3.1 Introduction

In 1927 a French mathematician Maurice Fréchet has introduced Fréchet distribution. It is a special case of generalized extreme value distribution and is also known as type II extreme value distribution, which is equivalent to taking the reciprocal of values from a standard Weibull distribution. Extreme Value distributions, are widely used in risk management, finance, insurance, economics, hydrology, material sciences, telecommunications and many other industries dealing with extreme events. The Fréchet distribution is useful for modeling and analysis of several extreme events ranging from accelerated life testing to earthquakes, floods, rainfall, sea currents and wind speeds, etc. More information about the Fréchet distribution can be found in Kotz and Nadarajah (2000), Coles (2001) and Johnson et al. (2004), Nadarajah and Kotz (2008), Mubarak (2012), Harlow

Nadarajah and Kotz (2003) introduced the exponentiated Fréchet distribution with distribution function

\[ F(x) = 1 - (1 - \exp\{-\left(\frac{\sigma}{x}\right)^{\lambda}\})^\alpha; \]

\[ x > 0, \alpha > 0, \beta > 0, \lambda > 0, \sigma > 0. \] For \( \alpha = 1 \), the exponentiated Fréchet distribution becomes the Fréchet distribution with parameters \( \lambda \) and \( \sigma \). Abd-Elfattah and Omima (2009) discussed estimation of parameters of the generalized Fréchet distribution. Nadarajah and Gupta (2004) introduced the Beta Fréchet distribution with the distribution function

\[ F(x) = \frac{1}{B(a, b)} \int_0^{e^{-\left(\frac{x}{\lambda}\right)^{\lambda}}} w^{a-1}(1 - w)^{b-1} dw; x, \sigma, \lambda, a, b > 0. \]

The Beta Fréchet distribution generalizes some well known distributions. For \( a = 1 \), we obtain the exponentiated Fréchet distribution with parameters \( \sigma, \lambda \) and \( \alpha = b \). For \( a = 1 \) and \( b = 1 \) we obtain the Fréchet distribution with parameters \( \sigma \) and \( \lambda \).

Recently, Krishna et al. (2013 a) introduced Marshall-Olkin Fréchet distribution with survival function given by

\[ \bar{G}(x) = \frac{\theta(1 - \exp\{-\left(\frac{x}{\lambda}\right)^{\lambda}\})}{\theta + (1 - \theta)\exp\{-\left(\frac{x}{\lambda}\right)^{\lambda}\}} \]

where \( x > 0, \theta > 0, \lambda > 0 \) and \( \sigma > 0 \). Krishna et al. (2013 b) also discussed the applications of Marshall-Olkin Fréchet distribution. Mahmoud et al. (2013) introduced the transmuted Fréchet distribution. Here we consider the EG class of distributions cor-
responding to Fréchet distribution and it is extended to the Marshall-Olkin exponentiated
generalized Fréchet distribution.

This chapter concentrates on Marshall-Olkin Exponentiated Generalized Fréchet Dis-
tribution and its Applications. In section 3.2 we discuss the important properties of the
Exponentiated generalized Fréchet distribution. Marshall-Olkin Exponentiated generalized
Fréchet Distribution and its properties are discussed in section 3.3. In section 3.4 we
consider the quantiles and distribution of order statistics. In section 3.5 the maximum
likelihood estimates are obtained and the results are applied to a real data set to com-
pare the new distribution with Exponentiated generalized Fréchet distribution. Reliability
of a system following Marshall-Olkin extended Fréchet distribution under stress-strength
model is estimated in section 3.6. Its validity is examined using average bias and average
mean square error calculated from the simulated values. Simulation studies are conducted
to compute the average length of the asymptotic 95\% confidence intervals and coverage
probability.

