4.1 GENERAL

The investigation on the impact behaviour of delaminated composite materials is motivated by practical situations encountered in the aerospace industry and other fields in which these materials are used extensively. The collision of a foreign object like a bird or a hail-stone on the body of an aircraft is very common during landing and take-off, as well as level flight. The situation is critical when it induces significant internal damage, undetectable by visual inspection which causes the reduction of strength and stability of the structure. Composite structures also pose special problems due to low transverse shear modulus and low inter-laminar shear properties. The delaminated structures under impact loading are vulnerable to damage due to low transverse shear strength and hence it exhibits a new dynamic characteristics depending on size, location of the delamination and impact parameters. The transient response of delaminated composites subjected to localized contact loading is of great concern in many advanced engineering structures and components, such as, the leading edge of an aircraft wing, fan blades in jet engine and turbine blades, because of the fact that impact situations with energies far below the penetration levels can cause severe damage to components made of composite materials. The turbomachinery blades are under a preload resulting from centrifugal forces and as a consequence, the initial stresses may aggravate the delamination damage due to impact. Hence, attention is needed for in-depth study of transient response of delaminated composite pretwisted conical shells subjected to low velocity normal impact. Numerical results are obtained for the case of centrally impacted shells and parametric studies are performed in respect of angle of twist, rotational speeds, laminate configuration, velocity of the impactor and relative position of delamination.

4.2 NORMAL IMPACT RESPONSE

The centrifugal forces arising out of rotation induce the initial stresses in composite conical shells. The pretwist of the composite shell also causes coupling in both bending
directions. Moreover, the delaminated composite shells are prone to incur damage by impacts especially those normal to the plane of the laminate. Hence, reliable and accurate prediction of normal impact response of delaminated composite pretwisted conical shells covers a wide range of parameters. The analyses herein are concerned with some important aspects in this context. The investigation primarily concentrates upon the low velocity normal impact which is said to occur for impactor speeds less than 100 m/sec [420]. The modified Hertzian contact law accounts for permanent indentation. The contact force between the impactor and the delaminated composite conical shell is computed utilizing this law. The Newmark’s time integration algorithm (constant average acceleration method) is employed for solving the time dependent equations of the conical shells and the impactor.

4.3 CONTACT FORCE

The contact force depends on a contact law which relates the contact force with the indentation. The present study considers composite shallow conical shells having a large ratio of the radius of curvature to its thickness as well as a high value of width to thickness ratio. Yang and Sun [414] proposed a power law based on static indentation tests using steel balls as indentors. This contact law accounted for permanent indentation after unloading cycles i.e. collisions upon the rebound of the target structure after the first period of contact were considered. The modified version of the above contact law obtained by Tan and Sun [124] was used in the analyses of Sun and Chen [128]. The centrifugal forces arising out of rotation generate initial stresses and therefore the contact force model following Sun and Chen [128] has been incorporated into the present finite element formulation as the study focuses numerical simulation of delaminated graphite-epoxy composite conical shells. The indentation parameter \( \alpha \) depends on the difference of the displacements of the impactor and the target structure at any instant of time, and so also the contact force. The values of \( \alpha \) are changing with time because of time varying displacements of both the rigid impactor and the target structure. So at an instant the maximum indentation takes place and as a result the maximum contact force is also obtained. At this instant the displacement of the impactor also attains the maximum value. Thereafter, the displacement of the impactor gradually decreases, but the target point displacement keeps on changing and finally increases to a maximum value and at some point of time these two displacements become equal. This leads to zero value of indentation and eventually the contact force becomes zero. At this instant the impactor looses contact with the target. The process after attaining the maximum contact
force till the reduction of contact force to zero value is essentially referred to as unloading. If the mass of the impactor is not very small, a second impact may occur upon the rebound of the target structure leading to a same phenomenon of contact deformation and attainment of maximum contact force. There is always a time gap between the end of the first contact period and the beginning of the second impact. This process is known as reloading.

4.4 COMPARISON OF RESULTS

In order to validate the computer codes developed for the transient dynamic analysis of delaminated composite conical shells under low velocity normal impact, the results obtained on the basis of present finite element modelling are compared with those of the reference solutions published in the open literature both in respect of impact and delamination model. The validation of delamination model [393] is explained in previous Chapter 3 (Section 3.2), depicted as Figure 3.1. To authenticate the present method in respect of impact response, computation has been carried out for flat isotropic and laminated composite plates (Figure 4.1). In this context, essentially two important aspects namely, analytical solution and finite element treatment have been taken into account. The analysis due to Goldsmith [421] for a simply supported 20 cm square and 0.8 cm deep steel plate impacted centrally by a steel sphere of 1 cm radius with initial velocity of 100 cm/sec, provides solution of an integral equation for a Hertzian type of contact law in which the contact force is expressed as

\[ F_c = k_1 \alpha^{3/2} \]

where

\[ k_1 = \frac{4}{3\pi} \sqrt{R} \left( \frac{1}{\delta_1 + \delta_2} \right), \quad \delta_1 = \frac{(1-\nu_1^2)}{E_1\pi}, \quad \delta_2 = \frac{(1-\nu_2^2)}{E_2\pi} \]

and \( R \) is the impactor’s radius and \( k_1 \) is the contact stiffness. In the present analysis the material parameters are considered as \( E_1 = E_2 = E_i = 200 \text{ GPa}, G_{12} = G_{23} = G_{13} = 77 \text{ GPa}, \rho = 7800 \text{ kg/m}^3, \nu_i = \nu_{12} = 0.3 \) and the time step (\( \Delta t \)) chosen is 1.0 \( \mu \text{-sec} \) with corresponding finite element mesh of 8 x 8 for full plate. Figure 4.1 shows the comparisons of contact force, central deflection of target plate, impactor’s displacement and impactor’s velocity considering the results obtained from present FEM and those of Goldsmith. The slight discrepancy in the present results can be attributed to the fact that the effect of rotary inertia and transverse shear deformation have been taken into account in the present formulation but
the nature of variation is same. The present analysis compares well with the existing analytical solution. Figure 4.2 and Figure 4.3 show time histories of contact force and deflection/thickness for ten layered symmetrically laminated cross ply \([0^\circ/90^\circ/0^\circ/90^\circ/0^\circ)s]\) composite plate under simply supported boundary condition as analyzed by Sun and Chen [128] using finite element technique.

