*Chapter – III**

Effect of Surface Roughness on Squeeze Film Characteristics between Parallel Stepped Plates with Rabinowitsch Fluid

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3.1 Introduction

The study of bearings with an assumption of smooth bearing surfaces will not predict the bearing performance accurately. Stresses generated when rough surfaces come in to contact and play an important role in most mechanism of friction and wear. Because of this, in recent years, the study of surface roughness has been studies with greater importance in the study of bearings, since the surface roughness is inherent to the process used in their manufacture. Height of the surface roughness may range from 0.05μm of less on polished surfaces to 10μm medium machined surfaces. Due to the random character of the surface roughness, a stochastic approach has been employed to mathematically model the surface roughness by several investigators. Christensen (1969-70) developed a stochastic model for the study of surface roughness on hydrodynamic lubrication of bearings. The stochastic concept of transverse and longitudinal roughness on the steady state behaviour of journal bearings is analyzed by Christensen and Tonder (1971, 1973). Hsu *et.al* (2009) studied the effects of surface roughness and rotating inertia on the squeeze film characteristics of parallel circular disks.

With the development of modern machine equipments the increasing use of non-Newtonian fluids as lubricants are becoming of great interest. According to recent experimental investigations (1988, 1995), base oil blended with long chained additives is found to improve lubricating properties and reduce friction and surface damage. Several micro continuum theories have been proposed to describe the rheological behaviours of such non-Newtonian lubrication in a better way. Recently, Naduvinamani *et.al* (2007) analyzed the combined effects of surface roughness and couple stresses on squeeze film lubrication between porous circular stepped plates. This theory has been widely used to investigate the effects of couple stresses on the performance of different type of fluid film bearings such as slider bearings (1979, 2005), journal bearings (2001, 2002).

Rabinowitsch fluid model is one of the models to establish the non-linear relationship between the shearing stress and shearing strain rate which can be described for one dimensional fluid flow as given in equation (2.1.1). Recently, several researchers have investigated the non-Newtonian effect of Rabinowitsch lubricants on various types of bearings. Lin *et.al.*(2001), studied the non-Newtonian effect of Rabinowitsch fluid model on the slider bearings, parallel annular disks by Lin, (2012) and parallel rectangular squeeze film plates by Lin *et.al.*(2013). A squeeze film characteristics between a long cylinder and a flat plate analyzed by Singh *et. al.* (2013), non-Newtonian effects on the squeeze film characteristics between a sphere and a flat plate lubricated with Rabinowitsch fluid model studied by Singh and Gupta (2012).

In this chapter, the effect of surface roughness on the squeeze film characteristic between stepped plates with Rabinowitsch fluid is analyzed which has not been studied so for.

3.2 Mathematical formulation of the problem.

Consider a squeeze film between two parallel stepped plates approaching each other with a normal velocity $\left(=\frac{-\partial H}{\partial t}\right)$ the bearing surface is rough (at $y = 0$) as shown in Figure 3.1. The lubricant in the film region is taken to be non-Newtonian Rabinowitsch fluid. Body forces and body couples are considered to be absent. According to hydrodynamic lubrication applicable to thin film (Dowson, 1961) the field equations governing the one dimensional motion of an incompressible non-Newtonian Rabinowitsch fluid model in Cartesian co-ordinates (x, y, z) system becomes

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}, \tag{3.2.1}$$

$$\frac{\partial p}{\partial x}=\frac{\partial \tau_{xy}}{\partial y}, \tag{3.2.2}$$

$$\frac{\partial p}{\partial y}=0. \tag{3.2.3}$$

which are solved under boundary conditions for velocity components are given by

i) At the upper surface $y = H$, $u = 0$ and $v = \left(-\frac{\partial H}{\partial t}\right)$ (3.2.4a)

ii) At the lower surface $y = 0$,

$u = 0$ and $v = 0.$ (3.2.4b)

The film thickness H is assumed to be made up of two parts and is given by

$$H_i = h_i + h_s(x, y, \xi) \quad \text{for i = 1,2}$$

where h denotes the nominal smooth part of the film geometry while h_s is the part due to the surface roughness is measured from the nominal level. Without loss of generality it may be assumed that the mean value of h_s over bearing surface is zero. The film thickness component h_s is the function of space co-ordinates x and y, and of the random variable ξ. Hence for a given value of ξ the surface component of the film thickness becomes deterministic function of the space variables.

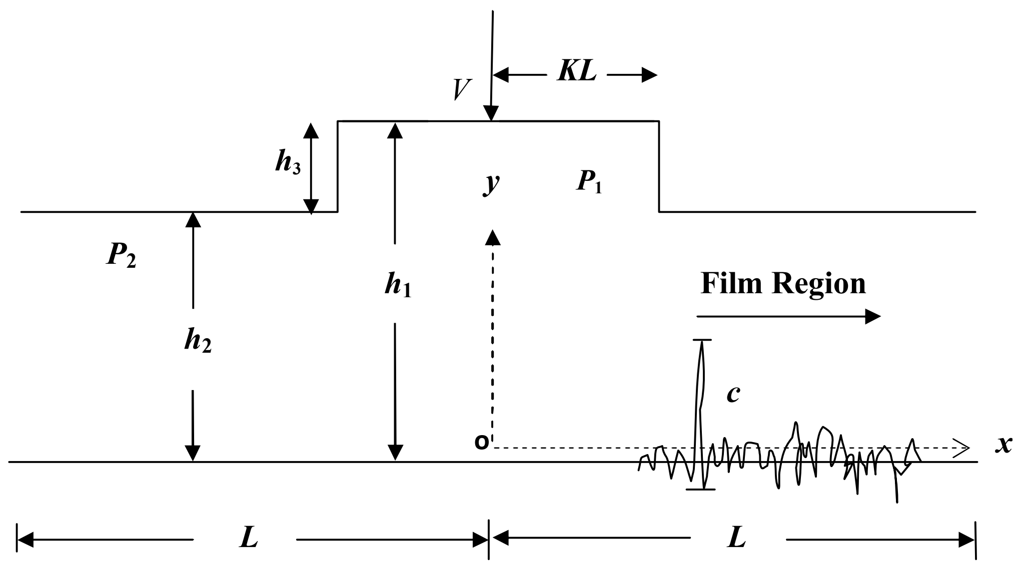

Figure 3.1 Squeeze film between rough parallel stepped plates.

