CHAPTER 2

VOLTAGE STABILITY

2.1 GENERAL

Environmental constraints limit the expansion of a transmission/distribution network and generation near the load centres are results in an increase in the electrical distance from the generator to the load. It further weakens the voltage support in the load area. Simultaneous growth in the use of electrical power without a corresponding increase of transmission/distribution capacity brings many power systems closer to their VS limits.

The heart of the VS problem is the inability of the system to meet its reactive power demands. Usually, but not always, voltage collapse involves system conditions with heavily loaded lines. When the transport of reactive power from neighbouring areas is difficult, any change that calls for additional reactive power support may lead to voltage collapse. The voltage collapse generally manifests itself as a slow decay of voltage. It is the result of an accumulative process involving the actions and interactions of many devices, controls and protective systems (Kundur 1993; Taylor 1994).

2.2 VOLTAGE STABILITY PHENOMENON

Voltage collapse is in general caused by either load variations or contingencies. The following illustration explains the voltage collapse due to the load variations. The basic configuration used to explain voltage collapse is shown in Figure. 2.1.

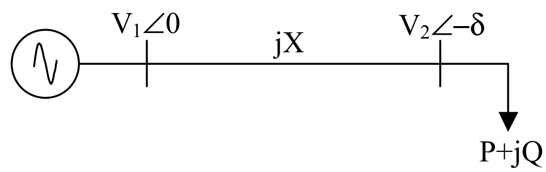

Figure 2.1 Sample two bus power system

In this circuit, a synchronous generator with a bus voltage of V1∠0 is connected to a load, described by its real and reactive powers P and Q, with a bus voltage of V2∠−δ through a loss-less transmission line reactance of jX. The governing algebraic relations are

$$P = \frac{|V_1||V_2|}{X}\sin\delta \tag{2.1}$$

$$Q = -\frac{|V_2|^2}{X} + \frac{|V_1||V_2|}{X}\cos\delta \tag{2.2}$$

Under steady-state conditions Equations (2.1) and (2.2) represent the voltage/power relation at the load end of the circuit. The PV curves are the plot of load voltage versus real power for several power factors and different sending-end voltages as shown in Figure.2.2. In the region corresponding to the top half of the curves, the load voltage decreases as the receiving-end power is allowed to increase. The nose point of these curves represents the maximum power that can theoretically be delivered to the load. If the load demand increases beyond the maximum capacity of the transmission line, the amount of actual load that can be supplied as well as the receiving-end voltage decreases. These curves indicate that there are two possible values of voltage for each value of load. The system cannot be operated in the lower portion of the curve, even though a mathematical solution exists. If the system operates at a point in the lower portion of the curve end and an additional quantity of load added, then under this condition the added load draws an increased current from the system. The resulting drop in voltage at this operating state offsets the increase in current so that the net effect is a decrease in the delivered power. However the load attempts to replenish the demand by some means, such as by increasing the current, because of which the voltage decreases even further and faster. The process eventually ends in voltage collapse, possibly leading in the loss of synchronism of the generating units and a major blackout.

The QV curves show the sensitivity and variation of bus voltages with respect to reactive power injections as depicted in Figure.2.3. They show the mega volt ampere reactive (MVAR) and voltage margins to instability and provide information on the effectiveness of the reactive power sources in controlling the voltage in different parts of the system.

