CHAPTER 3

AN ANALYSIS OF MANPOWER, MONEY AND BUSINESS WITH
RANDOM ENVIRONMENTS

3.1 Introduction

In this chapter consider a business organization under varying conditions which are restricted to depend on only three characteristics viz, manpower, money and business under fluctuating conditions of availability and shortage in all the three characteristics. The different states have been discussed under the assumptions that change from full availability to shortage and shortage to full availability occur in exponential times with different parameters. An expression for “Rate of crisis under steady state ($C_\infty$)” is arrived and the steady state costs have also been worked by assigning different cost for the characteristics under different conditions.

As loss and shortcomings are inevitable and fund management are to be done during busy period, one may have to borrow from private financial institutions or banks or individuals and to get the same at the busy period is again probabilistic. Labor and funds become dear during busy periods and one is impelled to pay a heavy cost. The occurrence of busy periods and lean periods are random and they occur alternately in a business organization. It may not become necessary to have full strength of staff and full availability of funds during busy periods. Their full usages are intermittent.

A portion of this chapter with the title “Analysis of Critical States and Total Cost under Varying Availabilities of Manpower, Money and Business” has been published in the proceedings of National Seminar on ‘Applications of Hidden Markov Models in Engineering Fields’, 23.4.2010, at Sri Sakthi Institute of Engineering and Technology, Coimbatore. The full content of this chapter in the present form has been published as a paper with the title “An Analysis of Manpower,
There are two models in this chapter. The first deals with the problem of steady state rate of crisis in which the crisis arises when there is either manpower shortage or money shortage or both during the busy period. The second model deals with the problem of steady state rate of crisis wherein the effect of randomly varying environments have been brought in. The steady state probabilities of the continuous time Markov chain describing the transitions in the various states are derived and critical states are identified for presenting the cost analysis. Numerical illustrations are provided.

3.2 Model - I

3.2.1 Assumptions

1. Busy and lean periods occur successively.
2. The time $T_1$ for which the staff strength remains full is exponentially distributed with parameter $\lambda$ and the time $R_1$ required to complete recruitment for filling up of vacancies is exponentially distributed with parameter $\mu$. $T_1$ and $R_1$ are independently distributed random variables.
3. The busy and lean periods of the business are exponentially distributed with parameter $\alpha$ and $\beta$ respectively.
4. The time $T_2$ for which the fund is available in full is taken to be exponentially distributed with parameter ‘a’ and the time $R_2$ required to supply of funds for meeting shortage is exponentially distributed with parameter ‘b’. $T_2$ and $R_2$ are independently distributed random variables.

3.2.2 System analysis
The stochastic process $X(t)$ describing the state of system is a continuous
time Markov chain with 8 points state space as given below.

$$S = [(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1)]$$

(3.2.1)

The various states are explained as follows.

<table>
<thead>
<tr>
<th>State</th>
<th>Money</th>
<th>Manpower</th>
<th>Business</th>
</tr>
</thead>
<tbody>
<tr>
<td>(111)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(110)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(101)</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(100)</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(011)</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(010)</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(001)</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(000)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

1 - refers full availability
0 - refers shortage / lean

The transition diagram of the system in the state space $S$ is shown by the diagram below:
The infinitesimal generator $Q$ of the continuous time Markov chain of the state space is given below using the transition “diagram”.

The infinitesimal generator $Q$ of the continuous time Markov chain of the state space is given below using the transition “diagram”.
\[
Q = \begin{pmatrix}
\eta_1 & \beta & \mu & 0 & b & 0 & 0 & 0 \\
\alpha & \eta_2 & 0 & \mu & 0 & b & 0 & 0 \\
\lambda & 0 & \eta_3 & \beta & 0 & 0 & b & 0 \\
0 & \lambda & \alpha & \eta_4 & 0 & 0 & 0 & b \\
a & 0 & 0 & 0 & \eta_5 & \beta & \mu & 0 \\
0 & a & 0 & 0 & \alpha & \eta_6 & 0 & \mu \\
0 & 0 & a & 0 & \lambda & 0 & \eta_7 & \beta \\
0 & 0 & 0 & a & 0 & \lambda & \alpha & \eta_8
\end{pmatrix}
\]