The present study is carried out to investigate the effects of relative position of delamination, velocity of impactor, angle of twist and rotational speed on the normal impact response of delaminated graphite-epoxy composite shallow conical shells. Accordingly the dimensions of length (L), width (b), radius of curvature (R) and thickness (h) of the shells are adopted as 0.4 m, 0.06732 m, 0.2 m and 0.001 m, respectively. Considering the complete planform of the shell a uniform mesh division of 8 × 8 has been used for the analyses. For all the cases shells are centrally impacted by a spherical steel ball of 0.0127 m diameter with an initial velocity of 3.0 m per second. The values of the contact stiffness coefficient, \((k)\) and mass density of the impactor used in this study are \(0.805 \times 10^9 \text{N/m}^{1.5}\) and 7960 kg/m\(^3\), respectively [128]. The material properties [128] for computation are adopted as: \(E_1 = 120\ \text{GPa},\ \ E_2 = 7.9\ \text{GPa},\ \ E_i = 210\ \text{GPa},\ \ G_{12} = G_{23} = G_{13} = 5.5\ \text{GPa},\ \ \rho = 1580\ \text{kg/m}^3,\ \ \nu_i = \nu_{12} = 0.3\)

### 4.5 RESULTS AND DISCUSSION

#### 4.5.1 ANGLE-PLY LAMINATES WITH SINGLE AND MULTIPLE DELAMINATION

The computational investigation is carried out for eight layered both single and quadruple delaminated graphite-epoxy composite angle-ply \([(0/-0/-0/-0)s]\) pretwisted conical shells subjected to low velocity normal impact with respect to degree of delamination, location of delamination, initial velocity of the impactor and angle of twist. The arrangement of layers with delamination is furnished in Figure 4.4. The effect of degree of delamination in terms of percentage of delamination is furnished in Figure 4.5. In quadruple 25%, 50% and 75% delamination, it is observed that the peak values of contact force have catastrophically slashed down compared to 12.5% delamination and undelaminated case. In contrast, for single delaminated cases, the peak values of contact force are found to decrease slightly as the percentage of delamination increases. Hence, it could be inferred that the increase in degree of delamination in higher percentage of delamination has significant influence in
reduction of elastic stiffness of the quadruple delaminated angle-ply \([(45°/-45°/45°/-45°)s]\) laminate irrespective of twist angle which is found absent in case of single delaminated cases.

The effect of twist and number of delamination on time histories of contact force for both untwisted and twisted cantilevered conical shells is furnished in Figure 4.6 to Figure 4.8 wherein delamination is considered with relative position of its centre line being 0.25 from the fixed end. It is observed that the peak value of contact force fluctuates with the increase of twist and number of delamination for all angle-ply configurations. The maximum contact force among all configurations is found to be lowest for angle-ply \([(45°/-45°/45°/-45°)s]\) configuration irrespective of twist angle and number of delamination. In general, as the twist angle increases, it is found to intensify the contact force and slightly tends to reduce the contact duration of the loading-unloading cycle. The time interval between completion of unloading and commencement of reloading is widened for cantilevered conical shells wherein the second impact does not occur within the time span of the analyses (e.g. 300 μ-sec). Dynamic response of composite conical shells under low velocity normal impact could be severe even though the maximum contact force is identified to be smallest one for \([(45°/-45°/45°/-45°)s]\) angle-ply configuration. This corroborates with the findings for cross-ply composite plate identified by Sun and Chen [128]. The contact duration is found to be sharply reduced as the degree of delamination increases for multiple delaminated angle-ply composite conical shells. Hence it can be inferred that number of delamination has significant effect on stiffness which is ultimately found responsible for variation of time-dependent parameters. The time histories of deflection/thickness curve (Figure 4.7) of angle-ply composite cantilever conical shells have no remarkable variation during the loading cycle but the loading time span is found lesser than unloading time for each cases. Earlier completion of unloading cycle for cantilever conical shell leads to persistence of stiffening effect and as a result the deflection/thickness curve shoots up high considerably with a linear trend. The variation of impactor displacement with respect to time is furnished in Figure 4.8. It is observed that an increase in number of delamination has significant effect on impactor displacement depending on the fibre orientation of the composite laminate.

The effect of relative position of delamination is studied considering eight layered quadruple delaminated angle-ply \([(45°/-45°/45°/-45°)s]\) composite pretwisted conical shells as furnished in Figure 4.9. Interestingly in this case, the unloading cycle time is found lower than loading cycle time duration irrespective of relative position of delamination. The time
histories of deflection/thickness curve are identified with a sharp hike immediately after completion of unloading cycle wherein the contact force dies down to null value irrespective of location of delamination and twist angle. The maximum value of contact force is found for relative position at 0.125 while the minimum value of contact force is identified for relative position at 0.5 and the maximum contact force at relative position 0.75 is found as intermediate one. In contrast, it is to be noted that the contact duration is found minimum value in case of relative position at 0.5, followed by relative position at 0.75 and the maximum contact duration is obtained for relative position at 0.125 irrespective of twist angle. Due to increase of elastic stiffness, the twisted conical shell shows comparatively higher peak value of contact force than untwisted one, invariant to location of delamination. In case of both untwisted and twisted cantilevered conical shells, the deflection/thickness curve builds up quickly when the contact period is over.

