Chapter – VII

Static Characteristics of Slider Bearings with an Exponential Film Profile: Rabinowitsch Fluid Model

7.1 Introduction

Nowadays, the use of Newtonian fluids blended with various additives increases due to the effective improvement in the bearing characteristics as compared to the Newtonian lubricants. Lubricants are introduced to the most of the mechanical systems where the relative motion occurs between two parts. The slider bearings are designed for supporting transverse load in engineering practice. The geometry of the contacting elements determines the shape of the lubricant film studied by Berlins *et al.(*2000). Several researchers have considered different configurations of the lubricating film in the clearance zone in their analysis. The contacting surfaces can be narrowed geometrically in linear style as considered by Ozalp, and Umur (2006) employed the iterative transfer matrix approach to suggest optimum film profile parameters for reduced friction coefficient. An understanding of the steady-state performance and dynamic characteristics of a bearing taking in to account the geometry and different operating conditions is important. By considering different film shapes, bearing characteristics have been analysed by Hamrock (1994), King and Taylor(1997), Launder, and Leschziner, 1987), Lin *et. al*.(2001) and Pinkus and Sternlicht (1961).

In recent studies, many authors have investigated the rheological effects of fluids on the lubrication problems. Also by considering different operating conditions, such as viscosity variation across the film studied by Qvale and Wiltshire (1972), the inertia force effects Launder and Leschziner, (1987), Hoshimoto *et. al.* (1994), Lin (2013) and Makay and Trumpler (1971), the turbulent flows by Richard *et. al.* (1971), the thermal effects by Dow and Bupara (1975), Zeinab Safasr (1979) and Safar (1980). All these studies however focus upon the performance of the slider bearing operating under the steady- state situation, in which the effects of dynamic squeezing motion are neglected. As the Rabinowitsch fluid model fits the viscosity data over a wide range of shear rate and its experimental verification is available (Wada and Hayashi, 1971), the analysis of the performance characteristics of film lubricated bearings with Rabinowitsch fluid is motivated. In the Rabinowitsch fluid model, the relationship for one –dimensional flows is as given in the equation (2.1.1). The theoretical results for fluid film pressure, load carrying capacity and squeezing time for journal bearing found to be in good agreement with the experimental results. The film pressure and load carrying capacity for pseudoplastic lubricant was found to be smaller than those for the Newtonian fluids. On the operation of journal bearings, the steady state and thermodynamic performance are calculated by Rajalingam.*et.al.* (1979). On the bases this model many researchers have found the effect of non-Newtonian rheology on the performance characteristics of various thin film lubricated bearings, such as the squeeze film bearing by Lin (2011) and Naduvinamani *et al*. (2014); the journal bearings by Bourging and Gay (1984); and one dimensional slider bearings by Hsu and Saibel (1965) and Lin (2001). According to the results obtained, the performance characteristics of thin film bearings are significantly affected by the non- Newtonian properties of dilatant fluids and pseudo-plastic fluids.

In the present chapter, the steady state characteristics of wide slider bearings with an exponential film shape, lubricated with Rabinowitsch fluid analyzed, which has not been studied so for. By applying a small perturbation technique steady state Reynolds equations are derived. The steady state performances at different inlet–outlet film ratios are then evaluated.

7.2 Mathematical formulation of the problem

Figure 7.1 shows the physical geometry of exponential slider bearing of length *L*. The bearing is consisting of two surfaces, a plane and curved slider separated by a lubricant film. The lower bearing surface is moving with a velocity *U* in the *x* –direction as shown in the Figure 7.1 Fluid in the film region is taken as non-Newtonian Rabinowitsch fluid. The body forces and body couples are assumed to be absent. According to the thin film theory of hydrodynamic lubrication, equations of continuity and motion in Cartesian coordinates reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{7.2.1}$$

$$\frac{\partial \tau_{xz}}{\partial z} = \frac{\partial p}{\partial x}, \tag{7.2.2}$$

$$\frac{\partial p}{\partial z} = 0, \tag{7.2.3}$$

where u and w are velocity components in x and z directions, p denotes the film pressure.