3.3 Solution of the problem

Integrating equation (3.2.2) with respect to y subject to the boundary conditions (3.2.4a) and (3.2.4b) and using constitutive equation (2.1.1), the expression for velocity component can be obtained as

$$u = \frac{1}{2\mu}\left[\frac{\partial p}{\partial x} y(y-H) + \kappa\left(\frac{\partial p}{\partial x}\right)^3 \left\{\frac{y^4}{2} - z^3 H + \frac{3}{4} y^2 H^2 - \frac{1}{4} y H^3\right\}\right] \tag{3.3.1}$$

Using equation (3.3.1) in the continuity equation (3.2.1) and integrating with respect to y under the relevant boundary conditions (3.2.4a) and (3.2.4b) for y, the modified Reynolds type equation for non-Newtonian Rabinowitsch fluid is obtained in the form

$$\frac{\partial}{\partial x}\left[H^3\left(\frac{\partial p}{\partial x}\right) + \frac{3}{20}\kappa\, H^5\left(\frac{\partial p}{\partial x}\right)^3\right] = -12\mu\frac{\partial H}{\partial t} \tag{3.3.2}$$

Let $f(h_s)$ be the probability density function of the stochastic film thickness h_s Taking stochastic average of the equation (3.3.2) with respect to $f(h_s)$ the averaged modified Reynolds type equation is obtained in the form

$$\frac{\partial}{\partial x}\left[E(H^3)\left(\frac{\partial E(p)}{\partial x}\right) + \frac{3}{20}\kappa\, E(H^5)\left(\frac{\partial E(p)}{\partial x}\right)^3\right] = -12\mu\frac{\partial H}{\partial t} \tag{3.3.3}$$

where the expectancy operator $E(\bullet)$ is defined by

$$E(\bullet) = \int_{-\infty}^{\infty} (\bullet) f(h_s) dh_s \tag{3.3.4}$$

Since the most of the engineering rough surfaces are in Gaussian nature. Hence, the Gaussian distribution is given by

$$f(h_s) = \begin{cases} \dfrac{35}{32c^7}(c^2 - h_s^2)^3. & -c < h_s < c \\ 0 & \text{elsewhere} \end{cases} \tag{3.3.5}$$

where $\sigma = c/3$ is the standard deviation.

In accordance with the Christensen (1970) stochastic theory for the hydrodynamic lubrication of rough surfaces, the analysis is done for two types of one- dimensional surface roughness pattern viz. one-dimensional longitudinal surface roughness pattern and transverse roughness pattern.

For one - dimensional longitudinal roughness pattern, the roughness is assumed to have the form of long, narrow ridges and valleys running in the *x*- direction. The film thickness therefore described by a function of the form

$$H_i = h_i + h_s(y, \xi) \quad \text{for } \mathrm{i} = 1, 2. \tag{3.3.6}$$

and stochastic modified Reynolds equation (3.3.3) takes the form

$$\frac{\partial}{\partial x}\left[E(H^3)\left(\frac{\partial E(p)}{\partial x}\right) + \frac{3}{20}\kappa\, E(H^5)\left(\frac{\partial E(p)}{\partial x}\right)^3 \right] = -12\mu \frac{\partial E(H)}{\partial t} \tag{3.3.7}$$

For one dimensional Transverse roughness, the roughness is assumed to have the form of long narrow ridges and furrows running in the direction perpendicular to the direction of sliding i.e. in the y-direction. The film thickness is therefore described by the function of the form

$$H_i = h_i + h_s\left(x,\xi\right) \quad \text{for} \quad i = 1, 2 \tag{3.3.8}$$

and the stochastic modified Reynolds equation takes the form

$$\frac{\partial}{\partial x}\left[\frac{1}{E\left(\frac{1}{H^3}\right)}\left(\frac{\partial E\left(p\right)}{\partial x}\right)+\frac{3}{20}\kappa\,\frac{1}{E\left(\frac{1}{H^5}\right)}\left(\frac{\partial E\left(p\right)}{\partial x}\right)^3\right]=-12\mu\frac{\partial E(H)}{\partial t} \tag{3.3.9}$$

Equations (3.3.7) and (3.3.9) together can be written as

$$\frac{\partial}{\partial x}\left[\left(G\left(H,c\right)\right)^3\left(\frac{\partial E(p)}{\partial x}\right)+\frac{3}{20}\kappa\left(G\left(H,c\right)\right)^5\left(\frac{\partial E(p)}{\partial x}\right)^3\right]=-12\mu\frac{\partial E(H)}{\partial t} \tag{3.3.10}$$

where

$$G\left(H,c\right)=\begin{cases} E\left(H\right) & \text{for longitudinal roughness} \\ \left[E\left(\frac{1}{H}\right)\right]^{-1} & \text{for transverse roughness} \end{cases}$$

$$E\left(H\right)=\frac{35}{32c^7}\int_{-c}^{c}H\left(c^2-h_s^2\right)^3 dh_s\,,$$

$$E\left(\frac{1}{H}\right)=\frac{35}{32c^7}\int_{-c}^{c}\frac{\left(c^2-h_s^2\right)^3}{H}dh_s.$$