### 3.2 Exponentiated Generalized Fréchet Distribution

The c.d.f. of the Fréchet distribution is $G(x) = \exp\left\{-\left(\frac{x}{\sigma}\right)^{\lambda}\right\}$ where $\sigma, \lambda > 0$. Then we
define the Exponentiated Generalized Fréchet (EGF) with cumulative distribution by

$$F(x) = \left[1 - \left(1 - \exp\left\{-\left(\frac{x}{\sigma}\right)^{\lambda}\right\}\right)^{\alpha}\right]^\beta \quad (3.2.1)$$

where $x > 0, \alpha > 0, \beta > 0, \lambda > 0$ and $\sigma > 0$. The EGF density can be obtained from
(1.3.4) as

$$f(x) = \alpha \beta \lambda \sigma \lambda x^{-(\lambda+1)} \exp\left\{-\left(\frac{x}{\sigma}\right)^{\lambda}\right\} \left(1 - \exp\left\{-\left(\frac{x}{\sigma}\right)^{\lambda}\right\}\right)^{\alpha-1} \left[1 - \left(1 - \exp\left\{-\left(\frac{x}{\sigma}\right)^{\lambda}\right\}\right)^{\alpha}\right]^{\beta-1}$$

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Here when $\beta = 1$, the distribution reduces to the Exponentiated Fréchet distribution and when $\alpha = 1, \beta = 1$, it reduces to the standard Fréchet distribution. The survival function and hazard rate function of the EGF distribution is given by

$$F(x) = 1 - \left[1 - \left(1 - \exp\left\{-\left(\frac{\sigma}{x}\right)^\lambda\right\}\right)^\alpha\right]^\beta$$

(3.2.2)

and

$$r(x) = \alpha\beta\lambda\sigma^\lambda x^{-(\lambda+1)}\exp\left\{-\left(\frac{\sigma}{x}\right)^\lambda\right\} \left(1 - \exp\left\{-(\frac{\sigma}{x})^\lambda\right\}\right)^{\alpha-1}$$

$$\left[1 - (1 - \exp\left\{-(\frac{\sigma}{x})^\lambda\right\})^\alpha\right]^{\beta-1}$$

$$\frac{1}{1 - [1 - (1 - \exp\left\{-(\frac{\sigma}{x})^\lambda\right\})^\alpha]^\beta}$$

Figure 3.1 shows the p.d.f. of EGF distribution for different values of parameters.

Figure 3.2 shows the hazard function of EGF distribution for different values of param-
The $p^{th}$ quantile function $x_p$ of the EGF distribution, which is the inverse of the distribution function $F(x_p) = p$ is given by

$$x_p = \left[ \frac{-\sigma \lambda}{\log[1 - (1 - p)^\frac{1}{\beta}]} \right]^{\frac{1}{\alpha}}$$

### 3.3 Marshall-Olkin Exponentiated Generalized Fréchet Distribution

Consider the survival function of exponentiated generalized Fréchet distribution $\bar{F}(x) = 1 - \left[1 - \left(1 - \exp\left\{-\left(\frac{x}{\lambda}\right)^\lambda\right\}\right)^\alpha\right]^\beta$. In this section we introduce the Marshall-Olkin Exponentiated Generalized Fréchet Distribution using Marshall-Olkin techniques. It is denoted by $\text{MOEGF}$
distribution. The survival function of MOEGF distribution is given by

\[
\bar{G}(x) = \frac{\theta [1 - (1 - \exp\{-\left(\frac{\sigma}{x}\right)^\lambda\})^{\alpha}]^\beta}{1 - (1 - \theta) [1 - (1 - \exp\{-\left(\frac{\sigma}{x}\right)^\lambda\})^{\alpha}]^\beta}
\]  

(3.3.1)

Then the corresponding probability density function is given by

\[
g(x) = \theta \alpha \beta \lambda \sigma^\lambda x^{-(\lambda+1)} \exp\{-\left(\frac{\sigma}{x}\right)^\lambda\} \left(1 - \exp\{-\left(\frac{\sigma}{x}\right)^\lambda\}\right)^{\alpha-1} \\
\frac{[1 - (1 - \exp\{-\left(\frac{\sigma}{x}\right)^\lambda\})^{\alpha}]^{\beta-1}}{(1 - \theta + \theta[1 - (1 - \exp\{-\left(\frac{\sigma}{x}\right)^\lambda\})^{\alpha}]^\beta)^2}
\]  

(3.3.2)

\( (3.3.3) \)

\[ \alpha > 0, \beta > 0, \lambda > 0, \sigma > 0, \theta > 0. \]