The no slip boundary conditions are

$$u = U, \quad w = 0, \qquad \text{at } z = 0 \tag{7.2.4a}$$

$$u = 0, \quad w = 0 \qquad \text{at } z = h \tag{7.2.4b}$$

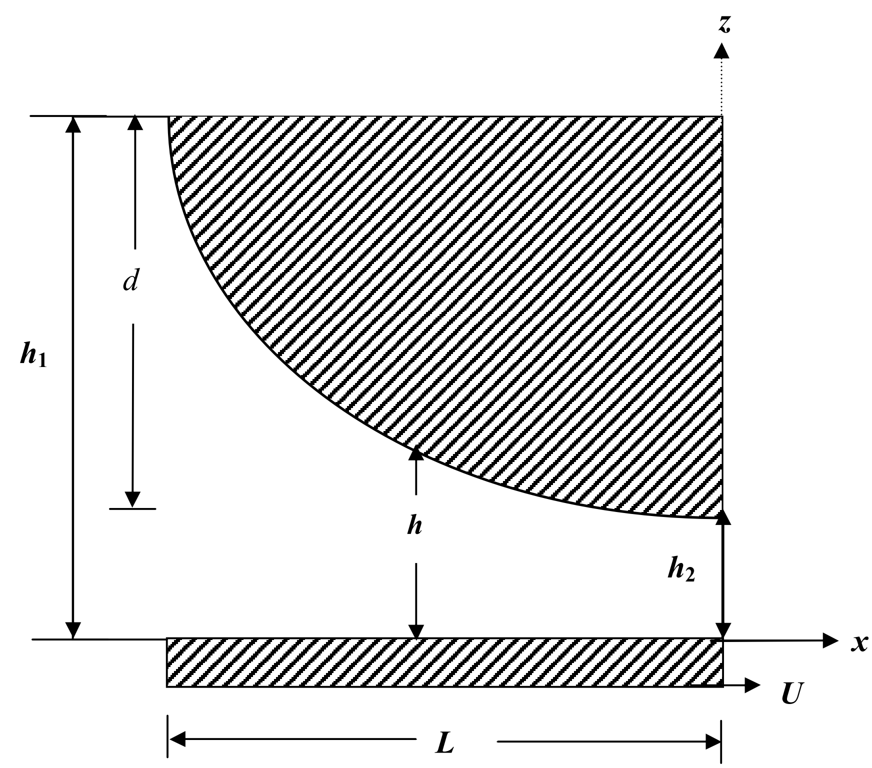

Figure 7.1 Physical Geometry of Exponential Slider Bearing

7.3 Solution of the problem

Integration of equation (7.2.2) with respect to z yields

$$\tau_{xz} = gz + c_1$$

where $g = \frac{\partial p}{\partial x}$, is the pressure gradient in x- direction. c_1 is the constant of integration to be determined. Substituting it into equation (2.1.1), the velocity component in x-direction is obtained in the form

$$u = \frac{1}{4\mu}\left[2gz^2 + 4c_1 z + \alpha(g^3 z^4 + 4g^2 c_1 z + 6gc_1^2 z^2 + 4c_1^3 z)\right] + c_2 \tag{7.3.1}$$

The integration constants c_1 and c_2 can be obtained by using the relevant boundary conditions given in (7.2.4a) and (7.2.4b). On simplifying (7.3.1) takes the form

$$u = \frac{1}{4\mu}\left[F_1 \frac{\partial p}{\partial x} + \frac{1}{2}\alpha F_2 \left(\frac{\partial p}{\partial x}\right)^3\right] + U\left\{1 - \frac{4z - \alpha F_3 \left(\frac{\partial p}{\partial x}\right)^2}{4h + \alpha h^3 \left(\frac{\partial p}{\partial x}\right)^2}\right\}, \tag{7.3.2}$$

where the functions F_1, F_2 and F_3 are described by

$$F_1 = 2z^2 - 2hz \tag{7.3.3a}$$

$$F_2 = 2z^2 - 4hz^3 + 3h^2 z^2 - h^3 z \tag{7.3.3b}$$

$$F_3 = 6hz^2 - 4z^3 - 3h^2 z \tag{7.3.3c}$$

Integrating the continuity equation (7.2.1) across the film height under the relevant boundary conditions (7.2.4a) and (7.2.4b), the modified Reynolds equation is obtained as

$$\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}+\frac{3}{20}\alpha h^5\left(\frac{\partial p}{\partial x}\right)^3\right]=6\mu U\frac{\partial h}{\partial x} \tag{7.3.4}$$

In the equation (7.3.4), p represent the fluid film pressure, μ denotes the lubricant viscosity, U is the sliding velocity of the lower surface.

