Preface

Historically, the step towards near rings was an axiomatic theory done by Dickson in 1905. The beginning of 30's saw the first proper near ring consideration. Since then the theory of near rings has been developed much and at present it becomes a sophisticated theory with numerous applications in various areas namely geometries, interpolation theory, group theory, polynomials and matrices, specially designs which are important applications of near rings. In recent years its connection with computer science, automata, dynamical systems, rooted trees, coding theory, cryptography etc. have also been dealt with. A near ring is exactly what is needed to describe the structure of the endomorphisms of various mathematical structures adequately.

Near rings are generalizations of rings. It is natural to generalize various concepts of rings to near rings. Betch, Beidleman, Ligh, Luh, Clay, Bell and others have generalized various concepts of near rings. Due to non-ring character of a near ring the results have their own beauty.

The present thesis entitled "Study of semigroup ideals and derivations in near rings" includes a part of research work carried out by the author during the last four years under the able guidance of Prof. Asma Ali at the Department of Mathematics, Aligarh Muslim University, Aligarh. The thesis comprises five chapters and each chapter is subdivided into various sections. The definitions, examples, results and remarks etc. have been specified with double decimal numbers. The first figure denotes the chapter, the second represents the section in the chapter and third points out the number of the definition, the example, the result or the remark as the case may be in a subsequent chapter. For example, Theorem 3.2.1 refers to the first theorem appearing in the second section of the third chapter.

Chapter 1 of the thesis contains some preliminary notions, basic definitions and important well-known results which may be needed for the development of the subsequent text. This chapter as a matter of fact, aims at making the present thesis as self contained as possible. However, the basic knowledge of the near ring theory has been presumed and no attempt is made to include the proofs of the results presented in this chapter.

In 1991 Bresar [62] introduced the concept of a generalized derivation in rings. As a motivation Golbasi [86] defined a generalized derivation in near rings. An additive mapping $F: N \rightarrow N$ is said to be a right generalized (resp. left generalized) derivation with associated derivation $d$ on a near ring $N$ if $F(xy) =
$F(x)y + xd(y)$ (resp. $F(xy) = d(x)y + xF(y)$) for all $x, y \in N$, $F$ is said to be a generalized derivation with associated derivation $d$ on $N$ if it is both a right generalized derivation and a left generalized derivation on $N$ with associated derivation $d$. All derivations are generalized derivations. In chapter 2, we prove some theorems in the setting of a semigroup ideal of a prime near ring admitting a generalized derivation and thereby extend some known results. In section 2.2 some basic definitions and preliminary results are included. Unlike known results of chapter 1, we prefer to give the outlines of the proofs of the results presented in this section in order to familiarize the reader with the techniques generally used in case of derivations that's why we designate them as propositions. Section 2.3 contains some important results proved on generalized derivations of a prime near ring which are required to develop the proofs of the main theorems of the chapter. Section 2.4 is devoted to the study of commutativity of prime near rings with generalized derivations. In fact we extend results of Bell [36, 38] and Bell and Mason [45] for a semigroup ideal of a prime near ring admitting a generalized derivation.

Chapter 3 deals with the study of product of generalized derivations of near rings. In 1957 E. C. Posner [144] established two very striking results (i) if $R$ is a prime ring admitting a nonzero derivation $d$ such that $[d(x), x] = 0$ for all $x \in R$, then $R$ must be commutative and (ii) if $R$ is a prime ring of characteristic not two and $d_1, d_2$ are derivations of $R$ such that the iterate $d_1 d_2$ is also a derivation, then at least one of $d_1, d_2$ is zero. A number of researchers generalized these results in several ways to mention a few; Bergen, Chebotar, Chuang, Hirano, Hvala, Jensen, Krempa, Lansi, Bresar, Vukman, Lee, Bell, Martindale. In section 3.3 we obtain extension of Posner's first theorem in case of a generalized derivation of a prime near ring which states that if $N$ is a prime near ring, $U$ a nonzero semigroup ideal of $N$ and $F$ is a nonzero generalized derivation with associated derivation $d$ such that $F(V) \subseteq U$ for some nonzero semigroup ideal $V$ contained in $U$, $a \in N$ for which $[F(U), a] = \{0\}$, then $a \in Z$, the centre of $N$. In section 3.4, we establish various generalizations/extensions of Posner's second theorem. Finally, we investigate some conditions of the form $F_1(x)F_2(y) + F_2(y)F_1(x) \in Z$ or $xF(y) + F(y)x \in Z$ for all $x, y \in U$ under which prime near ring $N$ with two generalized derivations $F_1$ and $F_2$ turns out to be a commutative ring.

Chapter 4 is devoted to the study of semiderivations of prime near rings. Throughout the chapter we consider right near rings. In 1983 Bergen [48] introduced the notion of a semiderivation in rings. We define semiderivation in near rings. An additive mapping $f : N \rightarrow N$ is said to be a semiderivation on a near ring $N$ if there exists a function $g : N \rightarrow N$ such that (i) $f(xy) = f(x)g(y) + xf(y) = f(x)y + g(x)f(y)$ and (ii) $f(g(x)) = f(x)$, for all $x, y \in N$. In case $g$ is the identity map on $N$, $f$ is of course just a derivation on $N$, so the notion of semiderivation generalizes that of derivation. In section 4.2 we prove some results on semiderivations of a prime near ring $N$ in case of semigroup ideal of $N$ which are necessary to develop the proofs of various theorems in the chapter. In section 4.2, we discuss commutativity of prime near
rings with semiderivations. In fact, we extend results of Bell [36], Bell and Martindale [44] and Bell and Mason [45] for a semigroup ideal of a prime near ring admitting a semiderivation.

In chapter 5, we discuss derivations which act as homomorphisms or as antihomomorphisms, a study initiated by Bell and Kappe [41]. In section 5.2, a study of generalized derivation has been made which acts as a homomorphism or as an antihomomorphism on a semigroup ideal of a prime near ring. Finally, we study semiderivation of a prime near ring $N$ acting as a homomorphism or antihomomorphism on a semigroup ideal of $N$ and prove that either semiderivation is zero or identity map or $N$ is a commutative ring.

The list of some papers based on the text which have already been published or accepted for publication in standard refereed Mathematical Journals/Research Volumes are given below.


In the end, an exhaustive bibliography of the existing material related to the subject matter of the thesis is included which may serve as source material for those, interested in the domain of this research area.