CHAPTER 4

INHOMOGENEOUS BIANCHI TYPE $VI_h$
STRING DUST COSMOLOGICAL MODEL
OF PERFECT FLUID DISTRIBUTION

4.1 INTRODUCTION

The early Universe is described by a model based on three hypothesis; homogeneity and isotropy, ordinary matter, and standard gravity. But Smoot et al. [1], Bennet et al. [2] and Spergel et al. [3] have emphasized that our Universe is neither exactly homogeneous nor isotropic and also there is not sufficient reason to believe that the behavior of its expansion was regular at early times. Thus, in order to understand the evolution and large scale structure of the Universe, it is required to consider a more general type of cosmologies obtained by removing the requirement of homogeneity and isotropy. Therefore, with the above considerations, there has been lot of interest in the research of anisotropic and inhomogeneous cosmologies.

Significant work has been done in obtaining various Bianchi type models and their inhomogeneous generalization. Stephani [4] has constructed a model which has an inhomogeneous generalization of FRW models. Wainwright
et al. [5] and Carmeli et al. [6] have obtained some exact solutions which generalize Bianchi type III, V and VI$_h$ models for vacuum as well as matter filled. Roy and Narain [7-8] have obtained solutions which generalize Bianchi type I, V and VI$_0$ models with perfect fluids. Roy and Prasad [9] have derived inhomogeneous generalization of Bianchi type VI$_h$ model with perfect fluid. Roy and Prasad [10-11] have derived inhomogeneous models filled with and without co-moving stiff fluid and radiation which generalize homogeneous models of Bianchi type VI$_h$.

The string theory is a useful concept before the creation of the particle in the Universe. The strings are nothing but the important topological stable defects due to the phase transition that occurs as the temperature lower below some critical temperature at the very early stages of the Universe. The present day configurations of the Universe are not contradicted by the large scale network of strings in the early Universe. Moreover, the galaxy formation can be explained by the density fluctuations of the vacuum strings. The general relativistic formalism of cosmic strings, are given by Letelier [12-13] and Stachel [14]. Krori et al. [15] have studied Letelier model in context of Bianchi type II, VI$_0$, VIII and XI space time filled with cloud of strings and have obtained a particular solution of the field equations. Shri Ram and Singh [16] have investigated Binachi type II string dust Universe. Bali et al. [17-21] have investigated Bianchi types I, V, IX string cosmological models in general relativity. Tripathi and Behra [22] have studied Bianchi type VI$_h$ string cloud cosmological models with bulk viscosity. Amirhashchi and Zainuddin [23] have obtained LRS Bianchi type II strings dust cosmological models for perfect fluids distribution in general relativity. H. Amirhashchi [24] has investigated string cosmology in Bianchi type-VI$_0$ dusty Universe with electromagnetic field. Tyagi et al. [25] have obtained the solutions of field equations for inhomogeneous Bianchi type VI$_0$ string dust cosmological model of perfect fluid distribution.
In this chapter we have investigated inhomogeneous Bianchi type $VI_h$ string dust cosmological model filled with perfect fluid in general relativity. To obtain the deterministic solution of Einstein’s field equations, we assume that string tension density $\lambda$ is equal to rest density $\rho$. Each of the cases $1+h = 0$ and $1+h \neq 0$ gives rise to families of Universe. Some of the models start with big bang and ultimately stops expanding. The various physical and geometrical features of the model are also discussed.

### 4.2 THE METRIC AND FIELD EQUATIONS

We consider the metric in the form

$$ds^2 = e^{2\alpha} (dx^2 - dt^2) + e^{\beta+\gamma+2x} dy^2 + e^{\beta-\gamma+2hx} dz^2, \quad \ldots \ (4.2.1)$$

where $\alpha = \alpha(x,t)$, $\beta = \beta(t)$ and $\gamma = \gamma(t)$, $h$ being constant.

