INHOMOGENEOUS BIANCHI TYPE VI$_0$ STRING DUST COSMOLOGICAL MODEL OF PERFECT FLUID DISTRIBUTION IN GENERAL RELATIVITY

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Abstract: We have investigated inhomogeneous Bianchi type VI$_0$ string dust cosmological model in general relativity. To obtain the deterministic solution of Einstein’s field equations, we assume that string tension density $\lambda$ is equal to rest density $\rho$ i.e. $\lambda = \rho$. The model obtained is expanding, shearing and non-rotating universe. Some physical and geometrical features of the model are also discussed.

Keywords: Bianchi – VI$_0$ space times, cosmic string and general relativity.

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1. Introduction
In recent year, there has been lot of interest in string cosmology because cosmic strings play an important role in study of the early universe. Cosmic string may have been created during phase transitions in the early era [7] and they act as a source of gravitational field [10]. It is also believed that strings may be one of the sources of density perturbations that are required for the formation of large scale structures of the universe. So far a considerable amount of work has been done on cosmic strings and string cosmological models by Krori et al. [9, 8], Tikekar and Patel [16], Bali et al.[1]. Ellis and MacCallum [6] obtained solutions of Einstein's field equations for a Bianchi type VI$_0$ space-time in the case of a stiff-fluid. Collins [5] and Ruban [14] presented some
exact solutions of Bianchi type VI$_0$ for perfect fluid distributions satisfying specific equations of state.

Wainwright et al.[18] have obtained some exact solutions which generalize Bianchi type-III, V, and VI$_h$ models for vacuum and for stiff perfect fluid. Carmeli et al.[3] have constructed new inhomogeneous generalisations of Bianchi-type-III, V, and VI$_h$ models for vacuum and for the case in which mass less scalar fields present. They have also generalized certain Bianchi-type models making use of the description that inhomogeneity propagates in the form of pulses considered by Matzner and Centrella[4]. Roy and Narain[12] have derived solutions generalizing the Bianchi type-I and V models for perfect fluid distribution. Roy and Narain[13] have also derived inhomogeneous generalization of Bianchi type VI$_0$ cosmological model of perfect fluid distribution. Pradhan and Bali[11] have investigated Bianchi type VI$_0$ string cosmological models in presence and absence of magnetic field. Bali et al.[2] have investigated some LRS Bianchi Type VI$_0$ Cosmological Models with Special Free Gravitational Fields. Tyagi et al.[17] have obtained Bianchi type IX string cosmological models for perfect fluid distribution. Verma and Shri Ram[15] have investigated Bianchi-Type VI$_0$ Bulk Viscous Fluid Models with Variable Gravitational and Cosmological Constants.

In this paper, we have investigated inhomogeneous Bianchi type VI$_0$ string dust cosmological model in general relativity. For the complete deterministic solution of Einstein’s field equations, we assume that string tension density $\lambda$ is equal to rest density $\rho$. Some physical and geometrical features of the model are also discussed.

2. Solution of Field Equations

We consider inhomogeneous Bianchi type VI$_0$ metric in the form:

$$ds^2 = A(x, t)\{dt^2 - dx^2\} - e^{2x}B^2(t)dy^2 - e^{-2x}C^2(t)dz^2 \quad \ldots (1)$$

The Einstein’s field equation for a cloud string takes the form

$$R^i_j - \frac{1}{2}Rg^i_j = -T^i_j = -\left(\rho v_i v^j - \lambda x_i x^j\right) \quad \ldots (2)$$

as given by Letelier [10].

Where $v_i$ is unit flow vector and $x_i$ satisfy conditions

$$v^iv_i = -x^ix_i = 1, \text{ and } v^iv_i = 0$$

Here, $\rho$ is the rest energy of the cloud of strings with massive particles attached to them. $\rho = \rho_p + \lambda$, $\rho_p$ being the rest energy density of particles attached to the strings and $\lambda$ the density of tension that characterizes the strings. The unit space-like vector $x^i$ represents the string direction in the cloud, i.e. the direction of anisotropy and the unit time-like vector $v^i$ describes the four-velocity vector of the matter satisfying the following conditions
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\[ g_{ij} v^i v^j = 1. \]

In the present scenario, the commoving coordinates are taken as

\[ v^i = \left(0,0,0, \frac{1}{A}\right) \]

and choose \( x^i \) parallel to \( x \)-axis so that

\[ x^i = \left(\frac{1}{A},0,0,0\right) \]

The Einstein’s field equations (2.2) for the line-element (2.1) lead to the following system of equations:

\[
\frac{1}{A^2} \left[ \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4}{A} \left( \frac{B_4}{B} + \frac{C_4}{C} \right) + \frac{B_4 C_4}{BC} + 1 \right] = \lambda \quad \text{.... (3)}
\]

\[
\left( \frac{A_4}{A} \right)_4 - \left( \frac{A_1}{A} \right)_1 + \frac{B_{44}}{B} - 1 = 0 \quad \text{.... (4)}
\]

\[
\left( \frac{A_4}{A} \right)_4 - \left( \frac{A_1}{A} \right)_1 + \frac{C_{44}}{C} - 1 = 0 \quad \text{....(5)}
\]

\[
\frac{1}{A^2} \left[ 1 - \frac{B_4 C_4}{BC} - \frac{A_4}{A} \left( \frac{B_4}{B} + \frac{C_4}{C} \right) \right] = -\rho \quad \text{....(6)}
\]

\[
\left( \frac{B_4}{B} - \frac{C_4}{C} \right) - \frac{A_1}{A} \left( \frac{B_4}{B} + \frac{C_4}{C} \right) = 0 \quad \text{.... (7)}
\]

The suffices 1 to 4 after \( A, B \) and \( C \) denote partial differentiation with respect to \( x \) and \( t \) respectively.

From equation (4) and (5) we have

\[
\frac{B_{44}}{B} = \frac{C_{44}}{C} \quad \text{.... (8)}
\]

Equation (7) leads to
\[ A_1 = \left( \frac{B_4 - C_4}{B + C} \right) \]
\[ A = \left( \frac{B_4 + C_4}{B + C} \right) \]

which leads to

\[ \log A = x F(t) + G(t) \]

\[ \text{Where } G(t) \text{ is a function of the time.} \]

From equation (8)

\[ CB_4 - BC_4 = b \]

\[ \text{Where } b \text{ is a constant.} \]

Equations (9) and (11) give

\[ F = \frac{b}{(BC)_4} \]

From equations (3), (4), (6), (10), and (11) and string tension density \( \lambda \) is equal to rest density \( \rho \) i.e. \( \lambda = \rho \), we have

\[ x \left[ F_{44} + F_4 \left( \frac{B_4}{B} + \frac{C_4}{C} \right) \right] + \left[ G_{44} + G_4 \left( \frac{B_4}{B} + \frac{C_4}{C} \right) - 2 \right] = 0 \]

from which we conclude that

\[ F_{44} + F_4 \left( \frac{B_4}{B} + \frac{C_4}{C} \right) = 0 \]

and

\[ G_{44} + G_4 \left( \frac{B_4}{B} + \frac{C_4}{C} \right) - 2 = 0 \]

Equation (14) on integration yields

\[ F_4 = \frac{a}{BC} \]

\[ \text{Where } a \text{ is a constant.} \]
Inhomogeneous Bianchi Type VI

From equations (12) and (16), we get

\[ F = \beta [BC]^b \]  

\[ \text{.... (17)} \]

Where \( \beta \) is a constant.