With reference to the film geometry shown in the Fig.7.1, the film thickness *h(x, t)* is described by the mathematical function

$$h(x,t)=h_2.\exp\left[-\frac{x}{L}\log\left(\frac{h_1}{h_2}\right)\right] \tag{7.3.5}$$

In the equation (7.3.5) h_2 denotes the minimum film thickness depending upon the time t only $d = h_1 - h_2$ is the difference between inlet and outlet film thickness, L is the length of the upper pad.

To analyze the bearing performance, the following non-dimensional variables and parameters are introduced.

$$\left.\begin{array}{llll} x^*=\dfrac{x}{L}, & p^*=\dfrac{ph^2}{\mu UL}, & r=\dfrac{h_1}{h_2}, & \beta=\alpha\dfrac{\mu^2U^2}{h_2^2}, \\ h^*=\dfrac{h}{h_2}=\exp\left(-x^*.\log(\delta+1)\right), & \text{where } \delta=\dfrac{d}{h_2} & \text{and } d = h_1-h_2 & \end{array}\right\} \tag{7.3.6}$$

where $d = h_1 - h_2$ the difference of inlet-outlet thickness, L is the inclined part of the bearing and $\delta=\dfrac{d}{h_2}$ is non-dimensional inlet-outlet thickness difference and β describes the non-linear parameter of non-Newtonian fluids. Equation (7.3.4) in non-dimensional form is obtained as

$$\frac{\partial}{\partial x^*}\left[h^{*3}\frac{\partial p^*}{\partial x^*}+\frac{3}{20}\beta h^{*5}\left(\frac{\partial p^*}{\partial x^*}\right)^3\right]=6\frac{\partial h^*}{\partial x^*}, \tag{7.3.7}$$

The relevant pressure boundary conditions are

$$p^*=0 \text{ at } x^*=-1 \text{ and } x^*=0 \tag{7.3.8}$$

The non-dimensional Reynolds equation (7.3.7) is highly non-linear ordinary differential equation, whose analytical solution is not possible hence the small perturbation method is applied for the solution of (7.3.7). The perturbed p^* is expressed as

$$p^*=p_0^*+\beta p_1^* \tag{7.3.9}$$

Where β is the small parameter. Substituting in to the non-linear equation (7.3.7) and dropping higher order terms, the zeroth order solution p_0^* and first order solution p_1^* obtained respectively as

$$\frac{d}{dx^*}\left\{h^{*3}\frac{dp_0^*}{dx^*}\right\}=6\frac{dh^*}{dx^*}, \tag{7.3.10}$$

$$\frac{d}{dx^*}\left\{h^{*3}\frac{dp_0^*}{dx^*}+\frac{3}{20}h^{*5}\left(\frac{dp_0^*}{dx^*}\right)^3\right\}=0. \tag{7.3.11}$$

Integrating equations (7.3.10) and (7.3.11) subject to the boundary conditions (7.3.8) yield the solution for zeroth order and first order film pressure p_0^* and p_1^*.Then the film pressure p^* for small values of the non-linear parameter can be obtained by

$$p^*=\frac{6\left(H_{B1}H_A-H_{A1}H_B\right)}{H_{B1}}+\frac{162}{5}\frac{\beta}{\left(H_{B1}\right)^3}\left(\frac{H_BH_{C1}-H_CH_{B1}}{H_{B1}}\right) \tag{7.3.12}$$

where, $H_{A1} = H_A(x^* = -1), \quad H_{B1} = H_B(x^* = -1), \quad H_{C1} = H_C(x^* = -1),$

$$H_A\left(x^*\right) = \int_0^{x^*} \frac{1}{h^{*2}} dx^*, \tag{7.3.13}$$

$$H_B\left(x^*\right) = \int_0^{x^*} \frac{1}{h^{*3}} dx^*, \tag{7.3.14}$$

$$H_C\left(x^*\right) = \int_0^{x^*} \frac{\left(H_{B1}h^* - H_{A1}\right)^3}{h^{*7}} dx^*, \tag{7.3.15}$$