where
\[
\eta_1 = -(b+\mu+\beta), \quad \eta_2 = -(b+\mu+\alpha), \quad \eta_3 = -(b+\lambda+\beta), \quad \eta_4 = -(b+\lambda+\alpha), \\
\eta_5 = -(a+\mu+\beta), \quad \eta_6 = -(a+\mu+\alpha), \quad \eta_7 = -(a+\lambda+\beta), \quad \eta_8 = -(a+\lambda+\alpha).
\]

The crises states are \([(001), (101), (011)]\) since the crises occur when the system is busy and shortages are felt in manpower or money or both.

The steady state probability vector of the matrix \(Q\) can be derived easily by noting \(\prod Q = 0\) and \(\prod e = 1\) where \(\prod\) is a vector of type \(1 \times 8\) given by,
\[
\prod = \begin{bmatrix}
\prod_{000} & \prod_{001} & \prod_{010} & \prod_{011} & \prod_{100} & \prod_{101} & \prod_{110} & \prod_{111}
\end{bmatrix}
\]

and \(e_1 = (1,1,1,\ldots,1)^t\) is a unit vector of type \(8 \times 1\).

They can be derived by noting the transition in the co-ordinates of the states space that occur independent of each other,
\[
\prod_{000} = \frac{a\lambda\beta}{(a+b)(\alpha+\beta)(\lambda+\mu)}
\]
\[
\prod_{010} = \frac{a\mu\beta}{(a+b)(\alpha+\beta)(\lambda+\mu)}
\]
\[
\prod_{001} = \frac{a\lambda\alpha}{(a+b)(\alpha+\beta)(\lambda+\mu)}
\]
\[
\prod_{011} = \frac{a\mu\lambda}{(a+b)(\alpha+\beta)(\lambda+\mu)}
\]
\[
\Pi_{100} = \frac{b\alpha}{(a + b)(\alpha + \beta)(\lambda + \mu)} \\
\Pi_{101} = \frac{b\alpha\beta}{(a + b)(\alpha + \beta)(\lambda + \mu)} \\
\Pi_{110} = \frac{b\beta}{(a + b)(\alpha + \beta)(\lambda + \mu)} \\
\Pi_{111} = \frac{b\beta\gamma}{(a + b)(\alpha + \beta)(\lambda + \mu)} 
\] (3.2.4)

Now the rate of crisis in steady state \( C_\infty \) is obtained as follows

\[
P[\text{crisis in (t, t+} \Delta t)] = P[X(t+\Delta t) = (001)| X(t) = (000)] P[X(t) = (000)] \\
+ P[X(t+\Delta t) = (101)| X(t) = (100)] P[X(t) = (100)] \\
+ P[X(t+\Delta t) = (011)| X(t) = (010)] P[X(t) = (010)] \\
+ P[X(t+\Delta t) = (101)| X(t) = (111)] P[X(t) = (111)] \\
+ P[X(t+\Delta t) = (011)| X(t) = (111)] P[X(t) = (111)] + O(\Delta t)
\]

Taking limit as \( \Delta t \rightarrow 0 \),

\[
C_1 = \beta P_{000}(t) + \beta P_{100}(t) + \beta P_{010}(t) + \lambda P_{111}(t) + a P_{111}(t) \\
C_\infty = \text{Lt}_{t \rightarrow \infty} \ [\beta P_{000}(t) + \beta P_{100}(t) + \beta P_{010}(t) + \lambda P_{111}(t) + a P_{111}(t)]. \\
\] (3.2.5)

\[
C_\infty = \beta \prod_{000} + \beta \prod_{100} + \beta \prod_{010} + \lambda \prod_{111} + a \prod_{111}. \\
\] (3.2.6)