From equations (12) and (17), we get

\[ BC = LT^{a+b} \]  

\[ \text{.... (18)} \]

Where \( L = \left( \frac{a+b}{\beta} \right)^{b} \), \( T = t + t_0 \), \( t_0 \) being a constant.

Equations (11) and (18) give

\[ \frac{B}{C} = \alpha \exp \left[ \frac{b(a+b)}{aL} \right] \]  

\[ \text{.... (19)} \]

Where \( \alpha \) is a constant.

Equation (15) on integration yields

\[ G = \frac{a+b}{a+2b} T^2 + \frac{l(a+b)}{aL} T^{a+b} + k \]  

\[ \text{.... (20)} \]

Where \( l \) and \( k \) are integration constant.

From equations (10), (17), (18) and (20), we have

\[ A = \exp \left[ (Xm+S)T^{l-n} + \frac{1}{n+1} T^2 + k \right] \]  

\[ \text{.... (21)} \]

Where

\[ X = x, \ m = \beta L^{\frac{a}{b}}, S = \frac{l(a+b)}{aL}, \ n = \frac{b}{a+b}. \]

From equations (18) and (19), we have
\[ B = (\alpha L)^{1/2} T^{n/2} \exp \left( \frac{b}{2L(1-n)} T^{1-n} \right) \] \hspace{1cm} \text{.... (22)}

and

\[ C = \left( \frac{L}{\alpha} \right)^{1/2} T^{n/2} \exp \left( -\frac{b}{2L(1-n)} T^{1-n} \right) \] \hspace{1cm} \text{.... (23)}

By suitable transformation of coordinates and remaining constants the line element (1) reduces to the form

\[ ds^2 = \exp \left[ 2 \left\{ (Xm + S)T^{1-n} + \frac{1}{n+1} T^2 + k \right\} \right] \left[ dT^2 - dX^2 \right] \]

\[ -T^n \exp \left( 2X + \frac{b}{L(1-n)} T^{1-n} \right) dY^2 \]

\[ -T^n \exp \left\{ -\left( 2X + \frac{b}{L(1-n)} T^{1-n} \right) \right\} dZ^2 \] \hspace{1cm} \text{.... (24)}

Which may be considered as an inhomogeneous generalization of Bianchi type-VI_0 string cosmological model of perfect fluid distribution.

3. Some Geometrical and Physical Properties of the Model

The physical and geometrical properties of the model are given as follows.

String tension density \( \lambda \) and rest density \( \rho \) are

\[ \rho = \frac{\left( \frac{n-1}{n+1} + \frac{n^2}{4T^2} + \frac{(Xm + S)n(1-n)}{T^{n+1}} - \frac{b^2}{4L^2T^{2n}} \right) \exp \left[ 2 \left\{ (Xm + S)T^{1-n} + \frac{1}{n+1} T^2 + k \right\} \right]}{\frac{n-1}{n+1} + \frac{n^2}{4T^2} + \frac{(Xm + S)n(1-n)}{T^{n+1}} - \frac{b^2}{4L^2T^{2n}}} = \lambda \] \hspace{1cm} \text{.... (25)}

Magnitude of rotation \( \omega \) is zero i.e.

\[ \omega = 0 \] \hspace{1cm} \text{.... (26)}

Scalar expansion (\( \theta \)) is given by
\[ \theta = \frac{\frac{(Xm + S)(1 - n)}{T^n} + \frac{2T}{n + 1} + \frac{n}{T}}{\exp\left[\frac{(Xm + S)T^{1-n} + \frac{1}{n + 1}T^2 + k}{T^n}\right]} \] 

Shear \((\sigma)\) is given by

\[ \sigma = \frac{\left[\frac{4T^2}{(n+1)^2} + \left(\frac{n-1}{Xm+S}\right) + \frac{3b^2}{4L^2}\right] \cdot \frac{1}{T^{2n}}}{\sqrt{3} \exp\left[\frac{(Xm + S)T^{1-n} + \frac{1}{n + 1}T^2 + k}{T^n}\right]} \]

\[ -\frac{4(n-1)(Xm + S)}{(n+1)T^{n-1}} + \frac{n(n-1)(Xm + S)}{T^{n+1}} + \frac{n^2}{4T^2} - \frac{2n}{n+1} \]^\frac{1}{2} \]

Deceleration parameter \((q)\) is given by

\[ q = -1 + \frac{3\left(\frac{2n(1-n)(Xm + S)}{T^{n+1}} + \frac{2(n-1)}{n + 1} + \frac{n}{T^2} + \frac{(n-1)^2(Xm + S)^2}{T^{2n}}\right)}{\left[(Xm + S)(1-n)T^{-n} + \frac{2T}{n + 1} + \frac{n^2}{T}\right]} \]

\[ + \frac{4T^2}{(n+1)^2} + \frac{4(1-n)(Xm + S)}{T^{n-1}} \right] \exp\left\{(Xm + S)T^{1-n} + \frac{1}{n + 1}T^2 + k\right\} \]

\[ \frac{1}{\sqrt{3}} \]

4. Conclusion

The model (24) starts with a big bang at \(T = 0\) and goes on expanding till \(T = \infty\) when \(\theta\) becomes zero. It is clear that as \(T\) increases, the ratio of the shear scalar \(\sigma\) and expansion \(\theta\) tends to finite value i.e.

\[ \frac{\sigma}{\theta} \rightarrow \frac{1}{\sqrt{3}} \text{ as } T \rightarrow \infty \]
Hence model does not approach isotropy for large value of T. However, we can make $\frac{\sigma}{\theta}$ less than any small arbitrary positive number by suitably choosing constants appearing in the models and for a finite period of time the models will be approximately FRW. Since the sign of the deceleration parameter $q$ is positive for $n = 1$, $T = 0$ and $X = -\left(\frac{S + k}{m}\right)$ that yields the decelerating phase of the universe. It is observed that for sufficiently large time $T$, $\lambda$ tends to zero. Therefore, the strings disappear from the Universe at a later time (i.e. present epoch). In general the model represents expanding, shearing and non-rotating universe.

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**References**


Inhomogeneous Bianchi type-VI_{0} String Cosmological Model for Stiff Perfect Fluid Distribution in General Relativity

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Abstract
We have investigated inhomogeneous Bianchi type VI\(_{0}\) string cosmological model for stiff perfect fluid in general relativity. To obtain the deterministic solution of Einstein’s field equations, we assume that the isotropic pressure is equal to string rest density i.e. \(p = \rho\). The model obtained is expanding, shearing and non-rotating universe. Some physical and geometrical features of the model are also discussed.

Keywords: Bianchi –VI\(_{0}\), space-time, cosmic string, stiff fluid, General Relativity.

