8.1 Introduction:

The free electron theory of Drude, which assumes that the interaction of the carriers with one another can be ignored, proves to be inadequate in explaining various phenomena observed in a wider variety of substances. With the evolution of the carrier transport theory, it has become apparent that the independent carrier approximation is an oversimplification of facts. Hence in order to overcome the deficiency of the free carrier theory, the carrier-carrier interaction is to be considered. The effect of carrier-carrier interaction is manifested in the phenomenon of screening. In this chapter, the effect of screening has been studied in bulk semiconductors and low-dimensional systems for both the static and dynamic limits. Section 8.2 develops the theory of screening in different formulations and the results obtained by applying the theory to bulk as well as one and two-dimensional hole gases are prescribed in section 8.3.

8.2 Theory of Screening(1):

When a charged particle is placed in a cloud of carrier gas having opposite charge, it attracts the carriers around it thus altering the charge density in its neighbourhood. This excess of carriers around the charged particle reduces or screens the field of the charged particle. Two potentials, therefore, come into play, the potential $\phi_{\text{ext}}(r)$ due to the charged particle itself and $\phi(r)$, the total potential due to the charged particle and the cloud of carriers. The two potentials are related by the equation.

$$\phi(q) = \frac{1}{\epsilon(q)} \phi_{\text{ext}}(q) \quad \text{.........(8.1)}$$

Where $\epsilon(q)$ is the wave vector dependent dielectric constant and $\phi(q)$ and $\phi_{\text{ext}}(q)$ are respectively the qth Fourier component of $\phi(r)$ and $\phi_{\text{ext}}(r)$. The dielectric constant $\epsilon(q)$ satisfies equation 2.30 of chapter 2 and may be obtained by either Thomas-Fermi method or Random Phase Approximation of
8.2.1 Thomas Fermi Method:

A) In bulk semiconductors:

The Poisson's equation in the presence of an external source of charge density $\rho_{\text{ext}}$ is given by (2,3)

$$\nabla^2 \phi = -\rho / \varepsilon_s$$ ..........(8.2)

Where $\rho = \rho_{\text{ext}} + \rho_{\text{ind}}$ and $\rho_{\text{ind}}$ is the induced charge density. In the long wavelength limit, the charge density at a point is a function of the local potential seen by the carriers.

$$\rho_{\text{ind}} = \varepsilon_s [n - n_0]$$ ..........(8.3a)

where $n = n_0 \exp(\varepsilon_s / K_B T)$

or, $\rho = n_0 \varepsilon_s [\exp(\varepsilon_s / K_B T) - 1] \approx n_0 \varepsilon_s^2 \phi / K_B T$ ..........(8.3b)

Therefore, equation (8.2) becomes

$$\nabla^2 \phi - (n_0 \varepsilon_s^2 \phi / K_B T) = \rho_{\text{ext}} / \varepsilon_s$$ ..........(8.4)

The above equation can be written as

$$\nabla^2 \phi - Q_s^2 \phi = \rho_{\text{ext}} / \varepsilon_s$$

Where $Q_s$ is the three-dimensional screening parameter given by (2).

$$Q_s = (n_0 \varepsilon_s^2 \phi / K_B T)^{1/2}$$ ..........(8.5)

B) For two-dimensional systems:

According to Thomas-Fermi model, the induced charge density in the two-dimensional limit is given by (2),
\[ \rho_{\text{ind}} = e[N_s(\phi) - N_s(0)] \delta(z) \] ........(8.6)

For a weak potential, the equation 8.6 may be linearised as

\[ \rho_{\text{ind}} = e\phi(dN_s/d\phi) \delta(z) \]
\[ = e^2 \phi (dN_s/dE_F) \delta(z) \] ........(8.7)

Therefore, the Poisson's equation becomes

\[ \nabla^2 \phi - (q_s/2) \phi \delta(z) = \rho_{\text{ext}} \epsilon_o \epsilon_s \] ........(8.8)