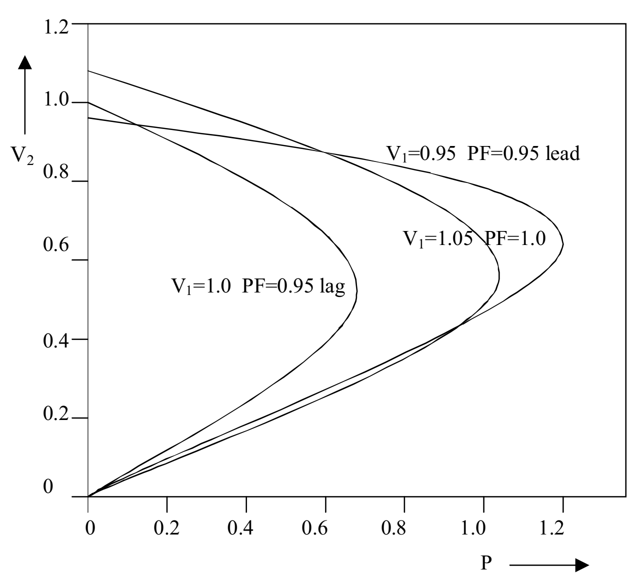

Figure 2.2 Voltage-Power characteristics for different power factors

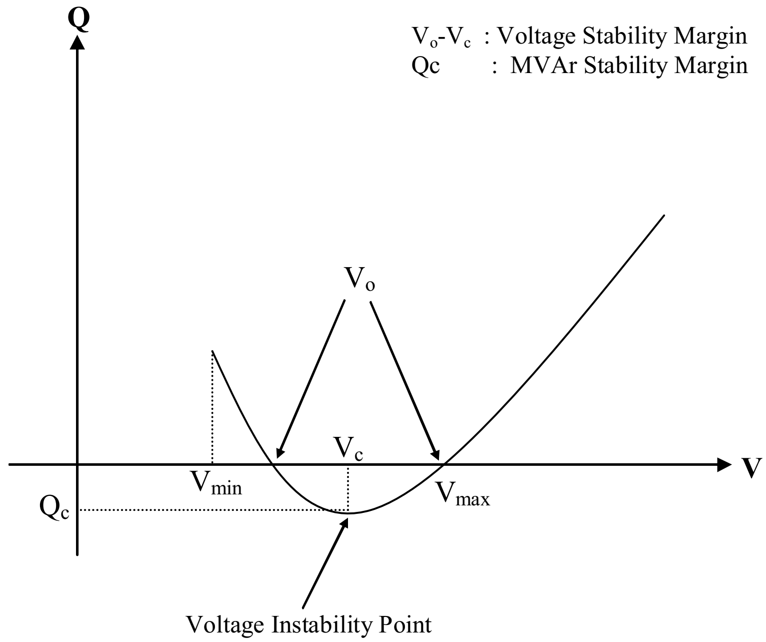

Figure 2.3 Sample QV Curve

For each QV curve a reactive power source is placed at the selected bus (QV bus) to move its voltage in a given range, V_{min} to V_{max}, by a given step size. At each voltage step, the power flow problem is solved to compute the required MVAR injection Q_i, at the QV bus for holding its voltage at V_i. The points of QV curves are computed by starting from the existing voltage, V_o and zero MVAR injection and increasing the voltage until V_{max} is reached or the associated power flow fails to converge. Then the system is reset to the initial condition at V_o and the QV computation is allowed to proceed in the opposite direction by decreasing the voltage until V_{min} is reached or the power flow becomes unsolvable. The voltage difference between V_o -V_c and the value of Q_c provides the voltage and MVAR stability margins at the bus and the slope of the curve gives the sensitivity information.

2.3 VOLTAGE STABILITY ANALYSIS

The incidents of voltage instability appear to be more common and the systems are continuing to be loaded closer to their stability limits. It therefore becomes imperative that the system operators be provided with tools that can identify potentially dangerous situations leading to voltage collapse. It is essential for the operator to obtain quickly the operating state of the system. There are a number of measures to ascertain the VS of system; the chief among them is the use of various VS indices. The most commonly used indices are:

- Singular value decomposition.
- Eigen value decomposition.
- Reduced jacobian determinant.
- Minimum singular value method
- Risk of voltage instability (RVI) indicator or L indicator
- Linear static VSI
- Radial VSI

2.3.1 Singular Value Decomposition

This technique is employed to assess whether or not the Jacobian matrix is singular (Canizares et al 2008). This evaluation is based on the fact that the singular values decomposition provides the rank of a matrix, given by the number of non-zero singular values. Therefore, the idea is, for a known operating point, the power flow jacobian is decomposed in such a way that:

$$J = U \in WT \qquad (2.3)$$

where U and W are the left and right singular vectors, respectively, and $\in$ is the matrix of singular values. On account of the fact that the matrix J represents the partial derivatives of the active and reactive power equations as a function of the state variables, it follows

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = W\,\Sigma^{1}\,U^{T} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad (2.4)$$

when the system jacobian becomes singular, the state variables experience large variations for small load disturbances.