Using steady state probabilities (3.2.4):

\[
C_\infty = \frac{1}{\text{Dr}} \ [ \beta a\alpha\lambda + \beta b\alpha\lambda + \beta a\alpha\mu + ab\beta\mu + ab\beta\mu ] \\
\] (3.2.7)

where \( \text{Dr} = (a + b)(\alpha + \beta)(\lambda + \mu) \) on simplification it gives

\[
C_\infty = \frac{a}{\text{Dr}} \ [ a\beta(\lambda + \mu)] + b[\beta\lambda(\alpha + \mu) + (a\beta\mu)] \\
\] (3.2.8)
3.2.3 Numerical illustration

In the expression obtained for $C_\infty$ the parametric values are assigned. Giving values $\mu = 1/50$, $\lambda = 1/80$, $\alpha = 1/80$, $\beta = 1/70$, $a = 1/75$ and when the fund shortage parameter ‘b’ takes values $1/100, 1/80, 1/40, 1/20$ and $1/10$, the $C_\infty$ values are obtained as below:

<table>
<thead>
<tr>
<th>b</th>
<th>$C_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/100</td>
<td>0.0070</td>
</tr>
<tr>
<td>1/80</td>
<td>0.0080</td>
</tr>
<tr>
<td>1/40</td>
<td>0.0090</td>
</tr>
<tr>
<td>1/20</td>
<td>0.0098</td>
</tr>
<tr>
<td>1/10</td>
<td>0.0100</td>
</tr>
</tbody>
</table>

(3.2.9)

Following graph shows an increase in shortage of fund, increases the rate of crisis.

[Graph showing $C_\infty$ values vs. $b$]

The steady state costs in different situations are determined by taking $c^1$ and $c^0$ as cost of funds when it is fully available and cost when there is shortage respectively; $c^1_m$ and $c^0_m$ as cost of manpower when it is fully available and when there is shortage of manpower; $c^1_B$ and $c^0_B$ as cost of busy service period.
and cost of lean service and \( c_1^F \) and \( c_0^F \) refer to cost of fund when it is fully available and when in shortage.

The steady state cost at \((i, j, k)\) is: 
\[
C_{iijk} = \Pi_{ijk} \left[ c_i^F + c_j^m + c_k^B \right],
\]
where \(i, j\) and \(k\) take values 0 or 1.

\[
c_1^F = 5 \quad c_1^m = 10 \quad c_1^B = 25
\]
\[
c_0^F = 8 \quad c_0^m = 15 \quad c_0^B = 30
\]

The cost table is given below:

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Steady State Probability</th>
<th>Cost of State</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.</td>
<td>( \Pi_{000} )</td>
<td>1.18</td>
</tr>
<tr>
<td>1.</td>
<td>( \Pi_{001} )</td>
<td>1.14</td>
</tr>
<tr>
<td>2.</td>
<td>( \Pi_{010} )</td>
<td>1.60</td>
</tr>
<tr>
<td>3.</td>
<td>( \Pi_{011} )</td>
<td>1.60</td>
</tr>
<tr>
<td>4.</td>
<td>( \Pi_{100} )</td>
<td>7.80</td>
</tr>
<tr>
<td>5.</td>
<td>( \Pi_{101} )</td>
<td>8.00</td>
</tr>
<tr>
<td>6.</td>
<td>( \Pi_{110} )</td>
<td>11.25</td>
</tr>
<tr>
<td>7.</td>
<td>( \Pi_{111} )</td>
<td>11.42</td>
</tr>
</tbody>
</table>

Total Expected cost = 43.99
3.3 Model – II

In the previous model is derived an expression for crisis state under steady state conditions and also the steady state probabilities considering availability and shortage in money and manpower in a business which is assumed to have busy periods and lean periods alternately, whereas in this model besides these three characteristics money, manpower and business, some environments are considered that affect the eight states. The different environments may be government policies, famine or drought, demand for the article of manufacture, scarcity of some raw materials that may be essential for the manufacture or war time, etc.