1. INTRODUCTION

Inhomogeneous generalizations of the Friedmann-Lemaitre–Robertson–Walker (FLRW) cosmological models have gained interest in the astrophysical community and are mostly applied to study cosmological phenomena. However, in many papers the inhomogeneous cosmological models are treated as an alternative to the FLRW models. In fact, they are not an alternative, but an exact perturbation of the latter, and are gradually becoming a necessity in modern cosmology. The assumption of homogeneity is just a first approximation introduced to simplify equations. So far this assumption is commonly believed to have worked well, but future and more precise observations will not be properly analysed unless inhomogeneities are taken into account.

The important part of the present research is devoted to investigation of inhomogeneous cosmological model which generalize Bianchi type models. Considerable work has been done in obtaining various Bianchi type models and their inhomogeneous generalization. Stephani [1] has constructed a model which an inhomogeneous generalization of FRW models. Wainwright \textit{et al.} [2] and Carmeli \textit{et al.} [3,4] have obtained some exact solutions which generalize Bianchi type III, V and VI\(_{h}\) models for vacuum as well as matter filled. Roy and Narain [5, 6] have obtained solutions which generalize Bianchi type I, V and VI\(_{0}\) models with perfect fluids.

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Stiff fluid cosmological models create more interest in the study of early universe because for these models velocity of sound is equal to the velocity of light so no material in this universe could be more stiff such stiffness is comprehensible at the very high densities just after the big bang. The equation of state \( \rho = p \) was first proposed by Zel'dovich [7] in the study of early universe. As importance of stiff fluid models, the cosmological models for stiff fluid distribution are also investigated by many authors, Barrow [8] Mak and Harko [9, 10] and Bali et al. [11,12] in different contexts.

String cosmology as cosmic strings play an important role in the study of the early universe. These strings appear after the big bang the universe may have experienced a number of phase transitions and its temperature lowered down. During the phase transition the symmetry of the universe is broken spontaneously. These phase transitions can produce vacuum domain structures such as domain walls, string and monopole Kibble [13].The existence of large scale network of strings in the early universe is not in contradicion with the present day observations of the universe. Moreover, galaxy formation can be explained by string theory Zel’dovich [14]. Letelier [15] has solved Einstein’s field equations for a cloud of massive strings and obtained cosmological models in Bianchi-I Kantowski-Sachs space times.

String cosmological models are also discussed by numbers of authors Banerjee et al. [16] Tikekarand Patel [17, 18], Roy and Banerjee [19], Patel and Maharaj [20], Bali et al. [21-25] in different context. Pradhan and Suman [26] have investigated Magnetized Bianchi type VI\(_0\) bulk viscous barotropic massive string universe with decaying vacuum energy density \( \Lambda \). Amirshchi [27] has derived String cosmology in Bianchi type-VI\(_0\) dusty Universe with electromagnetic field. Verma and Shri Ram [28] have investigated Bianchi-Type VI\(_0\) Bulk Viscous Fluid Models with Variable Gravitational and Cosmological Constants. Tyagi et al. [29] have obtained the solutions of field equations for inhomogeneous Bianchi type VI\(_0\) string dust cosmological model of perfect fluid distribution.

In this paper, we have investigated inhomogeneous Bianchi type VI\(_0\) string cosmological model for stiff perfect fluid in general relativity. For the complete deterministic solution of the Einstein’s field equations, we assume that the isotropic pressure \( p \) is equal to string rest density \( \rho \). The various physical and geometrical aspects of the models are also discussed.

2. SOLUTION OF THE FIELD EQUATIONS

We consider generalizes Bianchi type VI\(_0\) metric in the form:

\[
ds^2 = A^2(x,t) \left( dt^2 - dx^2 \right) - e^{2x} B^2(t) dy^2 - e^{-2x} C^2(t) dz^2,
\]

(1)

The energy momentum tensor for perfect fluid distribution in the presence of massive string proposed by Letelier [30] is taken in the form

\[
T_{ij}' = (\rho + p)\delta_{ij} - pg_{ij} - \lambda x_i x_j
\]

(2)
With $\rho = \rho_p + \lambda$ and $v_i$ and $x_i$ satisfy conditions

$$v_i v_i = -x_i x_i = 1, \text{ and } v^i x_i = 0$$

Here, $p$ denotes isotropic fluid pressure; $\rho$ denotes proper energy density and $\lambda$ the string tension density, $\rho_p$ enters into the stress energy tensor as simply an additional dust component.

The unit space-like vector $x^i$ represents the string direction in the cloud, i.e. the direction of anisotropy and the unit time-like vector $v^i$ describes the four-velocity vector of the matter satisfying the following conditions

$$g_{ij} v^i v^j = 1.$$  

We assume that the coordinates system is co-moving and so that

$$v^i = \left(0, 0, 0, \frac{1}{A}\right), \text{ and } x^i = \left(\frac{1}{A}, 0, 0, 0\right)$$

The Einstein’s field equations in the geometrized unit ($c=1, 8\pi G=1$)

$$R^i_j - \frac{1}{2} R g^i_j = -T^i_j$$  

(3)

The Einstein’s field equations (3) for the line-element (1) lead to the following system of equations:

$$\frac{1}{A^2} \left[ \frac{B_{44} + C_{44}}{B} - \frac{A^4}{A} \left( \frac{B_{44}}{B} + \frac{C_{44}}{C} \right) + \frac{B C_{44}}{B C} + 1 \right] = -p + \lambda$$  

(4)

$$\frac{1}{A^2} \left[ \frac{A^4}{A} \right] - \left( \frac{A^4}{A} \right) + \frac{B_{44}}{B} - 1 = -p$$  

(5)

$$\frac{1}{A^2} \left[ \frac{A^4}{A} \right] - \left( \frac{A^4}{A} \right) + \frac{C_{44}}{C} - 1 = -p$$  

(6)

$$\frac{1}{A^2} \left[ \frac{1}{BC} - \frac{B C_{44}}{A} - \frac{A^4}{A} \left( \frac{B_{44}}{B} + \frac{C_{44}}{C} \right) \right] = -p$$  

(7)

$$\left( \frac{B_{44}}{B} - \frac{C_{44}}{C} \right) - \frac{A^4}{A} \left( \frac{B_{44}}{B} + \frac{C_{44}}{C} \right) = 0$$  

(8)

The suffices 1 to 4 after $A$, $B$ and $C$ denote partial differentiation with respect to $x$ and $t$ respectively.

From equation (5) and (6) we have

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} = 0$$  

(9)

On integration of equation (9), we get
\[
\frac{B_4 - C_4}{B - C} = \frac{b}{BC}
\]  
\text{(10)}

where \(b\) is integration constant.

Equation (8) leads to

\[
\frac{A_4}{A} = \left(\frac{B_4 - C_4}{B - C}\right) = \gamma(t)
\]  
\text{(11)}

where \(\gamma(t)\) is some function of time \(t\) alone. Integrating equation (11), we have

\[
\log A = x\gamma(t) + \eta(t)
\]  
\text{(12)}

where \(\eta(t)\) is a function of the time.