The Universe described by the line element (4.2.1) is filled with co-moving string perfect fluid satisfying Einstein’s field equations

$$R_i^j - \frac{1}{2} Rg_i^j = -T_i^j = -(\rho v_i v^j - \lambda x_i x^j), \quad \ldots \ (4.2.2)$$

where $T_i^j$ is the energy-momentum tensor for a cloud of massive strings and perfect fluid distribution $v_i$ is unit flow vector and $x_i$ satisfy conditions

$$v_i v^i = -x_i x^i = -1, \quad \ldots \ (4.2.3)$$

and

$$v^j x_i = 0 \quad \ldots \ (4.2.4)$$

Here, $\rho$ is the rest energy of the cloud of strings with massive particles attached to them, $p$ is the pressure and $\lambda$ the density of tension that
characterizes the strings. The unit space-like vector $x^i$ represents the string direction in the cloud, i.e. the direction of anisotropy and the unit time-like vector $v^i$ describes the four-velocity vector of the matter satisfying the following conditions

$$g_{ij}v^iv^j = -1 \quad \ldots \ (4.2.5)$$

In a comoving coordinate system, we have

$$v^i = (0,0,0,e^{-\alpha}) \quad \ldots \ (4.2.6)$$

and choose $x^i$ parallel to x-axis so that

$$x^i = (e^{-\alpha},0,0,0) \quad \ldots \ (4.2.7)$$

The surviving components of mixed Ricci tensor $R^i_j$ and Ricci scalar $R$ are as follows:

$$R^1_1 = e^{-2\alpha} \left[ \alpha_{11} - \alpha_{44} - \alpha_4\beta_4 + 1 + h^2 - \alpha_1(1 + h) \right] \quad \ldots \ (4.2.8a)$$

$$R^2_2 = e^{-2\alpha} \left[ \beta_{44} + \gamma_{44} + \frac{1}{2} \beta_4(\beta_4 + \gamma_4) + 1 + h \right] \quad \ldots \ (4.2.8b)$$

$$R^3_3 = e^{-2\alpha} \left[ \beta_{44} - \gamma_{44} + \frac{1}{2} \beta_4(\beta_4 - \gamma_4) + h + h^2 \right] \quad \ldots \ (4.2.8c)$$

$$R^4_4 = e^{-2\alpha} \left[ \alpha_{11} - \alpha_{44} - \beta_{44} - \frac{1}{2}(\beta_4^2 - \gamma_4^2) + \alpha_4\beta_4 + \alpha_1(1 + h) \right] \quad \ldots \ (4.2.8d)$$

$$R^1_4 = e^{-2\alpha} \left[ \frac{1}{2}(1 + h)\beta_4 + \frac{1}{2}(1 - h)\gamma_4 - (1 + h)\alpha_4 - \alpha_1\beta_4 \right] \quad \ldots \ (4.2.8e)$$

$$R = R^1_1 + R^2_2 + R^3_3 + R^4_4$$
\[ e^{-2\alpha} \left[ 2\alpha_{11} - 2\alpha_{44} + \beta_{44} - \gamma_4^2 + 2(1 + h + h^2) \right] \] ... (4.2.9)

The non-vanishing components of Einstein’s tensor \( G_i^j \) are as follows

\[ G_1^1 = R_1^1 - \frac{R}{2} \]
\[ = e^{-2\alpha} \left[ \beta_{44} + \frac{3}{4} (\beta_4)^2 + \frac{1}{4} (\gamma_4)^2 - \alpha_4 \beta_4 - (1 + h) \alpha_1 - h \right] \] ... (4.2.10a)

\[ G_2^2 = R_2^2 - \frac{R}{2} \]
\[ = e^{-2\alpha} \left[ \alpha_{44} - \alpha_{11} + \frac{1}{2} (\beta_{44} - \gamma_4) + \frac{1}{4} (\beta_4 - \gamma_4)^2 - h^2 \right] \] ... (4.2.10b)

\[ G_3^3 = R_3^3 - \frac{R}{2} \]
\[ = e^{-2\alpha} \left[ \alpha_{44} - \alpha_{11} + \frac{1}{2} (\beta_{44} + \gamma_4) + \frac{1}{4} (\beta_4 + \gamma_4)^2 - 1 \right] \] ... (4.2.10c)