Figure 4.10 presents the effect of initial velocity of impactor on time histories of contact force, deflection/thickness and impactor displacement. In case of quadruple delaminated angle-ply, the peak values of contact force for untwisted conical shells are found to be slightly lower than the twisted one. On the other hand, the slope of deflection/thickness is identified to increase with the increase of time. It is also identified that the deflection/thickness increasingly rises with the increase of velocity of the impactor. These fluctuations could be attributed to plenty of waves of shorter wavelengths or higher modes of vibration induced by higher impact velocities. This corroborates with the findings of Sun and Chen [128]. For a particular velocity of the impactor, maximum contact force is identified for initial velocity of impactor (VOI) of 5 m/sec, followed by VOI of 3 m/sec and the lowest peak value of contact force is found for VOI of 1 m/sec irrespective of twist angle. Likewise the slope of both deflection/thickness and impactor displacement in both twisted and untwisted cases are identified to be maximum value for VOI of 5 m/sec and to be minimum value for VOI of 1 m/sec while the intermediate values are found for VOI of 3 m/sec. Hence, it can be inferred that for a particular velocity of impact, maximum contact force has inverse relationship with time required for unloading cycle irrespective of twist angles.

4.5.2 SINGLE AND MULTIPLE DELAMINATED CROSS-PLY LAMINATES

In this study, the similar parametric studies are carried out for eight layered delaminated graphite-epoxy composite cross-ply pretwisted conical shells with respect to location of delamination, velocity of the impactor and angle of twist. Figure 4.11 presents the
arrangement of layers of cross-ply laminate. For both untwisted ($\psi=0^\circ$) and twisted ($\psi=30^\circ$) cantilevered conical shells, the transient dynamic response subjected to low velocity normal impact is obtained with time step as 1.0 $\mu$s. In each case, 25% delamination is considered irrespective of the location of delamination. It is observed that the contact force history increases with the rise of twist angles for cross-ply composite shallow conical shells. In other words, as the twist angle increases, it is found to intensify the contact force and reduce the contact duration of the loading-unloading cycle (first impact). The time interval between completion of unloading and commencement of reloading is widened for cantilevered conical shells wherein the second impact does not occur within the time span of the analyses (e.g. 400 $\mu$-sec). In this case also, the softening effect on the laminate stiffness is increased due to initial stress and this may surmise that subsequent dynamic response of the composite conical shells could be more dangerous even though the maximum contact force is smaller for delaminated cross-ply composite conical shell configuration. This conjecture is confirmed by the time histories of deflection/thickness. The variation of contact force for different number of delaminations are furnished in Figure 4.12 while the detail histories of contact force, deflection/thickness and impactor displacement with respect to time for a quadruple delaminated cross-ply composite conical shells are furnished in Figure 4.13.

The effect of location of delamination on time histories of contact force, deflection/thickness and impactor’s displacement for quadruple mid-plane delaminated cross-ply composite cantilever conical shells are furnished in Figure 4.14 wherein relative positions of delamination at 0.125, 0.5 and 0.75 are considered from top surface. The elastic stiffness is also observed to change in the same fashion as that of natural frequency with respect to location of delamination along the span of the laminate. The peak value of contact force is obtained for relative position at 0.125 which is necked by relative position at 0.75 while on the other hand, the lowest peak value of contact force is identified for relative position at 0.5. It is to be noted that for cantilever conical shells contact period is completed earlier for the case of 0.75 and 0.125 compared to that of 0.5. In other words, the unloading cycle is found comparatively shorter for relative position at 0.75, followed by relative position of delamination at 0.125 and the maximum unloading cycle is observed in case of relative position of delamination at 0.5 invariant to laminate configuration and twist angle.

The effect of velocity of impactor is investigated considering a centrally point load impacted on target by the impactor with initial velocities of 1 m/sec, 3 m/sec and 5 m/sec,
respectively, furnished in Figure 4.15. In this case also, irrespective of initial velocity of impactor, contact force of twisted cases is found to be higher than the same of untwisted configurations. It is also found that there exist more fluctuations in contact force for higher initial velocity of impactor. In contrast, as the initial velocity of impactor increases, the slopes of deflection/thickness curves are identified to increase with time. It is also identified that the impactor’s displacement incrementally rises with the increase of velocity of the impactor. For a particular velocity of the impactor, maximum contact force is identified by twisted cross-ply compared to untwisted cross-ply while the time required for unloading cycle is observed to be slightly higher for untwisted cross-ply compared to twisted configuration. In this case also, the peak value of contact force has inverse relationship with time required for unloading cycle invariant to initial velocity of impactor and twist angle.

4.5.3 BENDING STIFF LAMINATES WITH SINGLE AND MULTIPLE DELAMINATION

Deflections at the impact point (L/2, b/2) are obtained and the optimum value of time step is chosen after performing convergence study. The arrangement of layers with delamination is furnished in Figure 4.16. Parametric studies are conducted for eight layered delaminated graphite-epoxy composite bending stiff [(0°/±30°)]s and pretwisted conical shells with respect to location of delamination, velocity of the impactor, rotational speed and angle of twist. The characteristic curves for time histories of contact force and deflection/thickness with respect to the number of delamination are depicted in Figure 4.17. It is observed that the transverse shear modulus has a major effect on the contact stiffness. Lowering the contact stiffness also lowers the contact forces and increases the contact area.

The initial stress is found to increase the softening effect on the laminate stiffness and this may surmise that subsequent dynamic response of the conical shells could be more dangerous even though the maximum contact force is smaller for bending stiff configuration at number of delamination (n_d)=2,4. This conjecture is confirmed by the defection/thickness with respect to time diagram wherein the stiffening effect of the tensile prestress is also clearly exhibited. The initial stress effect is found absent unless the deflection is large. The severe effects are identified while delamination is not located at the mid-plane, for instant, at n_d=2,4. The maximum contact force is found much lesser compared to n_d=1,3 wherein delamination is found to be located at the mid-plane. The sharp hike in deflection/thickness is identified for n_d=2,4 while a linear trend is observed for n_d=1,3. Hence, it can be inferred that
the mid-plane delamination has attributed to a significant catastrophic change in stiffening effect.