The cumulative distribution function is given by

\[
G(x) = \frac{[1 - (1 - \exp\{-\left(\frac{\sigma}{x}\right)^\lambda\})^{\alpha}]^\beta}{\theta + (1 - \theta)[1 - (1 - \exp\{-\left(\frac{\sigma}{x}\right)^\lambda\})^{\alpha}]^\beta}
\]

and the hazard rate function is given by

\[
h(x) = \frac{\alpha \beta \lambda \sigma^\lambda x^{-(\lambda+1)} \exp\{-\left(\frac{\sigma}{x}\right)^\lambda\} [A(x)]^{\alpha-1} [1 - [A(x)]^{\alpha}]^{\beta-1}}{K(x)[1 - \alpha K(x)]}
\]

\[ \text{where } A(x) = (1 - \exp\{-\left(\frac{\sigma}{x}\right)^\lambda\}) \text{ and } K(x) = [1 - (1 - \exp\{-\left(\frac{\sigma}{x}\right)^\lambda\})^{\alpha}]^\beta \]

Figure 3.3 shows the pdf of MOEGF distribution for different values of parameters.
3.4 Quantiles and Order statistics

The $p^{th}$ quantile function of the distribution, which is the inverse of the distribution function $F(x_p) = p$, is given by

$$x = -\frac{1}{\sigma} \left[ \log \left\{ \left[ 1 - \frac{p^\theta}{1 - p(1 - \theta)} \right]^\frac{1}{\theta} \right\} \right]^\frac{1}{\beta}$$

Let $X_1, X_2, ..., X_n$ be a random sample taken from the MOEGF distribution and $X_{1:n}, X_{2:n}, ..., X_{n:n}$ be the corresponding order statistics. The survival function of MOEGF distribution is given by (3.3.1). Then the c.d.f. of the first order statistic $X_{1:n}$
CHAPTER 3. MARSHALL-OLKIN EXPONENTIATED GENERALIZED FRÉCHET DISTRIBUTION AND ITS APPLICATIONS

is given by

\[ G_{1:n}(x) = 1 - (\bar{G}(x))^n = 1 - \left[ \frac{\theta \left[ 1 - \left( 1 - \exp\left\{ -\left( \frac{\sigma}{x} \right)^\lambda \right\} \right]^\beta \right]}{1 - (1 - \theta) \left[ 1 - \left( 1 - \exp\left\{ -\left( \frac{\sigma}{x} \right)^\lambda \right\} \right]^\alpha \right]^\beta} \right]^n \]

The c.d.f. of the \( n^{th} \) order statistic \( X_{n:n} \) is given by

\[ G_{n:n}(x) = \left[ 1 - \bar{G}(x) \right]^n = \left\{ \frac{[1 - (1 - \exp\{-(\frac{\sigma}{x})^\lambda\})^\alpha]^\beta}{\theta + (1 - \theta)[1 - (1 - \exp\{-(\frac{\sigma}{x})^\lambda\})^\alpha]^\beta} \right\}^n \]

The probability density function \( g_{i:n}(x) \) of the \( i^{th} \) order statistics \( X_{i:n} \) is given by

\[
g_{i:n}(x) = \frac{n!}{(i-1)!(n-1)!} \theta \alpha \beta \lambda \sigma^\lambda x^{-(\lambda+1)} \exp\{-\frac{\sigma}{x}\} \left( 1 - \exp\{-\frac{\sigma}{x}\} \right)^{\alpha-1} \left[ 1 - (1 - \exp\{-\left( \frac{\sigma}{x} \right)^\lambda \})^\alpha \right]^\beta-1 \left\{ \frac{1 - \exp\{-\left( \frac{\sigma}{x} \right)^\lambda \})^\alpha \right\}^\beta \left\{ \theta + (1 - \theta)[1 - (1 - \exp\{-\left( \frac{\sigma}{x} \right)^\lambda \})^\alpha]^\beta \right\}^{i-1} \left\{ \theta \left[ 1 - (1 - \exp\{-\left( \frac{\sigma}{x} \right)^\lambda \})^\alpha \right]^\beta \right\}^{n-i} \left\{ \frac{1 - (1 - \theta) \left[ 1 - (1 - \exp\{-\left( \frac{\sigma}{x} \right)^\lambda \})^\alpha \right]^\beta}{1 - (1 - \theta) \left[ 1 - (1 - \exp\{-\left( \frac{\sigma}{x} \right)^\lambda \})^\alpha \right]^\beta} \right\}^n \]

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3.5 Estimation of Parameters