3.3.1 Assumptions

1. Busy and lean periods occur successively.
2. The time $T_1$, $T_2$, ..., $T_n$ for which the staff strength remains full are exponentially distributed with parameter $\lambda_1$, $\lambda_2$, ..., $\lambda_n$ and the times $R_1$, $R_2$, ..., $R_n$ required to complete recruitment for filling up of vacancies are exponentially distributed with parameter $\mu_1$, $\mu_2$, ..., $\mu_n$. $T_1', T_2', ..., T_n'$ and $R_1$, $R_2$, ..., $R_n'$ are independently distributed random variables in the varying environments $\xi$ and $1 \leq \xi \leq n$.
3. The busy and lean periods of the business are exponentially distributed with parameter $\alpha_1$, $\alpha_2$, ..., $\alpha_n$ and $\beta_1$, $\beta_2$, ..., $\beta_n$ respectively in the varying environment $\xi$, $1 \leq \xi \leq n$.
4. The times $T_1'$, $T_2'$, ..., $T_n'$ for which the fund are available in full are taken to be exponentially distributed with parameter $a_1$, $a_2$, ..., $a_n$ respectively and
the times $R'_1, R'_2, \ldots, R'_n$ required to supply of funds for meeting shortage are exponentially distributed with parameter $b_1, b_2, \ldots, b_n$ respectively and that $T'_1, T'_2, \ldots, T'_n$ and $R'_1, R'_2, \ldots, R'_n$ are independently distributed random variables in the varying environments $\xi$, $1 \leq \xi \leq n$.

5. The change in environment occurs in accordance with the change in a continuous Markov chain $Q$ where

$$Q = \begin{bmatrix}
-q_{11} & q_{12} & \cdots & q_{1n} \\
q_{21} & -q_{22} & \cdots & q_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
q_{n1} & q_{n2} & \cdots & -q_{nn}
\end{bmatrix}$$

### 3.3.2 System analysis

The state of the system with money, manpower and business under random environment is given by

$$S = \{(\xi, i, j, k); 1 \leq \xi \leq n \text{ and } i, j, k \text{ take values } 0 \text{ or } 1\}. \quad (3.3.1)$$

The system is in the state $(\xi, i, j, k)$ if the system lies in the environment $(1 \leq \xi \leq n)$ when money availability is in state ‘i’, manpower availability is in state ‘j’ and availability of business in state ‘k’. $i, j$ and $k$ take values 0 or 1 according as they are not fully available (lean structure) or they are fully available (busy structure) respectively. The infinitesimal generator of the continuous time Markov chain is given by
\[ Q_1 = \begin{pmatrix} 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \\ 000 & \Delta \eta_1 & \Delta \beta & \Delta \mu & 0 & \Delta b & 0 & 0 & 0 \\ 001 & \Delta \alpha & \Delta \eta_2 & 0 & \Delta \mu & 0 & \Delta b & 0 & 0 \\ 010 & \Delta \lambda & 0 & \Delta \eta_3 & \Delta \beta & 0 & 0 & 0 & \Delta b \\ 011 & 0 & \Delta \lambda & \Delta \alpha & \Delta \eta_4 & 0 & 0 & 0 & \Delta b \\ 100 & \Delta \alpha & 0 & 0 & 0 & \Delta \eta_5 & \Delta \beta & \Delta \mu & 0 \\ 101 & 0 & \Delta \alpha & 0 & 0 & \Delta \alpha & \Delta \eta_6 & 0 & \Delta \mu \\ 110 & 0 & 0 & \Delta \alpha & 0 & \Delta \lambda & 0 & \Delta \eta_7 & \Delta \beta \\ 111 & 0 & 0 & 0 & \Delta \alpha & 0 & \Delta \lambda & \Delta \alpha & \Delta \eta_8 \end{pmatrix}, \quad (3.3.2) \]

