*Chapter – V**

On the Study of Rayleigh Step Slider Bearings Lubricated with Non-Newtonian Rabinowitsch Fluid

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5.1 Introduction

In recent studies, the problems of slider bearings have received considerable attention as these have wide application and are amenable to easy mathematical analysis. To ensure that no contact occurs between the opposing surfaces The Rayleigh step bearing was designed in the year 1918 by Lord Rayleigh. He was the first person to consider the concept of optimizing the design in lubrication applications and obtained an optimum design for an infinite-length stepped bearing using variation technique. Auloge, *et. al.* (1983) studied the optimum design of Rayleigh step bearings and determined the relationships between step location and height along with non-Newtonian lubricants. The same method was used by Fillon and Khonsari (1996) in tracing design charts for tilting-pad journal bearings. Jianming and Gaobing (1989) have presented the optimum design of a one dimensional Rayleigh step bearing with non-Newtonian lubricants. Tello (2003) has theoretically studied the regularity of the solution to the Reynolds equation in Rayleigh step-type bearings for both compressible and incompressible fluids by employing a rigorous mathematical approach. Pascovici *et.al*. (2011) presented an experimental evidence of cavitations effects in a Rayleigh step slider. Venkateswarlu and Rodkiewicz (1981) discussed the thrust bearing characteristics considering the terminal speed of the slider. The performance of hydrodynamic slider bearings for non-Newtonian lubricants analyzed by Williams and Symmons (1987) and an experimental comparison of the slider bearing performance for different shapes was presented by Sharma and Pandey (2009). During the last few decades, the researchers have shown that the performances of the bearings can be improved and enhanced pressure and load carrying capacity obtained using of non-Newtonian lubricants (2009, 1990). Naduvinamani *et.al.* (2010) analyzed inclined stepped composite bearings with micropolar fluids. Recently, inclined stepped composite bearings with Rabinowitsch fluids was studied by Naduvinamani *et. al*. (2013) and they observed that, the highest load carrying capacity and lowest coefficient of friction is observed for inclined stepped bearing as compared to the other types different of bearings.

Rabinowitsch fluid model is one of the models to establish the non-linear relationship between the shear stress and shear strain rate which can be described for one dimensional fluid flow as given in (2.1.1), where in μ denotes the zero shear rate viscosity and α denotes the non linear factor which describes the non-Newtonian effects of the lubricant which will be referred to as coefficient of pseudo plasticity. For Newtonian fluids $\alpha = 0$, dilatant fluids for $\alpha < 0$ and pseudoplastic lubricants for $\alpha > 0$. The advantage of this model lies in the fact that the theoretical predictions for present model was verified with experimental findings by Wada and Hayashi (1971).

In this chapter, an attempt has been made to analyze the steady state characteristics of Rayleigh step slider bearing lubricated with Rabinowitsch fluid which has not been studied so for.

5.2 Mathematical formulation of the problem

Figure 5.1 shows the geometry and co-ordinate system of one dimensional Rayleigh step bearing. The lower surface moves with a velocity U in the x- direction. The sudden change in the film thickness generates a hydrodynamic pressure field that support an applied load. The fluid in the film region is taken as non-Newtonian

Rabinowitsch fluid. Rayleigh step bearing is the junction of two separate bearings of length L_1 in the region-I ($0 \le x \le L_1$) and L_2 in the region-II ($L_1 \le x \le L$).

The fluid film thickness *h(x)* is described by

$$h(x) = \begin{cases} h_1 & \text{in region} - I \quad (0 \le x \le L_1) \\ h_2 & \text{in region} - II \quad (L_1 \le x \le L) \end{cases} \tag{5.2.1}$$

The body forces and body couples are assumed to be negligible. Under the usual assumptions of hydrodynamic lubrication theory, equations of continuity and motion in Cartesian coordinates reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = 0, \tag{5.2.2}$$

$$\frac{\partial \tau_{xz}}{\partial z} = \frac{\partial p}{\partial x}, \tag{5.2.3}$$

$$\frac{\partial p}{\partial z} = 0. \tag{5.2.4}$$

where u and v are velocity components in x and z directions, p denotes the film pressure. The no slip boundary conditions are

$$u = U, \quad v = 0, \quad \text{at } z = 0; \tag{5.2.5a}$$

$$u = 0, \quad v = 0 \quad \text{at } z = h. \tag{5.2.5b}$$

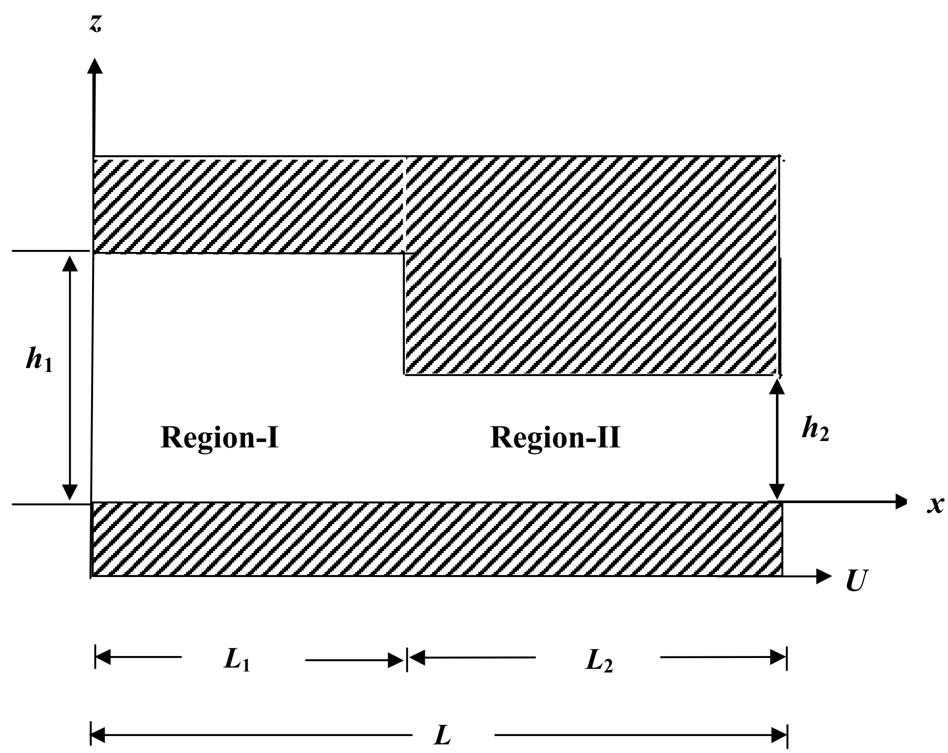

Figure 5.1 Physical geometry of Rayleigh step slider bearing

5.3 Solution of the problem

Integration of the equation (5.2.3) yields

$$\tau_{xz} = \frac{\partial p}{\partial x} z + c_1 \tag{5.3.1}$$

where c_1 is the constant of integration to be determined.