The effect of relative position of delamination on contact force and deflection/thickness with respect to time for double delaminated graphite-epoxy bending stiff $[(0^\circ/\pm30^\circ)_3]$ composite cantilever conical shells are shown in Figure 4.18 wherein delamination is considered with relative position of its centre line being 0.25 from the free end, and the relative locations of delamination across thickness are 0.125, 0.5 and 0.75 considered from top surface. It can be said that elastic stiffness also changes in the same manner as that of natural frequency with respect to location of delamination across the thickness. In conformity with this, the peak values of the contact force in the later part of the unloading cycle of twisted shells are also found to occur. The maximum value of contact force is found for relative position at 0.75, followed by relative position at 0.125 while the minimum value of contact force is identified for relative position at 0.5. It is observed that the contact period is completed earlier for the relative position at 0.5 compared to relative position at 0.75 and 0.125. The time span of unloading cycle is found to be minimum for relative position at 0.75, followed by relative position at 0.125 and the maximum unloading cycle time is identified for relative position at 0.5 invariant to twist angles. In contrast, the loading cycle time is found maximum for relative position at 0.75, followed by relative position at 0.125 and the minimum loading cycle time is found for relative position at 0.5 irrespective of twist angle. In twisted cases, a sharp hike in loading curves and a sharp drooping trend of unloading curves for contact force are identified for relative location of delamination at 0.75 and 0.125. Like the contact force, deflection/thickness is also influenced by the relative position of delamination in the later stages of unloading cycle, but no significant variation is noticed during the loading cycle.

For a particular location of delamination, the twisted conical shell shows higher peak value of contact force compared to untwisted one as expected as the elastic stiffness increases. The similar trend is also found in case of rotating cantilever shells. In case of both untwisted and twisted cantilevered conical shells, the deflection/thickness builds up quickly when the contact period is over. Natural frequencies of both untwisted and twisted delaminated cantilever conical shells have higher values when the delamination is located near the free surface (top or bottom) compared to its location at the mid-plane and accordingly the elastic stiffness also changes implying lower value corresponding to mid-
plane delamination. This is also reflected in the deflection history during the unloading process and the maximum values of deflection are obtained for the mid-plane delamination.

Rotational speed has a negligible effect on contact force and deflection/thickness for cantilever conical shell during the loading cycle. Contact duration of the unloading cycle for stationary cantilever conical shell is longer compared to loading cycle for bending stiff configuration irrespective of rotational speeds and twist angles. These findings corroborate the fact that stiffening because of higher initial stresses reduces the duration of contact period [128]. The effect of rotational speed on time histories of contact force and deflection/thickness for both untwisted and twisted cantilevered conical shells are shown in Figure 4.19 wherein mid-plane delamination is considered with relative position of its centre line being 0.25 from the fixed end. It is observed that the contact force history does not change with rotational speeds during the loading cycle of the laminate configurations while a slight fluctuation of deflection/thickness is identified during unloading cycle for twisted cases. The change in centrifugal stiffening (geometric stiffness) of the conical shell with increase of rotational speed does not appreciably encounter the maximum contact force, despite the fact that higher initial stresses result at higher speed of rotation. As the twist angle increases, it is found to intensify the contact force and reduce the contact duration of the loading-unloading cycle (first impact). The time interval between completion of unloading and commencement of reloading is widened for cantilevered conical shells wherein the second impact does not occur within the time span of the analyses (e.g. 30 μ-sec). For both stationary and rotating conditions, fundamental natural frequencies of the twisted conical shells are obtained and found to be less in comparison with the untwisted conical shells. These findings corroborate the fact that the time between two contacts is inversely related to the natural frequencies of the cantilevered conical shells [128]. The speed of rotation for cantilevered conical shell is found more and the displacement is observed due to the combined effect of steady state centrifugal force and contact force. Thus, the peak values of normalized deflection for cantilever conical shells are found to be more in comparison with that of twisted shells for each laminate configuration. The variation of deflection/thickness with respect to rotational speed is found negligible during the loading cycle while a slight variation is identified for twisted conical shells with higher value at stationary condition as expected due to the softening effect compared to rotating condition. Earlier completion of unloading cycle for rotating cantilever conical shell leads to persistence of centrifugal effect in the later stages and as a result the central deflection shoots up considerably, but the
stationary untwisted shell shows more or less steady values of deflection/thickness during the unloading cycle.

Another parametric study is carried out considering a centrally point loaded multiple delaminated bending stiff graphite-epoxy composite cantilever conical shells at stationary condition with 25% mid-plane delamination impacted on each laminate by the impactor with velocities of 1 m/sec, 3 m/sec and 5 m/sec, respectively. Since the impact velocity is the only variable, its effect on the transient dynamic impact responses with respect to change of velocity of impactor is clearly presented in Figure 4.20. Like earlier case, the maximum contact force for twisted cases is found to be higher than untwisted cases for a particular initial velocity of impactor. It is observed that higher initial velocity of the impactor encapsulates proportionally higher fluctuations in contact force. It is also found to increase the slope of deflection per thickness with the increase of to time. It is also identified that the deflection per thickness increasingly rises with the increase of velocity of the impactor. These fluctuations could be caused by the participation of more waves of shorter wavelengths or higher modes of vibration induced by higher initial velocity of impactor. This corroborates with the findings of Sun and Chen [128]. It can be inferred that for a particular initial velocity of impactor, maximum contact force has inverse relationship with time required for unloading cycle irrespective of twist angles.

4.5.4 COMPARATIVE STUDY OF BENDING STIFF (BS), TORSION STIFF (TS) AND CROSS-Ply (CP) LAMINATE CONFIGURATIONS

The present study is further extended for a centrally impacted eight layered graphite-epoxy bending stiff [(0°/±30°)s], torsion stiff [(±45°/±45°)s] and cross-ply [(0°/90°/0°/90°)s] composite pretwisted conical shells with single mid-plane delamination. The arrangement of layers indicating mid-plane delamination is furnished in Figure 4.21. Time histories of contact force and deflection/thickness are presented due to variation of angle of twist, rotational speeds, initial velocity of the impactor and location of delamination. The transient dynamic response subjected to low velocity normal impact is obtained for untwisted (ψ=0°) and twisted (ψ=30°) cantilevered conical shells corresponding to non-dimensional speeds of rotation, \( \Omega = \Omega / \omega_0 = 0.1 \) using the converged value of time step of 1.0 µs for all the above three laminate configurations (i.e., bending stiff, torsion stiff and cross-ply configurations), where \( \Omega \) and \( \omega_0 \) are the speed of rotation and fundamental natural
frequency of non-rotating shell, respectively. In each case, 25% delamination is considered irrespective of the location of delamination.