Where \( q_s \) is the two dimensional screening parameter given by

\[ q_s = (e^2/2\epsilon_o \epsilon_s)(dN_s/dE_F) \] ........(8.9)

At low temperatures where \( (E_o - E_F) >> K_B T \),

\[ q_s = 2me^2/4\pi \epsilon_o \epsilon_s \hbar^2 \] ........(8.10a)

and at higher temperatures

\[ q_s = e^2 N_s / 2 \epsilon_o \epsilon_s K_BT \] ........(8.10b)

**8.2.2 Lindhard Method:**

This method is an exact Hartree calculation of the charge density in the presence of the self consistent field of the carrier gas. The only simplification is that the induced charge density is required to be of linear order in \( \phi \).

The perturbing potential which causes the fluctuation in charge density is assumed to be a sinusoidal time varying potential (3).

\[ \delta U(q, \omega) = U_o \cos(q.r + \omega t)e^{in} \] ........(8.11)
\( \alpha \) is a small parameter that assumes that the potential vanishes at \( t \to -\infty \). This perturbing potential gives rise to a fluctuation in charge density given by

\[
\delta \rho = e^2 U_0 \sum_q [f(k)-f(k+q)]/[E(k+q)-E(k)+\hbar \omega - \imath \alpha] \quad \text{........(8.12)}
\]

This fluctuation in charge density produces, in turn, a fluctuation in the potential itself. Poisson's equation for the potential is

\[
\nabla^2 \phi = -\delta \rho \varepsilon \varepsilon_s
\quad \text{........(8.13)}
\]

Then the Fourier space representation of equation 8.13 is

\[
\phi = (\delta \rho / q^2 \varepsilon \varepsilon_s) \exp (i q \cdot r) \quad \text{........(8.14)}
\]

Therefore,

\[
\phi = (e^2 / \varepsilon \varepsilon_s q^3) \sum_k [f(k)-f(k+q)] U_0 /[E(k+q)-E(k)+\hbar \omega - \imath \alpha] \quad \text{........(8.15a)}
\]

Where \( U_0 \) is written as

\[
U_0 = \varepsilon_q V/\varepsilon(q, \omega)
\quad \text{........(8.15b)}
\]

Where \( \varepsilon(q, \omega) \) is the frequency dependent dielectric function. Therefore, \( \varepsilon(q, \omega) \) is obtained as

\[
\varepsilon(q, \omega) = 4\pi \varepsilon \varepsilon_s + (e^2 / q^2) \sum_k [f(k)-f(k+q)]/[E(k+q)-E(k)+\hbar \omega - \imath \alpha]
\quad \text{........(8.16)}
\]

### 8.2.3 Dynamic Screening Factor for 2DHG:

The dielectric function in a two-dimensional system is given by (2).

\[
\varepsilon(q, \omega, T) = 1+(e^2 / 2 \varepsilon \varepsilon_s q) F(q) \chi(q, \omega, T)
\quad \text{........(8.17)}
\]
Where $F(q)$ is the form factor defined in chapter 5 and $\chi(q, \omega, T)$ is the polarizibility under RPA.

$$\chi(q, \omega, T) = \sum_k [f_0(k) - f_0(k+q)]/[E_{k+q} - E_k + \hbar \omega - i\hbar \alpha] \quad \ldots(8.18)$$

The carrier distribution in the ground state is given by Fang and Howard variational wave function for a two-dimensional hole gas trapped in a triangular potential well of a heterojunction.

$$\xi_i(z) = (b_i/2)^{1/2} z \exp(-b_i z/2) \quad \ldots(8.19)$$

Where $b$ is the variational parameter given by

$$b_i^3 = 33 \pi m^* e^2 p / 8 \pi \epsilon \epsilon_0 \hbar^2 \quad \ldots(8.20)$$

Where $i$ stands for heavy holes and light holes. The screening parameter for such a system is given by

$$Q_{sr} = (e^2/2 \epsilon_\epsilon_0 \epsilon_\omega) F(q) \chi(q, \omega, T) \quad \ldots(8.21)$$

