \]

The equation has a good explanatory power as shown by the value of R-square. The sum of the b values = 1.6214 indicating an increasing returns to scale in this also.

---

<table>
<thead>
<tr>
<th>NS</th>
<th>-</th>
<th>Not significant</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>-</td>
<td>Significant at five percent level of probability</td>
</tr>
<tr>
<td>**</td>
<td>-</td>
<td>Significant at one percent level of probability</td>
</tr>
<tr>
<td>( )</td>
<td>-</td>
<td>Figure in the parenthesis gives the S.E. of estimates</td>
</tr>
</tbody>
</table>

Thus the results of production function analysis of primary data reveal much scope for improving economic efficiency in resource use. A comparative study of the performance of hosiery units of different sizes among themselves and with the performance of the industry as a whole is now possible. For convenience the ratios of marginal value of products (MVP) to the marginal cost (MC) are worked out and
compared in Table 1.4. For economically efficient use of resources for maximizing aggregate profit of the firms and industry, the ratio must be equal to unity. A large value indicating the need to use more of the resources and a smaller values showing the opposite.

The ratio presented in Table 1.4 reveal that in every group and for the industry as a whole the ratio for capital widely deviates from unity. Hence the capital is used less on optimality. The scope for increasing use of capital increase with the size of the hosiery units, the industry wise situation being close to the medium sized hosiery units. If all the size groups and the industry are seen to capital at less than the optimal level, it suggests that there may exist some firm of external or internal credit rationing in hosiery industry as the cost of capital is no cost for it. It may be due to (i) cautious approach to borrowing (Credit aversion of various degrees) (ii) difficult procedure for mobilising capital (iii) low credit worthiness of the firms (iv) lack of knowledge of the scope (efficiency creation) of the firms or (v) a combination of them, the policy must be to identify the causes and help release the constraint.

Table 1.4

Comparative study of Resources use Efficiency

<table>
<thead>
<tr>
<th>Resources</th>
<th>Ratio of MVP to MC for Resource in units</th>
</tr>
</thead>
</table>


<table>
<thead>
<tr>
<th></th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
<th>Total Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>2.08</td>
<td>3.89</td>
<td>4.20</td>
<td>3.05</td>
</tr>
<tr>
<td>Labour</td>
<td>1.06</td>
<td>0</td>
<td>1.21</td>
<td>1.46</td>
</tr>
<tr>
<td>Returns to scale</td>
<td>Constant</td>
<td>Increasing</td>
<td>Increasing</td>
<td>Increasing</td>
</tr>
</tbody>
</table>

In labour use a larger scope is seen for the industry as a whole but little scope in small and medium size hosiery enterprises. The large units have some scope for increased labour use, probably the influence the industry wise scope for employing more labour.

There is increasing returns to scale in medium, large groups and the industry as a whole, but there is constant returns in small units. Hence, new enterprises would gain from this increasing return to scale if they are organized at sizes represented by installed capacity exceeding Rs.120 lakhs of production. The general inference is that the hosiery industry of Tiruppur has scope for expansion.

**1.13. USE OF GOAL PROGRAMMING**

**1.13.1 Identification of Objectives**

When the possibility of multiple objectives is perceived for decision making by hosiery units of Tiruppur, the first step is to identify the specific goals set by individual units. Hence, while collecting Primary Data, the respondents, the decision makers or their nominees who are considered to be better informed by respondents themselves are asked to specifically state the organizational goals of their hosiery enterprises. Deliberately it is open – ended questions to evoke free response. When multiple goals
are stated the respondents were requested to indicate their order of preference. A study of the response by the respondents reveals that the following are the goals to be pursued.

