*Chapter – II**

On the Non-Newtonian Effects of Rabinowitsch Fluid on the Squeeze Film Characteristics Between Parallel Stepped Plates

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2.1 Introduction

In recent years, the use of squeeze film lubrication has received much attention. This type of lubrication is observed in many applications such as clutches, gears, bearings and machine tools etc. The relevant literature on squeeze film lubrication can be found in Moore (1968) and Archibald (1956). The squeeze film lubrication between two infinitely long parallel plates is studied by Cameron (1981). The flow of an incompressible fluid between two parallel plates due to normal motion of the plates is presented by Bujurke (1995). The squeeze film with Newtonian lubricants has been studied by Jackson (1963). Gupta and Gupta (1977) investigated the squeezing flow between parallel plates.

It has been found that, the use of additives can stabilize fluid properties and minimize the sensitivity of change in shearing strain rate (Spikes, H. A. 1994). For effective improvement in the bearing characteristics as compared to the Newtonian lubricants, the use of Newtonian fluids blended with various additives increases. The use of oils blended with high molecular weight as lubricants has received attention of several researchers.

The present theoretical analysis investigates the effect of non-Newtonian pseudoplastic and dilatant lubricants on the squeeze film characteristics between parallel stepped plates. There are several fluid models to study the non-Newtonian properties of the lubricants such as power law, couple stress and micro polar fluid model. The squeeze film between finite plates lubricated with couple stress fluids was studied by Ramanaiah (1979). The squeeze film lubrication between circular stepped plates of couple stress fluids was studied by Naduvinamani and Siddangouda (2009). They found that, the influence of couple stresses is to enhance the squeeze film pressure, load carrying capacity and decreases the response time as compared to the classical Newtonian fluid.. The squeeze film lubrication between parallel stepped plates with couple stress fluids was studied by Kashinath (2012).

Rabinowitsch fluid model is one of the models to establish the non-linear relationship between the shearing stress and shearing strain rate which can be described for one dimensional fluid flow as

$$\tau_{xy} + \kappa \tau_{xy}^{3} = \mu \frac{\partial u}{\partial y}, \tag{2.1.1}$$

where μ denote the zero shear rate viscosity and κ denotes the non linear factor which describes the non-Newtonian effects of the lubricant which will be referred to as coefficient of pseudo plasticity. This model can be applicable to Newtonian $(\kappa = 0)$, dilatant $(\kappa < 0)$ and pseudoplastic $(\kappa > 0)$ lubricants respectively. The advantage of this model lies in the fact that, the theoretical analysis for present model was verified with experimental justification by Wada and Hayashi (1971). Recently, several researchers have investigated the non-Newtonian effect of Rabinowitsch lubricants on various types of bearings. Lin *et.al.* (2001, 2012) studied the non-Newtonian effect of Rabinowitsch fluid model on the slider bearings and parallel annular disks. The effect of non-Newtonian Rabinowitsch fluids in wide parallel rectangular squeeze film plates is studied by Lin *et.al.* (2013). The Variation principle for non-Newtonian lubrication of Rabinowitsch fluid model was analyzed by He (2004). Singh *et.al.* (2011,2013) presented the effect of Rabinowitsch fluid model on the hydrostatic thrust bearings and the squeeze film characteristics between a long cylinder and a flat plate and a sphere and flat plate is also studied by Singh *et. al.*(2012). In this chapter an attempt is made to study the squeeze characteristics between parallel stepped plates lubricated with Rabinowitsch fluid.

