CHAPTER 1

INTRODUCTION

1.1 GENERAL

High efficiency solid-state power conversion has become possible through the continuous development of high power semiconductor devices. The operation of these devices as switches, which is necessary for high efficiency means that power electronic circuits are essentially nonlinear time varying dynamical systems. Though this makes the system difficult to study, the effort is well worthwhile because they have many practical applications and are becoming increasingly important in the delivery and utilization of electrical energy.

Power converters employ both switching components, such as transistors and diodes and reactive components, such as inductors and capacitors. Energy is steered around the circuit by the switching components while the reactive components act as intermediate energy stores and input/output reservoirs. The presence of both types of components implies that the circuits are nonlinear, time-varying dynamical systems. There are also several unavoidable sources of unwanted nonlinearity in practical power converters:

1. The semiconductor switching devices have intrinsically nonlinear DC characteristics. They also have nonlinear

capacitances and most suffer from minority carrier charge storage.

2. Nonlinear inductances abound: transformers, chokes, magnetic amplifiers and saturable inductors used in snubbers.

The basic operation of any power electronic circuit involves toggling among a set of linear or nonlinear topologies, under the control of a feedback system and is thereby regarded as piecewise switched dynamical systems. Through the use of the feedback control circuit, the relative durations of the various switching intervals are continuously adjusted so as to maintain the output variable at the reference level even under input and load variations. Such control circuits usually involve nonlinear components such as comparators, Pulse Width Modulators (PWM), multipliers, phase-locked loops, monostables and digital controllers. Thus, both the circuit topology and the control method determine the dynamical behaviour of a power electronic circuit.

These converters, being time-varying and nonlinear are found to exhibit several periodic steady state responses as well as chaotic. Chaos, a particular qualitative behaviour of nonlinear systems, characterized by an aperiodic and apparently random trajectory has a sensitive dependence on initial conditions. Chaos, in power electronics receives attention because it creates many undesirable effects like degrading the converter efficiency, causing power quality issues like distorting the line current and degrading the practical power factor.

In power electronic applications, the desired behaviour of the system is a stable periodic motion around a predefined value of periodicity equal to the period of the external clock, which is used to control the system. However, there are cases wherein some external parameters (like the input voltage or the load) fluctuate while the converter is in operation and cause the limit cycle to lose its stability and enter into other operating regimes. Such changes may also occur due to accidental loss of components or due to some design requirements. Such a phenomenon, where one mode fails to operate and another picks up, is termed as bifurcation. Hence it is of practical importance to know the conditions that ensure a desired operation and ways a desired operating mode may give way to an undesirable one. In reality, bifurcation is to be avoided, but it is also known that designing a system too remote from bifurcation boundaries may degrade other performance characteristics. Therefore, design-oriented bifurcation analysis has a significant impact on the practical methodologies taken to make design trade-off and performance optimizations.

Books devoted entirely to the subject of 'Nonlinear dynamics and chaos', have been published by Tse (2003), Banerjee and Verghese (2001), Thompson and Stewart (1986), Schuster and Just (2004), Lakshmanan and Murali (1996). The following section discusses the various bifurcations and instabilities observed in power electronic converters.

1.2 TYPES OF INSTABILITIES

The sudden change in the qualitative behaviour of the system, when a parameter is varied is termed as bifurcation. Successive bifurcations lead to instabilities in a power electronic converter. The bifurcation is classified according to the type of qualitative change that takes place when the parameter is varied. Bifurcations exhibited by power electronic converters can be mainly classified as standard/smooth bifurcations and non-standard/non-smooth bifurcations. Smooth bifurcations do not involve any structural change of the system and is normally associated with the loss of stability of one solution and the picking up of another. The eigenvalues at these bifurcations are associated with a zero real part in continuous-time systems and unitary modulus in discrete-time systems. Generally, two distinct types of smooth bifurcation behaviour have been identified for power electronic circuits, namely slow-scale bifurcation which leads to low frequency oscillation and fast-scale bifurcation that leads to fast-scale instability. Most common smooth bifurcations exhibited by power electronic converters are period-doubling bifurcation, Hopf bifurcation and Neimark-Sacker bifurcation.