By substituting (5.3.1) in equation (2.1.1), the expression for the velocity component u is obtained in the form

$$u = \frac{1}{4\mu}\left[2\left(\frac{\partial p}{\partial x}\right) z^2 + 4c_1 z + \alpha \left(\begin{array}{l} \left(\frac{\partial p}{\partial x}\right)^3 z^4 + 4\left(\frac{\partial p}{\partial x}\right)^2 c_1 z + \\ 6\left(\frac{\partial p}{\partial x}\right) c_1^2 z^2 + 4c_1^3 z \end{array} \right) \right] + c_2 \tag{5.3.2}$$

Integration constants c_1 and c_2 can be obtained by using relevant boundary conditions given in equation (5.2.5a) and (5.2.5b) and substituting in the equation (5.3.2) and on simplifying, equation (5.3.2) takes the form

$$u = \frac{1}{4\mu}\left[G_1 \frac{\partial p}{\partial x} + \frac{1}{2}\alpha G_2 \left(\frac{\partial p}{\partial x}\right)^3 \right] + U \left\{ 1 - \frac{4z - \alpha G_3 \left(\frac{\partial p}{\partial x}\right)^2}{4h + \alpha h^3 \left(\frac{\partial p}{\partial x}\right)^2} \right\} \tag{5.3.3}$$

where the functions G_1, G_2 and G_3 are described by

$$G_1 = 2z^2 - 2hz,,$$

$$G_2 = 2z^2 - 4hz^3 + 3h^2 z^2 - h^3 z,$$

$$G_3 = 6hz^2 - 4z^2 - 3h^2 z.$$

Integrating the continuity equation (5.2.2) across the film height and utilizing the relevant boundary conditions (5.2.5a) and (5.2.5b), the modified Reynolds equation for film pressure is obtained in the form

$$\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}+\frac{3}{20}\alpha h^5\left(\frac{\partial p}{\partial x}\right)^3\right]=6\mu U\frac{\partial h}{\partial x}. \tag{5.3.4}$$

Introducing non-dimensional variables and parameters are introduced in to equation (5.2.9)

$$\left.\begin{aligned} &x^*=\frac{x}{L};\quad p^*=\frac{ph^2}{\mu UL};\quad \beta=\alpha\frac{\mu^2U^2}{h_2^2};\\ &h_1^*=\frac{h_1}{h_2};\quad h^*=\frac{h}{h_2}\end{aligned}\right\} \tag{5.3.5}$$

where β describes the non-linear parameter of non-Newtonian fluids. The non-dimensional form of modified Reynolds equation is obtained in the form

$$\frac{\partial}{\partial x^*}\left[h^{*3}\frac{\partial p^*}{\partial x^*}+\frac{3}{20}\beta h^{*5}\left(\frac{\partial p^*}{\partial x^*}\right)^3\right]=6\frac{\partial h^*}{\partial x^*}. \tag{5.3.6}$$

The solution of equation (5.3.6) for the Rayleigh step bearing can be obtained by dividing the fluid film regions into Region-I ($0 \le x \le L_1$) and Region-II ($L_1 \le x \le L$). The non-dimensional Reynolds equation (5.3.6) is non-linear differential equation. The pressure in region-I and region-II can be obtained by applying small perturbation method.

$$p_i^*=p_{i0}^*+\beta p_{i1}^*,\quad i=1,2; \tag{5.3.7}$$

$$p_1^*=p_{10}^*+\beta p_{11}^*,\quad h_i^*=h_1^* \text{ In region- I } (0\le x^*\le L_1^*); \tag{5.3.8}$$

$$p_2^*=p_{20}^*+\beta p_{21}^*,\quad h_i^*=h_2^*=1 \text{ in region- II } (L_1^*\le x^*\le 1). \tag{5.3.9}$$

Substituting these in to equation (5.3.6) the non-dimensional Reynolds equation is obtained in region- I ($0 \le x^* \le L_1^*$) as

$$\frac{d}{dx^*}\left\{h_1^{*3}\frac{dp_{10}^*}{dx^*}\right\} = 6\frac{dh_1^*}{dx^*}, \tag{5.3.10}$$

$$\frac{d}{dx^*}\left\{h_1^{*3}\frac{dp_{10}^*}{dx^*} + \frac{3}{20}h_1^{*5}\left(\frac{dp_{11}^*}{dx^*}\right)^3\right\} = 0\ ; \tag{5.3.11}$$

and in region- II ($L_1^* \le x^* \le 1$) as

$$\frac{d}{dx^*}\left\{h_2^{*3}\frac{dp_{20}^*}{dx^*}\right\} = 6\frac{dh_2^*}{dx^*}, \tag{5.3.12}$$

$$\frac{d}{dx^*}\left\{h_2^{*3}\frac{dp_{20}^*}{dx^*} + \frac{3}{20}h_2^{*5}\left(\frac{dp_{21}^*}{dx^*}\right)^3\right\} = 0. \tag{5.3.13}$$

The relevant pressure boundary conditions are

$$p^* = 0 \quad \text{at} \quad x^* = 1 \quad \text{and} \quad x^* = 0 \tag{5.3.14}$$

$$p_1^* = p_2^* \quad \text{at} \quad x^* = L_1^* \tag{5.3.15}$$

Solving the equations (5.3.10), (5.3.11), (5.3.12) and (5.3.13) and using the boundary conditions (5.3.14), (5.3.15), the expressions for the non-dimensional pressure are obtained as

$$p_{10}^* = \left(\frac{6h_1^* + C_1}{h_1^{*3}}\right)x^*\ ; \tag{5.3.16a}$$

$$p_{11}^* = \left(\frac{6h_2^* + C_3}{h_2^{*3}}\right)(x^* - 1)\ ; \tag{5.3.16b}$$

$$p_{20}^{*}=\left\{\frac{C_5}{h_1^{*3}}-\frac{3}{20}\frac{\left(6h_1^{*}+C_1\right)^3}{h_1^{*7}}\right\}x^{*}; \tag{5.3.17a}$$