\[ G_4^4 = R_4^4 - \frac{R}{2} \]
\[ = e^{-2\alpha} \left[ \frac{1}{4} (\beta_4)^2 - (\gamma_4)^2 \right] \alpha_4 \beta_4 + (1 + h) \alpha_1 - (1 + h + h^2) \] ... (4.2.10d)

\[ G_4^1 = R_4^1 - \frac{R}{2} g_4^1 \]
\[ = R_4^1 \]
\[ = e^{-2\alpha} \left[ \frac{1}{2} (1 + h) \beta_4 + \frac{1}{2} (1 - h) \gamma_4 - (1 + h) \alpha_4 - \alpha_1 \beta_4 \right] \] ... (4.2.10e)
The energy momentum tensor for the line element (4.2.1) using (4.2.2) is obtained as

\[ T^1_\alpha = -\lambda \]  \quad \ldots (4.2.11a)

\[ T^2_\alpha = 0 \]  \quad \ldots (4.2.11b)

\[ T^3_\alpha = 0 \]  \quad \ldots (4.2.11c)

\[ T^4_\alpha = -\rho \]  \quad \ldots (4.2.11d)

The Einstein’s field equations (4.2.2) for the line-element (4.2.1) lead to the following system of equations:

\[ e^{-2\alpha} \left[ \beta_{44} + \frac{3}{4} (\beta_4)^2 + \frac{1}{4} (\gamma_4)^2 - \alpha_4 \beta_4 - (1+h)\alpha_1 - h \right] = \lambda \]  \quad \ldots (4.2.12)

\[ e^{-2\alpha} \left[ \alpha_{44} - \alpha_{11} + \frac{1}{2} (\beta_{44} - \gamma_{44}) + \frac{1}{4} (\beta_4 - \gamma_4)^2 - h^2 \right] = 0 \]  \quad \ldots (4.2.13)

\[ e^{-2\alpha} \left[ \alpha_{44} - \alpha_{11} + \frac{1}{2} (\beta_{44} + \gamma_{44}) + \frac{1}{4} (\beta_4 + \gamma_4)^2 - 1 \right] = 0 \]  \quad \ldots (4.2.14)

\[ e^{-2\alpha} \left[ \frac{1}{4} (\beta_4)^2 - (\gamma_4)^2 + \alpha_4 \beta_4 + (1+h)\alpha_1 - (1+h+h^2) \right] = \rho \]  \quad \ldots (4.2.15)

\[ \alpha_1 \beta_4 + (1+h)\alpha_4 - \frac{1}{2} (1+h)\beta_4 - \frac{1}{2} (1-h)\gamma_4 = 0 \]  \quad \ldots (4.2.16)

The suffices 1 to 4 after \( \alpha \), \( \beta \) and \( \gamma \) denote partial differentiation with respect to \( x \) and \( t \) respectively.
4.3 SOLUTION OF FIELD EQUATIONS

The field equations (4.2.12-4.2.16) are five equations in five unknowns \( \alpha, \beta, \gamma, \lambda \) and \( \rho \). We solve them in following ways:

If \( \lambda = \rho \) then equations (4.2.12), (4.2.13), (4.2.14) and (4.2.15) gives

\[
\alpha_{11} - \alpha_{44} - \alpha_4 \beta_4 - (1+h)\alpha_4 + (1+h^2) = 0
\]  

\[
\text{\ldots (4.3.1)}
\]

On subtracting equation (4.2.13) from (4.2.14), we get

\[
\gamma_{44} + \beta_4 \gamma_4 = (1-h^2)
\]  

\[
\text{\ldots (4.3.2)}
\]

**CASE-I:** \( 1+h \neq 0 \) and \( \beta_4 \neq 0 \) but any constant \( k \).

Equation (4.2.16) implies

\[
\alpha = \varphi + \frac{1}{2} \beta + \frac{1}{2} \frac{(1-h)}{(1+h)} \gamma
\]  

\[
\text{\ldots (4.3.3)}
\]

where \( \varphi = \varphi((1+h)x-\beta) \). Hence from the equations (4.3.1), (4.3.2) and (4.3.3), we get

\[
\varphi'' - \varphi' + \frac{1}{2} = 0
\]  

\[
\text{\ldots (4.3.4)}
\]

where a prime ('') denotes differentiation w.r.t. \((1+h)x-\beta\).