2.2 Mathematical formulation of the problem

Consider a squeeze film between parallel stepped plates approaching each other with a normal velocity $\left(=\frac{\partial h}{\partial t}\right)$ as shown in the figure 2.1. The lubricant in the film region is considered as non-Newtonian Rabinowitsch fluid. Under the assumptions of thin film lubrications, the basic equations governing the fluid flow are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.2.1}$$

$$\frac{\partial p}{\partial x}=\frac{\partial \tau_{xy}}{\partial y}, \tag{2.2.2}$$

$$\frac{\partial p}{\partial y}=0. \tag{2.2.3}$$

The relevant boundary conditions for velocity components are

i. At the upper surface $y = h$:

$$u=0 \text{ and } v=\left(-\frac{\partial h}{\partial t}\right). \tag{2.2.4a}$$

ii. At the lower surface $y=0$:

$$u=0 \text{ and } v=0. \tag{2.2.4b}$$

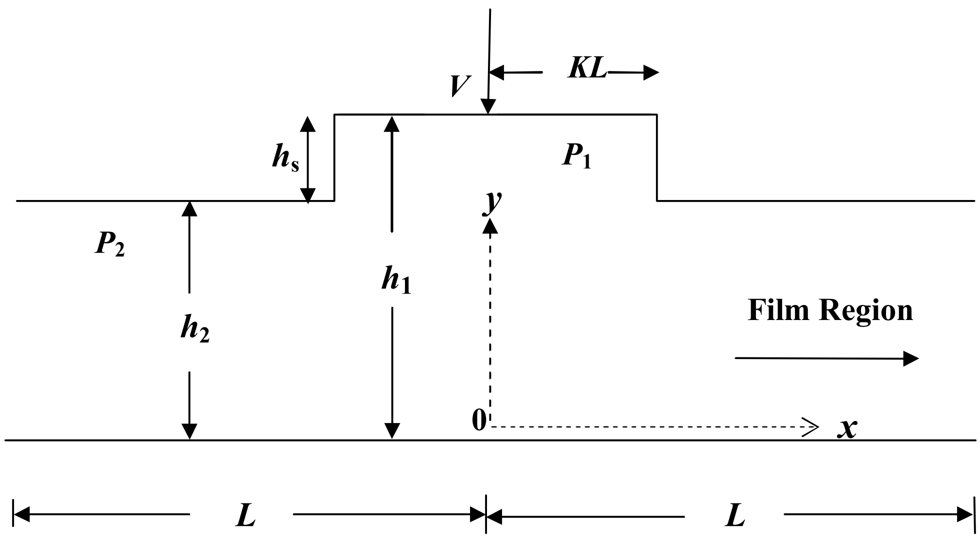

Figure 2.1 Squeeze film between parallel stepped plates.

2.3 Solution of the problem

Integrating equation (2.2.2) with respect to y subject to the boundary conditions (2.2.4a) and (2.2.4b) and using constitutive equation (2.1.1) the expression for velocity component is obtained as

$$u = \frac{1}{\mu}\left[\frac{1}{2} f F_1 + \kappa^* f^3 F_2\right] \tag{2.3.1}$$

where $F_1 = y(y-h)$, $F_2 = \frac{1}{4}y^4 - \frac{1}{2}y^3 h + \frac{3}{8}y^2 h^2 - \frac{1}{8}yh^3$ and

$f = \frac{\partial p}{\partial x}$, κ^* = non-linear factor of lubricants.

Using equation (2.3.1) in the continuity equation (2.2.1) and integrating with respect to y under the relevant boundary conditions (2.2.4a) and (2.2.4b) for y, the modified Reynolds type equation is obtained in the form

$$\frac{\partial}{\partial x}\left[h^3\left(\frac{\partial p}{\partial x}\right) + \frac{3}{20}\kappa^* h^5 \left(\frac{\partial p}{\partial x}\right)^3\right] = -12\mu\frac{\partial h}{\partial t}. \tag{2.3.2}$$

The equation (2.3.2) is observed to be highly non-linear equation and it is difficult to find the solution in closed from using analytical methods. Hence, the small perturbation method is used to find its solution. The squeeze film pressure can be perturbed as

$$p = p_0 + \kappa^* p_1 \tag{2.3.3}$$

Substituting into the Reynolds type equation (2.3.2) and neglecting the higher order terms of κ^*, the two separated equations governing the squeeze film pressure p_0 and p_1 can be derived respectively

$$\frac{\partial}{\partial x}\left[h^3\frac{\partial p_0}{\partial x}\right]=-12\mu\frac{\partial h}{\partial t} \tag{2.3.4}$$

$$\frac{\partial}{\partial x}\left[\frac{3}{20}h^5\left(\frac{\partial p_0}{\partial x}\right)^3+h^3\left(\frac{\partial p_1}{\partial x}\right)\right]=0. \tag{2.3.5}$$