Figure 4.22 presents the impact response with respect to different laminate configurations viz. bending stiff, torsion stiff and cross-ply laminates. It is found that the peak value of the contact force is maximum for torsion stiff laminate and minimum for bending stiff configuration irrespective of twist angle and the intermediate value is noted for cross-ply laminate. The contact force is found to intensify as the twist angle increases although loading-unloading cycles remain same as furnished in Figure 4.22(a) and Figure 4.22(b). This may be attributed to the fact that because of coupling effects the increase of twist angle increases the elastic stiffness resulting in rise of maximum contact force [422]. The time interval between completion of unloading and commencement of reloading is widened for cantilevered conical shells wherein the second impact does not occur within the time span of the analyses (e.g. 400 μ-sec). The contact duration of unloading cycle is found to be maximum for bending stiff ([0°/±30°]s), followed by cross-ply ([0°/90°/0°/90°]s) and the minimum time span of unloading cycle is identified for torsion stiff ([±45°/±45°]s) composite cantilever conical shells for both twisted (ψ=30°) and untwisted (ψ=0°) cases. It is also to be noted that the duration of unloading cycle for untwisted bending stiff configuration is found to be less than that of twisted case, but in case of cross-ply and torsion stiff configurations, there is no remarkable deviation observed in respect of unloading cycles while compared between their twisted and untwisted cases. For the same impact conditions, all three laminates are impacted with equal energy and the peak value of the contact force for bending stiff laminate being minimum value, the duration of the unloading cycle herein is observed to be maximum as expected.

The time histories of deflection/thickness are furnished in Figures 4.22(c) and 4.22(d) wherein equivalent linear trend is identified during loading cycles for all three cases irrespective of twist angle but in unloading cycle after contact force dies down to zero value, the slope of deflection/thickness is found maximum for torsion stiff followed by cross-ply and minimum slope is observed for bending stiff configuration. Hence, it could be inferred that among these three laminate configurations, the severe effect due to low velocity normal impact on delaminated conical shells could be found in case of bending stiff configuration during the loading cycle.
The effect of rotational speeds on contact force for both untwisted ($\psi=0^\circ$) and twisted ($\psi=30^\circ$) conical shells with mid-plane delamination subjected to low velocity normal impact at the centre are shown in Figure 4.23. Rotational speed has a negligible effect on contact force for cantilever conical shell. In other words, it is observed that the time histories of contact force does not appreciably change with rotational speeds during the loading cycle of all three basic laminate configurations while a tiny negligible fluctuation of contact force is identified during the unloading cycle irrespective of twist angle. Hence the change in centrifugal stiffening effect due to variation of geometric stiffness does not appreciably influence the maximum contact force, despite the fact that higher initial stresses result at higher speeds of rotation.

Parametric studies are also performed with different initial velocities of the impactor such as 1 m/sec, 3 m/sec and 5 m/sec, and the results are furnished in Figure 4.24 and Figure 4.25. In general, for a particular laminate configuration and velocity of impactor, the maximum contact force of twisted cases are found to be always higher than that of the untwisted cases for all the above three configurations. It is also observed that there exist more fluctuations in contact force for higher impact velocities of the impactor and this is strikingly found in case of bending stiff and cross-ply laminates compared to torsion stiff configuration. These fluctuations could be caused by the participation of more waves of shorter wavelengths (or higher modes of vibration) induced by higher impact velocities [128]. Similarly it is also identified that deflection/thickness increasingly rises with the increase of initial velocity of the impact. In this case, for a particular initial velocity of the impactor, the maximum value of deflection/thickness (at 400 $\mu$-sec) is identified to achieve for twisted cases for each laminate configuration. In this case also, for a particular initial velocity of the impactor, the maximum value of contact force is identified in torsion stiff configuration, followed by cross-ply laminate and the minimum value is found for bending stiff configuration irrespective of twist angle, while the time required for unloading cycle is observed to be maximum for bending stiff configuration, followed by cross-ply laminate and the minimum value being found for torsion stiff configuration. Hence, it can be inferred that for a particular velocity of impactor, maximum contact force has inverse relationship with contact duration of unloading cycle irrespective of twist angles. Thus, the slope of deflection/thickness is found maximum for torsion stiff configuration and minimum for bending stiff laminate while the same for cross-ply is always found as intermediate one. The unloading cycle time of twisted bending stiff
configuration at each initial velocity of impactor is noted to be higher than that of the same for untwisted one while there is no such variation found in case of cross-ply and torsion stiff configuration.