Integration of the fluid film pressure over the film region gives the load carrying capacity

$$W = -\int_0^{-L} p dx . \tag{7.3.16}$$

The non-dimensional load carrying capacity is given by

$$W^* = \frac{Wh_2^2}{\mu UL^2} = -\int_0^{-1} p^* dx^* \tag{7.3.17}$$

$$W^* = -\int_0^{-1} \left\{ \frac{6\left(H_{B1}.H_A - H_{A1}H_B\right)}{H_{B1}} + \frac{162}{5}\frac{\beta}{(H_{B1})^3}\left(\frac{H_B H_{C1} - H_C H_{B1}}{H_{B1}}\right)\right\} dx^* \tag{7.3.18}$$

$$W^* = \left[\left\{\frac{6\left(H_{A1}.G_B - H_{B1}G_A\right)}{H_{B1}}\right\} + \frac{162}{5}\frac{\beta}{(H_{B1})^3}\left\{\frac{H_{B1}G_C - H_{C1}G_B}{H_{B1}}\right\}\right] \tag{7.3.19}$$

where $G_A = \int_0^{-1}\int_0^{x^*} \frac{1}{h^{*2}}\left(dx^*\right)\left(dx^*\right),$

$$G_B = \int_0^{-1}\int_0^{x^*} \frac{1}{h^{*3}}\left(dx^*\right)\left(dx^*\right) \quad \text{and}$$

$$G_C = \int_0^{-1}\int_0^{x^*} \frac{\left(H_{B1}h^* - H_{A1}\right)^3}{h^{*7}}\left(dx^*\right)\left(dx^*\right).$$

The frictional force can be obtained by integrating the shear stress acting upon the sliding surface

$$f_f = -\int_{x=0}^{-L} \left(\tau_{xz}\right)_{at\ z=0} dx \ , \tag{7.3.20}$$

which is in the non-dimensional form

$$F_f = \frac{f_f h_2}{\mu UL} = -\int_{x^*=0}^{-1} \left\{ \frac{1}{h^*} + \frac{1}{2} h^* \frac{\partial p^*}{\partial x^*} - \frac{\beta}{16} h^* \left(\frac{\partial p^*}{\partial x^*} \right)^2 \right\} dx^* \ . \tag{7.3.21}$$

The coefficient of a friction is calculated by

$$C = F_f / W^* . \tag{7.3.22}$$

7.4 Results and discussion

In the present study, the non-Newtonian effects of Rabinowitsch fluid on the steady state characteristics of exponential slider bearing are analyzed. Figure 7.2 shows the variation of dimensionless steady state film pressure with the coordinate x^* for different values of non linear factor β. It is observed that, dimensionless pressure decreases as β increases from -0.1 to 0.1.i.e.on comparison with the Newtonian case, the dimensionless pressure decreases with the pseudoplaticity and increases with the dilatants nature of the lubricant. It is observed that, at the inlet and outlet of the bearing the pressure p^* is zero. As the value of x^* increases the pressure also increases. The critical value of x^* is observed at which p^* attains its maximum.

The variation of non-dimensional steady load carrying capacity W^* with the profile parameter δ for different values of nonlinear factor β is shown in the Figure 7.3.It is observed that dimensionless load carrying capacity increases with the increase in δ and decreases thereafter. It is further observed that, the load carrying capacity for $\beta = -0.1$ (Dilatants lubricants) is higher than that of the Newtonian case ($\beta = 0$) and for $\beta = 0.1$ (pseudoplastic lubricant) it is less than the Newtonian case. Also, on comparison with the Newtonian case, the deviation of load capacity due to pseudoplaticity and Dilatant effect is significant when $\delta > 0.7$.

Figure 7.4 show the variation of non-dimensional friction force verses the profile parameter δ for different values of nonlinear factor β. It is observed that, dimensionless friction force decreases as profile parameter δ increases. It is further observed that, the friction force for $\beta = -0.08$ (Dilatants lubricants) is higher than that in the Newtonian case ($\beta = 0$) and for $\beta = 0.08$ (pseudoplastic lubricant) it is less than the Newtonian case.

Figure 7.5 presents the variation of frictional coefficient C with profile parameter δ for different values of nonlinear factor β . It is found that the frictional coefficient decreases with increasing values of profile parameter δ. Further, it is also observed a higher value of C for pseudoplastic lubricant as compared to the Newtonian case but reverse trend is observed for dilatant lubricants.