Equation (3.3.10) is a non- linear equation, hence it is not easy to find its solution in closed form by using analytical methods. Hence, the small perturbation method is used to find its solution. The squeeze film pressure can be perturbed as,

$$p = p_0 + \kappa\, p_1 \tag{3.3.11}$$

Substituting into the Reynolds type equation (3.3.10) and neglecting the higher order terms of κ , the two separated equations governing the squeeze film pressure p_0 and p_1 are obtained respectively as.

$$\frac{\partial}{\partial x}\left[\left(G(H,c)\right)^3\frac{\partial E(p_0)}{\partial x}\right]=-12\mu\frac{\partial h}{\partial t}, \tag{3.3.12}$$

$$\frac{\partial}{\partial x}\left[\frac{3}{20}\left(G(H,c)\right)^5\left(\frac{\partial E(p_0)}{\partial x}\right)^3+\left(G(H,c)\right)^3\left(\frac{\partial E(p_1)}{\partial x}\right)\right]. \tag{3.3.13}$$

Using boundary conditions

$$\frac{\partial E(p)}{\partial x}=0 \quad \text{at} \quad x=0 \quad ,$$

the modified Reynolds type equation for determining the squeeze film pressure is obtained from the equation (3.3.12) and (3.3.13) as

$$\frac{\partial E(p_{0i})}{\partial x}=\frac{-12\mu\frac{\partial h}{\partial t}x}{\left(G_i(H,c)\right)^3} \quad \text{and} \tag{3.3.14}$$

$$\frac{\partial E\left(p_{1i}\right)}{\partial x}=\frac{1296\mu\frac{\partial h}{\partial t}x^3}{5\left(G_i\left(H,c\right)\right)^7} \tag{3.3.15}$$

where

$h_i = h_1$ and $p_i = p_1$ for the region $0 \le x \le KL$, (3.3.16a)

$h_i = h_2$ and $p_i = p_2$ for the region $KL \le x \le L$. (3.3.16b)

$$G_i\left(H_i,c\right)=\begin{cases}E\left(H_i\right) & \text{for longitudinal roughness}\\ \left[E\left(\dfrac{1}{H_i}\right)\right]^{-1} & \text{for transverse roughness}\end{cases}$$

where $H_i = h_i + h_s$ for i =1, 2.

The relevant boundary conditions for the pressure are

$$E(p_1)=E(p_2) \quad \text{at} \quad x=KL, \tag{3.3.17a}$$

$$E(p_2)=0 \quad \text{at} \quad x=L. \tag{3.3.17b}$$

The fluid film pressure for the Region- I:

$$\left.\begin{aligned}E(p_1)=6\mu\,\frac{\partial h}{\partial t}\left[\frac{K^2L^2-x^2}{\left(G_1(H_1,c)\right)^3}+\frac{L^2(1-K^2)}{\left(G_2(H_2,c)\right)^3}\right]-\kappa\,\mu\,\frac{\partial h}{\partial t}\frac{324}{5}\\ \left[\frac{K^4L^4-x^4}{\left(G_1(H_1,c)\right)^7}+\frac{L^4(1-K^4)}{\left(G_2(H_2,c)\right)^7}\right]\end{aligned}\right\} \tag{3.3.18}$$

The fluid film pressure for the Region - II:

$$E\left(p_2\right)=6\mu\,\frac{\partial h}{\partial t}\left[\frac{L^2-x^2}{\left(G_2\left(H_2,c\right)\right)^3}\right]-\kappa\,\mu\,\frac{\partial h}{\partial t}\frac{324}{5}\left[\frac{L^4-x^4}{\left(G_2\left(H_2,c\right)\right)^7}\right]. \tag{3.3.19}$$

The non-dimensional pressure in the region I (3.3.18) and in the region II (3.3.19) are obtained in the form

$$\left.\begin{aligned}E\left(p_1^*\right)=\frac{p_1}{6\mu\dfrac{\partial h}{\partial t}L^2}=\left\{\frac{\left(K^2-x^{*2}\right)}{\left(G_1\left(H_1^*,c\right)\right)^3}+\frac{\left(1-K^2\right)}{\left(G_2\left(H_2^*,c\right)\right)^3}\right\}-\frac{54\alpha}{5}\\ \left\{\frac{\left(K^4-x^{*4}\right)}{\left(G_1\left(H_1^*,c\right)\right)^7}+\frac{\left(1-K^4\right)}{\left(G_2\left(H_2^*,c\right)\right)^7}\right\}\end{aligned}\right\}, \tag{3.3.20}$$

$$E\left(p_2^*\right)=\frac{p_2}{6\mu\dfrac{\partial h}{\partial t}L^2}=\left\{\frac{\left(1-x^{*2}\right)}{\left(G_2\left(H_2^*,c\right)\right)^3}\right\}-\frac{54\alpha}{5}\left\{\frac{\left(1-x^{*4}\right)}{\left(G_2\left(H_2^*,c\right)\right)^7}\right\}, \tag{3.3.21}$$

where

$$\alpha = \kappa L^2,\ x^* = \frac{x}{L},\ h_1 = \frac{h_1}{h_0},\ h_2 = \frac{h_2}{h_0}.$$