Using the wave function of equation 8.19 the real part of $Q_{sr}$ is obtained as

$$Q_{sr} = (e^2 m^* / 2 \pi \epsilon \epsilon_0 \epsilon_\omega \hbar^2) \{(b/8)((8b_i^2 + 9b_i q + 3q^2)/(b_i + q)^3) \times \int_0^x \frac{dx}{(1 + \exp(\eta(x^2 - 1)))((Q_{sr}^2 + (2m^* \omega / \hbar k_{r_f}^2) - (2xQ_{sr}^2)^{1/2}) + \int_0^{x2} \frac{dx}{(1 + \exp(\eta(x^2 - 1)))((Q_{sr}^2 - (2xQ_{sr}^2)^{1/2})}} \quad \ldots(8.22)$$

where $x = k / k_{r_f}$, $Q_f = q / k_{r_f}$, $\eta = \hbar^2 k_{r_f}^2 / 2 m_i^* K_{b_f} T$, $x_1 = (Q_f^2 / 2) + (m^* \omega / \hbar^2 Q_{r_f} k_{r_f}^2)$, $x_2 = (Q_f^2 / 2) - (m^* \omega / \hbar^2 Q_{r_f} k_{r_f}^2)$

and $k_{r_f}$ is the fermi wave vector. The imaginary part of $Q_{sr}$ is neglected because it introduces an insignificant change in the computed value as seen in case of...
8.2.4 Screening Factor for 1DHG:

The screening factor in 1DHG is also obtained from the Lindhard potential. The dielectric function under Random Phase Approximation for a 1DHG at zero temperature is given by (5,6).

$$\epsilon_0(q, E_F) = 4\pi\epsilon_0 e^2 \frac{(4e^2m^*/\pi\hbar^2)\{F(q)/q\} \ln |(q+2k_F)/(q-2k_F)|}{(q^2)^{3/2}}$$

......(8.23)

The second term of equation 8.23 has a singularity at \( q = 2k_F \). This divergence may be avoided if the dielectric function is evaluated at non-zero temperature (6).

$$\epsilon_T(q) = \int_0^\infty dE \epsilon_0(q, E_F) [4K_B T \cosh^2[(E_F - E)/K_B T]]^{-1}$$

......(8.24)

The form factor \( F(q) \) is given by

$$F(q) = (2/(qR)^2)[1 - 2K_1(qR)J_1(qR)]$$

......(8.25)

Where \( R \) is the radius of the cylindrical wire that approximately replaces the rectangular wire and \( K_1 \) and \( J_1 \) are Bessel functions.

8.3 Results and Discussions:

The calculation of screening factor has been done in GaAs bulk, 2DHG and 1DHG system in GaAs heterostructures and also 2DHG system in a Si-SiGe-Si quantum well. The parameters used for computation in AlGaAs-GaAs and Si-SiGe systems are given below.

[A] GaAs:

$$\epsilon_s = 12.9, m^*_{3D(HH)} = 0.45m_o, m^*_{3D(L)} = 0.082m_o, m^*_{2D(HH)} = 0.44m_o$$

$$m^*_{2D(L)} = 0.17m_o, m^*_{1D(HH)} = 0.027m_o, \hbar\omega_o = 36.8\text{meV}, L_y = L_z = 100\text{Å}.$$
The variation of the screened dielectric constant $\varepsilon(2)$ with phonon wave vector $q$ obtained from the Thomas-Fermi model for bulk and 2DHG systems in GaAs are shown in fig. (8.1) and fig. (8.2) respectively.

The nature of variation is same in both the systems i.e, $\varepsilon(q)$ decreases with increase in $q$, but the screening in 2DHG systems is higher than that of the bulk.

The variation of dynamically screened permittivity with phonon wave vector $q$ for 2DHG system in a AlGaAs-GaAs heterojunction is shown in fig. (8.3). For a two-dimensional hole gas the screened permittivity is found to decrease sharply with increase in $q$ at low values of the phonon wave vector but for values of $q$ greater than $2k_F$ it does not change significantly.