Non-smooth bifurcations are characterized by an abrupt alteration of the detailed operating principle. These bifurcations manifest themselves as discontinuous transitions or sudden jumps. The occurrence of these transitions is commonly explained as due to interactions between system trajectories and state space boundaries where the system switches from one configuration to another. Non-standard bifurcations exhibited by power electronic converters are Border Collision Bifurcation (BCB) and grazing.

1.2.1 Fast-scale Instability

The phenomenon of a period-1 solution giving way to a period-2 solution is known as period-doubling bifurcation. Since the bifurcated orbit flips between two points, it is also known as the flip bifurcation. They are also referred as period-doubling or sub-harmonic bifurcation. In a discrete dynamical system, the system switches to a new behaviour with twice the period of the original system. That is, there exists two points such that applying the dynamics to each of the points yields the other point. In continuous dynamical systems, a new limit cycle emerges from an existing limit cycle and the period of the new limit cycle is twice that of the old one. It is characterized with a real eigenvalue crossing the boundary of stability at -1. Repeated period-doublings, known as the period-doubling cascade, result in aperiodic (chaotic) orbit. Thus, a period-1 solution exists for a parameter range and it loses stability at a critical value of the parameter giving birth to a period-2 cycle. Period-2 solution exists for a parameter range and it loses stability at a critical value of the parameter giving birth to a period-4 cycle. This process proceeds further ad infinitum (Feigenbaum scenario).

Nonlinear phenomena in elementary DC-DC converters, used to step down (buck converters), step up (boost converters) or both step down and step up (buck-boost converters) the voltages being the simplest, have been particularly subjected to extensive enquiry. This dynamic behavior exhibited by any converter varies for different control strategies and has been reported widely. Chaos and sub-harmonic instabilities in DC-DC buck converter have been observed and described by Brockett and Wood (1984), Wood (1989), Jefferies et al (1989), Chakrabarty et al (1996), Aroudi et al (2005), Maity et al (2007), Giaouris et al (2008), Deane and Hamill (1990), Hamill and Deane (1992), Yu et al (2012), Fossas and Olivar (1996). These works illustrate how chaos can occur in simple voltage-mode controlled DC-DC buck converters operating in the Continuous Conduction Mode (CCM). The transition from periodicity to chaos has also been investigated by Angulo et al (2005) and Angulo et al (2008) in a Zero Average Dynamics (ZAD) strategy controlled DC-DC buck converter. Chaos has also been studied for a current-programmed boost PWM DC-DC switching converter operating in CCM by Hamill and Jefferies (1988), Krein and Bass (1989), Zafrany and Ben-Yaakov (1995), Chan and Tse (1996, 1996 a), Marrero et al (1996), Deane (1992), Chan and Tse (1997), Cafagna and Grassi (2006), Giaouris et al (2012) and in the Discontinuous Conduction Mode (DCM) by Tse and Adams (1990) and Tse (1994).

Many of the theoretical investigations outlined have also been backed by experiments. Hamill and Deane (1992) and Deane and Hamill (1990 a) have reported the experimental observations of bifurcations in a voltage-mode controlled buck converter. Tse (1994 a) and Tse and Chan

(1997) have backed up their numerical investigation on the current-mode controlled DCM buck converter and CCM boost converter through experimental results. Chakrabarty et al (1996) and Banerjee and Chakrabarty (1998) have reported experimental studies on the buck converter and the boost converter.

Many papers in the literature have reported the analysis of period-doubling bifurcation in the elementary DC-DC converters like buck, boost and buck-boost converters with simple and advanced control techniques. However, detailed analysis of bifurcation in higher order converters like Ćuk converter with recent control techniques which has not been done extensively so far is presented in this thesis with a detailed mathematical analysis.