Equation (10) and (11) give

\[
\gamma = \frac{b}{(BC)_4}
\]  
\text{(13)}

For stiff fluid \(p = \rho \Rightarrow p - \rho = 0\)

From equations (6) and (7), we get

\[
\left(\frac{A_4}{A}\right)_4 - \left(\frac{A_4}{A}\right)_1 + \frac{C_{44}}{C} - 2 + \frac{B_4}{BC} + A_4 \left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 0
\]  
\text{(14)}

From equations (11), (12) and (14), we have

\[
x\left[\gamma_4 + \gamma_4 \left(\frac{B_4}{B} + \frac{C_4}{C}\right)\right] + \left[\eta_4 + \eta_4 \left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{B_4}{BC} + \frac{C_4}{C} - 2\right] = 0
\]  
\text{(15)}

From which we conclude that

\[
\gamma_4 + \gamma_4 \left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 0
\]  
\text{(16)}

and

\[
\eta_4 + \eta_4 \left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{B_4}{BC} + \frac{C_4}{C} - 2 = 0
\]  
\text{(17)}

Equation (16) on integration yields

\[
\gamma_4 = \frac{a}{BC}
\]  
\text{(18)}

where \(a\) is a constant.
From equations (13) and (18), we get
\[ \gamma = \beta [BC]^{\frac{a}{b}} \]  
\[ (19) \]
where \( \beta \) is a constant.

From equations (13) and (19), we get
\[ BC = LT^{\frac{b}{av+b}} \]  
\[ (20) \]
where \( L = \left\{ \frac{a+b}{\beta} \right\}, \) \( T = t + t_0, \) \( t_0 \) being a constant.

Equations (10) and (20) give
\[ \frac{B}{C} = \alpha \exp \left[ \frac{b(a+b)}{aL} \right] \]  
\[ (21) \]
where \( \alpha \) is a constant.

Equations (17) on integration yields
\[ \eta = \frac{a+b}{a+2b} T^2 + \frac{\ell(a+b)}{aL} T^{\frac{a}{av+b}} + \ell_1 - \log C \]  
\[ (22) \]
where \( \ell \) and \( \ell_1 \) are integration constant.

From equations (20) and (21), we get
\[ \log C = -\frac{b(a+b)}{2aL} T^{\frac{a}{av+b}} + \frac{b}{2(a+b)} \log T - \frac{1}{2} \log \frac{\alpha}{L} \]  
\[ (23) \]
From equation (22) and (23), we have
\[ \eta = \frac{a+b}{a+2b} T^2 + \left( \frac{\ell(a+b)}{aL} + \frac{b(a+b)}{2aL} \right) T^{\frac{a}{av+b}} - \frac{b}{2(a+b)} \log T + \ell_2 \]  
\[ (24) \]
where \( \ell_2 = \frac{1}{2} \log \frac{\alpha}{L} + \ell_1 \) is a new constant.

From equations (12), (19) and (24), we get
\[ A = T^{\frac{n}{2}} \exp \left[ (Xm+K)T^{\frac{2}{n+1}} + \frac{T^2}{n+1} + \ell_2 \right] \]  
\[ (25) \]
where \( X = x, \) \( m = \beta L^\frac{a}{b}, \) \( K = \frac{(b+2a\ell)(a+b)}{2aL} \) and \( n = \frac{b}{a+b} \).

From equations (18) and (19), we have
\[ B = (\alpha L)^{1/2} T^{n/2} \exp \left( \frac{b}{2L(1-n)} T^{1-n} \right), \quad n \neq 1 \]  
(26)

and

\[ C = \left( \frac{L}{\alpha} \right)^{1/2} T^{n/2} \exp \left( -\frac{b}{2L(1-n)} T^{1-n} \right), \quad n \neq 1 \]  
(27)

By suitable transformation of coordinates and remaining constants the line element (1) reduces to the form

\[ ds^2 = T^{-n} \exp \left[ 2 \left( (Xm+S)T^{1-n} + \frac{1}{n+1} T^2 + \ell_2^2 \right) \right] \left( dT^2 - dX^2 \right) - T^n \exp \left( 2X + \frac{b}{L(1-n)} T^{1-n} \right) dY^2 \]
\[ - T^n \exp \left[ - \left( 2X + \frac{b}{L(1-n)} T^{1-n} \right) \right] dZ^2 \]  
(28)

which may be considered as an inhomogeneous Bianchi type-\text{VI}_0 string cosmological model for stiff perfect fluid distribution.

### 3. SOME PHYSICAL AND GEOMETRICAL PROPERTIES

The physical and geometrical properties of the model are given as follows.

Pressure \( p \) and rest density \( \rho \) of the model are given by

\[
p = \frac{\left( (Xm+K)n(1-n) + n-1 \frac{n^2}{n+1} + \frac{b^2}{4L^2T^{2n}} \right)}{T^{n+1}} \quad \rho = \frac{T^{-n} \exp \left[ 2 \left( (Xm+K)T^{1-n} + \frac{1}{n+1} T^2 + \ell_2^2 \right) \right]}{T^n \exp \left[ - \left( 2X + \frac{b}{L(1-n)} T^{1-n} \right) \right]} \]
(29)

Magnitude of rotation \( \omega \) is zero i.e.

\[
\omega = 0
\]
(30)

The Expansion scalar \( \theta \) of the model is given by

\[
\theta = \frac{(Xm+K)(1-n)}{T^n} \quad + \frac{2T}{n+1} + \frac{n}{2T} \quad T^{-n} \exp \left[ (Xm+K)T^{1-n} + \frac{1}{n+1} T^2 + \ell_2^2 \right]
\]
(31)

The Shear \( \sigma \) of the model is given by
\[
\sigma = \frac{4T^2}{(n+1)^2} + \left( (n-1)^2(Xm + K)^2 + \frac{3b^2}{4L^2} \right) \frac{1}{T^{2n}} - \frac{4(n-1)(Xm + K)}{(n+1)T^{n+1}} - \frac{2(Xm + K)n(n-1)}{T^{2n+1}} + \frac{2n^2}{T^2} - \frac{4n}{n+1} \right]^{\frac{1}{2}}
\]

\[
\sqrt{3T^{n/2}} \exp \left[ (Xm + K)T^{1-n} + \frac{1}{n+1}T^2 + \ell_2 \right]
\]

(32)

String tension density \( \lambda \) of the model is given by

\[
\lambda = \frac{n(n-1)T^{n-2}}{\exp \left[ 2 \left\{ (Xm + K)T^{1-n} + \frac{1}{n+1}T^2 + \ell_2 \right\} \right]}
\]

(33)

The deceleration parameter \( q \) of the model is given by

\[
q = -1 + \frac{3}{T^2} \left[ \frac{(Xm + K)(n-1)}{T^{n+1}} - \frac{2(n-1)}{(n+1)} - \frac{3n^2}{4T^2} \frac{(n-1)^2(Xm + K)^2}{T^{2n}} - \frac{4T^2}{(n+1)^2} - \frac{4(1-n)(Xm + K)}{(n+1)T^{n+1}} \right]^{\frac{1}{2}}
\]

\[
\exp \left[ - \left\{ (Xm + K)T^{1-n} + \frac{1}{n+1}T^2 + \ell_2 \right\} \right]
\]

(34)