The modified Reynolds equation for determining the squeeze film pressure is obtained from of equation (2.3.4) and (2.3.5) in the form.

$$\frac{dp_{0i}}{dx}=\frac{-12\mu\frac{\partial h}{\partial t}x}{h_i^3} \tag{2.3.6}$$

$$\frac{dp_{1i}}{dx}=\frac{1296\mu\frac{\partial h}{\partial t}x^3}{5h_i^7} \tag{2.3.7}$$

where

$$h_i = h_1 \text{ for } 0\le x\le KL, \tag{2.3.8a}$$

$$= h_2 \text{ for } KL\le x\le L, \tag{2.3.8b}$$

The relevant boundary conditions for the pressure are

$$\left.\begin{array}{l} p_1=p_2 \text{ at } x=KL \\ p_2=0 \text{ at } x=L \end{array}\right\} \tag{2.2.13}$$

Solving equations (2.3.6) and (2.3.7) and using boundary conditions (2.3.9) the pressure in the Region-I and Region-II is obtained in the form

Pressure in the Region I:

$$p_1 = 6\mu \frac{\partial h}{\partial t}\left[\frac{K^2L^2 - x^2}{h_1^3} + \frac{L^2(1-\mathrm{K}^2)}{h_2^3}\right] + \kappa^* \mu \frac{\partial h}{\partial t}\frac{324}{5}\left[\frac{x^4 - K^4L^4}{h_1^7} + \frac{L^4(1-\mathrm{K}^4)}{h_2^7}\right] \tag{2.3.10}$$

Pressure in the Region II:

$$p_2 = 6\mu \frac{\partial h}{\partial t}\left[\frac{R^2 - x^2}{h_2^3}\right] + \kappa^* \mu \frac{\partial h}{\partial t}\frac{324}{5}\left[\frac{L^4 - x^4)}{h_2^7}\right] \tag{2.3.11}$$

The load carrying capacity W is obtained in the form

$$W = 2b\int_0^{KL} p_1 dx + 2b\int_{KL}^{L} p_2 dx \tag{2.3.12}$$

which in the non-dimensional form is

$$W^* = \frac{Wh_2^3}{8b\mu\left(\frac{\partial h}{\partial t}\right)L^3} = \left\{\frac{K^3}{H^{*3}} + \frac{(1-\mathrm{K}^3)}{1}\right\} + \frac{324}{25}\alpha\left\{\frac{-K^5}{H^{*3}} + \frac{(1-\mathrm{K}^5)}{1}\right\} \tag{2.3.13}$$

where $H^* = \frac{h_1}{h_2}$ and $\alpha = \kappa^*\left(\frac{L}{h_2}\right)^2$.

Writing $\frac{\partial h}{\partial t} = \frac{-\partial h_2}{\partial t}$ in the equation (2.3.13) the squeezing time for reducing the film thickness from the initial value h_0 of h_2 to a final value h_f is given by

$$t = \frac{-8\mu bL^3}{W} = \int_{h_0}^{h_f}\left[\left\{\frac{K^3}{h_1^3} + \frac{1-K^3}{h_2^3}\right\} + \frac{324}{25}\kappa^* L^3\left\{\frac{-K^5}{h_1^7} + \frac{1-K^5}{h_2^7}\right\}\right]dh_2 \tag{2.3.14}$$

which in the non- dimensional form is

$$t^* = \frac{Wh_0^2 t}{8\mu bL^3} = \int_{h_f^*}^{1} \left[\left\{ \frac{K^3}{(h_2^* + h_s^*)^3} + \frac{1-K^3}{h_2^{*3}} \right\} + \frac{324}{25}\alpha \left\{ \frac{-K^5}{(h_2^* + h_s^*)^7} + \frac{1-K^5}{h_2^{*7}} \right\} \right] dh_2^* \qquad (2.3.15)$$

where $h_f^* = \frac{h_f}{h_0}$; $h_2^* = \frac{h_2}{h_0}$; $h_s^* = \frac{h_s}{h_0}$.