Equation (4.3.4) gives

\[
\varphi = n_1 + n_2 e^{((1+h)x-\beta)} + \frac{1}{2}((1+h)x-\beta)
\]  

\[
\text{\ldots (4.3.5)}
\]

where \( n_1 \) and \( n_2 \) are constants of integration.
Now $\beta_4 = k$ on integration we get

$$\beta = kt + k_1.$$ ... (4.3.6)

where $k_1$ is constant of integration.

And equation (4.3.2) gives on integration

$$\gamma = c_1 + c_2 e^{-kt} + \frac{1-h^2}{k} t$$ ... (4.3.7)

where $c_1$ and $c_2$ are constants of integration.

From equations (4.3.5) and (4.3.6), we get

$$\varphi = n_1 + n_2 e^{(1+h)x-(kt+k_1)} + \frac{1}{2} \{(1+h)x-(kt+k_1)\}$$ ... (4.3.8)

and from (4.3.3), (4.3.6), (4.3.7) and (4.3.8), we get

$$\alpha = L_4 e^{(1+h)x-kt} + L_2 e^{-kt} + L_3 t + \frac{1}{2} (1+h)x + L_4$$ ... (4.3.9)

where

$$L_1 = n_2 e^{-k}; \quad L_2 = \frac{1}{2} \left(\frac{1-h}{1+h}\right) c_2; \quad L_3 = \frac{1}{2} \left(\frac{1-h}{k}\right)^2 \quad \text{and} \quad L_4 = n_1 + \frac{1}{2} \left(\frac{1-h}{1+h}\right) c_1$$

are new constants.

So the line element (4.2.1) becomes in this case

$$ds^2 = \exp 2 \left\{ L_4 e^{(1+h)x-kt} + L_2 e^{-kt} + L_3 t + \frac{1}{2} (1+h)x + L_4 \right\} (dx^2 - dt^2)$$

$$+ \exp \{ c_2 e^{-kt} + (2L_3 + k)t + L_5 + 2x \} dy^2$$

$$+ \exp \{ -c_2 e^{-kt} + (k - 2L_3)t + L_6 + 2hx \} dz^2$$ ... (4.3.10)
where $L_5 = k + c_1$ and $L_6 = k - c_1$.

**CASE II:** $1 + h \neq 0$ and $\beta_4 = 0$

Equation (4.3.2) gives on integration

$$\gamma = 1 - h^2 \frac{t^2}{2} + \ell_1 t + \ell_2$$

… (4.3.11)

Now $\beta_4 = 0$ on integration we get

$$\beta = \ell$$

… (4.3.12)

and hence equation (4.2.16) gives

$$\alpha = 1 - h^2 \frac{t^2}{4} + \frac{1}{2} \left( 1 - h \right) \ell_1 t + L$$

… (4.3.13)

$L$ being arbitrary functions of $x$ only and $\ell$, $\ell_1$ and $\ell_2$ are constants of integration.

Equation (4.3.1) determines $L$ as

$$L = m_1 + m_2 e^{(1+h)x} + \frac{1}{2} (1 + h)x$$

… (4.3.14)

where $m_1$ and $m_2$ are constants of integration

Therefore

$$\alpha = (1 - h) \frac{t^2}{4} + \frac{1}{2} \left( \frac{1 - h}{1 + h} \right) \ell_1 t + m_1 + m_2 e^{(1+h)x} + \frac{1}{2} (1 + h)x$$