Table 1.5

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Decision Goals</th>
<th>No. of Units Responding</th>
<th>Percentage</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Maximize Profit (A)</td>
<td>7</td>
<td>4.70</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Maximize Sales (B)</td>
<td>4</td>
<td>2.60</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>Minimize use of Capital (C)</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>Minimize Labour Use (D)</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>Minimize risk (E)</td>
<td>36</td>
<td>24.0</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>A + C</td>
<td>6</td>
<td>4.0</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>A + C + D</td>
<td>86</td>
<td>57.2</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>A + E</td>
<td>3</td>
<td>2.00</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>A + B</td>
<td>4</td>
<td>2.70</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>Other Combinations</td>
<td>4</td>
<td>2.70</td>
<td>4</td>
</tr>
</tbody>
</table>

Broadly there are five goals and their combination, however unique goal for decision making appear to be rare. Maximizing use of capital and minimum labour use are not considered important. However minimization of business risk is found to be important for nearly 36 hosiery units with the only exception, the traditional assumption
of single objective decision making stands rejected. Among the multiple goals the combination of profit maximization (A) and minimization of capital use (C) and labour use (D) has the largest numbers (86) of respondents and ranks first by the percentage share in the total respondents. Other combinations are not important as shown by low percentage of units reporting them. Hence only two situations need serious consideration. First one is the combination of A+C+D and the second one is unique goal of minimizing business risk. Ordinary programming methods cannot be used since the units refuse to give critical data on prices and sales for several years. Then the only situation that merits consideration is the multiple goal A+B+D. This Justifies the rejection of simple LP model and the choice of LGP model.

1.13.2 The Model

The first step in the specification of the LGP model is to know the preference ordering of the goals. There are three size groups of hosiery units and they may have different ordering. By the criteria of largest probability, the preference ordering of the firms in the three groups is decided and shown in Table. 1.6.

All the 50 units in the medium size group, 98% in small size group and just 10% in larger group preferred ordering A|C|D. It is to be read as profit maximation (A) first, minimization of capital use (C) second and minimization of labour use (D) third preferences. In contrast the ordering C|A|D is preferred by the large units. Since large units have to make large investment of capital, their first preference to minimize it is reasonable to expect. Thus by clear majority criterion by ordering A|C|D is used for small and medium size groups, while it is C|A|D for large units.
The objective of profit maximization refers to the maximization of aggregate profit of the units from different types of products produced by them. There are eight products in production of hosiery units. They are

1. One piece ladies nighty ($X_1$)
2. Banians ($X_2$)
3. Ladies top and bottom ($X_3$)
4. Chest printed T-Shirts for Ladies ($X_4$)
5. White rib neck, chest printed ladies T-shirts ($X_5$)
6. Folding neck, front three shades in black T-shirts for males ($X_6$)
7. Rib-neck T-shirts for males ($X_7$)
8. Neutral top and bottom printed children wears ($X_8$)
Not all the eight varieties are produced by all the firms, but they can be produced by them if there is market for them. Hence all the eight products are introduced as possible real activities in models for all three size groups. Then the aggregate profit from these real activities must be maximized. In LP model this is set as an objective function to be maximized. In LGP it is one of the goals, by adding deviational variables and setting an artificially high target (Specific numerical value) for the right hand side constant of this equation. The deviational variable measures the deviation from the target.

Accounting constraints for capital and labour use are turned in to goals by adding deviational variables and targets set for them. The target is set at current levels of aggregate use of capital and labour respectively. Then there are real constraints to satisfy technological limits such as the use of processing time for various operations, limit to the size of sales, minimum and maximum limits (upper and lower bounds) for products. With these additions the mathematical model for the LGP is specified as

Min \( g_g = [(n_1), (p_2 + p_3 + p_4), (n_5)] \)

subject to

**Targets**

1. Profit goal \( : \quad a_{11} X_1 + a_{12} X_2 + \ldots + a_{18} X_8 + n_1 - p_1 = \beta_1 \)
2. Capital use \( : \quad a_{21} X_1 + a_{22} X_2 + \ldots + a_{28} X_8 + n_2 - p_2 = \beta_2 \)
3. Labour use \( : \quad a_{31} X_1 + a_{32} X_2 + \ldots + a_{38} X_8 + n_3 - p_3 = \beta_3 \)
4. Process time \( : \quad a_{41} X_1 + a_{42} X_2 + \ldots + a_{48} X_8 + n_4 - p_4 = \beta_4 \)
5. Sales \( : \quad a_{51} X_1 + a_{52} X_2 + \ldots + a_{58} X_8 + n_5 - p_5 = \beta_5 \)
6. Upper bound for variables: \[ \leq \beta_j \]
7. Lower bound for variables: \[ \geq \beta_k \]