7.5 Conclusions

On the basis of Rabinowitsch fluid model, this chapter predicts the non-Newtonian effects on the exponential slider bearings. The modified Reynolds equation for the steady state characteristics has been obtained. The modified Reynolds equations have been solved by using small perturbation technique. The results are in well agreement with Newtonian results for $\beta = 0$.

Based on the results, so obtained the following conclusions have been drawn.

1. Load carrying capacity and fluid film pressure increases with dilatants lubricants, and decreases with respect to the pseudoplastic lubricants.
2. Load carrying capacity increases with profile parameter δ up to $\delta \approx 0.9$ and decreases thereafter.
3. The non-dimensional friction force F_f increases (decreases) as δ decreases (increases).
4. Coefficient of friction C is less for dilatant lubricants as compared with Newtonian lubricants.
5. As compared with the Newtonian lubricants, the load carrying capacity and friction force is more for dilatants lubricants and less for pseudoplastic lubricants

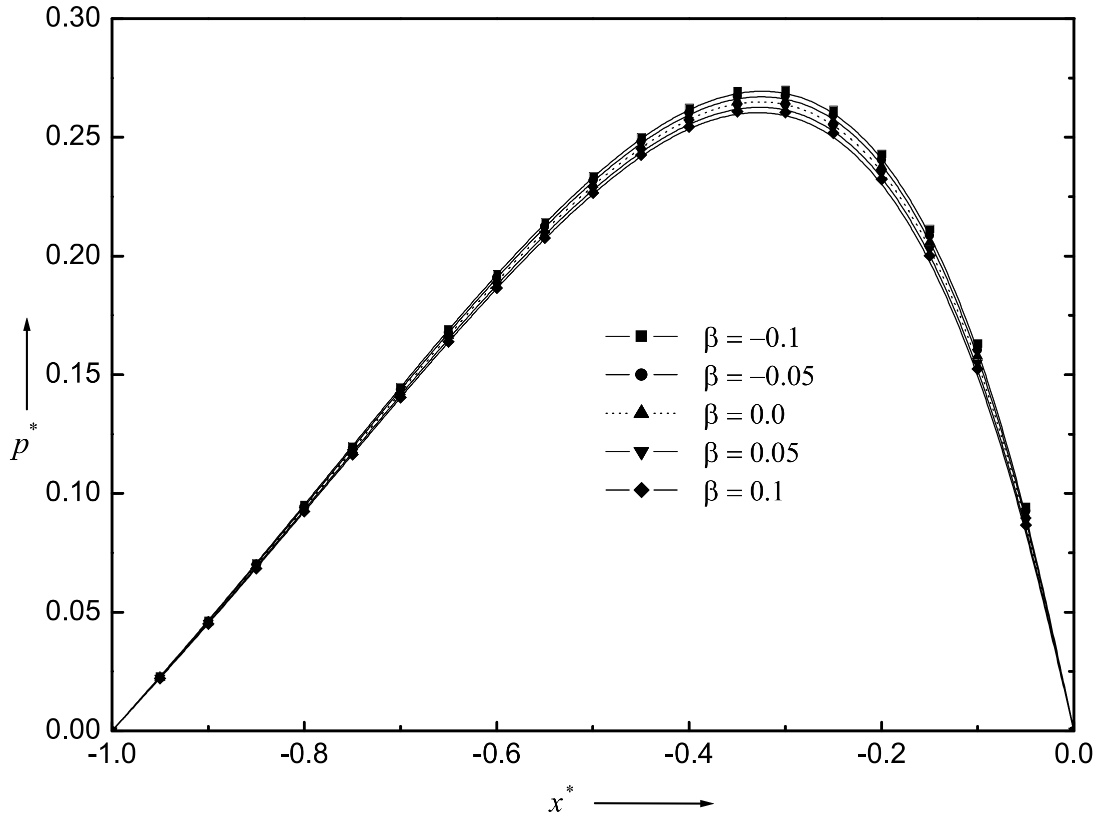

Figure 7.2 Film pressure p^* verses the coordinate x^* for different values of β

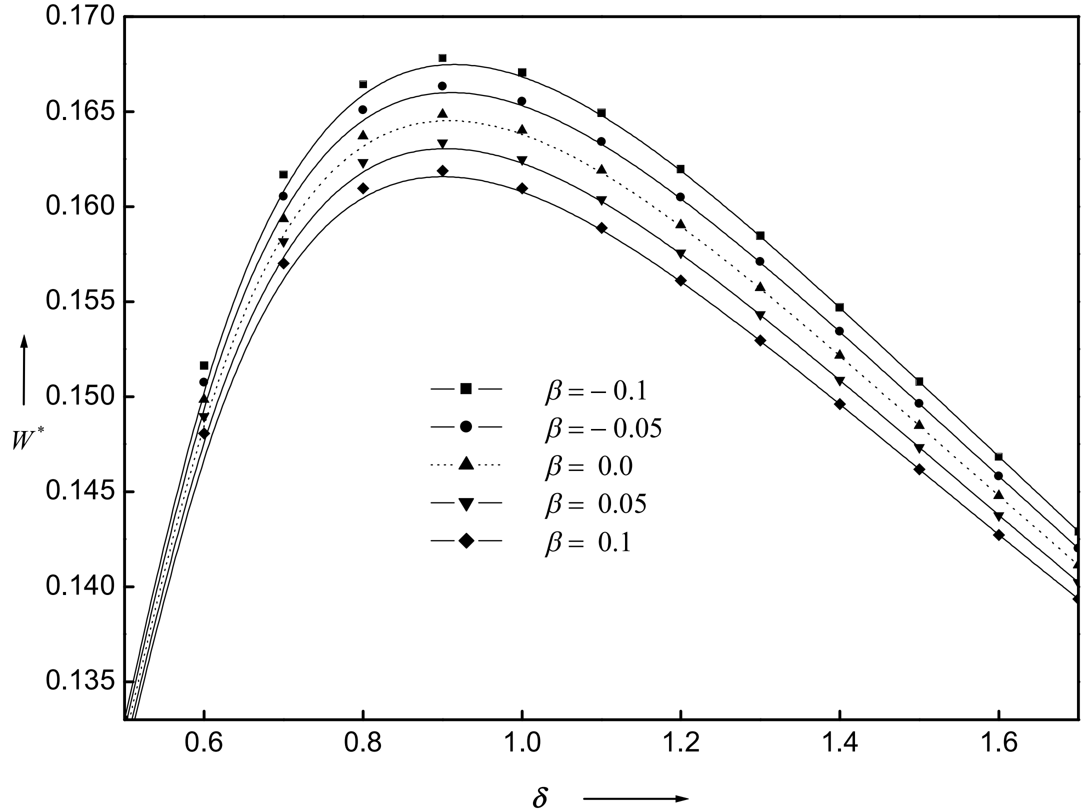

Figure 7.3 Load carrying capacity W^* profile parameter δ for different values of β