… (4.3.15)
So the line element (4.2.1) becomes in this case

\[
ds^2 = \exp 2 \left\{ (1-h)^2 \frac{t^2}{4} + \frac{1}{2} (1-h)^2 \left[ 3 + 2 + e^{(1+h)x} \right] + \frac{1}{2} (1+h)x \right\} (dx^2 - dt^2) \\
+ \exp \left\{ (1-h^2) \frac{t^2}{2} + \ell(t + 2x) \right\} dy^2 \\
+ \exp \left\{ -(1-h^2) \frac{t^2}{2} - \ell(t - 2x) + 2hx \right\} dz^2 \\
\]

… (4.3.16)

**CASE –III** \( 1 + h = 0 \) and \( \beta_4 \neq 0 \)

Equation (4.2.16) implies

\[
a = \frac{\gamma_4}{\beta_4} x + Q(t) \\
\]

… (4.3.17)

where \( Q \) is an arbitrary function of \( t \); whereas equation (4.3.2) implies

\[
\gamma_4 = s_1 e^{-\beta} \\
\]

… (4.3.18)

where \( s_1 \) is a constant of integration.

From equation (4.3.1), (4.3.17) and (4.3.18), we get

\[
x \left[ \frac{d^2}{dt^2} \left( \frac{\gamma_4}{\beta_4} \right) + \beta_4 \frac{d}{dt} \left( \frac{\gamma_4}{\beta_4} \right) \right] + \left[ \frac{d^2 Q}{dt^2} + \beta_4 \frac{dQ}{dt} - 2 \right] = 0 \\
\]

… (4.3.19)

This implies that

\[
\frac{d^2}{dt^2} \left( \frac{\gamma_4}{\beta_4} \right) + \beta_4 \frac{d}{dt} \left( \frac{\gamma_4}{\beta_4} \right) = 0 \\
\]

… (4.3.20)
From equations (4.3.18) and (4.3.20), we get

\[ \beta_4 = \frac{s_1}{a_2} e^{\left( \frac{1 + \alpha_1}{s_1} \right)^\beta} \] ...

and

\[ \exp \beta = \left[ \frac{s_1 + a_1}{a_2} t + a_4 \right]^{\left( \frac{s_1}{s_1 + a_1} \right)} \] ...

Again from (4.3.19) which implies that

\[ \frac{d^2 Q}{dt^2} + \beta_4 \frac{dQ}{dt} - 2 = 0 \] ...

Which implies that

\[ Q = \frac{2a_2}{2s_1 + a_1} \left[ \left( \frac{s_1 + a_1}{a_2} \right)^2 + a_4 t \right] + \frac{a_2 a_5}{a_1} \left[ \left( \frac{s_1 + a_1}{a_2} \right)^2 + a_4 t \right] + a_6 \] ...

From equations (4.3.17), (4.3.18), (4.3.21), (4.3.22) and (4.3.25), we get

\[ \alpha = \left( \frac{a_2 x + a_2 a_5}{a_1} \right) \left[ \left( \frac{s_1 + a_1}{a_2} \right)^2 + a_4 t + \frac{a_1}{s_1 + a_1} \right] + \frac{2a_2}{2s_1 + a_1} \left[ \left( \frac{s_1 + a_1}{a_2} \right)^2 + a_4 t \right] + a_6 \] ...

\[ \]
After using suitable coordinate transformations and renaming of constants, the line element (4.2.1) reduces to the form

\[ ds^2 = B \exp \left\{ 2(Cx + C')T \left( \frac{a_1}{s_1 + a_1} \right) + 2 \left( \frac{s_1 + a_1}{2s_1 + a_1} \right) T^2 \right\} (dx^2 - dT^2) \]

\[ + T^{s_1 + a_1} \exp \left\{ C' T^{s_1 + a_1} + 2x \right\} d\xi^2 \]

\[ + T^{s_1 + a_1} \exp \left\{ -C'' T^{s_1 + a_1} + 2hx \right\} d\eta^2 \]

... (4.3.27)

where

\[ B = \exp \left\{ a_6 - \frac{a_2^2 a_4^2}{(s_1 + a_1)(2s_1 + a_1)} \right\}, \quad C' = \frac{a_5 a_2}{a_1} \left( \frac{s_1 + a_1}{a_2} \right)^{a_1/s_1 + a_1} \]

and

\[ C'' = \frac{a_2 s_1}{a_1} \left( \frac{s_1 + a_1}{a_2} \right)^{a_1/s_1 + a_1} \]

are new constants.