In this section we consider maximum likelihood estimates of the parameters with respect to a given sample \((x_1, x_2, \ldots, x_n)\). Then the log likelihood function is given by

\[
\log L(\alpha, \beta, \sigma, \lambda, \theta) = n \left[ \log \theta + \log \alpha + \log \beta + \log \lambda + \lambda \log \sigma \right] - \\
(\lambda + 1) \sum_{i=1}^{n} \log x_i - \lambda \sum_{i=1}^{n} \left( \frac{\sigma}{x_i} \right) + (\alpha - 1) \sum_{i=1}^{n} \log \left[ 1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\} \right] + \\
(\beta - 1) \sum_{i=1}^{n} (1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\})^\alpha - \\
2 \sum_{i=1}^{n} \log \left\{ 1 - \tilde{\theta} + \tilde{\theta} \left[ 1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\} \right]^\beta \right\}
\]

The partial derivative of the log likelihood functions with respect to the parameters are

\[
\frac{\partial \log L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \log (1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\}) - (\beta - 1) \sum_{i=1}^{n} \log (1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\}) + \\
\frac{(1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\})^\alpha}{1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\})^\alpha + 2\theta \sum_{i=1}^{n} (1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\})^\alpha} \\
\log (1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\}) \left\{ 1 - (1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\})^\alpha \right\}^\beta - 1
\]

\[
\frac{\partial \log L}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \log [1 - (1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\})^\alpha] - \\
2\theta \sum_{i=1}^{n} \log [1 - (1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\})^\alpha] \left\{ 1 - \tilde{\theta} + \tilde{\theta} \left[ 1 - \exp \left\{ -\left( \frac{\sigma}{x_i} \right)^\lambda \right\} \right]^\beta \right\}
\]
The equations can be solved using nlm package in R software.
Table 3.1: Summary of fitting for the MOEGF and exponentiated generalized Fréchet distribution.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Estimates</th>
<th>-Log-likelihood</th>
<th>K-S statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGF</td>
<td>α</td>
<td>0.0828</td>
<td>416.6228</td>
<td>0.8078</td>
</tr>
<tr>
<td></td>
<td>β</td>
<td>0.0849</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>λ</td>
<td>0.0718</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>σ</td>
<td>0.0245</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MOEGF</td>
<td>α</td>
<td>0.0758</td>
<td>304.6260</td>
<td>0.6975</td>
</tr>
<tr>
<td></td>
<td>β</td>
<td>0.0763</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>λ</td>
<td>0.0849</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>σ</td>
<td>0.0508</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td>0.0654</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.5.1 Data Analysis

In this section we analyze some data sets and compare Marshall-Olkin exponentiated generalized Fréchet distribution with the exponentiated generalized Fréchet distribution. We consider the data from Lawless (1986). The data given here arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life test and they are 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40.

We estimate the unknown parameters of the distribution by the method of maximum likelihood estimation. Also, we draw the P-P plots and Q-Q plots for fitted distributions and are presented in Figure 3.4 and in Figure 3.5. We can see that the Marshall-Olkin exponentiated generalized Fréchet distribution is a good fit as compared to exponentiated generalized Fréchet distribution.
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Figure 3.4: QQ and PP plot for MOEGF Distribution

Figure 3.5: QQ and PP plot for EGF Distribution
3.6 Stress-Strength Analysis

In this section we consider the stress-strength reliability \( R = P(X < Y) \), where \( X \) represents stress and \( Y \) represents the strength. Gupta et al. (2010) obtained various results on the MO family in the context of reliability modeling and survival analysis.

\[
P(X < Y) = \int_{-\infty}^{\infty} P(Y > X | X = x) g_X(x) \, dx
\]

Consider the p.d.f. of Marshall-Olkin Exponentiated Generalized Fréchet distribution given by

\[
g(x) = \theta \alpha \beta \lambda x^{-(\lambda+1)} \exp\left\{-\left(\frac{\sigma}{x}\right)^\lambda\right\} \left(1 - \exp\left\{-\left(\frac{\sigma}{x}\right)^\lambda\right\}\right)^{\alpha - 1} \left[1 - \left(1 - \exp\left\{-\left(\frac{\sigma}{x}\right)^\lambda\right\}\right)^{\alpha \beta}\right]^{\frac{1}{\theta + \bar{\theta} + (1 - \exp\left\{-\left(\frac{\sigma}{x}\right)^\lambda\right\})^{\alpha \beta}}}
\]