where \((i, j, k) = \{(\xi, i, j, k); 1 \leq \xi \leq n\}\) for \(i, j, k\) is 0 or 1 and
\[
\begin{align*}
\Delta \eta_1 &= Q - \Delta (b + \mu + \beta), \quad \Delta \eta_2 = Q - \Delta (b + \mu + \alpha), \\
\Delta \eta_3 &= Q - \Delta (b + \lambda + \beta), \quad \Delta \eta_4 = Q - \Delta (b + \lambda + \alpha), \\
\Delta \eta_5 &= Q - \Delta (a + \mu + \beta), \quad \Delta \eta_6 = Q - \Delta (a + \mu + \alpha), \\
\Delta \eta_7 &= Q - \Delta (a + \lambda + \beta), \quad \Delta \eta_8 = Q - \Delta (a + \lambda + \alpha).
\end{align*}
\]
\[
\Delta b = \text{diag}(b_1, b_2, \cdot, \cdot, b_n), \quad \Delta \alpha = \text{diag}(a_1, a_2, \cdot, \cdot, a_n), \\
\Delta \lambda = \text{diag}(\lambda_1, \lambda_2, \cdot, \cdot, \lambda_n), \quad \Delta \mu = \text{diag}(\mu_1, \mu_2, \cdot, \cdot, \cdot, \mu_n), \\
\Delta \beta = \text{diag}(\beta_1, \beta_2, \cdot, \cdot, \cdot, \beta_n).
\]

\(Q_1\) is a matrix of order \(8n\). The steady state probability vector \(\Pi\) of the matrix \(Q_1\) satisfies the following equations:
\[
\Pi Q_1 = 0 \quad \text{and} \quad \Pi \varepsilon = 1, \quad (3.3.3)
\]
where
\[
\Pi = (\Pi_{000}, \Pi_{001}, \Pi_{010}, \Pi_{011}, \Pi_{100}, \Pi_{101}, \Pi_{110}, \Pi_{111})
\]
and \(\varepsilon = (1, 1, \cdot, \cdot, \cdot, \cdot, 1)^t\) are vectors of order \(1 \times 8n\) and \(8n \times 1\) respectively.

The vector \(\Pi_{ijk}\) means
\[
\Pi_{ijk} = (\Pi_{ijk}, \cdot, \cdot, \cdot, \Pi_{nijk}),
\]
where \(i, j, k\) take values 0 or 1. The matrix \(Q_1\) is partitioned as
\[
Q_1 = \begin{bmatrix} Q'_1 & \varepsilon \\ \tau & -q_m -a_n -\lambda_n - \alpha_n \end{bmatrix},
\]
where \( Q_1' = ((Q_1)_{ij}) \) for \( 1 \leq i, j \leq 8n - 1 \) is the sub matrix of \( Q_1 \) without considering the last row and the last column of \( Q_1 \).

\[
(\begin{array}{cccc}
0 & \text{for } j = 1,2,\ldots,4n-1,4n+1,\ldots,6n-1, \\
ad_n & \text{for } j = 4n \\
\lambda_n & \text{for } j = 6n \\
ad_n & \text{for } j = 7n
\end{array})
\]

\( \mathbf{r} = ((Q_1)_{8n,1}, (Q_1)_{8n,2}, \ldots, (Q_1)_{8n,8n-1}) \), is a vector of order \( 1 \times 8n - 1 \), where

\[
\mathbf{e} = (((Q_1)_{1,8n}, (Q_1)_{2,8n}, \ldots, (Q_1)_{8n-1,8n}))^t.
\]

This is of order \( (8n - 1) \times 1 \) and

\[
(\begin{array}{cccc}
0 & \text{for } j = 1,2,\ldots,4n-1,4n+1,\ldots,6n-1, \\
b_j & \text{for } j = 4n \\
\mu_j & \text{for } j = 6n \\
\beta_j & \text{for } j = 7n
\end{array})
\]

Using (3.3.3),

\[
\Pi' Q_1' + \Pi_{8n} \mathbf{r} = 0
\]

where \( \Pi = (\Pi', \Pi_{8n}) \), \( \Pi' \) and \( \Pi_{8n} \) represent partition of \( \Pi \) and \( \Pi' \) is a vector of order \( 1 \times 8n - 1 \).