$$p_{21}^{*}=\left\{\frac{C_7}{h_2^{*3}}-\frac{3}{20}\frac{\left(6h_2^{*}+C_1\right)^3}{h_2^{*7}}\right\}\left(x^{*}-1\right); \tag{5.3.17b}$$

substituting these in equations (5.3.8) and (5.3.9), the non-dimensional pressure in region-I and region-II are obtained respectively in the form

$$p_1^{*}=\left(\frac{6h_1^{*}+A}{h_1^{*3}}\right)x^{*}+\beta\left\{\frac{B}{h_1^{*3}}-\frac{3}{20}\frac{(6h_1^{*}+A)^3}{h_1^{*7}}\right\}x^{*} \quad \text{and} \tag{5.3.18}$$

$$p_2^{*}=(6+A)(x^{*}-1)+\beta\left\{B-\frac{3}{20}(6+A)^3\right\}(x^{*}-1) \ , \tag{5.3.19}$$

where

$$A=-\frac{6h_1^{*}\left\{h_1^{*2}\left(1-L_1^{*}\right)+L_1^{*}\right\}}{\left\{L_1^{*}+h_1^{*3}\left(1-L_1^{*}\right)\right\}}, \tag{5.3.20a}$$

$$B=\frac{3}{20}\frac{(6+A)^3 h_1^{*7}\left(1-L_1^{*}\right)+\left(6h_1^{*}+A\right)^3 L_1^{*}}{h_1^{*4}\left\{L_1^{*}+\left(1-L_1^{*}\right)h_1^{*3}\right\}}. \tag{5.3.20b}$$

The non- dimensional load carrying capacity is W^* is obtained in the form

$$W^{*}=\int_0^{L_1^{*}} p_1^{*}dx^{*}+\int_{L_1^{*}}^{1} p_2^{*}dx^{*} \ . \tag{5.3.21}$$

The use of expressions for p_1^*, p_2^* given in equations (5.3.18) and (5.3.19) in equation

(5.3.21) and integrating with respect to x^*, gives the non- dimensional load carrying capacity in the form

$$W^* = \left\{ \left(\frac{6h_1^* + A}{h_1^{*3}}\right)\frac{L_1^{*2}}{2} - (6+A)\frac{(1-L_1^*)^2}{2} + \beta\left[\left\{\frac{B}{h_1^{*3}} - \frac{3}{20}\frac{\left(6h_1^* + A\right)^3}{h_1^{*7}}\right\}\frac{L_1^{*2}}{2} + \left\{B - \frac{3}{20}(6+A)^3\right\}\frac{(1-L_1^*)^2}{2}\right] \right\}, \tag{5.3.22}$$

where the values of A and B are as given in equation (5.4.20a) and equation (5.4.20b).

The frictional force, F can be obtained by integrating the shear stress acting upon the sliding surface

$$F = \int_{x=0}^{L} \left(\tau_{xy}\right)_{z=0} dx, \tag{5.3.23}$$

which in the non-dimensional form obtained as

$$F^* = -\frac{f_f h_2}{\mu UL} = \int_0^{L_1^*} \left\{ \frac{1}{h_1^*} + \frac{h_1^*}{2}\frac{\partial p_1^*}{\partial x^*} - \frac{\beta h_1^*}{16}\left(\frac{\partial p_1^*}{\partial x^*}\right)^2 \right\} dx^* + \int_{L_1^*}^{1} \left\{ 1 + \frac{1}{2}\frac{\partial p_2^*}{\partial x^*} - \frac{\beta}{16}\left(\frac{\partial p_2^*}{\partial x^*}\right)^2 \right\} dx^*, \tag{5.3.24}$$

$$F^* = -\frac{f_f h_2}{\mu UL} = \left\{ \frac{1}{h_1^*} + \frac{h_1^*}{2}\frac{\partial p_1^*}{\partial x^*} - \frac{\beta h_1^*}{16}\left(\frac{\partial p_1^*}{\partial x^*}\right)^2 \right\} L_1^* + \left\{ 1 + \frac{1}{2}\frac{\partial p_2^*}{\partial x^*} - \frac{\beta}{16}\left(\frac{\partial p_2^*}{\partial x^*}\right)^2 \right\}\left(1 - L_1^*\right). \tag{5.3.25}$$

The coefficient of a friction is calculated by

$$C = \frac{F^*}{W^*} \tag{5.3.26}$$

5.4 Results and discussions

The non-Newtonian effects of Rabinowitsch fluid on the performance of Rayleigh step slider bearing are analyzed. The values of the non-dimensional parameters are chosen as β = - 0.1, - 0.05, 0, 0.05, 0.1; h_1^* = 1.1-2.0; $L_1^* = 0.4 - 0.9$ for the numerical computations of the results

5.4.1 Fluid film pressure

Figure 5.2 shows the variation of non-dimensional pressure p^* with x^* for different values of non-linear parameter, β with $L_1^* = 0.6$ and h_1^* =1.4. As the values of β increases, the pressure decreases. It is observed that, the larger fluid film pressure is attained for the dilatant fluids ($\beta < 0$) as compared with pseudoplastic fluids ($\beta > 0$). The variation of non-dimensional maximum pressure p_{max}^* with L_1^* for different values of β with h_1^* = 1.4 is depicted in Fig.5.3. It is important to note that, p_{max}^* attains its maximum for $0.6 \le L_1^* \le 0.73$ and $-0.1 \le \beta \le 0.1$.

5.4.2 Load carrying capacity

The variation of non-dimensional load carrying capacity, W^* with h_1^* for different values of β with L_1^* = 0.6 is shown Fig. 5.4. It is observed that, the load carrying capacity increases with increasing values of h_1^*, which attains its maximum for h_1^* =1.56 and decreases thereafter. Also it is observed that W^* is larger for dilatants fluids (β < 0) as compared with Newtonian case (β = 0). However, for pseudoplastic fluids (β > 0) load carrying capacity is less as compared with Newtonian case. (β = 0). Figure 5.5 depicts the variation of non-dimensional load carrying capacity W^* with L_1^* for different values of β with film height ratio h_1^* =2. It is observed that, as the values of β increases the load carrying capacity decreases. Table 5.1.shows the variation of non-dimensional load carrying capacity W^* with h_1^* for different values of L_1^* and with β = 0.1, 0.0 and -0.1. Clearly it is observed that, the load carrying capacity decreases with increasing values of L_1^*. Also it is observed that, for every value of L_1^* load carrying capacity is larger for dilatants as compared to pseudoplastic fluids. Further, it is observed that, the maximum load carrying capacity for pseudoplastic fluid is observed at h_1^* = 1.8 and L_1^*= 0.7 and for dilatants at h_1^* =1.9 and L_1^*= 0.72. Hence, the optimum load carrying capacity for the pseudoplastic lubricants is observed at slightly lower values of h_1^* and L_1^* as compared to the corresponding Newtonian lubricants (β = 0).