1.2.2 Slow-scale Instability

A Hopf or Poincaré-Andronov-Hopf bifurcation is a type of bifurcation characterized by a sudden expansion of a stable fixed point to a limit cycle as a control parameter is varied. As the same parameter continues to vary, the system admits another periodicity which is not in a rational ratio to that of the first limit cycle and under some circumstances become chaotic (Ruelle-Takens-Newhouse scenario). This bifurcation is characterized by a change of the real parts of the pair of complex conjugate eigenvalues associated with an equilibrium point from a negative to positive value while the imaginary part remains greater than zero. This implies that a Hopf bifurcation can only occur in systems of dimension two or higher. The bifurcation is called supercritical if the limit cycle is stable above the bifurcation point and subcritical if unstable.

Hopf bifurcation in a hysteresis current controlled Ćuk converter has been reported by Tse et al (2000) and the presence of slow-scale oscillation has also been experimentally verified in a positive output Luo converter by Kavitha and Uma (2008). Wang et al (2010) has analysed slow-scale instability in a boost Power Factor Correction (PFC) converter using the method of harmonic balance and Floquet theory whereas Dai et al (2007) has analysed the slow-scale instability in a single stage power factor correction (PFC) circuit using an averaged model and Jacobian theory.

Until now, much work has been reported on the fast-scale instabilities of non-autonomous switching converters, while very little work has been performed on the slow-scale instabilities of autonomous switching converters. In this thesis, slow-scale instability in a hysteresis current-controlled cascaded-boost converter which is widely used in solar energy systems is investigated with a detailed mathematical analysis. Various industrial applications and solar energy systems demand high level DC voltages as explained by Nabulsi et al (2009), Vighetti et al (2012) and Zhao et al (2012). In recent years, cascaded-boost converters with geometric progression voltage conversion ratio, but with simpler structure, higher power density and higher efficiency are widely used as reported by Luo and Ye (2003, 2005, 2006), Luo (1998), Ortiz Lopez et al (2008). Conventional linear control techniques based on small-signal models are found to be incapable of achieving the necessary regulation, dynamic response, and stability requirements under wide load variations, supply disturbances and parameter variations which paved the way for several modern nonlinear control techniques. Among these, free-running or hysteresis current control is often used, since they require no external clock, their constructions are relatively simple and can provide fast dynamic response with a wide regulation range.

1.2.3 Coexisting Fast and Slow Scale Instabilities

Generally, the slow-scale bifurcation can be regarded as a kind of low-frequency instability which is caused by the outer voltage feedback loop permitting low-frequency oscillation and the fast-scale bifurcation can be regarded as a kind of high-frequency instability which is caused by the inner current loop instability. The fast and slow scale bifurcations have been independently investigated as it has been generally believed that the outer voltage loop is much slower than the inner current loop and the two loops can be considered non-interacting. As a result, slow-scale bifurcation and fast-scale bifurcation have been studied separately. However, in practice, under certain conditions, slow and fast-scale bifurcations can occur simultaneously.

The theoretical and experimental investigation of fast and slow scale instabilities in a DC-DC buck converter has been reported by Mazumder et al (2001). Though the authors have investigated both the slow and fast scale instabilities, it has been done for two different parameter variations and not discussed the possibility of coexistence of both the instabilities. Chen et al (2008) have investigated the coexistence of slow and fast-scale instabilities in current-mode controlled DC-DC elementary buck, boost and buck-boost converters. However, coexistence of instabilities in PFC converters used to regulate the output voltage and to achieve a near Unity Power Factor (UPF) has not been identified so far in literature.

The stability of PFC converters have been studied in detail by Mohan et al (1984), Henze and Mohan (1986), Ridley (1989), Williams (1989), Zhou and Jovanović (1992). Mohan et al (1984) has used the concept of quasi-static analysis to analyse the current-loop stability of boost PFC circuits operating with hysteretic control. Zhou and Jovanović (1992) have demonstrated the occurrence of current-loop instabilities in the boost PFC circuit operating with peak current-mode control and with average Current-Mode Control (CMC). The nonlinear dynamics of an average current-mode controlled boost converter has also been analysed thoroughly by Orabi and Ninomiya (2003, 2004). By Herbert et al (2003), fast-scale instability has been detected in the PFC boost converter with average CMC which can cause distortion to the line current and degrade the practical power factor. Also, the slow-scale instability has been unveiled by Orabi and Ninomiya (2004 a), Wang et al (2010), Dai et al (2007), where the power factor reduced to less than half as the converter moves from a stable, fundamental period-1 operation to a period-2 operation.