2.3.2 Eigen-Value Decomposition

The eigen-value decomposition method can be applied to identify a singular matrix, since an eigen-value becomes zero at a saddle node bifurcation (Dobson 1992). The jacobian matrix is decomposed in this case as:

$$J = X \Lambda Y^{T} \qquad (2.5)$$

where X and Y are the left and right eigenvectors and Λ is the matrix with the system eigen-values. Therefore:

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = X \Lambda^{-1} Y^{T} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad (2.6)$$

A new matrix seen in Equation (2.7) is obtained if there is no active power variation at the operating point,

$$\begin{bmatrix} O \\ \Delta Q \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \qquad (2.7)$$

Thus,

$$\Delta Q = (D - CA\text{-}1B)\ \Delta V = [JQV]\Delta V \tag{2.8}$$

the matrix [JQV] is quasi-symmetric. This kind of matrix inherits similar eigen-values and singular values (Arnborg et al 1997). Hence, decomposing [JQV] through eigen-values and singular values offers the same results.

2.3.3 Reduced Jacobian determinant

This tool is used as a voltage security index (De Souza 2000). The Jacobian matrix associated with a known operating point is re-ordered in such a way that the rows and columns associated with the load bus of interest are the last ones, yielding

$$\begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta P_l \\ \Delta Q_l \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \\ \Delta \delta_l \\ \Delta V_l \end{bmatrix} \tag{2.9}$$

where “l” is the load bus that is analysed. If ΔP and ΔQ are set to zero in the above equation, the only incremental active and reactive power at bus l is considered. The Equation (2.9) thus becomes

$$\begin{bmatrix} 0 \\ 0 \\ \Delta P_l \\ \Delta Q_l \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \\ \Delta \delta_l \\ \Delta V_l \end{bmatrix} \tag{2.10}$$

The operating condition above is valid for small load variations at bus l. In this situation, the swing bus supplements the active and reactive power variations at bus l, as well as the system losses. However if a large load variation occurs these assumptions are no longer valid and a new operating point is required to be determined. The partial derivatives of active and reactive powers of bus l with respect to its state variables allows the Equation (2.10) to be reduced as

$$\begin{bmatrix} \Delta P_l \\ \Delta Q_l \end{bmatrix} = [D - CA^{-1}B] \begin{bmatrix} \Delta \delta_l \\ \Delta V_l \end{bmatrix} = D_{ll} \begin{bmatrix} \Delta \delta_l \\ \Delta V_l \end{bmatrix} \tag{2.11}$$

where matrix D_{ll} bears dimension 2 X 2. This reduction is carried out for all the system load buses. The load bus associated with the smallest determinant is considered as the critical one at the current operating point. One of the problems with this reduction is the identification of the critical bus, which is only possible at the bifurcation point. This problem is resolved through the tangent vector technique (De Souza et al 1997). If the system critical bus is known a priori, then this approach facilitates a smooth behaviour, as a function of the increase in load (Yabe et al 1996).

2.3.4 Minimum singular value method

The minimum singular value of load flow jacobian matrix is included as an index for quantifying the proximity to the voltage collapse point. It is an indicator available from the normal power flow calculations. The computation of minimum singular value can be done fast with special algorithms (Lof et al 1992). The method is based on the analysis of linear system Ax = b. The singular value decomposition of the matrix A is given in Equation (2.13), where $A \in R_{mxm}$ G and H are mxm orthonormal matrices, the left and right singular vectors G_i and H_i are the columns of the matrices G and H respectively, $\in$ is a diagonal matrix (E =diag {i}), and “a” is a singular value.

$$A = U\Sigma H^T = \sum_{i=1}^{m} \sigma_i g_i h_i^T \tag{2.12}$$

The singular value decomposition is applied to linearised load-flow equations to analyse the power system VS. The analysis studies the influence of a small change in the active and reactive power injections $[\Delta P\ \Delta Q]^T$ to the change of angle and voltage $[\Delta\ \delta\ \ \Delta V]^T$. The solution of the linearised load-flow equations using the singular value decomposition is given in Equation (2.13).