The effect of location of delamination on time histories of contact force and deflection/thickness are shown in Figure 4.26 and Figure 4.27 wherein the relative locations of centerline of delamination along span are considered as 0.125, 0.5 and 0.75 from the clamped end. The peak values of the contact force for both untwisted and twisted cases are found to occur at relative position of 0.125 and 0.75 while the minimum value of contact force is identified for relative location at 0.5. It is observed that for all the above three configurations in both twisted and untwisted cases, the duration of contact period is completed much earlier for the case of relative positions at 0.75 and 0.125 compared to that of relative position at 0.5. In other words, the time duration of unloading cycle is found shortest for relative location at 0.75, necked by relative location at 0.125 and the maximum unloading cycle is observed for mid-plane delamination with relative position at 0.5 irrespective of laminate configurations and twist angles. The contact duration of unloading cycle of twisted bending stiff is found to be significantly higher than that of untwisted one for a particular initial velocity of impactor and relative position of delamination whereas the similar effect is found absent in case of torsion stiff and cross-ply configuration. In contrast, the variation of deflection/thickness linked with change in location of delamination is found negligible during the loading cycle for all three laminates irrespective of twist angles but the effect of change in location of delamination could be observed during the later part of the unloading cycle. From the time histories of contact force and deflection/thickness, it could be inferred that for bending stiff laminate the location of centerline of delamination at mid-span is very critical during the loading cycle.
Fig. 4.1 (a) Contact force and Displacement of target plate (b) Impactor’s displacement and Velocity of impactor with respect to time for a simply supported plate (isotropic) impacted at the centre considering laminate dimension $a=b=20$ cm, $h=0.8$ cm, time step $(\Delta t)=1.0$ μ-sec, $E_1=E_2=E_i=200$ GPa, $G_{12}=G_{23}=G_{13}=77$ GPa, $\rho=7800$ kg/m$^3$, $\nu_{i}=\nu_{12}=0.3$, initial velocity of impactor (VOI)=1 m/s, radius of impact=0.01 m [421]
**Fig. 4.2** Contact force with respect to time for centrally impacted ten layered cross-ply 
\([0^\circ/90^\circ/0^\circ/90^\circ/0^\circ]_S\) composite plate under simply supported boundary condition, considering laminate dimension 20 cm x 20 cm x 0.269 cm, mass density of impactor=7.96 x 10^{-5} N-sec^2/cm^4, time step (Δt) =1.0 μ-sec, \(E_1=120\) GPa, \(E_2=7.9\) GPa, \(E_i=210\) GPa, \(G_{12}=G_{23}=G_{13}=5.5\) GPa, \(\rho=1580\) kg/m^3, \(v_{12}=0.3\), \(VOI=3\) m/s [128]

**Fig. 4.3** Deflection/thickness with respect to time for centrally impacted ten layered cross-ply 
\([0^\circ/90^\circ/0^\circ/90^\circ/0^\circ]_S\) composite plate under simply supported boundary condition, considering laminate dimension 20 cm x 20 cm x 0.269 cm, mass density of impactor=7.96 x 10^{-5} N-sec^2/cm^4, time step (Δt) =1.0 μ-sec, \(E_1=120\) GPa, \(E_2=7.9\) GPa, \(E_i=210\) GPa, \(G_{12}=G_{23}=G_{13}=5.5\) GPa, \(\rho=1580\) kg/m^3, \(v_{12}=0.3\), \(VOI=3\) m/s [128]
Fig. 4.4 Arrangement of layers of angle-ply laminates indicating location of delamination
Fig. 4.5 Variation of contact force with respect to time for undelaminated and delaminated (in the form of percentage) angle-ply [(45°/-45°/45°/-45°)s] composite pretwisted conical shells, considering n=8, time step (Δt)=1.0 μ-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_3=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $\nu_1=\nu_{12}=0.3$, $V_{Oi}=3$ m/s, $h=0.005$ m, $L/s=0.7$, $a/L=0.25$, $d/L=0.5$, $\theta_o=45^\circ$, $\theta_v=20^\circ$. 
Fig. 4.6 (a) to (h) Contact force with respect to time for $\psi=0^\circ$, $30^\circ$ delaminated angle-ply $[(0/-\theta/-\theta/-\theta/s)]$ composite pretwisted conical shell at stationary condition with 25% mid-plane delamination, considering $n=8$, time step $(\Delta t)=1.0$ $\mu$-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_3=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $\nu_i=\nu_{12}=0.3$, VOI=3 m/s, $h=0.005$ m, $L/s=0.7$, $d/L=0.5$, $\theta_o=45^\circ$, $\theta_v=20^\circ$. 
\[ \Psi = 0^\circ \quad \Psi = 30^\circ \]

(a) \[ (\theta - \theta - \theta)_{\text{s}} (\theta - 0^\circ) \]

(b) \[ (\theta - \theta - \theta)_{\text{s}} (\theta - 0^\circ) \]

(c) \[ (\theta - \theta - \theta - \theta)_{\text{s}} (\theta - 0^\circ) \]

(d) \[ (\theta - \theta - \theta - \theta)_{\text{s}} (\theta - 0^\circ) \]

(e) \[ (\theta - \theta - \theta - \theta)_{\text{s}} (\theta - 0^\circ) \]

(f) \[ (\theta - \theta - \theta - \theta)_{\text{s}} (\theta - 0^\circ) \]

(g) \[ (\theta - \theta - \theta - \theta)_{\text{s}} (\theta - 0^\circ) \]

(h) \[ (\theta - \theta - \theta - \theta)_{\text{s}} (\theta - 0^\circ) \]

Fig. 4.7 (a) to (h) Variation of deflection/thickness with respect to time for \( \psi = 0^\circ, 30^\circ \) delaminated angle-ply [(0/-\theta/0/-\theta)s] composite pretwisted conical shell at stationary condition with 25% mid-plane delamination, considering \( n=8 \), time step \( (\Delta t) = 1.0 \) µ-sec, \( E_1=120 \) GPa, \( E_2=7.9 \) GPa, \( E_3=210 \) GPa, \( G_{12}=G_{23}=G_{13}=5.5 \) GPa, \( \rho=1580 \) kg/m\(^3\), \( v_{12}=0.3 \), VOI=3 m/s, \( h=0.005 \) m, \( L/s=0.7 \), \( d/L=0.5 \), \( \theta_o=45^\circ \), \( \theta_v=20^\circ \).
Fig. 4.8 (a) to (h) Variation of imapactor’s displacement with respect to time for twist angle ($\psi=0^\circ$, $30^\circ$) of delaminated angle-ply [(0/0/0)/0] composite pretwisted conical shells at stationary condition with 25% mid-plane delamination, considering $n=8$, time step ($\Delta t$)=1.0 $\mu$-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_t=210$ GPa, $G_{13}=G_{23}=G_{12}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $\nu_1=\nu_{12}=0.3$, VOI=3 m/s, $h=0.005$ m, $L/s=0.7$, $d/L=0.5$, $\theta_o=45^\circ$, $\theta_v=20^\circ$. 