The necessity for a multiplier-divider in the integrated circuit, to ensure power balance under changes in load and line voltages is a major drawback in average CMC PFC converters. On the other hand, it is advantageous, if the reference current is not derived explicitly from the line voltage, resulting in reduced passive components in the integrated circuit which paved the way for Nonlinear Carrier (NLC) control. The NLC control for high power factor boost rectifiers and other converter based rectifiers have been proposed by Maksimovic et al (1996) and Regan Zane and Maksimovic (1998). The presence of nonlinearity in the control necessitates a better understanding of the system dynamics. In this thesis, the coexistence of slow and fast scale instabilities in a NLC controlled PFC Ćuk converter, as explained by Erickson and Maksimovic (2004), Tse (2003), Banerjee and Verghese (2001) is studied.

1.2.4 Intermittent Instability

Intermittency means that a signal which is regular in time becomes interrupted by statically distributed periods of irregular motion (intermittent bursts). Some nonlinear systems exhibit this behaviour as a certain parameter is varied. The average number of these bursts increases with the variation of an external control parameter. As the parameter is varied further, bursts of chaotic behaviour separated by long intervals of periodic behaviour exhibited intermittently reaches a critical value beyond which the motion becomes completely chaotic. The intermittency route to chaos (Pomeau-Manneville scenario), has been discovered by Manneville and Pomeau (1979).

Intermittent operation, heard as a sizzle with rather long period which clearly distinguishes it from other noise signals, is a phenomenon which may arise frequently in periodically driven nonlinear systems, where the frequency of the interference signal may not be consistent with the system's driving frequency. It has been first reported by Zhou et al (2008). Such intermittency has been observed in switching power supplies which are not protected against the intrusion of spurious signals or when parasitic inductances and capacitances are present, causing unwanted oscillations (ringing) of the essential control signals. The intrusion can also take the form of coupling via conducted or radiated paths present on the same circuit board or its close proximity as has been studied by Wong et al (2004), Ferreira et al (1997).

Intermittent chaotic operation in a current-mode controlled boost converter with the sinusoidal spurious signal injected directly to the compensation ramp signal has been studied by Wong et al (2004). Furthermore intermittency has been studied in a voltage-mode controlled buck converter which superposes the spurious perturbation directly on the control voltage by Tse et al (2003). In particular, only sinusoidal interference is considered for analysis. It has also been shown by Zhou et al (2003), Kavitha and Uma (2010) that the resonant parametric perturbation is suitable for controlling chaos and the control effort can be reduced dramatically by introducing an appropriate phase shift. Moreover, when frequency mismatch exists in the perturbation, it is shown that the system exhibits intermittent chaos. Dai et al (2005) has presented a general approach for inducing chaos by using small, resonant perturbations. The quasi-periodic behaviour due to sinusoidal disturbance in the input voltage of boost converter has also been reported by Bernardo et al (1996) and the same has also been reported by Aroudi et al (2000), Aroudi and Leyva (2001) in DC-DC converters without a distribution in the input voltage or the reference signal. In order to enhance the understanding of the reliable operation of a converter, it is significant to consider different types of periodic signals and also the influence of interference signals in different vital places. In this connection, the effect of sinusoidal, triangular and saw-tooth interference signals in input and control voltages of a voltage-mode controlled buck converter has been studied in this thesis.