In the limiting case of $\alpha \to 0$, the equation (2.3.13) and (2.3.15) reduces to their corresponding Newtonian case presented by Naduvinamani and Siddangouda (2009) (When the couple stress parameter tends to zero)

2.4 Results and discussions

Based on the Rabinowitsch fluid model the effect of non-Newtonian rheology on the squeeze film characteristics between parallel stepped plates is presented. In the present analysis we study the characteristics between parallel stepped plates lubricated with the Newtonian fluids ($\alpha = 0$), dilatants fluids ($\alpha < 0$) and pseudoplastic fluids ($\alpha > 0$).

2.4.1 Load carrying capacity

From the figure 2.2, it is observed that, the dimensionless load carrying capacity W^* increases as the height of the fluid film thickness decreases. It is also observed that the maximum load is delivered for dilatant fluids for smaller values of H^*. The increase in W^* is more accentuated for pseudoplastic fluids as compared to the Newtonian fluids. But for the dilatant fluids the reverse trend is observed. The dotted curve represents the Newtonian case. The variation of W^* with H^* for different values of K with dilatant $(\alpha > 0)$ and Newtonian $(\alpha = 0)$ is presented in figure 2.3. It is observed that, as the value of K increases the load carrying capacity decreases. The relative percentage increase in W^* $R_{W^*}\left[=\left(\frac{W^*_{Rabinowitsch}-W^*_{Newtonian}}{W^*_{Newtonian}}\right)\times 100\right]$ for different values of K is given in the Table 2.1.

2.4.2 Time-height relationship

The most important characteristics of the squeeze film bearing is the squeeze film time i.e. the time required for reducing the initial film thickness h_0 of h_2 to a final value h_f. The figure 2.4 depict the variation of non-dimensional squeeze-film time t^* as a function of h_f^* for different values of α. It is observed that the response time t^* increases for decreasing film thickness. For $\alpha=-0.01$ and $\alpha=-0.005$, as the film thickness h_f^* decreases the response time also decreases. An increase in t^* is more accentuated for dilatant fluids than the corresponding Newtonian fluids. The relative increase in non-dimensional squeeze film time t^*, $R_{t^*}\left[=\left(\frac{t^*_{Rabinowitsch}\pm t^*_{Newtonian}}{t^*_{Newtonian}}\right)\times 100\right]$ for different values of K and α is given in the Table 2.1. It is found that an increase of nearly 30% is observed for the dilatant fluids.

The variation of t^* with h_f^* for different values of K for both Newtonian fluids ($\alpha=0$), dilatant fluids ($\alpha<0$) and pseudoplastic fluids ($\alpha>0$) is depicted in the figure 2.5. It is observed that t^* increases as the value K decreases.

The variation of t^* with h_f^* for different values of step height h_s^* with K = 0.7 is shown in the figure 2.6. It is observed that t^* decreases for increasing value of step height h_s^* the decrease in t^* is observed more for dilatant fluids than Newtonian fluids where as reverse trend is observed for pseudoplastic fluids.

2.5 Conclusions

Based on the Rabinowitsch fluid model this chapter predicts the non-Newtonian effects on squeeze film lubrication between parallel stepped plates. Based on the present theoretical analysis and the presented one can draw the following conclusions are drawn:

1. The effect of dilatant fluids is to increase the load carrying capacity as compared to the corresponding Newtonian case. Whereas the reverse trend is observed for the pseudoplastic fluids.
2. The relative load response time R_{t^*} and relative load carrying capacity R_{W^*} are dependent on the step size (*K*) and the non-linear factor of the lubricant (α).
3. As the squeeze film thickness decreases the non-dimensional response time t^* increases for dilatant fluids.