\(\alpha > 0, \beta > 0, \lambda > 0, \theta > 0, \sigma > 0.\)

Let \((x_1, x_2, ..., x_m)\) and \((y_1, y_2, ..., y_n)\) be two independent random samples of sizes \(m\) and \(n\) from Marshall-Olkin Exponentiated Generalized Fréchet distribution with tilt parameters \(\alpha_1\) and \(\alpha_2\) respectively, and common unknown parameters \(\beta, \lambda\) and \(\theta\). The log likelihood function is given by

\[L(\alpha_1, \alpha_2, \beta, \lambda, \theta) = \sum_{i=1}^{m} \log g(x_i; \alpha_1, \beta, \lambda, \theta, \sigma) + \sum_{j=1}^{n} \log g(y_j; \alpha_2, \beta, \lambda, \theta, \sigma)\]

The maximum likelihood estimates of the unknown parameters \(\alpha_1, \alpha_2\) are the solutions of non linear equations \(\frac{\partial L}{\partial \alpha_1} = 0\) and \(\frac{\partial L}{\partial \alpha_2} = 0\) respectively.
3.6.1 Simulation Study

We generate $N = 10000$ values of $X$ and $Y$ observations from Marshall-Olkin Exponentiated Generalized Fréchet distribution with parameters $\alpha_1, \beta, \lambda, \theta, \sigma$ and $\alpha_2, \beta, \lambda, \theta, \sigma$ respectively. The combinations of samples of sizes $m = 20, 25, 30$ and $n = 20, 25, 30$ are considered. The estimates of $\alpha_1$ and $\alpha_2$ are obtained from each sample to obtain $\hat{R}$. The validity of the estimate of $R$ is examined as in section 2.6. The average bias and average mean square error of the simulated estimates of $R$ for various values of parameters are given in Table 3.2. The average confidence length and coverage probability of the simulated estimates are given in Table 3.3.

3.7 Conclusion

In this chapter we have introduced a new distribution namely, Marshall-Olkin Exponentiated Generalized Fréchet Distribution and its properties are discussed. We analyze a real data set and compare MOEGF distribution with the exponentiated generalized Fréchet distribution. We conclude that the MOEGF distribution is a better fit as compared to exponentiated generalized Fréchet distribution. The results are given in Table 3.1. Estimation of stress-strength reliability is also done. The average bias and average mean square error of the simulated estimates of $R$ for various values of parameters are given in Table 3.2. The average confidence length and coverage probability of various values of parameters are given in Table 3.3. The R program developed is given as Appendix.

3.8 Appendix

```r
# MOEGF DISTRIBUTION
# Probability density function
dMOEGF<-function(x,alpha,beta,lambda,sigma,theta) {
  temp <- exp(-(sigma/x)^lambda)
  theta*alpha*beta*lambda*sigma^lambda*x^(lambda+1)*temp*(1-temp)^(alpha-1)*(1-(1-temp)^alpha)^(beta-1)
  /((1-(1-theta))+(1-theta)*(1-(1-temp)^alpha)^beta)^2}
# Distribution function
pMOEGF<-function(x,alpha,beta,lambda,sigma,theta) {
```

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Table 3.2: Average bias and average MSE of the simulated estimates of $R$ for $\beta = 4, \lambda = 3, \sigma = 2$ and $\theta = 2$

<table>
<thead>
<tr>
<th>$(m,n)$</th>
<th>$(a_1,a_2)$</th>
<th>Average bias ($\bar{b}$)</th>
<th>Average Mean Square Error (AMSE)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.8,0.9)</td>
<td>(1.2,1.4)</td>
<td>(1.5,1.9)</td>
</tr>
<tr>
<td>(20,20)</td>
<td>0.0378</td>
<td>0.0509</td>
<td>0.0771</td>
</tr>
<tr>
<td>(20,25)</td>
<td>0.0384</td>
<td>0.0496</td>
<td>0.0767</td>
</tr>
<tr>
<td>(20,30)</td>
<td>0.0380</td>
<td>0.0508</td>
<td>0.0772</td>
</tr>
<tr>
<td>(25,20)</td>
<td>0.0395</td>
<td>0.0530</td>
<td>0.0788</td>
</tr>
<tr>
<td>(25,25)</td>
<td>0.0406</td>
<td>0.0527</td>
<td>0.0803</td>
</tr>
<tr>
<td>(25,30)</td>
<td>0.0402</td>
<td>0.0521</td>
<td>0.0801</td>
</tr>
<tr>
<td>(30,20)</td>
<td>0.0415</td>
<td>0.0541</td>
<td>0.0810</td>
</tr>
<tr>
<td>(30,25)</td>
<td>0.0421</td>
<td>0.0539</td>
<td>0.0815</td>
</tr>
<tr>
<td>(30,30)</td>
<td>0.0408</td>
<td>0.0539</td>
<td>0.0813</td>
</tr>
</tbody>
</table>