**Non-negativity Constraint**

\[
X_j \geq 0 \quad \text{for } j = 1,2, \ldots, 8 \\
n_i \geq 0 \quad \text{for } i = 1,2,3,4,5 \\
p_i \geq 0 \quad \text{for } i = 1,2,3,4,5
\]

This model is solved by sequential linear programming method (SLM) which uses simplex method in an interactive way and solves a sequence of LP problems as illustrated below.

**Problem 1. First Preference goal of Profit maximization**

\[
\text{Min } g_{g_1} = [n_1] \\
\text{subject to constraints 1,5,6,7}
\]

**Problem 2. Second Preference of Minimizing of Capital use**

\[
\text{Min } g_{g_2} = [p_2] \\
\text{subject to constraints 1,2,5,6,7}
\]

**Problem 3. Third Preference of Minimizing of Labour use**

\[
\text{Min } g_{g_3} = [p_3] \\
\text{subject to constraints 1,2,3,5,6,7}
\]

**Problem 4. Maximize capacity utilization**
This refers to maximum use of installed processing facilities

\[ \text{Min } g_{4} = [n_{4}] \]

subject to constraints 1,2,3,4,5,6,7

It should be noted that 6 and 7 represent a set of single or more than one constraints.

1.13.3 Representative unit

Above model was specified for each size group and solved separately. As there were 50 units in each group the model was applied only to one representative unit selected on the criterion that the units has installed capacity as close to the mean installed capacity of the group.

1.13.4 Values of the parameters

If available the value of parameters were the same as observed in the representative unit of the size group for \( \beta_{i} \) (right handside constant). For \( a_{ij} \) it is the average of all the sample units in the group which have the particular activity. If the values of these parameters were not available within the group (because some of real activities are possible but not currently in use) the values of the parameters were obtained from other size groups. Targets for the goals equations were decided in consultation with the representative unit. Thus the exercise is a mathematical programming for a synthetic (bench mark) situation - a practice that is very common in normative analysis at micro level.

1.13.5 Sensitivity analysis
The SLM used in this study is not a very convenient algorithm for this, hence sensitivity analysis was done only for the limit sales (market size) by solving LGP three times for three different values of $\beta_5$. The models were solved separately for each of the three groups and the results are discussed below:

1.13.6 Small Hosiery units

The LGP model for the representative hosiery units of small size groups is presented below:

Min $gg = [(n_1), (p_2), (p_3) (n_4)]$

subject to

Targets

1. Profits (Rs.)

$$5.30 X_1 + 8.85 X_2 + 4.15 X_3 + 5.20 X_4 + 4.0 X_5 + 4.3 X_6$$

$$+ 3.2 X_7 + 5.4 X_8 + n_1 - p_1 = 2,00,000$$

2. Capital use (Rs.)

$$3.89 X_1 + 4.78 X_2 + 2.84 X_3 + 2.66 X_4 + 2.38 X_5 + 2.7 X_6$$

$$+ 1.81 X_7 + 4.1 X_8 + n_2 - p_2 = 2,00,000$$

3. Labour use (Man hours)

$$2.10 X_1 + 2.07 X_2 + 1.96 X_3 + 0.98 X_4 + 1.10 X_5 + 1.62 X_6$$

$$+ 0.97 X_7 + 2.05 X_8 + n_3 - p_3 = 1,00,000$$

4. Processing time (hours)

$$1.52 X_1 + 2.02 X_2 + 1.69 X_3 + 1.91 X_4 + 1.75 X_5 + 1.75 X_6$$

$$+ 1.50 X_7 + 1.63 X_8 + n_4 - p_4 = 70,000$$
Constraints

5. Sales (no. of pieces)

\[ X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 \leq 50,000 \]

6. \[ X_2 \]

7. \[ X_4 \]

8. \[ X_8 \leq 16,000 \]

\[ X_j > 0, \text{ for } j=1,2,\ldots,8 \quad n_i > 0 \text{ for } i=1,2,3,4 \quad p_i > 0 \text{ for } i=1,2,3,4 \]

Sensitivity analysis was done for sales raised to 80,000 pieces and 1,00,000 pieces.