1.3 MODELING APPROACHES

In practical power electronic systems, since closed-loop stability and transient responses are basic design concerns, models that permit the direct application of conventional small-signal approaches are advantageous. However, these linearised models fall short of predicting any nonlinear behaviour. Hence in this thesis, two modeling approaches have been used for bifurcation analysis:

1. Continuous-time models or averaged models,
2. Discrete-time models or sampled-data models.

The analysis method based on state-space averaged continuous-time model gives a useful representation of the system and allows simple design procedures for operation in certain regimes. However, the averaged model is of little or no use in predicting and analyzing fast-scale dynamics like subharmonics and chaos, as in this modeling approach the time-varying dependence from the original time-varying model is removed. It effectively acts as a low-pass filter and ignores all phenomena that take place at clock frequency. On the other hand, the exact discrete-time sampled data modeling technique where the state variables are discretely observed at specific instants of time captures both the fast and slow-scale dynamics. In this method, the Poincaré map that describes the evolution of the state from one clock instant to the next, is explicitly derived and is then locally linearised at the fixed point. However, in many converters the map cannot be derived in closed form because of the transcendental form of the equations involved and therefore this approach cannot directly be applied. Alternatively, by truncating the transition matrix series in this model, a simplified discrete-time model is obtained, which can be regarded as an averaged discrete-time model. This explicit simplified model is useful in reducing the computational load and in explaining dependence of the interacting instability on parameters of the system, since the analytical expressions for equilibrium points and solutions of characteristic polynomial is obtained readily.

The initial investigations in nonlinear dynamics have been done with the most widely used small-signal analysis approach based on state-space averaging as reported by Middlebrook and Ćuk (1977, 1977 a), circuit averaging reported by Wester and Middlebrook (1972) and using the PWM switch model reported by Tymerski et al (1988). Simple analytical expressions in terms of the circuit components were derived to characterize the low-frequency response of such systems. Middlebrook and Ćuk (1977) have generalized the above technique by introducing the state-space averaging method. They replaced the state-space description of the switched network by a single description, hence eliminating the switching process from consideration and representing the average effect of the switched networks in one cycle of operation. By means of state-space averaging, the original nonlinear discrete system was simplified into a nonlinear continuous system. But with this approach, it was difficult to go beyond empirical observation of the phenomena.

Based on the sampled-data-modeling techniques of power electronic circuits presented in the textbook by Kassakian et al (1991), Hamill and Deane (1992) have proposed nonlinear map-based modeling. In this method, the state variables are discretely observed at those clock instants that result in a switching event and the skipped cycles are ignored. Deane and Hamill (1991) have later applied this method to analyse the current-mode controlled boost converter. Di Bernardo et al (1996, 1997, 1998, 1998 a), Di Bernardo (1996) have observed the state variables discretely at each switching event. On the other hand, Banerjee and Chakrabarty (1998), Chan and Tse (1997), Marrero et al (1996) have sampled at the beginning of each ramp cycle (known as stroboscopic sampling) in studying the dynamics of current-mode controlled converters.

1.4 OBJECTIVES OF THE THESIS

The objectives of the investigations carried out hitherto as presented in this section are summarized as follows:

- To analyse the fast-scale instability in higher order power electronic converters controlled with PWM techniques by sketching the phase portraits, Poincaré sections and bifurcation diagrams.
- To analyse the slow-scale instability in higher order autonomous power electronic converters by deriving the Jacobian matrix in continuous time domain and to identify the critical point at which bifurcation occurs from the loci of eigenvalues.
- To identify the coexistence of fast and slow scale instabilities in PFC converters using simulations and prototypes.
- To analyse intermittent bifurcation, period-bubblings and mode-locking instabilities in DC-DC and DC-AC converters and to verify the analytical results using simulations and prototypes.

1.5 ORGANISATION OF THE THESIS

The thesis is organised into six chapters. The organisation of each chapter is briefly described below:

Chapter 1 presents the general introduction to the problems and previous investigations reported in the literature. This chapter discusses the different types of bifurcations and the various modeling strategies available for nonlinear analysis. It concludes with the main objectives of the research work.

In **Chapter 2,** the nonlinear dynamical behaviour in a ZAD controlled DC-DC Ćuk converter is investigated. Effects of varying the control parameters on the qualitative behaviour of the system are studied in detail. Bifurcation analysis of a ZAD controlled DC-DC Ćuk converter has not been reported so far in literature as it involves complex modeling and implementation of the overall control system. To reduce the complexity in deriving the map dynamics and computing the ZAD control parameters, a reduced order model is derived. Moment matching technique is used to obtain the reduced order model. It is shown that for even small control parameter variations, the system exhibits period-doubling bifurcation. The dynamics of this converter system is mathematically described and analysed with a simple discrete map. Computer simulations as well as experimental investigations are performed to study the qualitative behaviour of the system under variations of converter and control parameters. The results offer useful information of parameter space for the design and operation of the converter in the desired fundamental stable regime.