5.4.3 Frictional force and coefficient of friction

The variation of non-dimensional frictional force F^* with h_1^* is depicted in the Fig. 5.6 with fixed values of L_1^*=0.6. It is observed that, F^* is larger for dilatant fluids as compared to the pseudoplastic fluids. Figure 5.7 shows that the effect of non-linear parameter β on the variations of F^* with L_1^*. It is observed that, F^* decreases for increasing values of L_1^*.

The effect of Rabinowitsch fluid on the variations of the coefficient of friction C with h_1^* is shown in Figure 5.8 for the fixed value of L_1^* = 0.6. The dotted curve in the graph corresponds to the Newtonian case. The coefficient of friction C decreases for increasing values of h_1^* even though the frictional force is more for dilatants as compared with pseudoplastic fluids. The variation of C with L_1^* for different values of β for a fixed value of h_1^* = 1.4 is shown in Figure 5.9. It is observed that, as the values of L_1^* increases the values of C decreases. Further, it is observed that, the Rayleigh step bearing with L_1^* = 0.7 gives minimum C for pseudoplastic fluids.

The relative percentage of the non-dimensional load carrying capacity R_{W^*} non-dimensional frictional force R_{F^*} and coefficient of friction R_C are defined by

$$R_{W^*} = \left\{ \left(W^*_{Rabinowitsch} - W^*_{Newtonian} \right) / W^*_{Newtonian} \right\} \times 100$$

$$R_{F^*} = \left\{ \left(F^*_{Rabinowitsch} - F^*_{Newtonian} \right) / F^*_{Newtonian} \right\} \times 100 \quad \text{and}$$

$$R_C = \left\{ \left(C_{Rabinowitsch} - C_{Newtonian} \right) / C_{Newtonian} \right\} \times 100$$

The values of R_{W^*} R_{F^*} and R_C are computed and listed in the Table 5.2 for different values of L_1^* and β .It is clear that, as the values of L_1^* and β increases, R_{W^*} R_{F^*} and R_C decreases. It is interesting to note that, nearly the significant increase in W^* is observed for pseudoplastic fluids $\beta = -0.1$ for the optimum step length L_1^*=0.7 with a marginal increase in C.

5.5 Conclusions

On the basis of Rabinowitsch fluid model, this chapter predicts the non-Newtonian effects of Rabinowitsch fluid on Rayleigh step slider bearings. On the basis of the results obtained, the following conclusions can be drawn:

1. The load carrying capacity, frictional force and coefficient of friction of Rayleigh step slider bearing are functions of non-dimensional non-linear factor β of the lubricant.
2. There is a significant increase in the load carrying capacity, and a marginal increase in coefficient of friction is observed for dilatant fluids as compared to the corresponding Newtonian lubricants. However, the reverse trend is observed for pseudoplastic lubricants.
3. The relative load R_{W^*}, the relative frictional force R_{F^*} and relative coefficient of friction R_C are found to be functions of L_1^* and β .
4. The optimum load carrying capacity and minimum coefficient of friction are found to occur for the slightly smaller values of L_1^* for the dilatant lubricants as compared to Newtonian and pseudoplastic lubricants (Table-5.1).

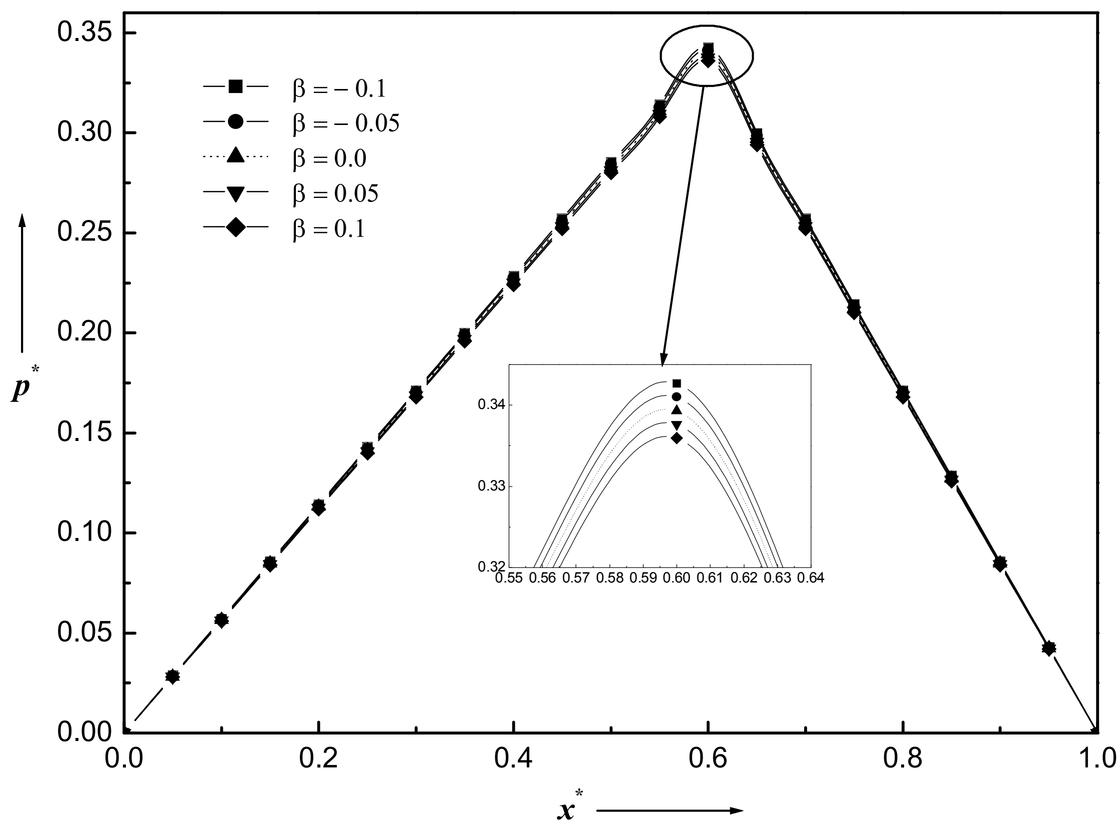

Figure 5.2 Variation of non-dimensional pressure p^* with x^* for different values of β with $h_1^* = 1.4$ and L_1^*=0.6.