4. CONCLUSIONS

The model (28) starts with a big bang at \( T = 0 \) when \( 0 < n < 1 \) and goes on expanding till \( T \to \infty \), \( \theta \) becomes zero when \(-1 < n < 1\). It is clear that as \( T \) increases, the ratio of the shear \( \sigma \) and expansion \( \theta \) tends to finite value i.e. \( \frac{\sigma}{\theta} \to \frac{1}{\sqrt{3}} \) as \( T \to \infty \). Hence model does not approach isotropy for large value of \( T \). The fluid flow is irrotational and it is observed that as \( T \to 0 \) and \( X \to 0 \), the energy density \( \rho \) \( \to \infty \) when \( n < -2 \) and as \( T \to \infty \) and \( X \to \infty \), \( \rho \to 0 \). As \( T \to 0 \) and \( X \to 0 \) string tension density \( \lambda \) \( \to \infty \) when \( n < 1 \) and as \( T \to \infty \) & \( X \to \infty \), \( \lambda \to 0 \) when \( n < 1 \) therefore the string will be disappear from the universe at later time. As \( T \to 0 \) the deceleration parameter \( q \) approaches to \(-1\) when \( n < -4 \) as in De-Sitter universe. The model represents accelerating universe. In general the model represents expanding, shearing and non-rotating universe.

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References

Inhomogeneous Bianchi Type VIh String Dust Cosmological Model of Perfect Fluid Distribution in General Relativity

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Abstract: Inhomogeneous Bianchi type VIh string dust cosmological model filled with prefect fluid in general relativity is investigated. To obtain the deterministic solution of Einstein field equations, we assume that string tension density \( \lambda \) is equal to rest density \( \rho \) i.e. \( \lambda = \rho \). The various physical and geometrical features of the model are also discussed.

Keywords: Bianchi –VIh space times, cosmic string and general relativity.

INTRODUCTION

The early universe is described by a model based on three hypothesis homogeneity and isotropy, ordinary matter and standard gravity. But Smoot et al. Bennet et al.2 and Spergel et al.3 have emphasized that our universe is neither exactly homogeneous nor isotropic and also there is not sufficient reason to believe that the behavior of its expansion was regular at early times. Thus, in order to understand the evolution and large scale structure of the universe, it is required to consider a more general type of cosmologies obtained by removing the requirement of homogeneity and isotropy. Therefore with the above considerations, there has been lot of interest in the research of anisotropic and inhomogeneous cosmologies.
Significant work has been done in obtaining various Bianchi type models and their inhomogeneous generalization. Stephani\textsuperscript{4} has constructed a model which an inhomogeneous generalization of FRW models. Wainwright \textit{et al.} \textsuperscript{5} and Carmeli \textit{et al.}\textsuperscript{6} have obtained some exact solutions which generalize Bianchi type III, V and VI\textsubscript{h} models for vacuum as well as matter filled. Roy and Narain\textsuperscript{7,8} have obtained solutions which generalize Bianchi type I, V and VI\textsubscript{0} models with perfect fluids. Roy and Prasad\textsuperscript{9} have derived inhomogeneous generalization of Bianchi type VI\textsubscript{0} model with perfect fluid. Roy and Prasad\textsuperscript{10,11} have derived inhomogeneous models filled with and without co-moving stiff fluid and radiation which generalize homogeneous models of Bianchi type VI\textsubscript{h}.

The string theory is a useful concept before the creation of the particle in the universe. The string are nothing but the important topological stable defects due to the phase transition that occurs as the temperature lower below some critical temperature at the very early stages of the universe. The present day configurations of the universe are not contradicted by the large scale network of strings in the early universe. Moreover, the galaxy formation can be explained by the density fluctuations of the vacuum strings. The general relativistic formalism of cosmic strings, are given by Letelier\textsuperscript{12,13} and Stachel\textsuperscript{14}. Krori \textit{et al.}\textsuperscript{15} have studied Letelier model in context of Bianchi type II, VI\textsubscript{0}, VIII and XI space time filled with cloud of strings and have obtained a particular solution of the field equations. Shri Ram and Singh\textsuperscript{16} have investigated Binachi type II string dust universe. Bali \textit{et al.}\textsuperscript{17-21} have investigated Bianchi Types I, V, IX string cosmological models in General Relativity. Tripathi and Behra\textsuperscript{22} have studied Bianchi type VI\textsubscript{h} string cloud cosmological models with bulk viscosity. Amirhashchi and Zainuddin\textsuperscript{23} have obtained LRS Bianchi type II strings dust cosmological models for perfect fluids distribution in general relativity. H. Amirhashchi\textsuperscript{24} have investigated String cosmology in Bianchi type-VI\textsubscript{0} dusty Universe with electromagnetic field. Tyagi \textit{et al.}\textsuperscript{25} have obtained the solutions of field equations for inhomogeneous Bianchi type VI\textsubscript{0} string dust cosmological model of perfect fluid distribution.

In this paper we have investigated inhomogeneous Bianchi type VI\textsubscript{h} string dust cosmological model filled with prefect fluid in general relativity. To obtain the deterministic solution of Einstein’s field equations, we assume that string tension density $\lambda$ is equal to rest density $\rho$. Each of the cases $1+h = 0$ and $1+h \neq 0$ gives rise to families of universe. Some of the models start with big bang and ultimately stop expanding. The various physical and geometrical features of the model are also discussed.

**SOLUTION OF FIELD EQUATIONS**

We consider the metric in the form

$$ds^2 = e^{2\alpha} (dx^2 - dt^2) + e^{\beta+\gamma+2x} dy^2 + e^{\beta-\gamma+2z} dz^2,$$

... (1)

Where $\alpha = \alpha(x, t)$, $\beta = \beta(t)$ and $\gamma = \gamma(t)$, $h$ being constant. The universe described by the line element (1) is filled with co-moving string perfect fluid satisfying Einstein’s field equations

$$R^i_i - \frac{1}{2} R g^i_i = -T^i_i = -(\rho v^i v^j - \lambda x_i x^j),$$

... (2)

Where $T^i_i$ is the energy-momentum tensor for a cloud of massive strings and perfect fluid distribution, $v_i$ is unit flow vector and $x_i$ satisfy conditions $v_i v^i = -x_i x^i = -1, v^i x_i = 0$.  

Here, $\rho$ is the rest energy of the cloud of strings with massive particles attached to them. $\rho = \rho_p + \lambda$, $\rho_p$ being the rest energy density of particles attached to the strings and $\lambda$ the density of tension that characterizes the strings. The unit space-like vector $x^i$ represents the string direction in the cloud, i.e. the direction of anisotropy and the unit time-like vector $v^i$ describes the four-velocity vector of the matter satisfying the following conditions $g_{ij}v^iv^j = -1$.