The solution values for the model 1 are presented in Table 1.7

Table 1.7

Optimal Solution for small size Hosiery units.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Variables</th>
<th>For Sales limit set at (No.of pieces.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>50,000</td>
</tr>
<tr>
<td>1</td>
<td>Product X₁</td>
<td>6422</td>
</tr>
<tr>
<td>2</td>
<td>Product X₂</td>
<td>22500</td>
</tr>
<tr>
<td>3</td>
<td>Product X₃</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Product X₄</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Product X₅</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>Product X₆</td>
<td>928</td>
</tr>
<tr>
<td>7</td>
<td>Product X₇</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>Product X₈</td>
<td>14970</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>44820</td>
</tr>
<tr>
<td></td>
<td>Deviations $n_i$</td>
<td></td>
</tr>
<tr>
<td>----</td>
<td>-----------------</td>
<td>------</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>118028</td>
</tr>
<tr>
<td>10</td>
<td>Deviations $n_2$</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Deviations $n_3$</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Deviations $n_4$</td>
<td>11248</td>
</tr>
<tr>
<td>13</td>
<td>Deviations $p_1$</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Deviations $p_2$</td>
<td>4700</td>
</tr>
<tr>
<td>15</td>
<td>Deviations $p_3$</td>
<td>7756</td>
</tr>
<tr>
<td>16</td>
<td>Deviations $p_4$</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** N.F. Not feasible.

Only four products are in the final solution. They are 6,422 pieces of $X_1$, 22,500 pieces of $X_2$, 928 pieces of $X_6$ and 14,970 pieces $X_8$ adding up 44,820 pieces well within the sales limit of 50,000 pieces. The objective of maximizing profit is first realized. The value of $n_1$ is 118028 showing that the total profit will exceed the target Rs.2,00,000. Optimum yields nearly 59 percent more profit than the target set. Once, this target is realized next goal is to minimize the use of capital. The value of $p_2$ being 4,700 the capital use will be reduced by Rs.4,700 from the target of Rs.2,00,000 only a marginal reduction is thus possible. The value of $p_3$ is 7,756. Hence the labour will be reduced by 7756 hours from the target level of 1,00,000 hours. To utilize the installed capacity of 70,000 hours of processing time, the hosiery unit sets the target of
maximizing processing time use. The value of $n_4$ being 11,248 hours, the use of
processing time will exceed the target by 11,248 hours to be equal to nearly 80,000 hours.
The upper bounds of $X_4$ and $X_8$ and lower bound for $X_2$ are also met with. Thus the
solution values meet all the targets and sequentially satisfies the constraints also. The
targets are sequentially achieved so that the preference ordering (lexicographic orders) of
goals is also met with.

When the scope of sales increased to 80,000 and 1,00,000 pieces the problem
becomes infeasible, even the present limit of 50,000 pieces could not be reached only by
relaxing (revising upward) the target for capital use, labour use and processing time the
problem will become feasible. The results show that the small units gain by specializing
in production of banians ($X_2$) with limited diversification to children wear ($X_8$) ladies
nightee ($X_1$) and T-shirts for males ($X_6$).