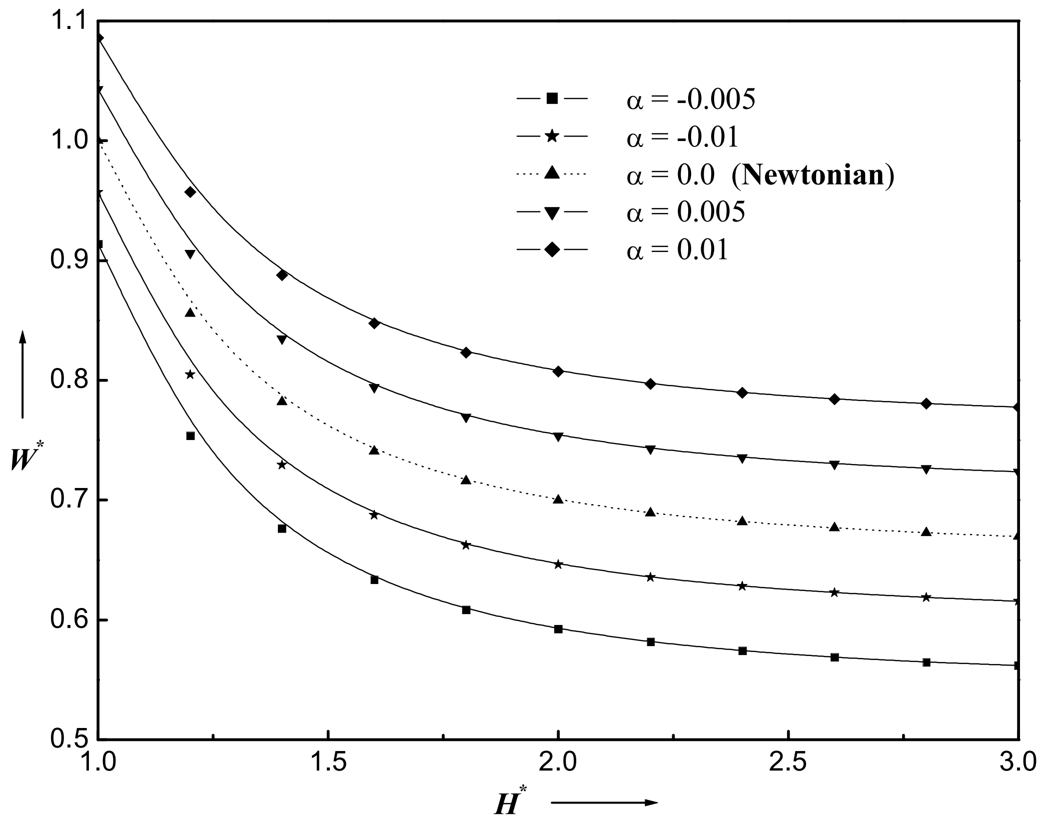

Figure 2.2 Variation of non-dimensional load carrying capacity W^* with H^* for different values of α with $K = 0.7$.