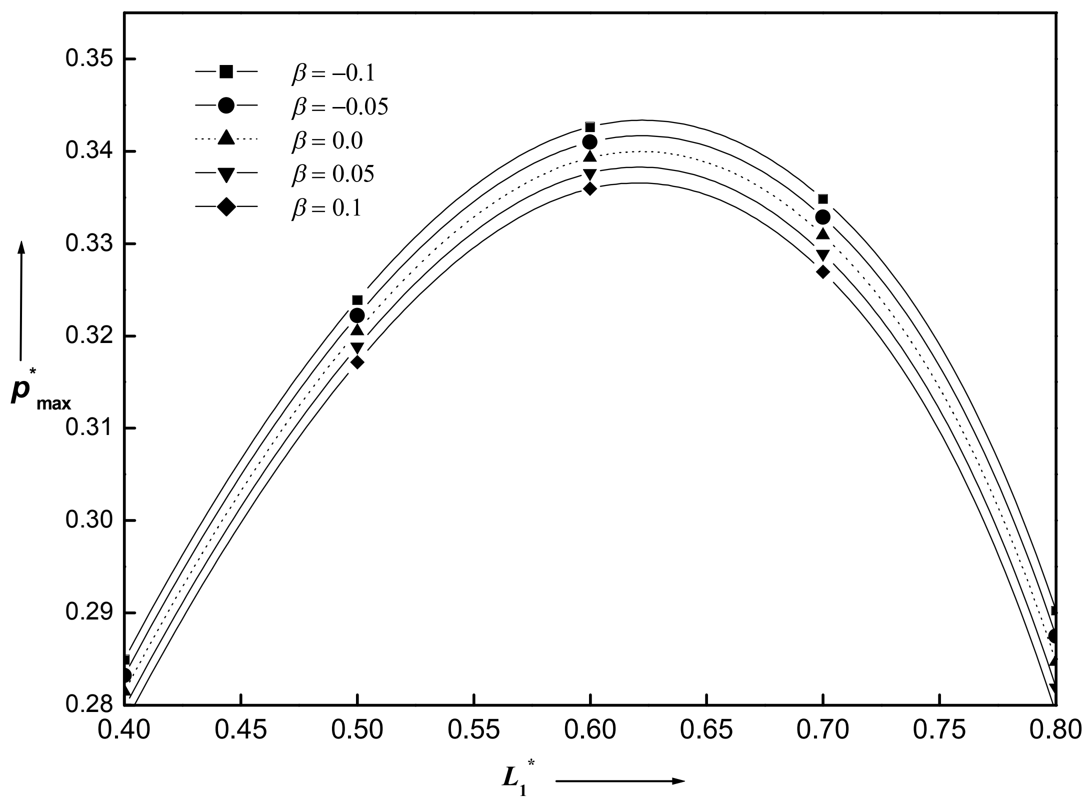

Figure 5.3 Variation of non-dimensional pressure p^*_{max} with L_1^* for different values of β with h_1^*=1.4.

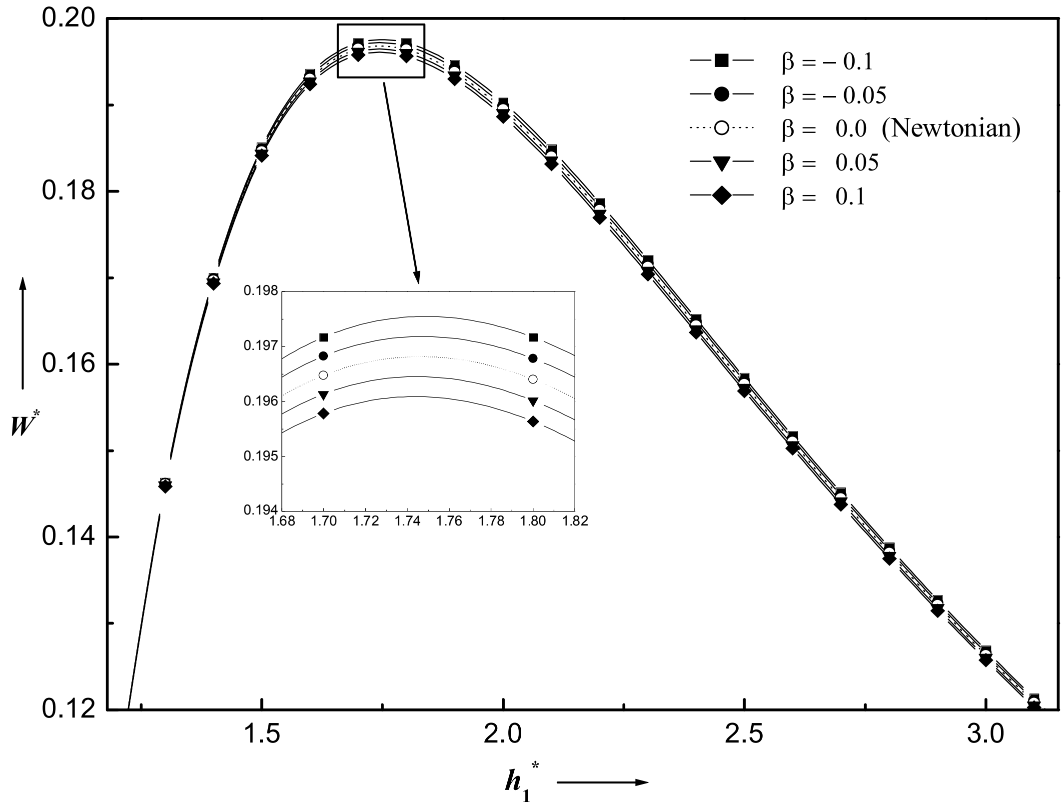

Figure 5.4 Variation of non-dimensional load carrying capacity W^* with h_1^* for different values of β with L_1^*=0.6.