The load carrying capacity $E(W)$ is obtained in the form

$$E(W) = 2b\int_0^{KL} E(p_1)dx + 2b\int_{KL}^{L} E(p_2)dx. \tag{3.3.22}$$

The non-dimensional mean load carrying capacity W^* is obtained in the form

$$W^* = \frac{E(W)h_2^3}{8b\mu \frac{\partial h}{\partial t} L^3} = \left. \begin{matrix} \left\{ \dfrac{K^3}{(G_1(H^*, c))^3} + \dfrac{(1-\mathrm{K}^3)}{(G_2(1, c))^3} \right\} - \dfrac{324}{25}\alpha \\ \overline{\left\{ \dfrac{K^5}{(G_1(H^*, c))^7} + \dfrac{(1-\mathrm{K}^5)}{(G_2(1, c))^7} \right\}} \end{matrix} \right\}. \tag{3.3.23}$$

Writing $\frac{\partial h}{\partial t} = \frac{-\partial h_2}{\partial t}$ in the equation (3.3.22) the squeezing time for reducing the film thickness from the initial value h_0 of h_2 to a final value h_f is given by

$$t = \frac{-8\mu b L^3}{E(W)} = \int_{h_0}^{h_f} \left[\frac{\left\{ \dfrac{K^3}{(G_1(H_1, c))^3} + \dfrac{1-K^3}{(G_2(H_2, c))^3} \right\} - \dfrac{324}{25}\kappa L^3}{\left\{ \dfrac{K^5}{(G_1(H_1, c))^7} + \dfrac{1-K^5}{(G_2(H_2, c))^7} \right\}} \right] dh_2 \tag{3.3.24}$$

which in the non- dimensional form is

$$t^* = \frac{E(W)h_0^2 t}{8\mu b L^3} = \int_{h_f^*}^{1} \left[\frac{\left\{ \dfrac{K^3}{(G_1(h_2^*, h_3^* h_s^*, C))^3} + \dfrac{1-K^3}{(G_2(h_2^*, h_s^*, C))^3} \right\} - \dfrac{324}{25}\alpha}{\left\{ \dfrac{K^5}{(G_1(h_2^*, h_3^* h_s^*, C))^7} + \dfrac{1-K^5}{(G_2(h_2^*, h_s^*, C))^7} \right\}} \right] dh_2^* \tag{3.3.25}$$

Where $h_f^* = \frac{h_f}{h_0};$; $h_2^* = \frac{h_2}{h_0};$ $h_s^* = \frac{h_s}{h_0};$ $C = \frac{c}{h_0};$ $h_3^* = \frac{h_3}{h_0}.$

3.4 Results and discussion

In this chapter, the effect of surface roughness on the squeeze film lubrication between parallel stepped plates with Rabinowitsch fluid is analysed, and on the basis of the Christensen stochastic theory for the study of rough surfaces. By considering two different types of one dimensional roughness pattern *viz.* longitudinal roughness pattern and transverse roughness pattern. The effect of surface roughness is characterised by roughness parameter *C*. The limiting case of $C \rightarrow 0$ corresponds to smooth case studied by Naduvinamani *et. al*. (2015).

3.4.1 Load carrying capacity

The variation of non-dimensional load carrying capacity W^* with H^* for different values of α with $K = 0.5$ and $C = 0.1$ is shown in Fig. 3.2 for both the longitudinal and transverse roughness patterns. It is observed that, the load carrying capacity W^* decreases for increasing value of H^* for both longitudinal and transverse roughness patterns. Further, it is also observed that W^* increases for decreasing values of α, i.e. load carrying capacity increases for dilatant fluids and decreases for pseudoplastic lubricants for both type of roughness patterns. The variation of non-dimensional loads carrying capacity W^* with H^* for different values of *K* with $\alpha = 0.01$ and $C = 0.2$ is shown in Fig. 3.3 for both types of roughness patterns. It is observed that W^* decreases for increasing values of *K* the increasing value of *K* leads to increase the step region and hence increase in the area of the fluid film and hence the decrease in pressure and load carrying capacity. From the Fig. 3.4 it is observed that increasing values of roughness parameter *C*, the load carrying capacity W^* increases for transverse roughness pattern and decreases for the longitudinal of roughness pattern.

3.4.2 Squeeze film time

The variation of non dimensional squeeze film time t^* with h_f^* for different values of α with $C=0.2$, $K=0.5$ and. $h_3^*=0.15$ is shown in the Fig.3.5. For both type of surface roughness patterns. A significant increase in t^* is observed for dilatant fluids as compared to the Newtonian case. Further, the increase (decrease) in t^* is more for transverse (longitudinal) roughness pattern. Figure 3.6 depicts the variation of non-dimensional squeeze film time t^* with h_f^* for different values of K with $\alpha=0.01, C=0.2$ and $h_3^*=0.15$ as value of K increases the squeeze film time decreases in both longitudinal and transverse roughness patterns. Figure 3.7 depicts the variation of non-dimensional squeeze film time t^* with h_f^* for different values of C with $\alpha=0.01$, $K=0.4$ and $h_3^*=0.15$, it is observed that as C increases t^* increases (decreases) for longitudinal (transverse) roughness pattern in case of pseudoplastic fluids ($\alpha=0.01$) where as the reverse trend is observed for the dilatant lubricants ($\alpha=-0.01$) which is evident in Fig.3.8.