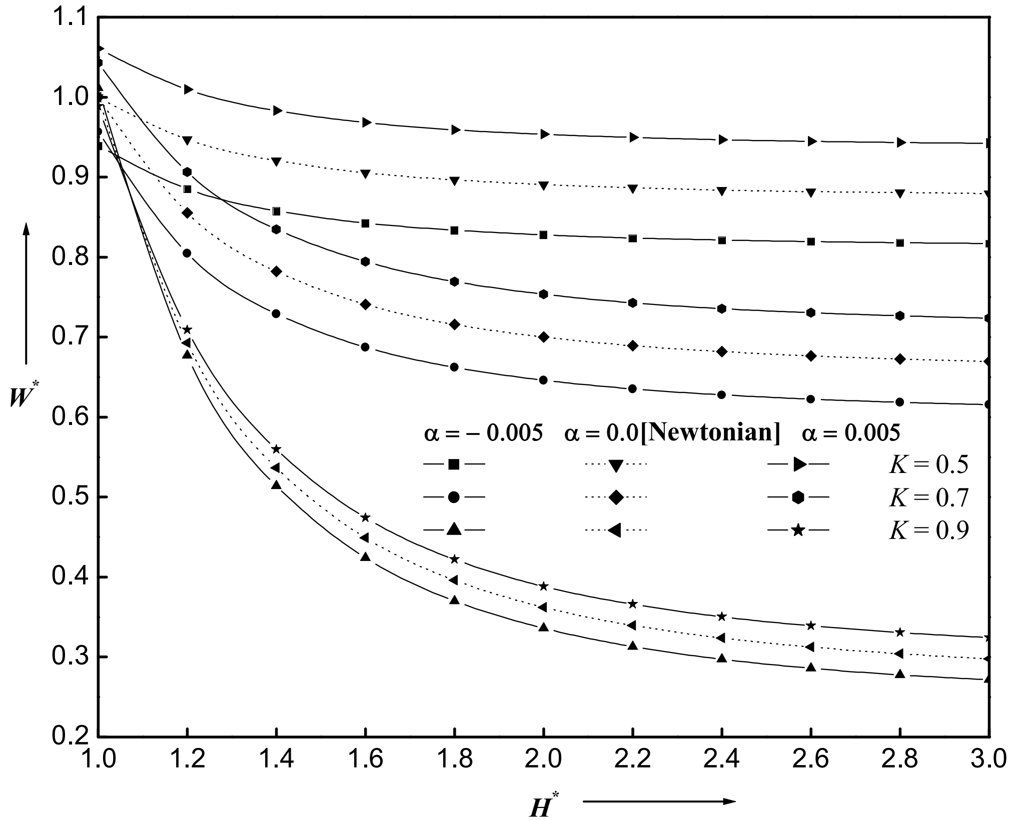

Figure 2.3 Variation of non-dimensional load carrying capacity W^* with H^* for different values of K.