In a comoving coordinate system, we have

$$v^i = (0, 0, 0, e^{-\alpha}) \quad \ldots (3)$$

and choose $x^i$ parallel to x-axis so that

$$x^i = (e^{-\alpha}, 0, 0, 0) \quad \ldots (4)$$

The Einstein's field equations (2) for the line-element (1) lead to the following system of equations:

$$e^{-2\alpha} \left[ \beta_{44} + \frac{3}{4} (\beta_4)^2 + \frac{1}{4} (\gamma_4)^2 - \alpha_4 \beta_4 - (1 + h) \alpha_1 - h \right] = \lambda \quad \ldots (5)$$

$$e^{-2\alpha} \left[ \alpha_{44} - \alpha_{11} + \frac{1}{2} (\beta_{44} - \gamma_{44}) + \frac{1}{4} (\beta_4 - \gamma_4)^2 - h^2 \right] = 0 \quad \ldots (6)$$

$$e^{-2\alpha} \left[ \alpha_{44} - \alpha_{11} + \frac{1}{2} (\beta_{44} + \gamma_{44}) + \frac{1}{4} (\beta_4 + \gamma_4)^2 - 1 \right] = 0 \quad \ldots (7)$$

$$e^{-2\alpha} \left[ \frac{1}{4} (\beta_4)^2 - (\gamma_4)^2 \right] + \alpha_4 \beta_4 + (1 + h) \alpha_1 - (1 + h + h^2) = \rho \quad \ldots (8)$$

$$\alpha_1 \beta_4 + (1 + h) \alpha_4 - \frac{1}{2} (1 + h) \beta_4 - \frac{1}{2} (1 - h) \gamma_4 = 0 \quad \ldots (9)$$

The suffixes ‘1’ and ‘4’ after $\alpha$, $\beta$ and $\gamma$ denote partial differentiation with respect to x and t respectively.

If $\lambda = \rho$ then equations (5), (6), (7) and (8) gives

$$\alpha_{11} - \alpha_{44} - \alpha_4 \beta_4 - (1 + h) \alpha_1 + (1 + h^2) = 0 \quad \ldots (10)$$

On subtracting equation (6) from (7), we get

$$\gamma_{44} + \beta_4 \gamma_4 = (1 - h^2) \quad \ldots (11)$$

CASE I: $1 + h \neq 0$ and $\beta_4 \neq 0$ any constant $k$.

Equation (9) implies
\[ \alpha = \varphi + \frac{1}{2} \beta + \frac{1}{2} \left( \frac{1-h}{1+h} \right) \gamma \] ... (12)

Where \( \varphi = \varphi\{(1+h)x - \beta\} \). Hence from the equations (10), (11) and (12), we get

\[ \varphi'' - \varphi' + \frac{1}{2} = 0 \] ... (13)

Where a prime (‘) denotes differentiation w.r.t. \((1+h)x - \beta\)

Equation (13) gives

\[ \varphi = n_1 + n_2 e^{(1+h)x - \beta} + \frac{1}{2} \{(1+h)x - \beta\} \] ... (14)

Where \( n_1 \) and \( n_2 \) are constants of integrations.

Now \( \beta_4 = k \) On integration we get

\[ \beta = kt + k_1 \] ... (15)

Where \( k_1 \) are constant of integration.

And equation (11) gives on integration

\[ \gamma = c_1 + c_2 e^{-kt} + \frac{(1-h^2)}{k} t \] ... (16)

Where \( c_1 \) and \( c_2 \) are constants of integration.

Therefore

\[ \varphi = n_1 + n_2 e^{(1+h)x - (kt + k_1)} + \frac{1}{2} \{(1+h)x - (kt + k_1)\} \] ... (17)

and

\[ \alpha = L_1 e^{(1+h)x - k_1} + L_2 e^{-kt} + L_3 t + \frac{1}{2} (1+h)x + L_4 \] ... (18)

Where \( L_1 = n_2 e^{-k}; L_2 = \frac{1}{2} \left( \frac{1-h}{1+h} \right) c_2; L_3 = \frac{1}{2} \left( \frac{1-h^2}{k} \right) \) and \( L_4 = n_1 + \frac{1}{2} \left( \frac{1-h}{1+h} \right) c_1 \) are new constants.

So the line element (1) becomes in this case

\[ ds^2 = \exp 2 \left\{ L_1 e^{(1+h)x - k_1} + L_2 e^{-kt} + L_3 t + \frac{1}{2} (1+h)x + L_4 \right\} (dx^2 - dt^2) + \exp \{ c_2 e^{-kt} + (2L_3 + k)t + L_5 + 2x \} dy^2 + \exp \{-c_2 e^{-kt} + (k - 2L_3)t + L_6 + 2hx\} dz^2 \] ... (19)
Inhomogeneous …

Where \( L_5 = k + c_1 \) and \( L_6 = k - c_1 \).

CASE II: \( l + h \neq 0 \) and \( \beta_4 = 0 \)

From equation (11) gives on integration

\[
\gamma = \left(1 - h^2\right) \frac{t^2}{2} + \ell_1 t + \ell_2 \quad \ldots (20)
\]

Now \( \beta_4 = 0 \) on integration we get

\[
\beta = \ell 
\quad \ldots (21)
\]

and hence equation (9) gives

\[
\alpha = \left(1 - h^2\right) \frac{t^2}{4} + \frac{1}{2} \left(\frac{1 - h}{1 + h}\right) \ell_1 t + L 
\quad \ldots (22)
\]

L being arbitrary functions of \( x \) only and \( \ell \), \( \ell_1 \) and \( \ell_2 \) are constants of integration. Equation (10) determines \( L \) as

\[
L = m_1 + m_2 e^{(1+h)x} + \frac{1}{2} (1+h)x 
\quad \ldots (23)
\]

Where \( m_1 \) and \( m_2 \) are constants of integration.

Therefore

\[
\alpha = \left(1 - h^2\right) \frac{t^2}{4} + \frac{1}{2} \left(\frac{1 - h}{1 + h}\right) \ell_1 t + m_1 + m_2 e^{(1+h)x} + \frac{1}{2} (1+h)x 
\quad \ldots (24)
\]

So the line element (1) becomes in this case

\[
ds^2 = \exp 2 \left\{ \left(1 - h^2\right) \frac{t^2}{4} + \frac{1}{2} \left(\frac{1 - h}{1 + h}\right) \ell_1 t + m_1 + m_2 e^{(1+h)x} + \frac{1}{2} (1+h)x \right\} \left( dx^2 - dt^2 \right) + \\
\exp \left\{ \left(1 - h^2\right) \frac{t^2}{2} + \ell_1 t + \ell_2 + \ell + 2x \right\} dy^2 + \exp \left\{ -(1-h^2) \frac{t^2}{2} - \ell_1 t - \ell_2 + \ell + 2hx \right\} dz^2 
\quad \ldots (25)
\]

CASE III: \( l + h = 0 \) and \( \beta_4 \neq 0 \)

Equation (9) implies
\[ \alpha = \frac{\gamma_4}{\beta_4} x + Q(t) \]  

... (26)

Where \( Q \) is an arbitrary function of \( t \); whereas equation (11) implies

\[ \gamma_4 = s_1 e^{-\beta} \]  

... (27)

Where \( s_1 \) is a constant of integration.