3.5 Conclusions

The squeeze film lubrication between rough stepped plates with Rabinowitsch fluid is presented in this chapter. It is found that there is significant increase in load carrying capacity for dilatant fluids as compared to the corresponding Newtonian fluids for both longitudinal and transverse roughness pattern whereas the reverse trend is observed for the pseudoplastic lubricants. The response time t^* increases for decreasing values of α, i.e. the response time is more for dilatant lubricants and less for pseudoplastic lubricants

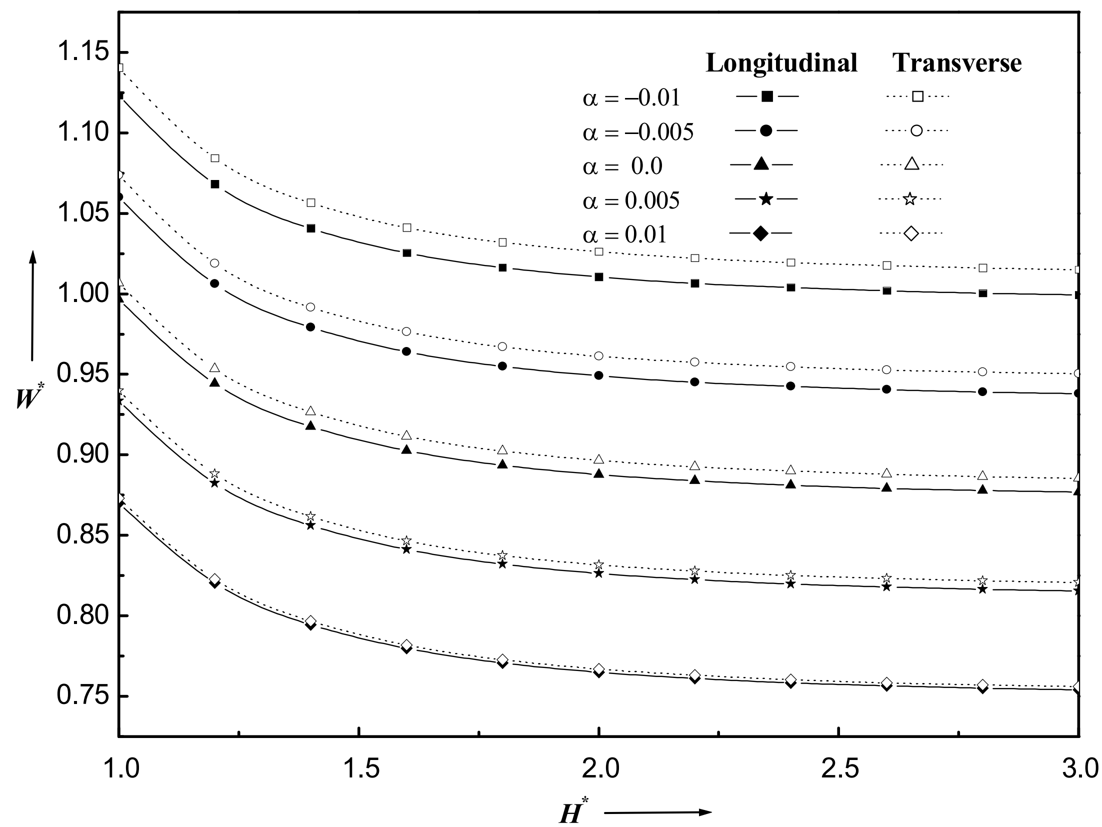

Figure 3.2 Variation of non-dimensional load carrying capacity W^* with H^* for different values of α with $K = 0.5$ and $C = 0.1$