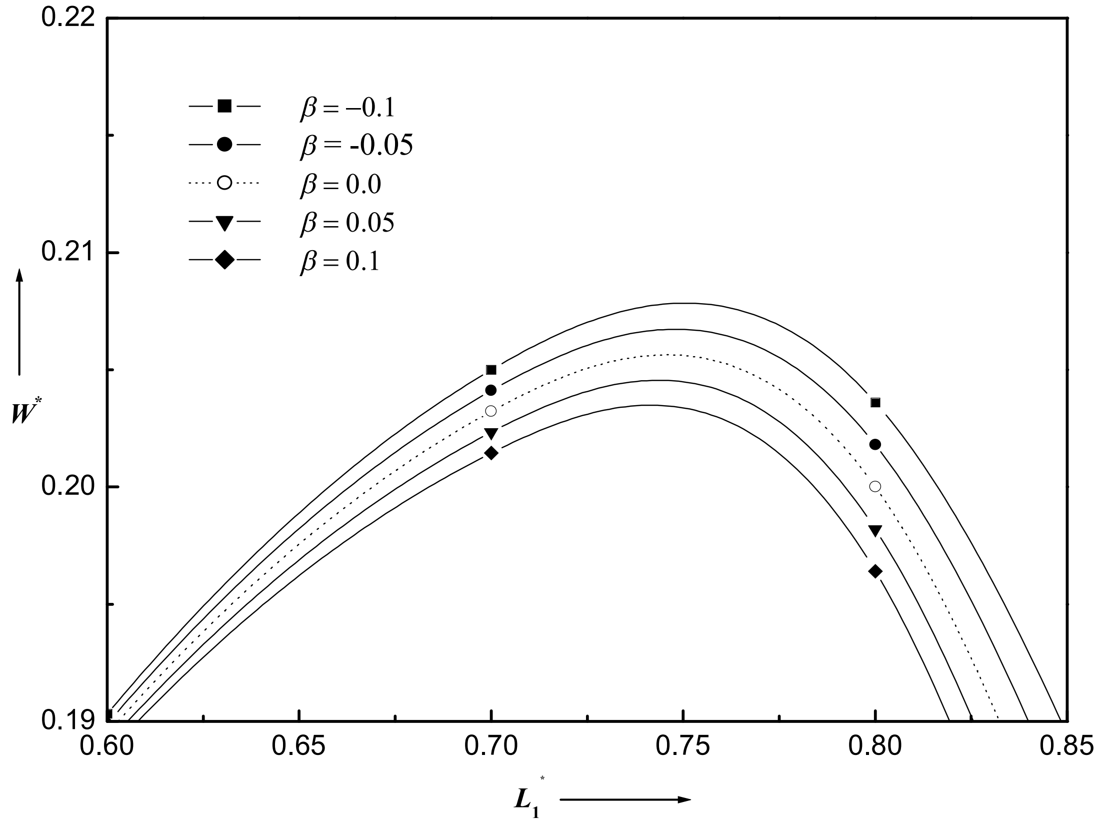

Figure 5.5 Variation of non-dimensional load carrying capacity W^* with L_1^* for different values of β with h_1^*=2.

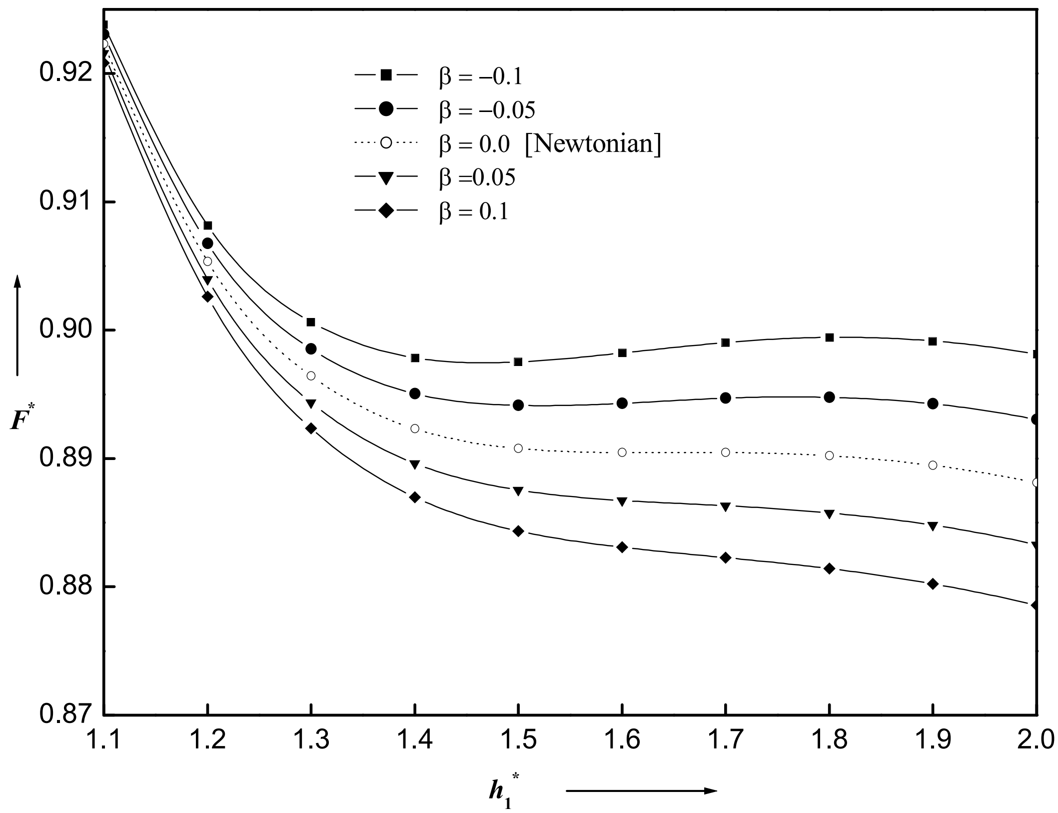

Figure 5.6 Variation of non-dimensional frictional force F^* with h_1^* for different values of β with L_1^*=0.6.