1.13.7 **Medium size units**

The algebraic structure of the model is the same as that for small units but the
value of the parameters are different. Empirical model is presented below

\[
\text{Min } g = [(n_1), (p_2), (p_3), (n_4)]
\]

subject to

**Targets,**

1. Profits (Rs.)

\[
5.40 X_1 + 8.90 X_2 + 0 X_3 + 5.65 X_4 + 4.15 X_5 + 4.3 X_6 \\
+ 3.8 X_7 + 6.1 X_8 + n_1-p_1 = 4,00,000
\]
2. Capital Use (Rs.)
\[ 3.61 X_1 + 4.80 X_2 + 0 X_3 + 2.60 X_4 + 2.47 X_5 + 2.65 X_6 + 1.70 X_7 + 4.25 X_8 + n_2-p_2 = 3,60,000 \]

3. Labour use (Man hours)
\[ 2.10 X_1 + 2.07 X_2 + 0 X_3 + 1.02 X_4 + 1.20 X_5 + 1.60 X_6 + 0.95 X_7 + 1.90 X_8 + n_3-p_3 = 1,60,000 \]

4. Processing time (hours)
\[ 1.50 X_1 + 2.02 X_2 + 0 X_3 + 1.91 X_4 + 1.70 X_5 + 1.80 X_6 + 1.48 X_7 + 1.60 X_8 + n_4-p_4 = 1,20,000 \]

Constraints

5. Sales (no. of pieces)
\[ X_1+X_2+0+X_4+X_5+X_6+X_7+X_8 \leq 80,000 \]

6. \( X_2 \) > 40,000

7. \( X_4 \) \leq 10,000

8. \( X_8 \) \leq 30,000

\( X_j \geq 0, \text{ for } j=1,2,3,\ldots 8 \)

\( n_i \geq 0 \text{ for } i=1,2,3,4 \)

\( p_i \geq 0 \text{ for } i=1,2,3,4 \)

Sensitivity analysis was done for sales limit of 50,000 pieces and 1,00,000 pieces. The details of solution are given in Table 1.8
Table 1.8
Optimal solution for medium size Hosiery units

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Variables</th>
<th>For Sales limit set at (No. of pieces.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>50,000</td>
</tr>
<tr>
<td>1</td>
<td>Product X₁</td>
<td>3310</td>
</tr>
<tr>
<td>2</td>
<td>Product X₂</td>
<td>38420</td>
</tr>
<tr>
<td>3</td>
<td>Product X₃</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Product X₄</td>
<td>3490</td>
</tr>
<tr>
<td>5</td>
<td>Product X₅</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Product X₆</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>-----------</td>
<td>---</td>
</tr>
<tr>
<td>7</td>
<td>Product X₇</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Product X₈</td>
<td>4780</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>50000</strong></td>
<td><strong>72900</strong></td>
</tr>
<tr>
<td>9</td>
<td>Deviations n₁</td>
<td>8688</td>
</tr>
<tr>
<td>10</td>
<td>Deviations n₂</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>Deviations n₃</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>Deviations n₄</td>
<td>23112</td>
</tr>
<tr>
<td>13</td>
<td>Deviations p₁</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>Deviations p₂</td>
<td>134246</td>
</tr>
<tr>
<td>15</td>
<td>Deviations p₃</td>
<td>60878</td>
</tr>
<tr>
<td>16</td>
<td>Deviations p₄</td>
<td>0</td>
</tr>
</tbody>
</table>

The medium sized hosiery units do not produce the ladies top and bottom suit sets (X₃). Hence it was dropped from the deviation variable. Of the remaining, among the seven products only four viz X₁, X₂, X₄, and X₈ found place in the optimal solution sets with values 12,060, 45,620, 8,020 and 7,200 pieces respectively. Thus the production of banians is the major activity followed by ladies nighties (X₁), ladies T-shirts (X₄) and children’s wear (X₈) in that order. Even though children’s wear ranked second in unit profits, their production ranks last because of their demand, labour and processing time.
The profit is maximized exceeding the target of Rs. four lakhs by nearly Rs.160,376(X₁). The processing is maximized by exceeding the target of 1,00,000 hrs by 17,080 hours (X₄). The goals of minimizing capital use and labour used were realised to the extent of Rs.46,036 and 18,380 hour below the target set. The targets and their priority ordering have been realized fully. The lower bound for X₁ and the upper bound for X₄ and X₈ are also satisfied. Total production of 72,900 pieces was less than the sales limit of 80,000 units.