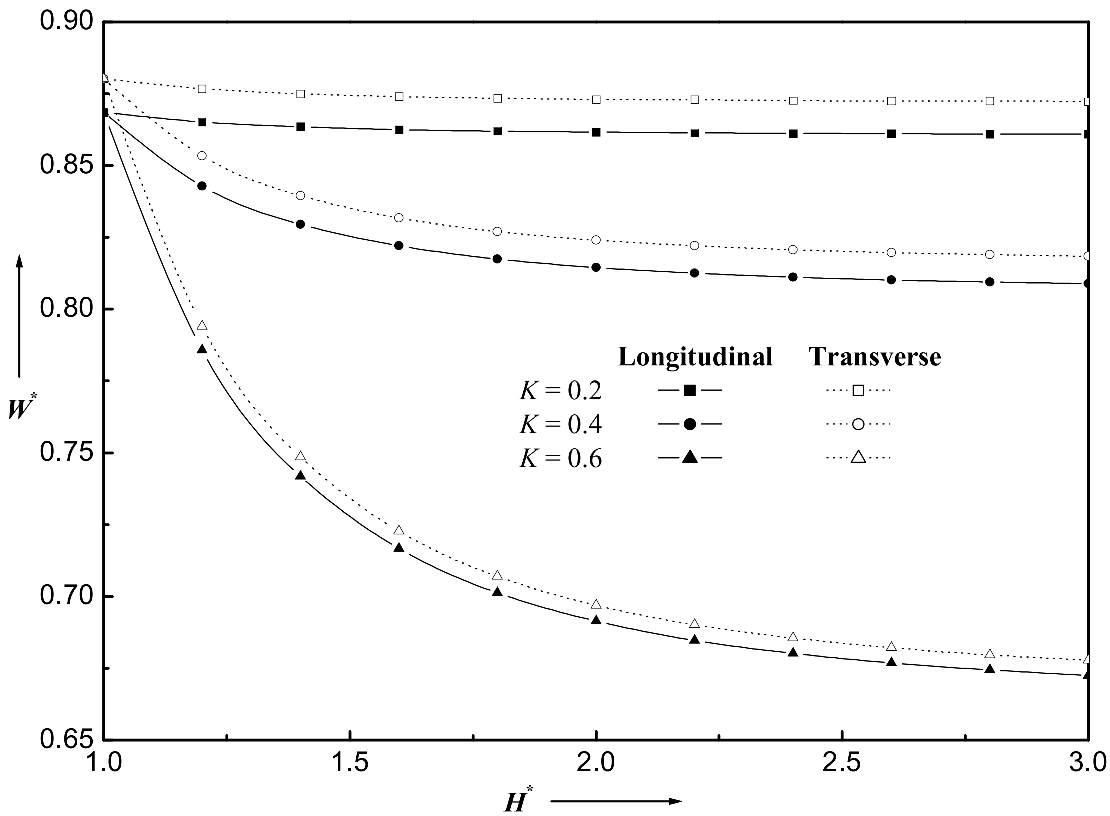

Figure 3.3. Variation of non-dimensional load carrying capacity W^* with H^* for different values of K with α = 0.01, C = 0.2

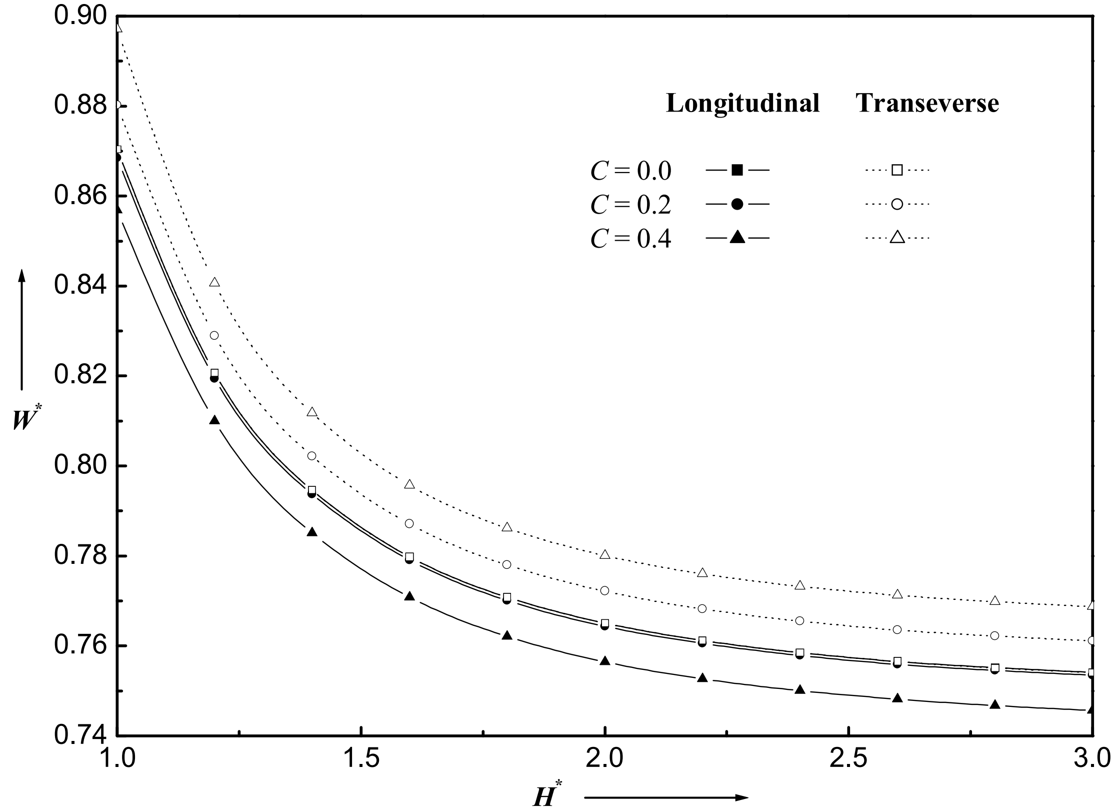

Figure 3.4 Variation of non-dimensional load carrying capacity W^{*} with H^{*} for different values of C with K= 0.5 and $\alpha = 0.01$