Table 3.3: Average confidence length and coverage probability of the simulated 95% percentage confidence intervals of $R$ for $\beta = 4, \lambda = 3, \sigma = 2$ and $\theta = 2$

<table>
<thead>
<tr>
<th>$(m,n)$</th>
<th>$(a_1,a_2)$</th>
<th>Average Confidence Length</th>
<th>Coverage Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.8,0.9)</td>
<td>(1.2,1.4)</td>
<td>(1.5,1.9)</td>
</tr>
<tr>
<td>(20,20)</td>
<td>0.3544</td>
<td>0.3537</td>
<td>0.3525</td>
</tr>
<tr>
<td>(20,25)</td>
<td>0.3363</td>
<td>0.3356</td>
<td>0.3344</td>
</tr>
<tr>
<td>(20,30)</td>
<td>0.3236</td>
<td>0.3229</td>
<td>0.3218</td>
</tr>
<tr>
<td>(25,20)</td>
<td>0.3335</td>
<td>0.3357</td>
<td>0.3346</td>
</tr>
<tr>
<td>(25,25)</td>
<td>0.3172</td>
<td>0.3166</td>
<td>0.3153</td>
</tr>
<tr>
<td>(25,30)</td>
<td>0.3037</td>
<td>0.3031</td>
<td>0.3019</td>
</tr>
<tr>
<td>(30,20)</td>
<td>0.3238</td>
<td>0.3232</td>
<td>0.3217</td>
</tr>
<tr>
<td>(30,25)</td>
<td>0.3038</td>
<td>0.3032</td>
<td>0.3020</td>
</tr>
<tr>
<td>(30,30)</td>
<td>0.2869</td>
<td>0.2891</td>
<td>0.2880</td>
</tr>
</tbody>
</table>
Chapter 3. Marshall-Okin Exponentiated Generalized Fréchet Distribution and Its Applications

```r
temp <- -(sigma/x)^lambda
temp <- (1-exp(temp))^alpha
theta*(1-(1-temp)^beta)/(1-(1-theta)*(1-(1-temp)^beta))
}
# Quantile function
qMOEGF<-function(p,alpha,beta,lambda,sigma,theta) {
  -1/sigma*(log((1-p*theta/(1-p*(1-theta))))"^(1/beta))"^(1/alpha))"^(1/lambda)
}
# Random generation
rMOEGF<-function(alpha,beta,lambda,sigma,theta,nobs) {
  varSample<-double(nobs)
  for (i in 1:nobs) {
    p<-runif(1)
    varSample[i]<- -1/sigma*(log((1-p*theta/(1-p*(1-theta))))"^(1/beta))"^(1/alpha))"^(1/lambda)
  }
  varSample
}
# Log-likelihood function
logLikelihoodMOEGF<-function(x) {
  alpha<-x[1]
  beta<-x[2]
  lambda<-x[3]
  sigma<-x[4]
  theta<-x[5]
  nobs<-length(y);
  temp <- 1-exp(-(sigma/x)^lambda)
  nobs*log(theta)+nobs*log(alpha)+nobs*log(beta)+nobs*log(lambda)+nobs*log(sigma)
  -(lambda+1)*sum(log(x))-lambda*sum(sigma/x)+(alpha-1)*sum(log(temp))+(beta-1)*sum(1-temp^alpha)
  -2*sum(log(1-(1-theta)+(1-theta)*(1-temp^alpha)^beta))
}
gradientMOEGF <- function(x) {
  alpha<-x[1]
  beta<-x[2]
  lambda<-x[3]
  sigma<-x[4]
  theta<-x[5]
  nobs<-length(y);
  temp <- exp(-(sigma/x)^lambda)
  der1 <- n/alpha+sum(log((1-temp)^alpha)-beta*(alpha-1)*sum(log((1-temp)^alpha)"^(1-temp))"^(1-temp))"^(1-temp)*"^alpha/(1-(1-temp)"^alpha))"*2*theta*beta*
  nobs*sum(log((1-temp)^alpha)*log((1-temp)^alpha)"^(1-temp))"^(1-temp)"^(1-temp)"^(alpha)"^(beta-1)/(1-(1-theta)*"^alpha)"/(1-(1-theta)*"^alpha)"^(beta))
  der2 <- n/beta*sum(log(1-temp)^alpha))"-2*theta*sum(log((1-temp)^alpha)*log((1-temp)^alpha))"^alpha/(1-(1-temp)"^alpha)"*beta
  /((1-(1-theta)*"^alpha)"^beta)
  der3 <- n*(1/lambda*sum(log(sigma))-sum(log(x))"^alpha*sum(1/x)"^alpha^-1*sigma*sum((1/x)*temp*log(sigma/x)/(1-temp))
```
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\[-(\beta-1)\alpha \sigma^\lambda \sum \left( \frac{1}{x^\lambda} \text{temp} \log \left( \frac{\sigma}{x} \right) (1-\text{temp})^{\alpha-1} \right) (1-(1-\text{temp})^{\alpha})^{-\beta} \]