<table>
<thead>
<tr>
<th>$\Psi = 0^\circ$</th>
<th>$\Psi = 30^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
</tr>
</tbody>
</table>
Fig. 4.9 (a) to (f) Contact force, plate displacement and impactor displacement with respect to time at different relative position of delamination with twist angle (Ψ=0°, 30°) of multiple delaminated angle-ply [(45°/-45°)/45°/-45°]s composite pretwisted conical shell, considering n=8, time step (Δt)=1.0 μ-sec, E₁=120 GPa, E₂=7.9 GPa, E₃=210 GPa, G₁₂=G₂₃=G₁₃=5.5 GPa, p=1580 kg/m³, v₁₁=v₁₂=0.3, VOI=3 m/s, nₐ=4, h=0.005 m, L/s=0.7, d/L=0.5, a/L=0.25, θ₀=45°, θᵥ=20°.
Fig. 4.10 (a) to (f) Contact force, deflection/thickness and impactor displacement with respect to time at different of velocity of impactor for twist angle (ψ=0°, 30°) of delaminated angle-ply [(45°/-45°)/45°/-45°]s composite pretwisted conical shell at stationary condition, considering n=8, time step (Δt)=1.0 μ-sec, E₁=120 GPa, E₂=7.9 GPa, E₃=210 GPa, G₁₂=G₂₃=G₁₃=5.5 GPa, ρ=1580 kg/m³, ν₁₂ = ν₁₃ = 0.30, nₜ=4, h=0.005 m, L/s=0.7, d/L=0.5, a/L=0.25, θₒ=45°, θᵣ =20°.
Fig. 4.11 Arrangement of layers of cross-ply laminates indicating location of delamination

![Arrangement of layers of cross-ply laminates](image)

(a) n_d=1  (b) n_d=4

Fig. 4.12 Time histories of contact force for different twist angles ($\psi=0^\circ, 30^\circ$) with different number of delamination ($n_d=0, 1, 4$) of eight layered cross-ply $[(0^{\circ}/90^{\circ}/0^{\circ}/90^{\circ})_8]$ composite pretwisted conical shells, considering $n=8$, time step ($\Delta t=1.0 \mu$-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_3=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $\nu_{12}=\nu_{13}=0.3$, VOI=3 m/s, $h=0.002$ m, $L/s=0.7$, $d/L=0.5$, $a/L=0.25$, $\theta_o=45^\circ$, $\theta_v=20^\circ$.}
Fig. 4.13 (a) Contact force (b) Deflection/thickness and (c) Impactor’s displacement with respect to time for different twist angles ($\psi=0^\circ, 30^\circ$) of multiple delaminated cross-ply [(0º/90º/0º/90º)s] composite pretwisted conical shells, considering $n=8$, time step ($\Delta t$)=1.0 $\mu$-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_3=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $\nu_i=\nu_{12}=0.3$, VOI=3 m/s, $n_d=4$, $h=0.002$ m, $L/s=0.7$, $d/L=0.5$, $a/L=0.25$, $\theta_0=45^\circ$, $\theta_v=20^\circ$. 
Fig. 4.14 (a,b) Contact force (c,d) Deflection/thickness and (e,f) Impactor’s displacement with respect to time at different relative position of delamination at $\psi=0^\circ$, $30^\circ$ for multiple delaminated cross-ply $[(0^\circ/90^\circ/0^\circ/90^\circ)s]$ composite pretwisted conical shells, considering $n=8$, time step ($\Delta t$) =1.0 $\mu$-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_t=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $v_i=v_{12}=0.3$, VOI=3 m/s, $n_d=4$, $h=0.002$ m, $L/s=0.7$, $d/L=0.5$, $a/L=0.25$, $\theta_o=45^\circ$, $\theta_v=20^\circ$. 
Fig. 4.15 (a,b) Contact force (c,d) Deflection/thickness and (e,f) Impactor’s displacement with respect to time at different initial velocity of impactor with $\psi=0^\circ$, $30^\circ$ for multiple delaminated cross-ply [(0º/90º/0º/90º)s] composite pretwisted conical shells, considering $n=8$, time step ($\Delta t$)=1.0 $\mu$-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $v_i=v_{12}=0.3$, VOI=3 m/s, $n_d=4$, $h=0.002$ m, $L/s=0.7$, $d/L=0.5$, $a/L=0.25$, $\theta_o=45^\circ$, $\theta_v=20^\circ$. 
Fig. 4.16 Arrangement of layers of bending stiff laminate indicating location of delamination
Fig. 4.17 (a) Contact force (b) Deflection/thickness with respect to time for undelaminated and multiple delaminated bending stiff [(0°/±30°)₈] composite pretwisted conical shells at stationary condition (at ψ=0°) with 25% mid-plane delamination considering n=8, time step (Δt)=1.0 μ-sec, E₁=120 GPa, E₂=7.9 GPa, Eᵢ=210 GPa, G₁₂=G₂₃=G₁₃=5.5 GPa, ρ=1580 kg/m³, νᵢ=ν₁₂=0.3, VOI=3 m/s, h=0.005 m, L/s=0.7, d/L=0.5, a/L=0.25, θ₀=45°, θᵥ=20°.
Fig. 4.18 (a,b) Contact force (c,d) Deflection/thickness with respect to time at different
relative positions delaminated bending stiff [(0°/±30°)₈] composite pretwisted conical shells
at stationary condition with 25% mid-plane delamination considering n=8, time step (Δt)=1.0
μ-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_i=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$,
$\nu_i=\nu_{12}=0.3$, VOI=3 m/s, $n_d=2$, $h=0.005$ m, $L/s=0.7$, $d/L=0.5$, $a/L=0.25$, $\theta_o=45^\circ$, $\theta_v=20^\circ$. 
Fig. 4.19 (a,b) Contact force (c,d) Deflection/thickness with respect to time at different rotational speeds of double delaminated bending stiff $[(0^\circ/\pm30^\circ)_S]$ composite pretwisted conical shells with 25% mid-plane delamination considering $n=8$, time step $(\Delta t)=1.0$ μ-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_3=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $\nu_i=\nu_{12}=0.3$, $V_{0}=3$ m/s, $n_d=2$, $h=0.005$ m, $L/s=0.7$, $d/L=0.5$, $a/L=0.25$, $\theta_0=45^\circ$, $\theta_\psi=20^\circ$. 
Fig. 4.20 (a,b) Contact force and (c,d) Deflection/thickness with respect to time at different initial velocities of impactor for both twisted ($\psi=30^\circ$) and untwisted ($\psi=0^\circ$) double delaminated bending stiff $[(0^\circ/30^\circ)\mp]$ composite pretwisted conical shells considering $n=8$, time step ($\Delta t$)=1.0 $\mu$-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_i=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $v_i=v_{12}=0.3$, $n_d=2$, $h=0.005$ m, $L/s=0.7$, $d/L=0.5$, $a/L=0.25$, $\theta_o=45^\circ$, $\theta_v=20^\circ$.