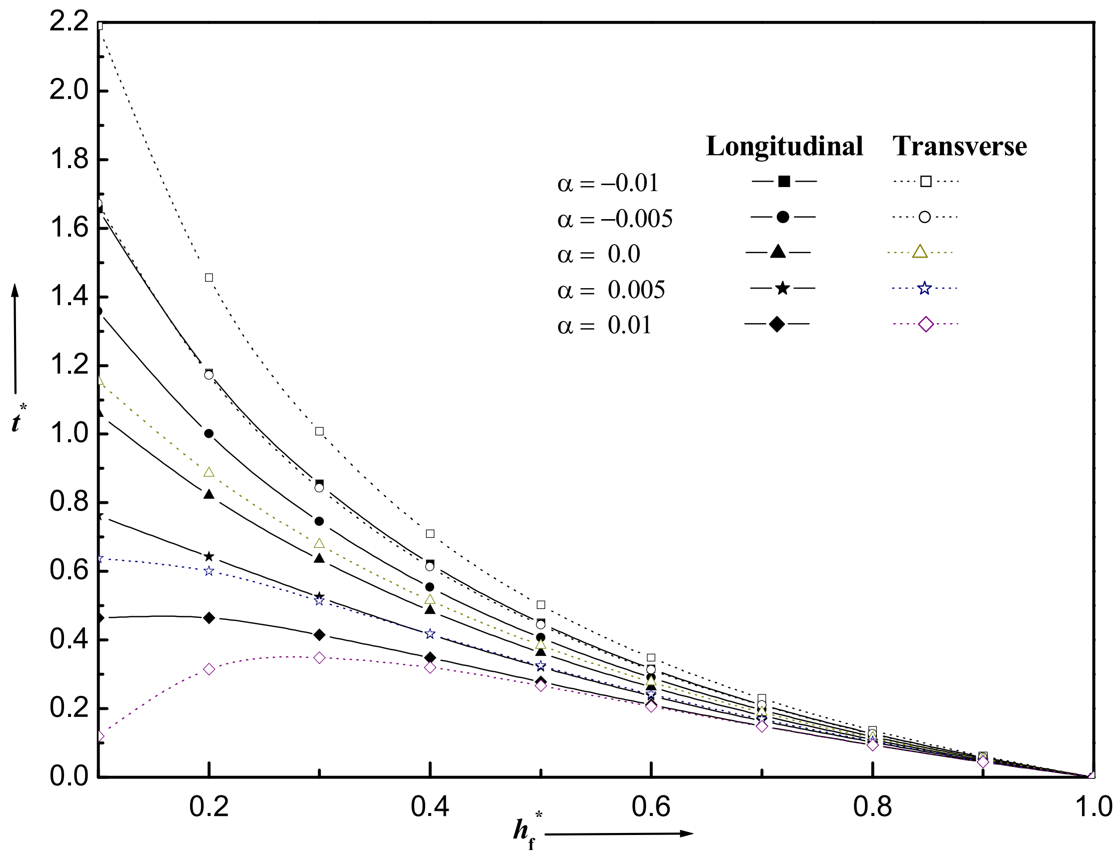

Figure 3.5 Variation of non-dimensional time of approach t^* with h_f^* for different values of α with $C = 0.2$, K = 0.5, $h_2^* = 0.15$.

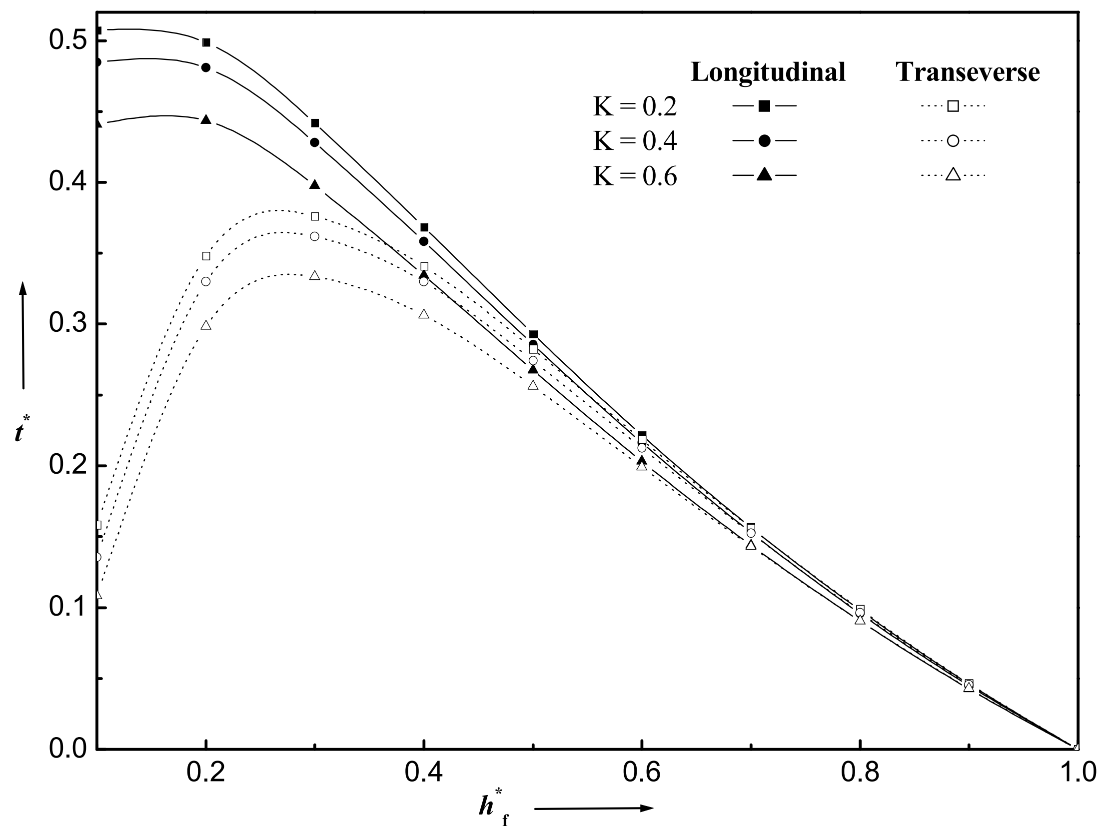

Figure.3.6 Variation of non-dimensional time of approach t^* with h_f^* for different values of K with $\alpha = 0.01$, $C = 0.2$, $h_3^* = 0.15$.