When the sales limit was lowered to 50,000 production was reduced to that limit exactly and there was no change in the structure (composition of) productions, only the optimal solution values have come down. The value of (n₄) was 23,112 hours of processing showing that if sales are limited, the use of processing time is reduced lowering the capacity utilization, even though other targets and constraints were satisfied. Hence the medium sized hosiery units should pay attention to sales promotion to achieve sales of at least 80,000 pieces.

When sales potential was raised to 1,00,000 pieces the result show that the profit will grow by Rs.29,544 over the target of Rs.4,00,000. The use of processing time will go up by 51,366 hours. Capital use and labour use would still not exceed targets, because the minimization of their use is possible within the target set even when production increase to meet the new relaxed sales constraint. It implies that at present limit to sales limit production and capacity utilization is not full.

1.13.8 Large Hosiery units
For the large hosiery units the first priority is to minimize use of capital, followed by profit maximization, minimization of labour use and maximizing processing time. Hence, the sequence of iteration is in that order.

The model is:

Min $g = [(p_1), (n_2), (p_3), (n_4)]$

subject to

**Targets**

1. Profits (Rs.)

\[3.69 X_1 + 3.33 X_2 + 2.61 X_3 + 2.60 X_4 + 2.31 X_5 + 2.45 X_6 + 1.72 X_7 + 4.10 X_8 + n_1-p_1 = 6,00,000\]

2. Capital Use (Rs.)

\[5.46 X_1 + 8.79 X_2 + 4.15 X_3 + 5.20 X_4 + 4.10 X_5 + 4.3 X_6 + 3.2 X_7 + 5.51 X_8 + n_2-p_2 = 12,00,000\]

3. Labour use (Man hours)

\[2.15 X_1 + 2.07 X_2 + 1.90 X_3 + 1.12 X_4 + 1.58 X_5 + 1.49 X_6 + 1.05 X_7 + 1.87 X_8 + n_3-p_3 = 4,00,000\]

4. Processing time (hours)

\[1.50 X_1 + 2.03 X_2 + 1.71 X_3 + 1.90 X_4 + 1.76 X_5 + 1.81 X_6 + 1.50 X_7 + 1.60 X_8 + n_4-p_4 = 3,60,000\]

**Constraints**

5. Sales (no. of pieces)

\[X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8 \leq 2,00,000\]
6. \( X_2 \geq 80,000 \)

7. \( X_1 \leq 20,000 \)

8. \( X_4 \leq 10,000 \)

9. \( X_8 \leq 30,000 \)

\( X_j \geq 0, \text{ for } j=1,2,3,...8 \)

\( x_i \geq 0 \text{ for } i=1,2,3,4 \)

\( p_i \geq 0 \text{ for } i=1,2,3,4 \)

The above problem was solved in the sequence indicated in the objective function and the optimal solution is presented in Table 1.9.

### Table 1.9

Optimal solution plan for Large size Hosiery Units

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Variables</th>
<th>For Sales limit set at (No.of pieces.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1,60,000</td>
</tr>
<tr>
<td></td>
<td>Product</td>
<td>Demand</td>
</tr>
<tr>
<td>---</td>
<td>----------</td>
<td>---------</td>
</tr>
<tr>
<td>1</td>
<td>Product X₁</td>
<td>12,840</td>
</tr>
<tr>
<td>2</td>
<td>Product X₂</td>
<td>85,210</td>
</tr>
<tr>
<td>3</td>
<td>Product X₃</td>
<td>34220</td>
</tr>
<tr>
<td>4</td>
<td>Product X₄</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Product X₅</td>
<td>8,416</td>
</tr>
<tr>
<td>6</td>
<td>Product X₆</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Product X₇</td>
<td>19,314</td>
</tr>
<tr>
<td>8</td>
<td>Product X₈</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>1,60,000</td>
</tr>
<tr>
<td>9</td>
<td>Deviations n₁</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>Deviations n₂</td>
<td>17,298</td>
</tr>
<tr>
<td>11</td>
<td>Deviations n₃</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>Deviations n₄</td>
<td>10,536</td>
</tr>
<tr>
<td>13</td>
<td>Deviations p₁</td>
<td>1,26,896</td>
</tr>
<tr>
<td>14</td>
<td>Deviations p₂</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>Deviations p₃</td>
<td>97,414</td>
</tr>
<tr>
<td>16</td>
<td>Deviations p₄</td>
<td>0</td>
</tr>
</tbody>
</table>

