Preface

The theory of variational inequalities is well established subject in the area of nonlinear analysis and optimization. It started with the study of minimization of an infinite dimensional functional associated with calculus of variations. It has tremendous applications in many areas of science, social science, engineering and management. It emerged as a powerful tool to investigate and study a wide class of problems arising in industrial management, optimization, partial differential equations, game theory, etc. During the last three decades, it has been extended and generalized in several directions. There are significant developments of variational inequalities related with multi-valued maps, nonmonotone operators, nonconvex optimization problems and structural analysis.

In Chapter 1, we give a brief introduction to the theory of variational inequalities, hierarchical variational inequalities, system of variational inequalities, variational-hemivariational inequalities and split feasibility problems.

In Chapter 2, we collect some basic definitions and results from analysis, metric projection, geometry of Banach spaces and nonsmooth analysis. These definitions and results will be used in this work.

Chapter 3 deals with several kinds of well-posedness of variational-hemivariational inequalities and metric characterizations for the well-posedness. We show that the well-posedness by perturbations of a variational-hemivariational inequality is closely related to the well-posedness by perturbations of the corresponding inclusion problem. We introduce the notion of strongly generalized well-posed-like by perturbation and derive some conditions under which the variational-hemivariational inequalities are strongly generalized the well-posed-like by perturbations.

In Chapter 4, we consider a general system of variational inequalities and establish its equivalence with a system of fixed point problems. We propose implicit and explicit Mann-type algorithms for solving general system of
variational inequalities. We establish the strong convergence of the sequences generated by the proposed algorithms.

The Mann-type extragradient iterative algorithms with regularization for finding a common element of the solution set of a general system of variational inequalities, the solution set of a split feasibility problem and the fixed point set of a strictly pseudo-contractive mapping are presented in Chapter 5. We also give weak convergence result for the sequences generated by the proposed iterative algorithms.

In Chapter 6, we consider triple hierarchical variational inequality problems (THVIP) and system of hierarchical variational inequalities (SHVI). We propose several iterative methods for computing a fixed point of a pseudo-contractive mapping and a solution of a THVIP. We prove that the sequences generated by the proposed algorithm converge strongly to a fixed point of a pseudo-contraction mapping which is also a solution of a THVIP. We also propose multi-step explicit and implicit hybrid extragradient-like methods to compute the approximate solutions of THVIP and present the convergence analysis of the sequences generated by the proposed methods. We derive explicit and implicit solution methods for solving a system of hierarchical variational inequalities (SHVI). Under some mild conditions, we prove that the sequences generated by the proposed methods converge strongly to a unique solution of the SHVI.

Last chapter deals with the triple hierarchical variational inequality problems (THVIP) defined over the set of solutions of a hierarchical variational inequality problem which is defined over the intersection of the fixed point sets of a finite family of nonexpansive mappings. We suggest a hybrid type iterative algorithm for solving this class of THVIP. The strong convergence of the suggested algorithm is also studied.

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