From (3.3.4),

\[
\Pi' = \Pi_{8n} \mathbf{r} (-Q_1')^{-1}
\]

As \( \Pi \mathbf{e} = 1 \), \( \Pi_{8n} \mathbf{r} (-Q_1')^{-1} \mathbf{e} + \Pi_{8n} = 1 \)

\[
\Pi_{8n} = [ 1 + \mathbf{r} (-Q_1')^{-1} \mathbf{e} ]^{-1}.
\]

Substituting this, (3.3.5) becomes

\[
\Pi' = \frac{1}{\mathbf{X}} \mathbf{r} (-Q_1')^{-1} \\
\]
The critical states are given by 
\[ C = [0, 0, 1, 0, 1, 0, 1], \]
where \( C \) has 3n coordinates.

The rate of crisis in the steady state is given by the dot product 
\[ C_\infty = \beta \cdot \Pi_{000} + \beta \cdot \Pi_{100} + \beta \cdot \Pi_{010} + \lambda \cdot \Pi_{111} + a \cdot \Pi_{111}, \] (3.3.7)
where
\[ \beta = (\beta_1, \beta_2, ..., \beta_n) \]
\[ \lambda = (\lambda_1, \lambda_2, ..., \lambda_n) \]
\[ a = (a_1, a_2, ..., a_n). \]

The steady state cost of the system in different situations and environments are determined by taking \( C_F^{\xi} \) and \( C_F^{0\xi} \) as cost of funds when it is fully available and cost when there is shortage respectively in the \( \xi \)th environment for \( 1 \leq \xi \leq n \).

Similarly, take \( C_M^{\xi} \) and \( C_M^{0\xi} \) as cost of manpower when it is fully available and when there is shortage in manpower in the \( \xi \)th environment respectively and \( C_B^{\xi} \) and \( C_B^{0\xi} \) as busy service period cost and lean service period cost in the \( \xi \)th environment respectively. The cost with respect to the state \((\xi, i, j, k)\) in the \( \xi \)-th environment is 
\[ C_{\xi ijk} = \Pi_{\xi ijk} [ C_F^{\xi i} + C_M^{\xi j} + C_B^{\xi k} ] \] (3.3.8)
For \( 1 \leq \xi \leq n \) and \( i, j, k \) takes value 0 or 1.

3.4 Numerical Example
Consider the case of two environments \((n = 2)\), when the business fluctuates between the periods busy and lean. Assume that in case of \(\xi_1\) (first environment) the values of the parameters are:

\[
q_{11} = 1, \lambda_1 = 4, \mu_1 = 2, \alpha_1 = 2, \beta_1 = 3, a_1 = 5, b_1 = 6
\]

and in the case of \(\xi_2\) (second environment) the values of parameters are assumed to be \(q_{21} = 3, \lambda_2 = 8, \mu_2 = 5, \alpha_2 = 4, \beta_2 = 7, a_2 = 10, b_2 = 12\). The infinitesimal generator \(Q_1\) for the above set of values can be got using (3.3.2). Using the infinitesimal generator the following values are obtained.

\[
\begin{align*}
\mathbf{r} \mathcal{L} (-Q_1)^{-1} \mathbf{1} &= (0.528599, 0.470001, 1.197480, 0.935373, 0.369403, 0.328173, \\
&\quad 0.701138, 0.679816, 0.306123, 0.532455, 1.382236, 1.158757, \\
&\quad 0.411164, 0.407831, 0.850339, 0.088818)
\end{align*}
\]

and

\[
\mathbf{r} \mathcal{L} (-Q_1')^{-1} \mathbf{1} = 10.25895.
\]

Now using (3.3.6), the value of steady state probabilities in environments \(\xi_1\) and \(\xi_2\) are as follows:

\[
\begin{align*}
\Pi_{\xi_1000} &= 0.046949, \Pi_{\xi_1001} = 0.041744, \Pi_{\xi_1010} = 0.106359, \Pi_{\xi_1011} = 0.083077, \\
\Pi_{\xi_1100} &= 0.032809, \Pi_{\xi_1101} = 0.029147, \Pi_{\xi_1110} = 0.062273, \Pi_{\xi_1111} = 0.060379, \\
\Pi_{\xi_2000} &= 0.027189, \Pi_{\xi_2001} = 0.047291, \Pi_{\xi_2010} = 0.122767, \Pi_{\xi_2011} = 0.102918, \\
\Pi_{\xi_2100} &= 0.036518, \Pi_{\xi_2101} = 0.036223, \Pi_{\xi_2110} = 0.075525, \Pi_{\xi_2111} = 0.088818.
\end{align*}
\]