Fig. 4.21 Arrangement of layers indicating location of delamination for bending stiff $[(0^\circ/30^\circ)\mp]$ or torsion stiff $[(45^\circ/45^\circ)\pm]$ and cross-ply $[(0^\circ/90^\circ/0^\circ/90^\circ)\pm]$ laminates.
Fig. 4.22 (a,b) Contact force and (c,d) Deflection/thickness with respect to time at $\psi=0^\circ$, $30^\circ$ of delaminated bending stiff $[(0^\circ/\pm 30^\circ)_{\mathrm{s}}]$, torsion stiff $[(\pm 45^\circ/\mp 45^\circ)_{\mathrm{s}}]$ and cross-ply $[(0^\circ/90^\circ/0^\circ/90^\circ)_{\mathrm{s}}]$ composite pretwisted conical shells with 25% mid-plane delamination considering $n=8$, time step $(\Delta t)=1.0$ $\mu$-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_i=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $\nu_i = \nu_{12} = 0.3$, initial velocity of impactor (VOI)=3 m/s, $h=0.001$ m, $s/h=500$, L/s=0.8, d/L=0.5, $\theta_0=45^\circ$, $\theta_v=20^\circ$. 
Fig. 4.23 Effect of rotational speeds ($\Omega = \Omega'/\omega_0 = 0.1$) on contact force with respect to time at $\psi=0^\circ$, $30^\circ$ of delaminated bending stiff [(0°/±30°)]$_s$, torsion stiff [(±45°/±45°)]$_s$ and cross-ply [(0°/90°/0°/90°)]$_s$ composite pretwisted conical shells with 25% mid-plane delamination considering $n=8$, time step ($\Delta t$)=1.0 $\mu$-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_i=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $v_i=v_{12}=0.3$, VOI=3 m/s, $h=0.001$ m, $s/h=500$, $L/s=0.8$, $d/L=0.5$, $\theta_o=45^\circ$, $\theta_v=20^\circ$. 
Fig. 4.24 Time histories of contact force with different initial velocity of impactor at $\psi=0^\circ$, $30^\circ$ of delaminated bending stiff $[(0^\circ/\pm30^\circ)],$ torsion stiff $[(\pm45^\circ/\pm45^\circ)]$ and cross-ply $[(0^\circ/90^\circ/0^\circ/90^\circ)]$ composite pretwisted conical shells at stationary condition considering $n=8$, time step $(\Delta t)=1.0$ $\mu$-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_i=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $p=1580$ kg/m$^3$, $v_i=v_{12}=0.3$, $h=0.001$ m, $s/h=500$, $L/s=0.8$, $d/L=0.5$, $a/L=0.25$, $\theta_o=45^\circ$, $\theta_v=20^\circ$. 
Fig. 4.25 Time histories of deflection/thickness with different initial velocity of impactor and twist angle (ψ=0°, 30°) of delaminated bending stiff [0°/±30°]s, torsion stiff [±45°/±45°]s and cross-ply [(0°/90°/0°/90°)s] composite pretwisted conical shells at stationary condition considering n=8, time step (Δt)=1.0 μ-sec, E₁=120 GPa, E₂=7.9 GPa, E₃=210 GPa, G₁₂=G₂₃=G₁₃=5.5 GPa, ρ=1580 kg/m³, ν₁=ν₁₂=0.3, h=0.001 m, s/h=500, L/s=0.8, d/L=0.5, a/L=0.25, θ₀=45°, θ_v=20°.
Fig. 4.26 Contact force with respect to time with respect to different relative position of delamination and twist angle ($\psi=0^\circ, 30^\circ$) for delaminated bending stiff $[(0^\circ/\pm30^\circ)_3]$, torsion stiff $[(\pm45^\circ/\mp45^\circ)_3]$ and cross-ply $[(0^\circ/90^\circ/0^\circ/90^\circ)_s]$ composite pretwisted conical shells at stationary condition considering $n=8$, time step ($\Delta t$)=1.0 $\mu$-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_i=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $v_1=v_{12}=0.3$, VOI=3 m/s, $h=0.001$ m, $s/h=500$, $L/s=0.8$, $d/L=0.5$, $a/L=0.25$, $\theta_o=45^\circ$, $\theta_0=20^\circ$. 
Fig. 4.27 Deflection/thickness with respect to time at different relative position of delamination and twist angle ($\psi=0^\circ, 30^\circ$) for delaminated bending stiff $([0^\circ/\pm 30^\circ]_s)$, torsion stiff $([\pm 45^\circ/\mp 45^\circ]_s)$ and cross-ply $([0^\circ/90^\circ/0^\circ/90^\circ]_s)$ composite pretwisted conical shells at stationary condition considering $n=8$, time step ($\Delta t$)=1.0 μ-sec, $E_1=120$ GPa, $E_2=7.9$ GPa, $E_3=210$ GPa, $G_{12}=G_{23}=G_{13}=5.5$ GPa, $\rho=1580$ kg/m$^3$, $\nu_{12}=0.3$, VOI=3 m/s, $h=0.001$ m, $s/h=500$, $L/s=0.8$, $a/L=0.5$, $\theta_0=45^\circ$, $\phi_v=20^\circ$. 