\[-2\alpha \beta \theta \sigma^\lambda \sum \left( \frac{1}{x^\lambda} \text{temp} \log \left( \frac{\sigma}{x} \right) (1-\text{temp})^{\alpha-1} (1-(1-\text{temp})^{\alpha})^{\beta-1} \right) \]

\[\text{der4} \leftarrow n \lambda \sigma^\lambda \sum \left( \frac{1}{x^\lambda} \text{temp} \log \left( \frac{\sigma}{x} \right) (1-\text{temp})^{\alpha-1} \right) - (\beta-1)\alpha \lambda \sigma^{\lambda-1} \sum \left( \frac{\text{temp}}{x^\lambda} (1-\text{temp})^{\alpha-1} \right) \]

\[\text{der5} \leftarrow n \theta^{-2} \sum \left( \frac{1}{x^\lambda} \right)^{\beta} \left( 1-(1-\text{temp})^{\alpha} \right)^{\beta} \]

\[c(\text{der1}, \text{der2}, \text{der3}, \text{der4}, \text{der5}) \]

# We minimize the following function

\[\text{functionMOEGF}\leftarrow\text{function}(x)\{\]
\[\text{res}<-\text{logLikelihoodMOEGF}(x);\]
\[\text{# attr(res,"gradient") }<- \text{-gradientMOEGF}(x);\]
\[\text{res}\]

# QQ plot

\[\text{qqMOEGF}\leftarrow\text{function}(y,\alpha,\beta,\lambda,\sigma,\theta)\{\]
\[\text{nn}\leftarrow\text{length}(y);\]
\[\text{x<-qMOEGF}(\text{points(nn)},\alpha,\beta,\lambda,\sigma,\theta)[\text{order(order(y))}];\]
\[\text{plot}(y, x, \text{main}="MOEGF Q-Q Plot", \text{xlab}="Theoretical Quantile", \text{ylab}="Sample Quantiles");\]
\[\text{z<-quantile(y,c(0.25,0.75))};\]
\[\text{xx<-qMOEGF}(\text{c(0.25,0.75)},\alpha,\beta,\lambda,\sigma,\theta);\]
\[\text{slope}\leftarrow\text{diff}(z)/\text{diff}(xx);\]
\[\text{int}\leftarrow z[1]-\text{slope}\times xx[1];\]
\[\text{abline(int, slope)}\]

# PP plot

\[\text{ppMOEGF}\leftarrow\text{function}(y,\alpha,\beta,\lambda,\sigma,\theta)\{\]
\[\text{nn}\leftarrow\text{length}(y);\]
\[\text{yvar}\leftarrow\text{pMOEGF}(\text{sort(y)},\alpha,\beta,\lambda,\sigma,\theta);\]
\[\text{xvar}\leftarrow\text{1:nn};\]
\[\text{x<-xvar-0.5}/\text{nn};\]
\[\text{plot}(\text{yvar}, x, \text{main}="MOEGF P-P Plot", \text{xlab}="Expected", \text{ylab}="Observed");\]
\[\text{curve}(\text{i}\times 0,1, \text{add}\leftarrow T)\]