The critical states in the two environments \(\xi_1\) and \(\xi_2\) are

\[
C = [(\xi_1 001), (\xi_1 101), (\xi_1 011), (\xi_2 001), (\xi_2 101), (\xi_2 011)].
\]

Using (3.3.7), rate of crisis in steady state in environments \(\xi_1\) and \(\xi_2\) are

\[
C(\xi_1 \infty) = 1.101756, C(\xi_2 \infty) = 2.904092.
\]

Let the cost under various situations (busy and lean) under the two environments \(\xi_1\) and \(\xi_2\) be

\[
\begin{align*}
C_{F1}^{1\xi_1} &= 5, C_{F1}^{1\xi_2} = 10, C_{M1}^{1\xi_1} = 10, C_{M1}^{1\xi_2} = 20, C_{B1}^{1\xi_1} = 25, C_{B1}^{1\xi_2} = 50, \\
C_{F0}^{0\xi_1} &= 8, C_{F0}^{0\xi_2} = 16, C_{M0}^{0\xi_1} = 15, C_{M0}^{0\xi_2} = 30, C_{B0}^{0\xi_1} = 30, C_{B0}^{0\xi_2} = 60.
\end{align*}
\]
then, using (3.3.8), it is found
\[
\begin{align*}
C(\xi_1 000) &= 2.477697, C(\xi_1 001) = 2.003712, C(\xi_1 010) = 5.105136, \\
C(\xi_1 011) &= 3.572311, C(\xi_1 100) = 1.640450, C(\xi_1 101) = 1.311615, \\
C(\xi_1 110) &= 2.802285, C(\xi_1 111) = 2.415160, C(\xi_2 000) = 2.882034, \\
C(\xi_2 001) &= 4.539936, C(\xi_2 010) = 11.785632, C(\xi_2 011) = 8.850948, \\
C(\xi_2 100) &= 3.651800, C(\xi_2 101) = 3.260070, C(\xi_2 110) = 6.77025, \\
C(\xi_2 111) &= .10544.
\end{align*}
\] (3.3.10)

3.5 Conclusion

It is observed from (3.3.9) that the rate of crisis in steady state in environments \(\xi_2\) is different from the rate of crisis in steady state in environment \(\xi_1\). There is a difference as the intensity of environment 2 may be worse than that of environment 1 which is clear from the values assigned for the parameters. The rate of crisis cannot be compared between the two environments as they are different from each other. Also it is observed from (3.3.10) that in general the cost of steady state probabilities in different states in environments \(\xi_1\) and \(\xi_2\) are such that the cost is more for a particular state in environment \(\xi_2\) than for its corresponding state in environment \(\xi_1\) for same reasons. This fact is obvious as crisis on account of environment 2 is worse than the crisis on account of environment 1, naturally a heavy price has to be paid. By considering more number of environments higher order infinitesimal generator (matrix) is obtained which can easily be inverted by using Matlab facility for determining the steady state probabilities.

**REASONS FOR SELECTION OF MODELS AND THEIR APPLICATION TO INDUSTRIES / ORGANIZATIONS**
Any business depends mainly on manpower and money and they have probabilistic behaviour hence stochastic processes is applied to determine probability of various states under steady state conditions. But effect of environments affecting business cannot be overlooked, therefore a model involving n environments has been worked to determine in general rate of crisis, steady state probabilities and total expected cost and numerical examples are worked to justify the impact of environment in business. In general any industry or business requires minimum of two characteristics namely money and manpower and it is very common that manpower is volatile nowadays that any better opportunity is clinched, manpower goes off and consequently affects the Institution or organization. Sudden scarcity of raw materials or power crisis, like what is prevailing in Southern region of Tamil Nadu where the Weavers suffer because of power cut will result in workers leaving, sales getting affected, shortage of funds, etc. This is an apt real life situation where the models can be applied.