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = J^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = (G\Sigma H^T)^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = H\,\Sigma^{-1}\,G^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{2.13}$$

The inverse of the minimum singular value, (min {σi})-1 indicates the largest change in the state variables. Small changes in either matrix J or vector $[\Delta P\ \Delta Q]^T$ may cause major changes in $[\Delta\delta\ \Delta V]^T$, if (min $\{\sigma_i\}$) is small enough. The minimum singular value is a measure of how close to singularity the load-flow jacobian matrix is. If the minimum singular value of the load-flow jacobian matrix is zero, then this matrix is singular i.e. the inverse of the jacobian matrix does not exist. The operating point ceases to have a load-flow solution in that case, because the sensitivity of the load-flow solution to small disturbances is infinite. The singularity point is also called the saddle node bifurcation point. Singular vectors can provide information about a power system's critical areas and components. The right singular vector corresponding to the smallest singular value indicates the sensitive voltages and angles, i.e. critical areas. The left singular vector corresponding to the smallest singular value indicates the sensitive directions for the changes of active and reactive power injections (Lof et al 1992).

If the operating point is far away from the voltage collapse point, then the minimum singular value does not describe the state of the system accurately. The minimum singular value of a load-flow jacobian matrix is also sensitive to the limitations of the generator reactive power, transformer tap changer and compensating device. The minimum singular value method may be applied to a reduced jacobian matrix in order to improve the profile of the index.

2.3.5 RVI Indicator

One of the globally accepted VS indices is the RVI indicator, also called L-index, suggested by (Kessel and Glavitsch 1986). This indicator involves the solutions of a solved power flow, from where variables and parameters are taken to compute the indicator for VS. For any load bus j, the indicator is defined as

$$L_j = \left| 1 - \frac{\sum_{i \in \alpha G} F_{ji} V_i}{V_j} \right| \qquad j \in \alpha L \tag{2.14}$$

where

αL = set of load buses

αG = set of generator buses

V_j = complex voltage at load bus-j

V_i = complex voltage at generator bus-i

$F_{ji} = |F_{ji}| \angle \theta_{ji}$ = elements of hybrid-F matrix

The $[F]$ is computed using

$$[F] = -[Y_{LL}]^{-1}[Y_{LG}] \tag{2.15}$$

where $[\mathbf{Y}_{LL}]$ and $[\mathbf{Y}_{LG}]$ are sub matrices of the Y-bus matrix.

The maximum value of this indicator, close to one, is indicative of the proximity to power flow divergence, which is prone to voltage collapse. The minimum value, close to zero, is indicative of the most stable state.

2.3.6 Linear Static VSI

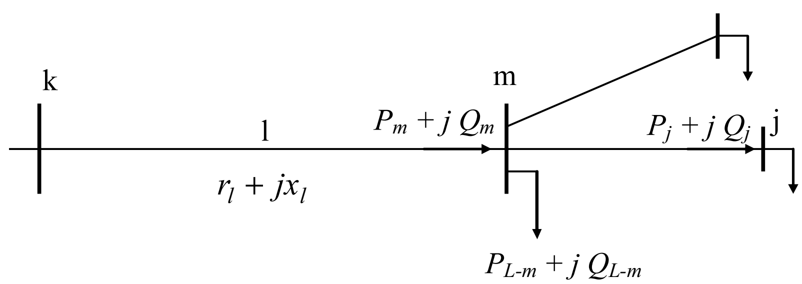

Figure.2.4 Sample Distribution Line

In low voltage distribution systems, a number of branches are usually connected in series to form a radial feeder. The VS margin (Haque 2006) of node- m or branch- l of Figure 2.4, which is connected between nodes-, $k\&m$ can be written from knowledge of the complex node voltages as

$$VSI_m = \left[2\,\frac{V_m}{V_k}\cos(\delta_k - \delta_m) - 1\right]^2 \tag{2.16}$$

This index uses only the complex node voltages and it does not require any knowledge on the source or load impedance. The variation of this index against the load power at any power factor is more or less linear in the entire operating region. The VS margin of the other branches can be calculated from the above equation. The feeder that acquires the lowest value of loading margin can be considered as the weakest feeder of the system and is prone to voltage collapse.

2.3.7 Radial VSI

The load flow equation of a feeder- m of Figure 2.4 can be written as

$$V_m^{\,4} - \alpha V_m^{\,2} + \beta = 0 \tag{2.17}$$

where

$$\alpha = V_k^{\,2} - 2P_m r_l - 2Q_m x_l$$
$$\beta = \left\{P_m^{\,2} + Q_m^{\,2}\right\}\left\{r_l^{\,2} + x_l^{\,2}\right\}$$

Using a realistic solution of the above load flow equation, the VSI (Chakravorty and Das 2001) can be obtained as

$$VSI_m = V_k^{\,4} - 4\{P_m x_l - Q_m r_l\}^2 - 4\{P_m r_l + Q_m x_l\}V_k^{\,2} \tag{2.18}$$

the minimum value of this indicator, close to zero, is indicative of the proximity to power flow divergence and found to be prone to voltage collapse, while the maximum value, close to one, indicates the most stable state.