From equation (10), (26) and (27), we get

\[
x \left[ \frac{d^2}{dt^2} \left( \frac{\gamma_4}{\beta_4} \right) + \beta_4 \frac{d}{dt} \left( \frac{\gamma_4}{\beta_4} \right) \right] + \left[ \frac{d^2 Q}{dt^2} + \beta_4 \frac{dQ}{dt} - 2 \right] = 0
\]  

... (28)

This implies that

\[ \frac{d^2}{dt^2} \left( \frac{\gamma_4}{\beta_4} \right) + \beta_4 \frac{d}{dt} \left( \frac{\gamma_4}{\beta_4} \right) = 0 \]  

... (29)

From equations (27) and (29), we get

\[ \beta_4 = \frac{s_1}{a_2} e^{\left\{ -\frac{\gamma_4}{a_1} \right\} \beta_4} \]  

... (30)

\[ \exp \beta = \left[ \frac{s_1 + a_1 t + a_4}{a_2 + a_4} \right]^{\frac{s_1}{\gamma_4 + a_4}} \]  

... (31)

and

\[ \gamma = \frac{a_1 s_1}{a_2} e^{\frac{\gamma_4}{a_1}} + a_4 \]  

... (32)

Again from (28) which implies that

\[ \frac{d^2 Q}{dt^2} + \beta_4 \frac{dQ}{dt} - 2 = 0 \]  

... (33)

Which implies that
\[ Q = \frac{2a_2}{2s_1 + a_1} \left[ \left( \frac{s_1 + a_1}{a_2} \right) t + a_4 \right] + \frac{a_2a_5}{a_1} \left[ \left( \frac{s_1 + a_1}{a_2} \right) T + a_4 \right] + \frac{a_6}{a_1} \] … (34)

From equations (26), (27), (30), (31) and (34), we get

\[ \alpha = \left( a_2x + \frac{a_2a_5}{a_1} \right) \left[ \left( \frac{s_1 + a_1}{a_2} \right) t + a_4 \right] + \frac{2a_2}{2s_1 + a_1} \left[ \left( \frac{s_1 + a_1}{a_2} \right) t^2 + a_4 \right] + a_6 \] … (35)

After using suitable coordinate transformations and renaming of constants, the line element (1) reduces to the form

\[ ds^2 = B \exp \left\{ \left( 2Cx + C' \right) T^{-\frac{a_1}{s_1 + a_1}} + 2 \left( \frac{s_1 + a_1}{2s_1 + a_1} \right) T^2 \right\} \left( dx^2 - dT^2 \right) + \]

\[ \frac{\eta}{T^{s_1 + a_1}} \exp \left\{ C'' T^{s_1 + a_1} + 2x \right\} dY^2 + \frac{\eta}{T^{s_1 + a_1}} \exp \left\{ -C'' T^{s_1 + a_1} + 2hx \right\} dZ^2 \] … (36)

Where \( B = \exp \left\{ \frac{a_2a_5}{s_1 + a_1} \right\} \), \( C = \left( \frac{s_1 + a_1}{a_2} \right) \), \( C' = \frac{a_2a_5}{a_1} \left( \frac{s_1 + a_1}{a_2} \right) \) and \( C'' = \frac{a_2s_1}{a_1} \left( \frac{s_1 + a_1}{s_1 + a_1} \right) \) are new constants.

**PHYSICAL AND GEOMETRICAL PROPERTIES**

The physical and geometrical properties of the model are given as follows:

**Case–I**

Magnitude of rotation \( \omega \) is zero i.e.

\( \omega = 0 \) … (37)

The Expansion scalar \( \theta \) of the model is given by

\[ \theta = \frac{\left[ \left( -kL_4 \right) e^{(1+h)x - kt} - kL_2 e^{-kt} + L_3 + k \right]}{\exp \left[ L_4 e^{(1+h)x - kt} + L_2 e^{-kt} + L_3 t + \frac{1}{2} \left( 1 + h \right) x + L_4 \right]} \] … (38)
The Shear $\sigma$ of the model is given by

$$\sigma = \left[ \frac{2\left( (1+h)x - k t \right) e^{(1+h)x - k t} - L_k e^{-k t} + L_3}{2\sqrt{3} e^{(1+h)x - k t}} \right]^2 \left[ \frac{3c_k e^{-k t} + 2L_3}{2} \right]^{1/2}$$

\[ \text{(39)} \]

String tension density $\lambda$ and rest density $\rho$ of the model are given by

$$\lambda = \rho = \frac{k^2 - (1+h)^2}{2 e^{(1+h)x - k t} + L_3 e^{-k t} + L_3 t + \frac{1}{2}(1+h)x + L_4}$$

\[ \text{(40)} \]

The deceleration parameter $q$ of the model is given by

$$q = 1 - 3e^\theta \left[ e^{b(2L_k e^{(1+h)x} + L_3 e^{-k t} - L_3 e^{(1+h)x} + L_3 - \frac{L_3 e^{(1+h)x}}{k})^2} - \left[ L_k e^{(1+h)x} + L_3 - \frac{L_3 e^{(1+h)x}}{k} \right]^2 \right]$$

\[ \text{(41)} \]

Case-II

Magnitude of rotation $\omega$ is zero i.e.

$$\omega = 0$$

\[ \text{(42)} \]

The Expansion scalar $\theta$ of the model is given by

$$\theta = \exp \left[ \frac{t^2}{4} + \frac{1}{2} \left( \frac{1-h}{1+h} \right) \ell_1 t + m_1 + m_2 e^{(1+h)x} + \frac{1}{2}(1+h)x \right]$$

\[ \text{(43)} \]

The Shear $\sigma$ of the model is given by

$$\sigma = \left[ \frac{\left\{ (1-h)^2 t + \left( \frac{1-h}{1+h} \right) \ell_1 \right\}^2 + 3(1-h^2) t + \ell_1^2}{2\sqrt{3}} \right]^{1/2} \left[ \frac{3c_k e^{-k t} + 2L_3}{2} \right]^{1/2}$$

\[ \text{(44)} \]
String tension density $\lambda$ and rest density $\rho$ of the model are given by
\[
\lambda = \rho = \frac{-(1 + h)^2}{2 \exp \left\{ (1 - h)^2 \frac{t^2}{2} + \frac{1 - h}{1 + h} \left[ \ell \, t + 2m_1 + 2m_2 e^{(1+h)x} + (1+h)x \right] \right\}} \quad \text{... (45)}
\]

The deceleration parameter $q$ of the model is given by
\[
q = -1 - 3e^a \left[ \frac{2(1 + h)^2}{\{1 - h^2 \} t + \ell_1} - 1 \right] \quad \text{... (46)}
\]

Case-III

Magnitude of rotation $\omega$ is zero i.e.
\[
\omega = 0 \quad \text{... (47)}
\]

The Expansion scalar $\theta$ of the model is given by
\[
\theta = \frac{(R_1 x + R_1') T^{-\left( \frac{a_1}{s_1 + a_1} \right)}}{\sqrt{B} \exp \left\{ (C \omega + C') T^{\left( \frac{s_1}{s_1 + a_1} \right)} + R_2 T^2 \right\}} \quad \text{... (48)}
\]

Where $R_1 = a_1 \left( \frac{s_1 + a_1}{a_2} \right)^{-\frac{s_1}{s_1 + a_1}}$, $R_1' = a_2 \left( \frac{s_1 + a_1}{a_2} \right)^{-\frac{s_1}{s_1 + a_1}}$, $R_2 = \frac{s_1 + a_1}{2s_1 + a_1}$ and $R_3 = \frac{s_1}{2(s_1 + a_1)}$.