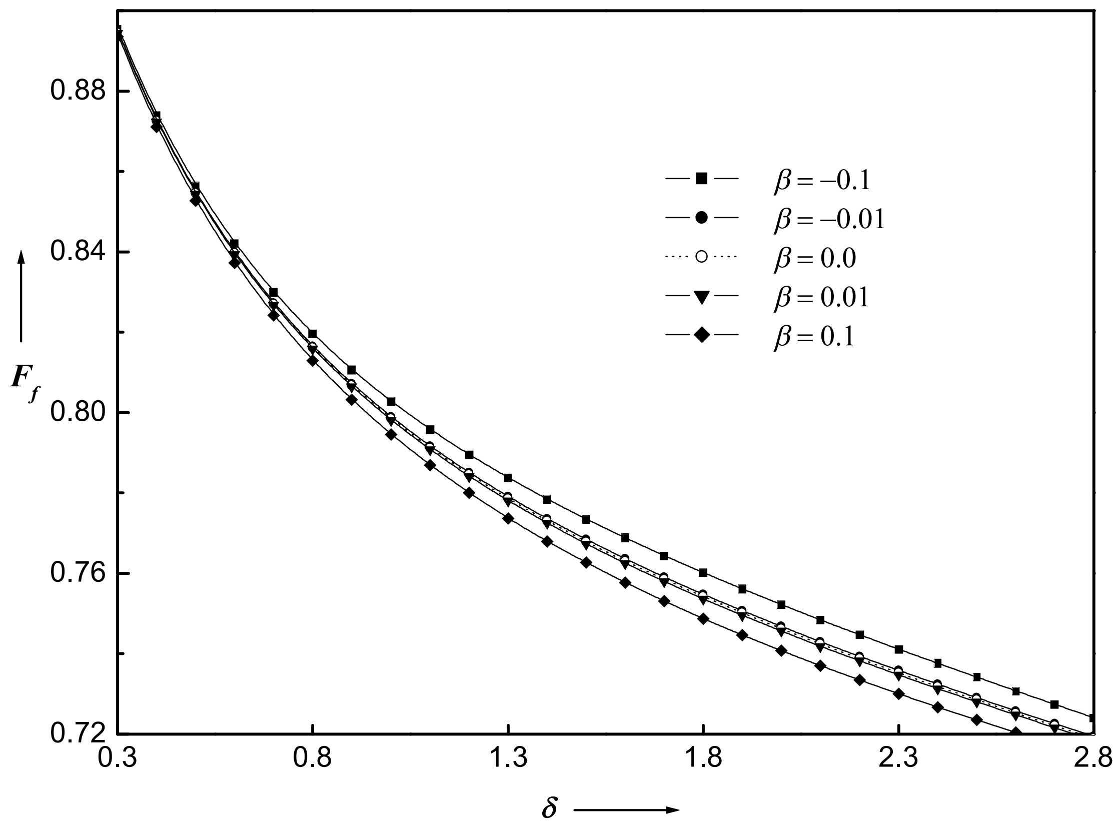

Figure 7.4 Friction force F_f verses the inlet- outlet height ratio *r* for different β

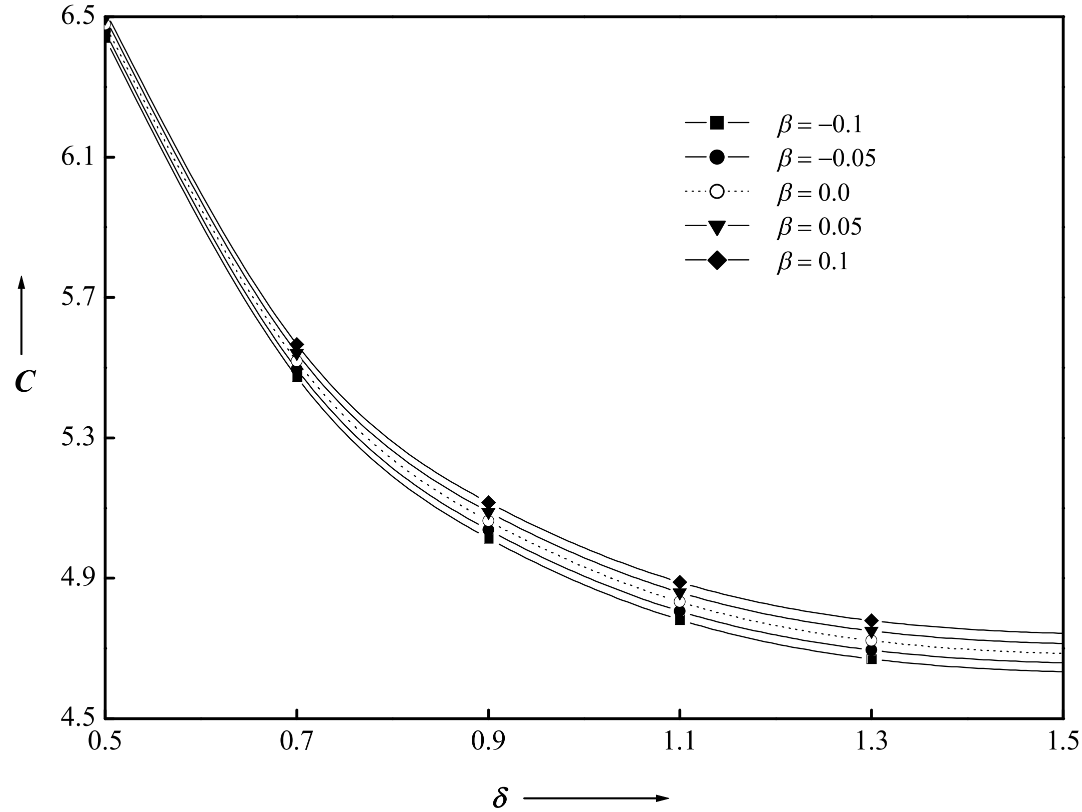

Figure 7.5 Variation of C verses profile parameter δ for different values of β