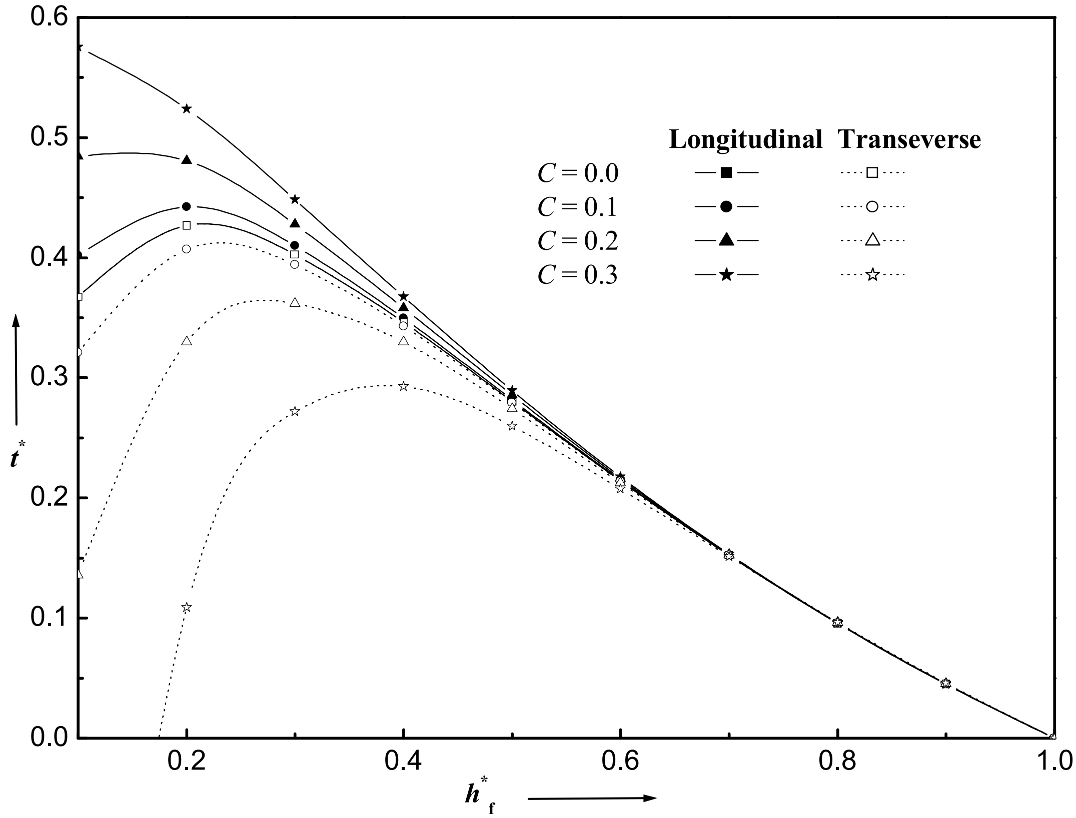

Figure.3.7 Variation of non-dimensional time of approach t^* with h_f^* for different values of C with $\alpha = 0.01$, $K = 0.4$, $h_3^* = 0.15$

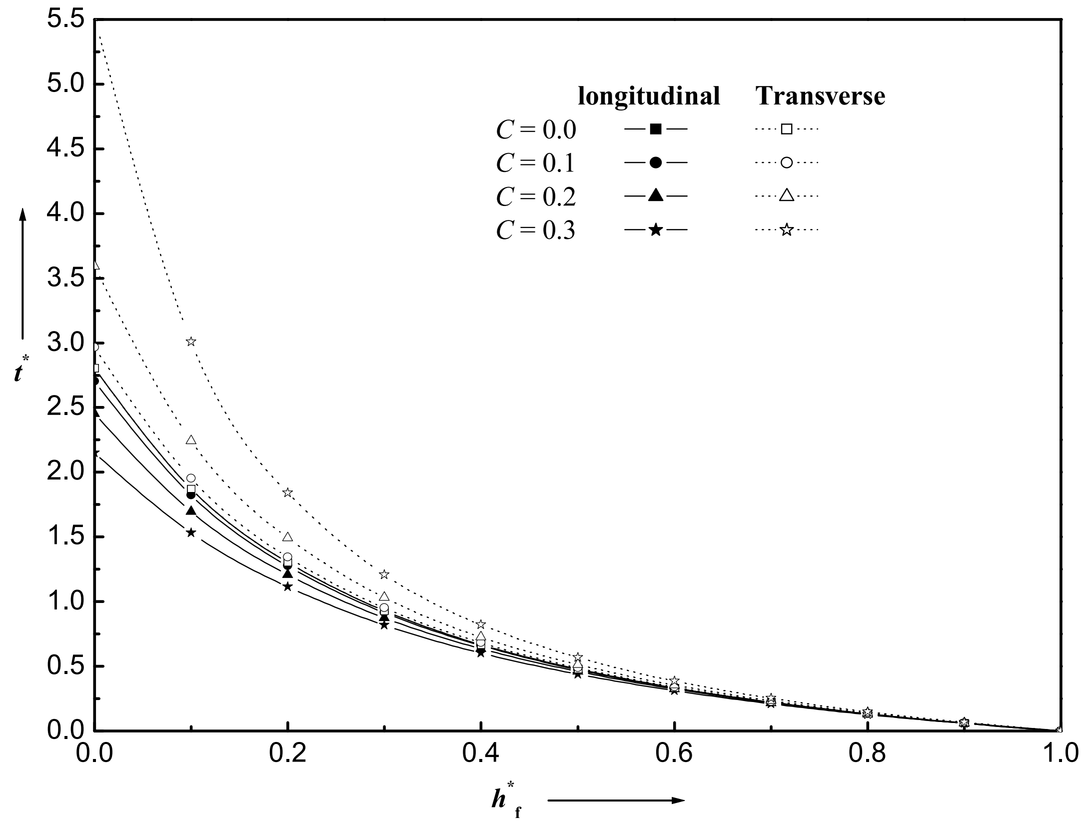

Figure 3.8 Variation of non-dimensional time of approach t^* with h_f^* for different values of C with $\alpha = -0.01$, $K = 0.4$, $h_3^* = 0.15$