# EGF DISTRIBUTION:

# Probability density function

\[\text{dEGF}\leftarrow\text{function}(x,\alpha,\beta,\lambda,\sigma)\{\]
\[\text{temp}\leftarrow\exp(-\left(\frac{\sigma}{x}\right)\lambda);\]
\[\alpha \beta \lambda \sigma^\lambda x^{(\lambda+1)} \left( 1-(1-\text{temp})^{\alpha-1} \right) (1-(1-\text{temp})^{\alpha}) \]

# Distribution function
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```r
# Probability function
pEGF <- function(x, alpha, beta, lambda, sigma) {
  temp <- -(sigma/x)^lambda
  temp <- (1-exp(temp))^-alpha
  (1-(1-temp)^beta)
}
# Quantile function
qEGF <- function(p, alpha, beta, lambda, sigma) {
  (-sigma^lambda/log(1-(1-p)^((1/beta)*(1/alpha))))^(1/lambda)
}
# Random generation
rEGF <- function(alpha, beta, lambda, sigma, nobs) {
  varSample <- double(nobs)
  for (i in 1:nobs) {
    p <- runif(1)
    varSample[i] <- (-sigma^lambda/log(1-(1-p)^((1/beta)*(1/alpha))))^(1/lambda)
  }
  varSample
}
# Log-likelihood function
logLikelihoodEGF <- function(x) {
  alpha <- x[1]
  beta <- x[2]
  lambda <- x[3]
  sigma <- x[4]
  nobs <- length(y);
  temp <- 1-exp(-(sigma/x)^lambda)
  nobs*log(alpha)+nobs*log(beta)+nobs*log(lambda)+nobs*lambda*log(sigma)-
  (lambda+1)*sum(log(x))-lambda*sum(sigma/x)+(alpha-1)*sum(log(temp))-
  (beta-1)*sum(1-temp^alpha)
}
# gradientEGF <- function(x) {
  alpha <- x[1]
  beta <- x[2]
  lambda <- x[3]
  sigma <- x[4]
  nobs <- length(y);
  temp <- exp(-(sigma/x)^lambda)
  der1 <- n/alpha+n*sum(log(1-temp))-(beta-1)*n*sum(log(1-temp)^alpha)+(1-temp)^alpha
  der2 <- n/beta+n*sum(log(1-temp)^alpha)
  der3 <- n*(1/lambda+n*lambda*log(sigma)+n*sum(log(x))-sigma*sum(1/x)+(alpha-1)*sigma*sum((1/x)*log(x))+(1-temp))-
  (beta-1)*alpha+n*lambda*log(sigma)+n*sum((1/x)*alpha*lambda*temp*log(x)/(1-temp)+
  lambda*temp*sigma+alpha*lambda*temp)/(1-temp))-(beta-1)*alpha+n*lambda*lambda*temp*sigma
  der4 <- n*lambda^2/lambda+n*lambda^2+alpha*lambda*temp*sigma+n*lambda*alpha*temp*sigma
}
```

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c(der1,der2,der3,der4)
# We minimize the following function
functionEGF<-function(x) {
  res<-logLikelihoodEGF(x);
  # attr(res,"gradient") <- -gradientMOEGF(x);
  res
}
-loglikelihoodEGF(x);
# QQ plot
qqEGF<-function(y,alpha,beta,lambda,sigma) {
  nn<-length(y);
  x<-qEGF(ppoints(nn),alpha,beta,lambda,sigma)[order(order(y))];
  plot(y,x,main="EGF Q-Q Plot",xlab="Theoretical Quantile",ylab="Sample Quantiles");
  z<-quantile(y,c(0.25,0.75));
  xx<-qEGF(c(0.25,0.75),alpha,beta,lambda,sigma);
  slope<-diff(z)/diff(xx);
  int<-z[1]-slope*xx[1];
  abline(int,slope)
}
# PP plot
ppEGF<-function(y,alpha,beta,lambda,sigma) {
  nn<-length(y);
  yvar<pEGF(sort(y),alpha,beta,lambda,sigma);
  xvar<1:nn;
  x<(xvar-0.5)/nn;
  plot(yvar,x,main="EGF P-P Plot",xlab="Expected",ylab="Observed");
  curve(i*x,0,1,add=T)
}

References