### 4.4 PHYSICAL AND GEOMETRICAL PROPERTIES

The physical and geometrical properties of the model are given as follows:

**CASE-I**

Magnitude of rotation \( \omega \) is zero i.e.

\[ \omega = 0 \]

... (4.4.1)
The Expansion scalar $\theta$ of the model is given by

$$\theta = \frac{\left[ (-kL_1)e^{(1+h)x-kt} - kL_2e^{-kt} + L_3 + k \right]}{\exp\left[ L_4e^{(1+h)x-kt} + L_2e^{-kt} + L_3t + \frac{1}{2}(1+h)x + L_4 \right]} \quad \ldots (4.4.2)$$

The Shear $\sigma$ of the model is given by

$$\sigma = \frac{\left[ 2(-kL_1)e^{(1+h)x-kt} - kL_2e^{-kt} + L_3 - k^2 + 3\{-c_2ke^{-kt} + 2L_3\}^2 \right]^{\frac{1}{2}}}{2\sqrt{3}\exp\left[ L_4e^{(1+h)x-kt} + L_2e^{-kt} + L_3t + \frac{1}{2}(1+h)x + L_4 \right]} \quad \ldots (4.4.3)$$

String tension density $\lambda$ and rest density $\rho$ of the model are given by

$$\lambda = \rho = \frac{\left[ k^2 - (1+h)^2 \right]}{2\exp\left[ 2\left( L_4e^{(1+h)x-kt} + L_2e^{-kt} + L_3t + \frac{1}{2}(1+h)x + L_4 \right) \right]} \quad \ldots (4.4.4)$$

The deceleration parameter $q$ of the model is given by

$$q = -1 - 3e^{\alpha\left[ e^{kt}\left[ 2L_4e^{(1+h)x} + L_2 - \frac{L_3}{k}e^{-kt}\right] - \left( L_4e^{(1+h)x} + L_2 - \frac{L_3}{k}e^{-kt}\right)^2 \right]}
\left[ L_4e^{(1+h)x} + L_2 - \left( \frac{L_3}{k} - 1 \right)e^{-kt}\right]^2 \quad \ldots (4.4.5)$$

**CASE-II**

Magnitude of rotation $\omega$ is zero i.e.

$$\omega = 0 \quad \ldots (4.4.6)$$
The Expansion scalar $\theta$ of the model is given by

$$
\theta = \frac{(1-h)\frac{t^2}{2} + \frac{1}{2}\left(\frac{1-h}{1+h}\right)\ell_1}{\exp\left[(1-h)\frac{t^2}{4} + \frac{1}{2}\left(\frac{1-h}{1+h}\right)\ell_1 t + m_1 + m_2e^{(1+h)x} + \frac{1}{2}(1+h)x\right]} \quad \text{(4.4.7)}
$$

The Shear $\sigma$ of the model is given by

$$
\sigma = \frac{\left\{(1-h)\frac{t^2}{2} + \frac{1}{2}\left(\frac{1-h}{1+h}\right)\ell_1\right\}^2 + 3\left((1-h^2)t + \ell_1\right)^2}{2\sqrt{3}\exp\left[(1-h)\frac{t^2}{4} + \frac{1}{2}\left(\frac{1-h}{1+h}\right)\ell_1 t + m_1 + m_2e^{(1+h)x} + \frac{1}{2}(1+h)x\right]} \quad \text{(4.4.8)}
$$

String tension density $\lambda$ and rest density $\rho$ of the model are given by

$$
\lambda = \rho = \frac{-(1+h)^2}{2\exp\left[(1-h)\frac{t^2}{2} + \frac{1}{2}\left(\frac{1-h}{1+h}\right)\ell_1 t + 2m_1 + 2m_2e^{(1+h)x} + (1+h)x\right]} \quad \text{(4.4.9)}
$$