As shown by table a special feature of the optimal plan for the large hosiery units is that X₃ enters the optimal solution set for the two variations in the sales limit and X₃ finds entry only when sales target is revised upto 2.40 lakhs pieces. Further sales limits
are the binding constraints in option 2 and 1, while the inequality constraints is satisfied in option 3. Actual production 2.28 lakh pieces is less than the sales limit of four lakhs pieces. In option 2 (of sales limit of two lakh pieces) the optimal solution set consist of only $X_2$, $X_5$ and $X_7$ and $X_2$ is the largest activity with 1,24,668 pieces followed by $X_7$ with 54,102 pieces and $X_5$ with 21,230 pieces.

Capital use is minimized at a level which is largest by Rs.42,758 than the target of Rs.6,00,000. Profit falls short of the target of Rs.12,00,000 by Rs.1,56,000. The processing time is maximized by reducing short fall in target by 10,994 hours, while deviation in labour use is minimized to 51,586 hours.

Option 1 (sales target of 1,60,000 pieces) was infeasible and became feasible only when the target for processing time was scaled down to 2.80 lakhs hours. Here again sales limit is seen to be the binding constraints. All other constraints are satisfied. The deviation in profit is maximized to Rs.17,298 and processing time by 10,536 hours. The deviations in capital and labour sue are minimized to Rs.1,26,896 and 97,414 hours respectively.

Similarly the option 3 (sales limit of 2.40 lakh pieces) was infeasible. It became feasible only by revising the target for capital use to Rs.8 lakhs and for labour to 4.5 lakhs hours. In the final optimal solution production was 2.28 lakhs pieces, much less than the sales limit of 2.40 lakhs pieces, but all other constraints are satisfied. Target for profits is executed by about Rs.3.00 lakhs while that for processing time 55,578 hours. The use of capital was less by Rs.91,658 to the revised target of Rs.Eight lakhs. The use of labour was less by 20,668 hours to the revised target of 4.50 lakh hours. To realise the
sales limit of 2.40 lakh piece further relaxation of target for capital use, labour use are necessary. Thus further expansion of production in large hosiery unit would require still high investment in material and human capital.

1.13.9 A comparative study of performance of size groups

If the optimal production plans are implemented, how will the performance of the representative unit of the size groups of the hosiery units would change? For this purpose we worked out profit/capital ratio, profit/labour ratio and capital/labour ratio, for the three options for the representative unit of each of the three size group of hosiery units of Tiruppur and compared them with the ratios for the existing solutions. Data for calculating the ratios are brought together.

As shown in the table there was no feasible solution for options 2 and 3 for the small hosiery units. This is due to very tight investment opportunities and difficulty in expanding market potential. In the medium and large unit also these problems exist, but they were less restrictive. From the details presented in Table 1.10 important ratios to measure the performance efficiency of the optimal plan vis-a-vis existing situation and between different size groups. Important measure of performance were profit rate, (ie.) profit/capital ratio, labour productivity ratio ($\pi/L$), capital| labour ratio, capacity utilizations as measured by the percentage of processing time utilised and sales realization which was measured by the percentage of realised production (number of piece) to the assumed level of sales potential on the RHS of first constraint (fifth row) below the objective function in the model. The value of RHS was changed for two levels, one below the initial level and one above it and the problem analysis of the problem was
analysed for the final optimal solution. The optimal production of all the products (total number of pieces) was then expressed as a percentage of assumed level of sales potential. In making this measure, the basic assumption is that there is no problem on inventory accumulation and hence optimal production is the realized sales. The measures of performance efficiency are presented in Table 1.10.