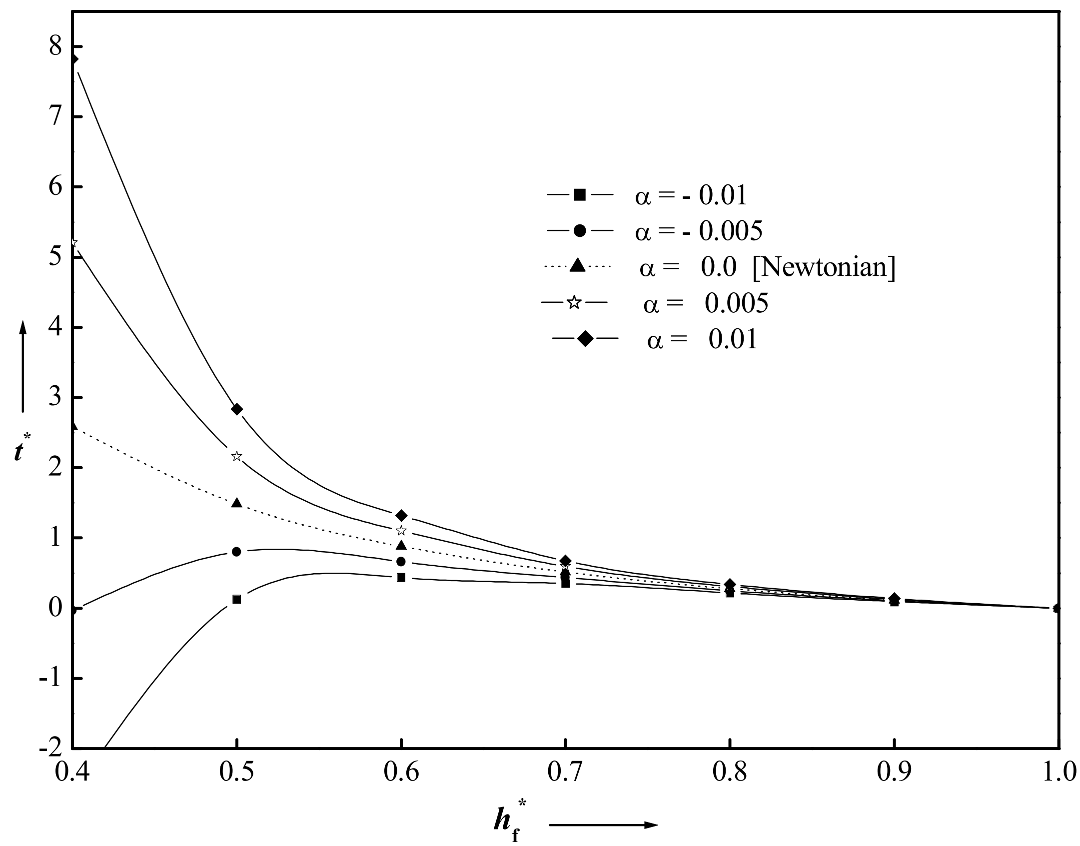

Figure 2.4 Variation of non-dimensional response time t^* with h_f^* for different values of α with $h_s^* = 0.15$

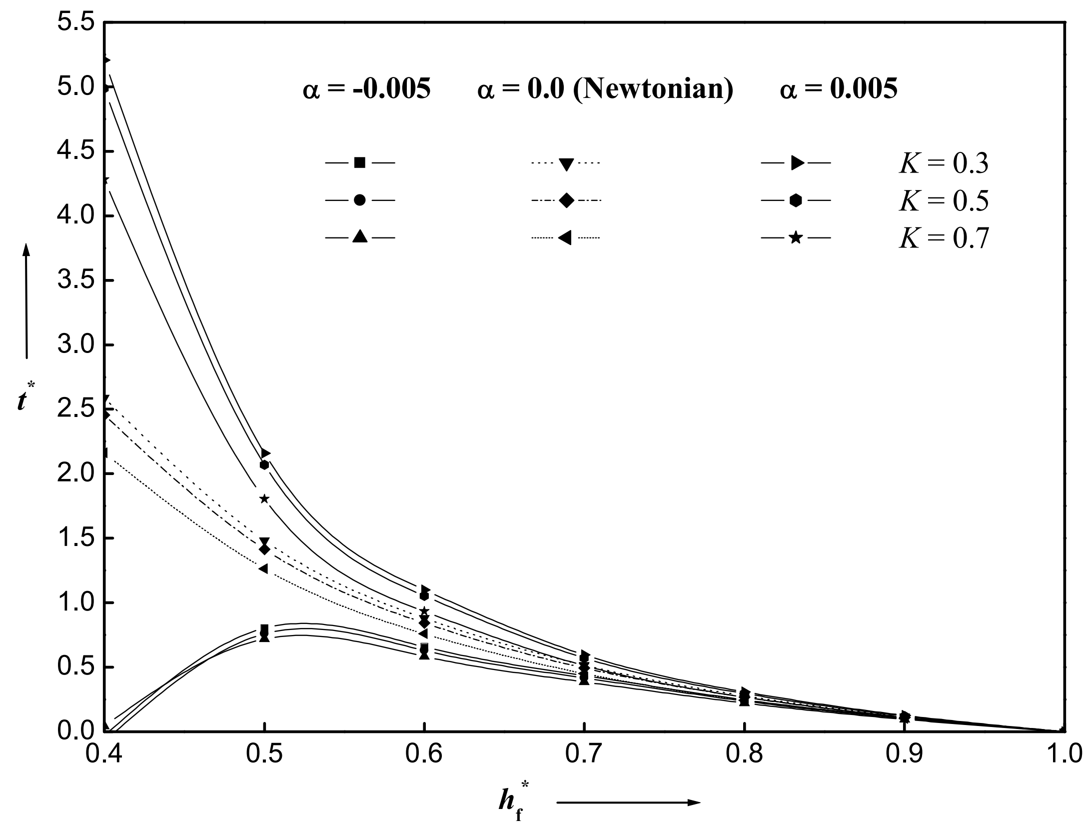

Figure 2.5 Variation of non-dimensional response time t^* with h_f^* for different Values of K with $h_s^* = 0.15$.