The Shear $\sigma$ of the model is given by
\[
\sigma = \frac{2 \left[ (2(R_1 x + R_1') T^{-\left( \frac{a_1}{s_1 + a_1} \right)} + 4R_2 T - R_3 T^{-1}) \right]^2 + 3 \left( R_3 T^{-\left( \frac{a_1}{s_1 + a_1} \right)} \right)^2}{2\sqrt{3} \sqrt{B} \exp \left\{ (C \omega + C') T^{\left( \frac{s_1}{s_1 + a_1} \right)} + R_2 T^2 \right\}} \quad \text{... (49)}
\]

String tension density $\lambda$ and rest density $\rho$ of the model are given by
\[
\lambda = \rho = \frac{2R_3 \left[ 2R_3 - 1 \right]}{BT^2 \exp \left\{ 2(C \omega + C') T^{\left( \frac{s_1}{s_1 + a_1} \right)} + 2R_2 T^2 \right\}} \quad \text{... (50)}
\]
The deceleration parameter \( q \) of the model is given by

\[
q = -1 - 3e^\alpha \left\{ \left( R_1 x + R'_1 \right) \left( \frac{-S_1}{s_1 + a_1} \right) T \left( \frac{2s_1 + \alpha t}{s_1 + \alpha t} \right) + 2R_2 - 2R_3 T^{-2} \right\}
\]

\[
\left( R_1 x + R'_1 \right) T \left( \frac{-h}{s_1 + \alpha t} \right) + 2R_2 T + 2R_3 T^{-1}\right)^2
\]

\[
= \left( R_1 x + R'_1 \right) T \left( \frac{\alpha t}{s_1 + \alpha t} \right) + 2R_2 T + 2R_3 T^{-1}\right)^2
\]

\[
\left( R_1 x + R'_1 \right) T \left( \frac{\alpha t}{s_1 + \alpha t} \right) + 2R_2 T + 2R_3 T^{-1}\right)^2
\]

CONCLUSION

We have investigated inhomogeneous Bianchi type VI\( h \) string dust cosmological model filled with perfect fluid. We have studied for (i) \( 1 + h \neq 0, \beta_4 \neq 0 \) (but any constant \( k \)), (ii) \( 1 + h \neq 0, \beta_4 = 0 \) and (iii) \( 1 + h = 0, \beta_4 \neq 0 \).

In Case-I the model (19) starts expanding at \( t = 0 \) and goes on expanding indefinitely when \( k > 0 \) and \( L_3 < 0 \). However, \( 0 \) becomes zero as \( t \to \infty \) when \( k > 0 \) and \( L_3 > 0 \). For large value of \( t \) the ratio of the shear \( \sigma \) and expansion \( \theta \) tends to a finite value i.e. \( \frac{\sigma}{\theta} = \left[ \frac{(2L_3 - k)^2 + 3L_5^2}{2\sqrt{3}(L_3 + k)} \right]^{\frac{1}{2}} \) is a constant. Hence model does not approach isotropy for large value of \( t \). The fluid flow is irrotational and it is observed that the energy density \( \rho \) and string tension density \( \lambda \) tends to constant values as \( t \to 0 \) and \( x \to 0 \). At a later stage both \( \rho \) and \( \lambda \) approach zero when \( t \to \infty \), \( x \to \infty \) and \( k < 0, 1 + h > 0 \), as expected therefore the string will be disappear from the universe at later time. As \( t \to 0 \) and \( x \to 0 \) the deceleration parameter \( q \) approaches to \(-1\) when \( L_1 = -L_2 \) and \( L_3 = 0 \), as in De-sitter universe therefore the model presenting accelerating phase of the universe. In general the model represents expanding, shearing and non-rotating universe.

In Case-II, the model (25) starts expanding at \( t = 0 \) and as \( t \to \infty \), \( 0 \) becomes zero i.e. the expansion stops. For large value of \( t \) the ratio of the shear \( \sigma \) and expansion \( \theta \) tends to a finite value i.e. \( \frac{\sigma}{\theta} = \frac{1}{\sqrt{3}} \left[ 1 + \frac{1 + h}{1 - h} \right]^2 \) where \( 1 - h \neq 0 \). Hence model does not approach isotropy for large value of \( t \).

The fluid flow is irrotational and it is observed that the energy density \( \rho \) and string tension density \( \lambda \) tends to constant values as \( t \to 0 \) and \( x \to 0 \). At a later stage both \( \rho \) and \( \lambda \) approach zero when \( t \to \infty \) and \( x \to \infty \) and \(-1 < h < 1\), as expected therefore the string will be disappear from the universe at later time.

As \( t \to 0 \) and \( x \to 0 \), deceleration parameter \( q \) approach to 0 when \( m_1 = -m_2 \) and \( h = \pm \frac{\ell}{\sqrt{3}} - 1 \) therefore the Universe expands at a constant rate, \( q \) approach to a value greater than zero when \( m_1 = -m_2 \) and \( \frac{(1+h)^2}{\ell^2} < \frac{1}{3} \) therefore the model presenting decelerating phase of the universe, \( q < -1 \) when \( m_1 = -m_2 \) and
\[ \frac{(1 + h)^2}{\ell^2} > \frac{1}{2} \]

therefore model representing super-exponential expansion of the universe. \( q = -1 \) when \( m_1 = -m_2 \) and \( h = \pm \frac{\ell}{\sqrt{2}} - 1 \) therefore model representing exponential expansion of the universe (also known as de Sitter expansion). In general the model represents expanding, shearing and non-rotating universe.

In Case-III, the model (36) starts with a big bang at \( T = 0 \) when \( -1 < \frac{S_1}{S_1 + a_1} < 1 \) and goes on expanding till \( T = \infty \). When \( -1 < \frac{S_1}{S_1 + a_1} < 1 \) and \( T \to \infty \), \( \theta \) becomes zero. It is clear that as \( T \) increases, the ratio of the shear \( \sigma \) and expansion \( \theta \) tends to finite value i.e. \( \frac{\sigma}{\theta} \to \frac{1}{\sqrt{3}} \) as \( T \to \infty \) when \( \frac{a_1}{S_1 + a_1} > -1 \). Hence model does not approach isotropy for large value of \( T \). The fluid flow is irrotational and it is observed that as \( T \to 0 \) and \( x \to 0 \), the energy density \( \rho \) and string tension \( \lambda \) both approach to \( \infty \) when \( \frac{a_1}{S_1 + a_1} > 0 \) and as \( T \to \infty \), and \( x \to \infty \), \( \rho \) and \( \lambda \) both tends to \( 0 \) when \( \frac{a_1}{S_1 + a_1} > 0 \). Therefore the string will be disappear from the universe at later time. As \( T \to 0 \) the deceleration parameter \( q < -1 \), when \( \frac{S_1}{S_1 + a_1} < 0 \) therefore model representing super exponential expansion of the universe. The model represents accelerating universe. In general the model represents expanding, shearing and non-rotating universe.

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REFERENCES


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