Table 1.10

Comparative study of performance of hosiery units by size and options

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Performance</th>
<th>Small Units</th>
<th>Medium Size Units</th>
<th>Large size units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>EX. OP₁</td>
<td>EX. OP₁ OP₂ OP₃</td>
<td>EX. OP₁ OP₂ OP₃</td>
</tr>
<tr>
<td>1</td>
<td>π/K</td>
<td>1.51 1.63</td>
<td>1.65 1.78 1.81 1.83</td>
<td>2.19 2.57 2.43 2.12</td>
</tr>
<tr>
<td>2</td>
<td>π/L</td>
<td>3.25 3.45</td>
<td>3.58 3.96 4.12 4.27</td>
<td>3.28 4.02 3.89 3.49</td>
</tr>
<tr>
<td>3</td>
<td>K/L</td>
<td>2.15 2.11</td>
<td>2.17 2.22 2.28 2.33</td>
<td>1.48 1.56 1.60 1.65</td>
</tr>
<tr>
<td>4</td>
<td>Capacity Utilization(%)</td>
<td>66.70 79.10</td>
<td>64.20 91.39 64.59 75.68</td>
<td>56.32 64.4 82.2 91.1</td>
</tr>
<tr>
<td>5</td>
<td>Sales Potential realized (%)</td>
<td>78.24 89.60</td>
<td>75.00 91.13 100.00 89.26</td>
<td>81.10 100 100 75</td>
</tr>
</tbody>
</table>
Note:

1. Small units with installed capacity of Rs.15 lakhs to Rs.120 lakhs production mean.
2. Medium units with installed capacity of Rs.121 lakhs to Rs.240 lakhs production mean.
3. Large units with installed capacity of Rs.241 lakhs to Rs.360 lakhs production mean.

Mean values represent the representative farm situations.

4. \( \pi = \text{Net Profit}, K = \text{Capital used}, L=\text{Labour used.} \)
5. Capacity utilization percentage of processing time utilized
6. Percentage of realized sales to the sales limit perceived
7. EX-existing OP\(_1\), OP\(_2\), OP\(_3\) optimal plans 1,2,3 respectively.

As could be seen in the table, for the small hosiery units optimal plan has larger values of capital and labour productivity and better capacity utilization and sales realization but the capital labour ratio is marginally smaller. It means that optimal plan satisfies the goals set by the hosiery unit and priority ordering in them and improve performance efficiency of the unit, even while reducing capital intensity.

For the medium sized units there were three optimal plans by the plan 2 sets a lower sales limit and correspondingly limits the scope for production as compared to plan 1. Thus, the scope of production plans were arranged in the ascending order as plan2, plan 1 and plan 3 as shown in the above table.
As could be seen in the table, capital productivity ($\pi /K$), labour productivity ($\pi /L$) and factor intensity ($K/L$) increased from existing plan to plan 3 in the order prescribed. Thus, the inference is clear: By increasing the sales potential, the medium sized units could steadily improve their performance efficiency. However, due to the differences in the production structure. (Composition of output) there is a decline in capacity utilization plan was not less than in the existing plan. Similar is the result in sales realization also.

For the large units, capital productivity, labour productivity declined from plan 2 to plan 1 and then to plan 3. In contrast factor intensity ($K/L$) increased steadily. Thus, increasing factor intensity had occurred with lower factor productivity and it would require attention to the technology of production. However in all cases, the ratios were larger than in the existing to the existing plans. The optimal plans also improved capacity utilization and sales realization significantly.

A comparative study of the three size groups show that optimal plans presented opportunities to improve performance efficiency in all the groups but the gain was larger, when the size of the units became larger. This was in agreement with returns to scale for medium and large units and constant returns to small units.