Fig. (8.4) shows the variation of screened permittivity $\varepsilon(q,T)$ with phonon wave vector $q$ for a one-dimensional hole gas system in a GaAs QWW. Here also $\varepsilon(q,T)$ is found to decrease with increase in $q$ but shows a spike at $q = 2k_F$.

The comparison of $\varepsilon(q,T)$ for two and one-dimensional hole gases has been shown in fig. (8.5). For 2DHG systems screening is higher than that in 1DHG systems. Thus the screening effect seems to be most prominent in case of two-dimensional systems. However, the values of static screening are higher than the dynamic screening.

The unscreened mobility of the 2DHG limited by polar optic phonon scattering is compared with the dynamically screened values in fig. (8.6). It is found that the screened values are ten times higher than the unscreened values. At low temperatures i.e, 30K the screened mobilities are 12 times higher than the unscreened values whereas at 100K the former is 15 times higher than the later one.

The screened and unscreened mobilities of the 2DHG in the Si-SiGe-Si symmetric quantum well has been shown in fig. (8.7). Here static screening has been considered. The effect of screening is found to decrease with increase in temperature. If screening is not included impurity scattering appears to be more dominant than alloy scattering. However, screening plays a significant role in lowering impurity scattering. If screening is included, alloy scattering becomes the dominant mechanism.
Fig. 8.1 Normalized permittivity Vs. normalized wave vector in bulk GaAs for static screening with 3D concentration, \( p_{3D} = 1 \times 10^{24} m^{-3} \).
Fig. 8.2 Normalized permittivity Vs. normalised wave vector in AlGaAs-GaAs heterojunction for static screening with 2D hole density $p_{2D} = 1 \times 10^{16} \text{ m}^{-2}$. 
Fig. 8.3 Normalised permittivity Vs. normalised wave vector in AlGaAs-GaAs heterojunction for dynamic screening with 2D hole density $p_{2D} = 1 \times 10^{16} m^{-2}$
Fig. 8.4 Normalised permittivity Vs. normalised wave vector in GaAs QWW with 1D hole density $p_{1D} = 1 \times 10^8 m^{-1}$
Fig. 8.5 Normalised permittivity Vs. normalised wave vector for 1-2DHG with $p_{2D} = 1 \times 10^{16} m^{-2}$, 2-1DHG with $p_{1D} = 1 \times 10^{9} m^{-1}$
The screened and unscreened mobility of 1DHG in a GaAs quantum wire are compared in fig (8.8). The screening causes an increase in mobility by about 50%. At low temperatures (i.e., below 40K) where impurity scattering dominates, the screening effect decreases with increase in temperature. At higher temperatures where polar optic phonon scattering dominates, the screening effect increases slightly with increase in temperature. This is also observed in case of dynamic screening in 2DHG.

8.4 Conclusion:

From the present studies it may be concluded that screening has a significant effect in the transport properties of the carriers. The screening effect increases in lower dimensions but it seems to be most prominent in two-dimensional systems. The static screening has a greater influence than dynamic screening. The screening effect becomes weaker at higher phonon wave vectors.
Fig. 8.6 Polar optic phonon scattering limited hole mobility vs. temperature for 2DHG in GaAs-AlGaAs heterojunction with 2D hole density $p_{2D} = 9.2 \times 10^{15} \text{m}^{-2}$. Sc-screened mobility, Un-unscreened mobility.
Fig. 8.7. 2DHG mobility Vs. temperature in Si-SiGe-Si quantum well for 2D hole density $p_{2D} = 8 \times 10^{15} \text{m}^{-2}$ Sc-Screened, UN-Unscreened.
Fig. 8.8. 1DHG mobility Vs. temperature for a GaAs QWW with 1D hole density $p_{1D} = 1 \times 10^8 \text{m}^{-1}$. SC-Screened, UN-Unscreened.
References