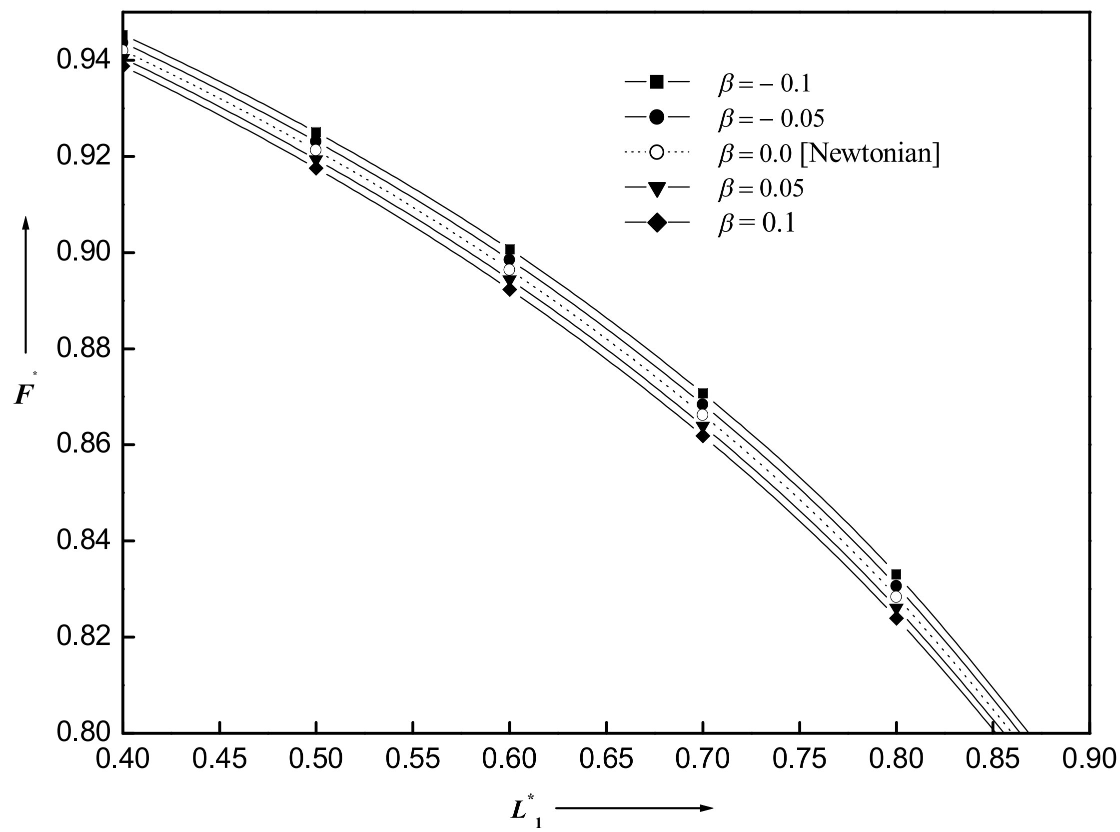

Figure 5.7 Variation of non-dimensional frictional force F^* with L_1^* for different values of β with h_1^* =1.4.

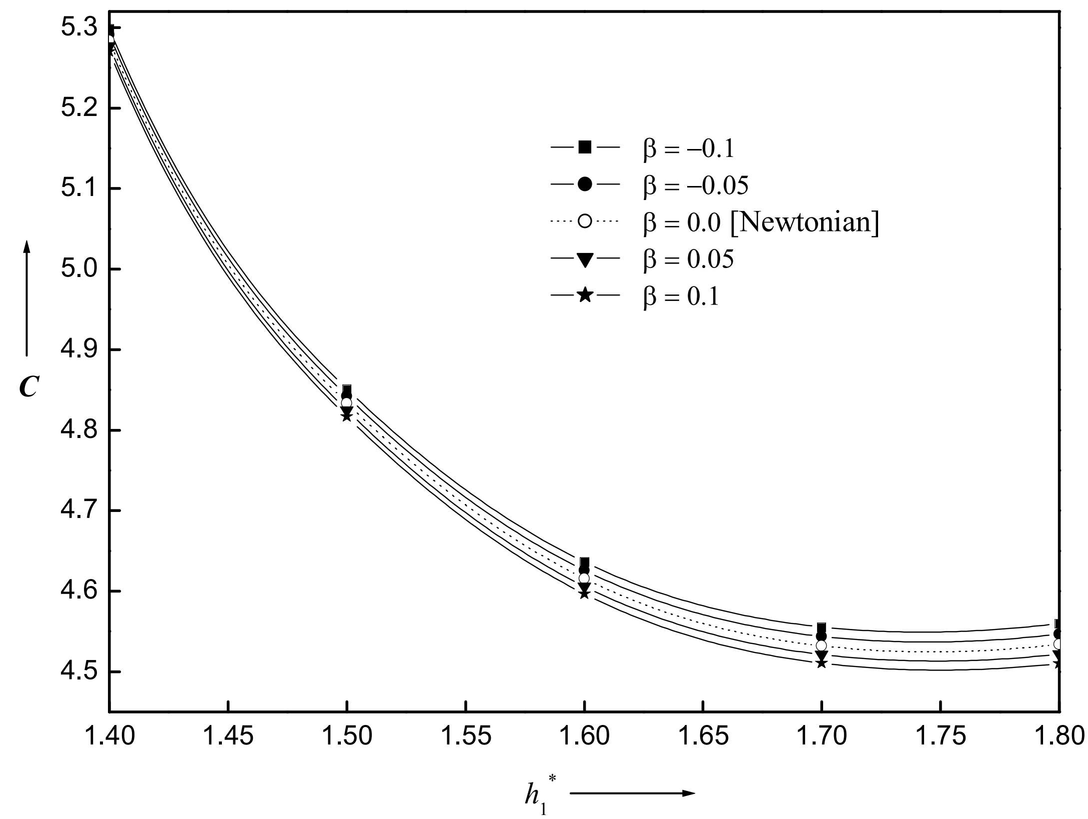

Figure 5.8 Variation of C with h_1^* for different values of β with L_1^*=0.6.