In **Chapter 3,** the nonlinear dynamics of an autonomous hysteresis current-controlled two stage cascaded-boost converter which is a popular choice, for solar energy systems is investigated. The stability of the system is studied with the aid of nonlinear state equations derived from an averaged continuous model. Analysis of the describing autonomous equations reveals that the system loses stability via supercritical Hopf bifurcation. Extensive computer simulations are performed to capture the system's dynamic behaviour and demarcate the bifurcation boundaries. The detailed dynamical behaviour as well as its potential influence on the system's performance is investigated. Furthermore, the possibility of subcritical Hopf bifurcation for variation in input voltage is also examined. Trajectory and Poincaré section before and after the bifurcation are shown. Experimental results are also provided to confirm the observed bifurcation scenario.

In **Chapter 4,** both the fast-scale and slow-scale instabilities in a PFC Ćuk converter under NLC control which is proved advantageous compared to other conventional control strategies is investigated. In this NLC control, the nearly unity power factor is achieved by using a nonlinear carrier signal, without sensing the input voltage. Bifurcation studies of NLC controlled higher order converters have not been reported so far in literature. It is shown that the fundamental periodic orbit loses its stability via period-doubling bifurcation and later bifurcates to chaos. Computer simulations as well as experimental investigations are performed to study the qualitative behaviour of the system under variations of different parameters. It is also shown that both the fast and slow scale instabilities may cause distortion in the line current and degrade the practical power factor. The bifurcation phenomena related to the existence of instabilities are also analysed through mathematical derivations. The mathematical analyses are validated with the simulation results and the stability margins are defined in different dimensions.

In **Chapter 5,** the effect of different types of periodic interference signals like sinusoidal, triangular and saw-tooth waveforms is considered and the influence of these signals in the input and control voltages of the buck converter is investigated. It is shown that the presence of sinusoidal or triangular interference signal whose frequency is comparable with the switching frequency of the converter or its rational multiples manifests as symmetrical period-doubling bifurcation in intermittent periods. However, the presence of either sinusoidal or triangular interference signal having irrational frequency ratios with the switching frequency of the converter leads to quasi-periodic route to chaos. Also it is shown that the influence of saw-tooth interference signal results with intermittent chaos in the dynamics of the system. Further, the effect of simultaneous presence of more than one interference signal is considered. The dynamics of this converter system is mathematically described and analysed with a simple discrete map. The ordered and the chaotic dynamics of this system is investigated with suitable analytical, numerical and experimental means.

In **Chapter 6,** the nonlinear dynamics of a full bridge DC-AC inverter controlled by fixed frequency PWM which is widely used in solar energy systems is investigated. The main results are illustrated with the aid of time domain simulations obtained from an accurate nonlinear time varying model of the system derived without making any quasi-static approximation. It is shown that for high filter time-constants, the system loses stability via Hopf bifurcation and exhibits mode-locked periodic motion and for low filter time-constants, via period-doubling bifurcation resulting in period-bubbling structures and intermittent chaos. The mode-locked instability is also theoretically verified using Jacobian matrix derived from an averaged model and that of period-bubbling instability is verified using monodromy matrix based on Filippov's method of differential inclusions. Furthermore, extensive analyses are performed to study the mechanism of the emergence of intermittency and remerging chaotic band attractors (or Feigenbaum sequences) for variation in filter parameters and to demarcate the bifurcation boundaries. Phase portraits and Poincaré sections before and after the bifurcations are shown. Experimental results are also provided to confirm the observed bifurcation scenario.

In **Chapter 7,** a review of the work reported in the thesis and major contributions are presented. This chapter also briefly presents the future work that can be carried out in this area.