2.4 PREVENTIVE MEASURES

There are two lines of defence against incidents, which jeopardize the VS of power systems:

The system security margins with respect to credible contingencies, i.e. incidents with a reasonable probability of occurrence, are analyzed and appropriate preventive actions taken to provide sufficient margins when needed.

The second approach is to implement automatic corrective actions, through system protection schemes to face the more severe, but less likely incidents.

The preventive security criteria usually require that the system remains stable after any credible contingency, without the help of corrective actions. The main reason is that these actions usually affect the system generation and/or load, which is acceptable only in the presence of severe disturbances. The most frequently used strategies are

- On load tap changer control
- Reactive power compensation
- Load shedding

2.4.1 On Load Tap Changer

It is well known that the on-load tap changer has a significant influence on VS. The secondary voltage of a transformer is usually maintained by the automatic on load tap changer (OLTC), even if the voltage of primary transmission system drops. If the load demand on the secondary of the transformer increases the current increases and the secondary voltage falls due to drop in the system as well as in the transformer itself. The OLTC then raises the tap position to keep the secondary voltage constant to restore the load under normal operating condition. However, if the load demand is excessively heavy, the action of OLTC to raise the secondary voltage may further increase the load demand and causes the secondary voltage to drop in which case the voltage may become unstable. This reverse action caused by OLTC may deteriorate the operating conditions in a heavily loaded system and results in voltage collapse. When the secondary voltage is pulled down by raising the tap position during voltage collapse, it may be required to reduce the tap position or lock the tap changer to reduce the demand of the load. When the system is found to be closer to the stability limits, one of the preventive measures is to block the action of OLTC in transformers.

2.4.2 Reactive Power Compensation

Series and shunt VAR compensation are used to modify the inherent electrical characteristics of the ac systems. The VAR support improves the stability of the system by increasing the maximum active power that can be transmitted. It also attempts to improve the system power factor, eliminate current harmonic components produced by large and fluctuating nonlinear industrial loads and maintain a substantially flat VP.

Traditionally, rotating synchronous condensers and fixed or mechanically switched capacitors or inductors are used for reactive power compensation. However, in recent years, static VAR compensators employing thyristor switched capacitors and thyristor controlled reactors to provide or absorb the required reactive power are in action. Besides the use of self-commutated PWM converters with an appropriate control scheme permits the implementation of static compensators capable of generating or absorbing reactive current components with a time response faster than the fundamental power network cycle. Based on the use of reliable high-speed powerful analytical tools, advanced control and microcomputer technologies, Flexible AC Transmission Systems, also known as FACTS introduce a new concept for the operation of power transmission systems. In these systems, the use of static VAR compensators with fast response times play an important role, facilitating the increase in the amount of apparent power transfer through an existing line, close to its thermal capacity, without compromising on its stability limits.

The inability of the system to supply reactive power is the main factor for initiating voltage collapse. The provision of VAR support at appropriate bus locations in right quantity at the right time inhibits the occurrence of voltage collapse in power systems.

2.4.3 Load Shedding

If all the other available options prove futile, then perhaps the operator may resort to the process of shedding a portion of load to avert voltage collapse. It is believed that LS is an economical solution to address the VS problem (Arnborg et al 1997). LS is a well established procedure to mitigate voltage collapse in power systems. Under voltage LS is a low cost, powerful counter measure to maintain VS. In order to benefit from this control action, the analysis requires the information of when, where and the amount of load is to be shed. The amount of LS is required to be minimal. It is important to manage the LS scheme in an appropriate fashion. Previous attempts to arrive at satisfactory settings for a LS scheme seem to rely heavily on simulation, usually a 'trial and error' approach. However efforts are on to identify a systematic methodology to pre-calculate the amount of load to be shed, with the view to maintain a flat VP and thus avoid voltage instability.

2.5 SUMMARY

An overview to VS, various definitions and phenomenon of VS have been discussed. The methods of assessing the VS of a system, suitable for on and off-line applications have been outlined. A few corrective measures to enhance the VS have also been briefed.