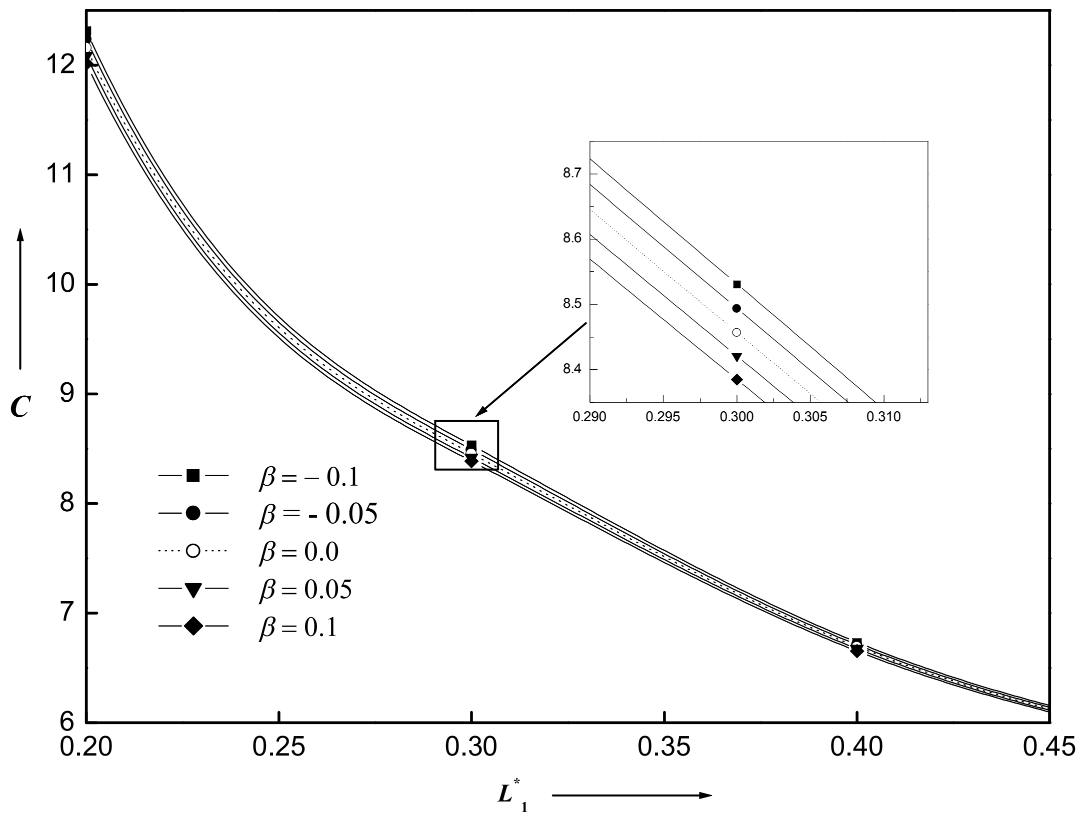

Figure 5. 9 Variation of C with L_1^* for different values of β with h_1^*=1.4.

Table 5.1 Variation of non-dimensional load carrying capacity W^* with h_1^* for different L_1^* and with β =0.1, 0.0 and -0.1

Fluid	h_1^*	$L_1^*=0.65$	$L_1^*=0.68$	$L_1^*=0.7$	$L_1^*=0.71$	$L_1^*=0.72$	$L_1^*=0.73$	$L_1^*=0.74$
$\beta=0.1$ = β (Psuedoplastic)	1.4	0.1693156	0.1669147	0.1646513	0.1632369	0.1616248	0.1598078	0.1577784
	1.5	0.1841435	0.1845233	0.1828577	0.1817174	0.1803596	0.1787746	0.1769517
	1.6	0.1924015	0.1956044	0.1946471	0.1938538	0.1928358	0.191581	0.1900764
	1.7	0.1957831	0.2015794	0.2013432	0.2009171	0.200269	0.1993853	0.198251
	1.8	0.1956323	0.2036593	**0.204097**	0.2040239	0.2037388	0.203227	0.2024726
	1.9	0.1929864	0.2028386	0.203871	0.2041171	0.2041656	0.2040017	0.2036091
	2	0.1886294	0.1999119	0.2014473	0.2019694	0.2023107	0.2024568	0.2023916
	2.1	0.1831446	0.1955001	0.1974464	0.1981989	0.1987883	0.1992009	0.1994213
	2.2	0.17696	0.1900804	0.1923517	0.1932911	0.1940847	0.1947198	0.195182
$\beta=0.0$ (Newtonian)	1.4	0.1695231	0.1675909	0.1654412	0.1640899	0.1625457	0.1608017	0.1588507
	1.5	0.1863481	0.1854545	0.1839416	0.1828868	0.1816216	0.1801371	0.1784235
	1.6	0.1965348	0.1967529	0.1959768	0.1952852	0.1943778	0.1932437	0.1918713
	1.7	0.2016206	0.2028986	0.2028612	0.2025464	0.2020194	0.2012682	0.2002796
	1.8	0.2028835	0.2051024	0.2057479	0.2057902	0.2056304	0.2052555	0.2046518
	1.9	0.2013505	0.2043633	0.2056061	0.2059679	**0.206141**	0.2061133	0.20587
	2	0.1978261	0.2014815	0.2032258	0.2038614	0.2043243	0.2046021	0.2046809
	2.1	0.1929279	0.1970841	0.1992353	0.2000978	0.2008041	0.2013422	0.2016989
	2.2	0.187123	0.1916543	0.1941249	0.19517	0.196075	0.1968288	0.1974188
$\beta=-0.1$ (Dilatants)	1.4	0.1700546	0.1682671	0.166231	0.1649429	0.1634666	0.1617956	0.159923
	1.5	0.1870858	0.1863858	0.1850255	0.1840561	0.1828836	0.1814996	0.1798952
	1.6	0.1974525	0.1979014	0.1973065	0.1967165	0.1959197	0.1949064	0.1936662
	1.7	0.2026832	0.2042177	0.2043793	0.2041757	0.2037698	0.2031511	0.2023082
	1.8	0.2040542	0.2065456	0.2073988	0.2075566	0.2075219	0.207284	0.206831
	1.9	0.2025941	0.205888	0.2073412	0.2078187	0.2081168	**0.208225**	0.2081309
	2	0.1991112	0.2030511	0.2050043	0.2057534	0.206338	0.2067474	0.2069701
	2.1	0.194228	0.1986682	0.2010242	0.2019967	0.2028199	0.2034835	0.2039765
	2.2	0.1884164	0.1932281	0.195898	0.1970489	0.1980654	0.1989379	0.1996557

Table 5.2. The variation of relative load (R_{W^*}), relative frictional force (R_{F^*}) and relative coefficient of friction (R_C) for different values of β and different values of L_1^* with $h_1^* = 2$

L_1^*	β	R_{W^*}	R_{F^*}	R_C
0.6	- 0.1	0.445444	1.089745	0.447824
	- 0.05	0.222722	78.6508	0.219904
	0.05	-0.22272	-78.7456	-0.21254
	0.1	-0.44597	-1.03735	-0.41728
0.65	- 0.1	0.649561	1.185116	0.347353
	- 0.05	0.325033	0.585225	0.169517
	0.05	-0.32453	-0.57079	-0.1612
	0.1	-0.64956	-1.12749	-0.31385
0.7	- 0.1	0.874888	1.303406	0.206041
	- 0.05	0.437444	0.643323	0.097864
	0.05	-0.43744	-0.62703	-0.08778
	0.1	-0.87538	-1.23789	-0.1657