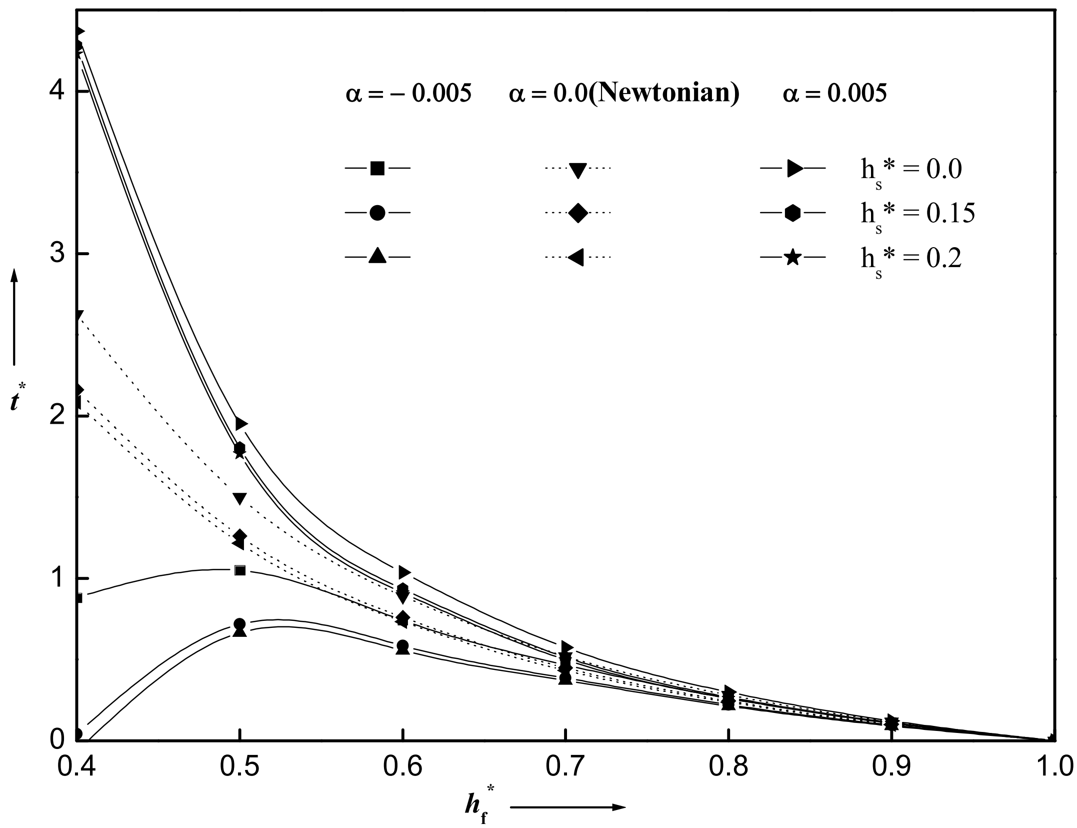

Figure 2.6 Variation of non-dimensional response time t^* with h_f^* for different values of h_s^* with *K*= 0.7

Table 2.1 The variation of load (R_{W^*}) and relative time (R_{t^*}) for different values of α and K

K	A	R_{W^*}	R_{t^*}
0.5	-0.01	-14.093294	-31.4265239
	-0.005	-7.0466448	-15.7132618
	0.005	14.093289	15.7132633
	0.01	7.0466470	31.4265259
0.7	-0.01	-15.381026	-28.1837266
	-0.005	-7.6905162	-14.0918644
	0.005	15.381032	14.0918622
	0.01	7.6905132	28.18372438
0.9	-0.01	-14.490749	-10.28214628
	-0.005	-7.2453738	-5.141071788
	0.005	14.490748	5.141079898
	0.01	7.2453744	10.28215439