The deceleration parameter $q$ of the model is given by

$$
q = -1 - 3e^\alpha\left[\frac{2(1+h)^2}{\left\{(1-h^2)t + \ell_1\right\}^2 - 1}\right] \quad \text{(4.4.10)}
$$

**CASE-III**

Magnitude of rotation $\omega$ is zero i.e.

$$
\omega = 0 \quad \text{(4.4.11)}
$$
The Expansion scalar \( \theta \) of the model is given by

\[
\theta = \frac{(R_1 x + R_1')}{{R_1}^2} \left( \frac{s_1}{s_1 + a_1} \right) + 2R_2T + R_3T^{-1} \\
\sqrt{B} \exp \left\{ (C x + C')T\left( \frac{s_1}{s_1 + a_1} \right) + R_2T^2 \right\}
\]

where

\[
R_1 = a_1 \left( \frac{s_1 + a_1}{a_2} \right)^{s_1/s_1 + a_1}, \quad R'_1 = a_2 \left( \frac{s_1 + a_1}{a_2} \right)^{-s_1/s_1 + a_1}, \quad R_2 = \frac{s_1 + a_1}{2s_1 + a_1}
\]

and

\[
R_3 = \frac{s_1}{2(s_1 + a_1)}.
\]

The Shear \( \sigma \) of the model is given by

\[
\sigma = \frac{\left[ 2(R_1 x + R_1')T \left( \frac{s_1}{s_1 + a_1} \right) + 4R_2T - R_3T^{-1} \right]^2}{2(2s_1 + a_1)} + 3 \left( R_4T \left( \frac{s_1}{s_1 + a_1} \right) \right)^2 \frac{1}{2}
\]

\[
\sqrt{3} \sqrt{B} \exp \left\{ (C x + C')T\left( \frac{s_1}{s_1 + a_1} \right) + R_2T^2 \right\}
\]

\[
\ldots \ (4.4.13)
\]

String tension density \( \lambda \) and rest density \( \rho \) of the model are given by

\[
\lambda = \rho = \frac{2R_3 - 2R_3 - 1}{BT^2 \exp \left\{ 2(C x + C')T \left( \frac{s_1}{s_1 + a_1} \right) + 2R_2T^2 \right\}}
\]

\[
\ldots \ (4.4.14)
\]

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The deceleration parameter \( q \) of the model is given by

\[
q = -1 - 3e^{\alpha q} \left\{ \frac{(R_1 x + R_1') \left( \frac{-s_1}{s_1 + a_1} \right) T \left( \frac{2s_1 + a_1}{s_1 + a_1} \right) + 2R_2 - 2R_3 T^{-2}}{(R_1 x + R_1') T \left( \frac{s_1}{s_1 + a_1} \right) + 2R_2 T + 2R_3 T^{-1}} \right\}^2
\]

\[
- \frac{(R_1 x + R_1') T \left( \frac{s_1}{s_1 + a_1} \right) + 2R_2 T}{(R_1 x + R_1') T \left( \frac{s_1}{s_1 + a_1} \right) + 2R_2 T + 2R_3 T^{-1}} \}
\]  

\[\ldots (4.4.15)\]

4.5 CONCLUSION

We have investigated inhomogeneous Bianchi type VI \( h \) string dust cosmological model filled with perfect fluid. We have studied for (i) \(1 + h \neq 0\) and \( \beta_4 \neq 0 \) (but any constant \(k\)), (ii) \(1 + h \neq 0\), \( \beta_4 = 0 \) and (iii) \(1 + h = 0\), \( \beta_4 \neq 0 \).

In Case-I, the model (4.3.10) starts expanding at \( t = 0 \) and goes on expanding indefinitely when \( k > 0 \) and \( L_3 < 0 \). However, \( \theta \) becomes zero as \( t \to \infty \) when \( k > 0 \) and \( L_3 > 0 \). For large value of \( t \), the ratio of the shear \( \sigma \) and expansion \( \theta \) tends to a finite value i.e.

\[
\frac{\sigma}{\theta} = \left[ \frac{(2L_3 - k)^2 + 3L_5^2}{2\sqrt{3}(L_3 + k)} \right] \frac{1}{2} \text{ is a constant.}
\]
Hence, model does not approach isotropy for large value of $t$. The fluid flow is irrotational and it is observed that the energy density $\rho$ and string tension density $\lambda$ tends to constant values as $t \to 0$ and $x \to 0$. At a later stage both $\rho$ and $\lambda$ approach zero when $t \to \infty$ and $x \to \infty$ and $k < 0, 1 + h > 0$ as expected; therefore, the string will disappear from the Universe at later time. As $t \to 0$ and $x \to 0$ the deceleration parameter $q$ approaches to $-1$ when $L_1 = -L_2$ and $L_3 = 0$, as in De-Sitter Universe, therefore, the model presenting accelerating phase of the Universe. In general the model represents expanding, shearing, and non-rotating Universe.

In Case-II, the model (4.3.16) starts expanding at $t = 0$ and as $t \to \infty$, $\theta$ becomes zero i.e. the expansion stops. For large value of $t$ the ratio of the shear $\sigma$ and expansion $\theta$ tends to a finite value i.e.

$$
\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}} \left[ 1 + 3 \left( \frac{1 + h}{1 - h} \right)^2 \right] \text{ where } 1 - h \neq 0.
$$

Hence, model does not approach isotropy for large value of $t$. The fluid flow is irrotational and it is observed that the energy density $\rho$ and string tension density $\lambda$ tends to constant values as $t \to 0$ and $x \to 0$. At a later stage both $\rho$ and $\lambda$ approach zero when $t \to \infty$ and $x \to \infty$, and $-1 < h < 1$, as expected therefore the string will disappear from the Universe at later time. As $t \to 0$ and $x \to 0$, deceleration parameter $q$ approach to 0, when $m_1 = -m_2$ and $h = \pm \frac{\ell}{\sqrt{3}} - 1$; therefore, the Universe expands at a constant rate, $q$ approach to a value greater than zero; when $m_1 = -m_2$ and $\frac{1 + h^2}{\ell^2} < \frac{1}{3}$; therefore, the model presenting decelerating phase of the Universe, $q < -1$ when $m_1 = -m_2$ and $\frac{1 + h^2}{\ell^2} > \frac{1}{2}$; therefore, model representing super-exponential expansion.
of the Universe. \( q = -1 \) when \( m_i = -m_2 \) and \( h = \pm \frac{\ell}{\sqrt{2}} - 1 \); therefore, model representing exponential expansion of the Universe (also known as De-Sitter expansion). In general the model represents expanding, shearing and non-rotating Universe.

In Case-III, the model (4.3.27) starts with a big bang at \( T = 0 \) when 
\[-1 < \frac{s_i}{s_i + a_i} < 1\]
and goes on expanding till \( T = \infty \). When 
\[-1 < \frac{s_i}{s_i + a_i} < 1\]
and \( T \to \infty \), \( \theta \) becomes zero. It is clear that as \( T \) increases, the ratio of the shear \( \sigma \) and expansion \( \theta \) tends to finite value i.e.

\[
\frac{\sigma}{\theta} \to \frac{1}{\sqrt{3}} \quad \text{as} \quad T \to \infty \quad \text{when} \quad \frac{s_i}{s_i + a_i} > -1.
\]

Hence, model does not approach isotropy for large value of \( T \). The fluid flow is irrotational and it is observed that as \( T \to 0 \) and \( x \to 0 \), the energy density \( \rho \) and string tension \( \lambda \) both approach to \( \infty \) when \( \frac{a_i}{s_i + a_i} > 0 \) and as \( T \to \infty \) and \( x \to \infty \), \( \rho \) and \( \lambda \) both tends to 0 when \( \frac{a_i}{s_i + a_i} > 0 \). Therefore the string will disappear from the Universe at later time. As \( T \to 0 \) the deceleration parameter \( q < -1 \), when \( \frac{s_i}{s_i + a_i} < 0 \) therefore, model representing super exponential expansion of the Universe. In general the model represents expanding, shearing and non-rotating Universe.
REFERENCES


