CHAPTER 8

STOCHASTIC ANALYSIS OF THREE LEVELS OF MANPOWER AFFECTING BUSINESS – CONTINUOUS TIME MARKOV CHAIN APPROACH

8.1 Introduction

In this chapter is considered a business organization under varying conditions which are restricted to depend on manpower, money and business under fluctuating conditions of availability of manpower and business with a special emphasis given to a new and prevailing idea of frequent changes taking place in manpower. Considered are three phases of manpower viz manpower is fully available, moderately or insufficiently available and manpower not at all available or scarcely available and the transitions take place from full availability to nil availability and from nil availability to full availability in case of money and business but in the case of manpower it is assumed that the transitions can be from any state to any state among the three possibilities. The different states have been discussed under the assumption that changes from availability to shortage and shortage to availability in the case business and money and the changes from nil to moderately available or full availability and moderately available to full availability and reverse cases occur in exponential times with different parameters. An expression for “Rate of Crisis under steady state (C∞)” is arrived and steady state cost have also been worked by assuming different costs for the parameters under different conditions.

The results obtained for the model has been published as a paper with the title “Stochastic Analysis of Three Levels of Manpower Affecting Business — Continuous Time Markov Chain Approach”– in the proceedings of the Fourth National
Employer and employee outlook has undergone a sea change over the years. Though they are interdependent for each one’s growth and survival, both have independence. Companies can retrench employees if their presence do not add to companies’ profit or their contribution for the company’s growth is not appreciable or their service is available cheap and in abundance. Equally true is with the employees. They may change their employers for reasons such as better emolument, working condition, perks, proximity to their residence, etc. Retrenchment may lead to financial suffering for employees or find it difficult to get another employment and this may result in family disturbance and mental agony. If employees leave the business the normal working may be affected. The company would be worst affected if skilled and knowledgeable persons leave. Sometimes the leaving experts may carry the good will of the company.

8.1.1 Assumptions

1. The time $T_1$ for which the staff strength is assumed to remain insufficient is exponentially distributed with parameter $\lambda_{10}$ and the time $T_1'$ for recruitment for filling up of vacancies insufficiently (under staff) because of scarcity of skilled manpower or the manpower becoming costly is exponentially distributed with parameter $\mu_{01}$. The time for $T_2$ for which staff strength is assumed to remain insufficient (minimal optimal) from full is also exponentially distributed with parameter $\lambda_{21}$. The time $T_2'$ required for recruitment to wipe out insufficiency to become full is also exponentially distributed with parameter $\mu_{12}$. The time $T_3$ for which staff strength to remain full till becoming nil is exponentially distributed with parameter $\lambda_{20}$ and the
time $T_2'$ required for recruitment to fill up fully from zero strength is also distributed exponentially with parameter $\mu_{02}$. $T_1, T_1', T_2, T_2', T_3, T_3'$ are all independently distributed random variables.

2. The time $R_1$ for which the fund is available in full is taken to be exponentially distributed with parameter $\alpha$ and the time $R_2$ for supply of funds required to meet shortage is exponentially distributed with parameter $\beta$. $R_1$ and $R_2$ are independently distributed random variables.

3. The busy and lean periods of the business are exponentially distributed with parameters $a$ and $b$ respectively.

4. Busy and lean periods are assumed to occur in cycles.

8.1.2 System analysis

The stochastic process $X(t)$ describing the state of the system is a continuous time Markov Chain with 12 points state space as given below

$$S = [(000), (001), (010), (011), (100), (101), (110), (111), (200), (201), (210), (211)]$$

(8.1.1)

0 refers to nil availability or shortages in resources, 1 refers to moderate availability and 2 refers to full availability. Every state space is in the order Manpower, Money and Business.
<table>
<thead>
<tr>
<th></th>
<th>000</th>
<th>001</th>
<th>010</th>
<th>011</th>
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<td>(0 0 0)</td>
<td>(0 0 1)</td>
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<td>(0 1 1)</td>
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<td>ε₅</td>
<td>b</td>
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<td>λ₁₀</td>
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<td>a</td>
<td>ε₈</td>
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<td>λ₂₁</td>
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<td>0</td>
<td>ε₉</td>
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<td>λ₂₀</td>
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<td>0</td>
<td>0</td>
<td>λ₂₁</td>
<td>α</td>
<td>ε₁₀</td>
<td>0</td>
<td>β</td>
</tr>
</tbody>
</table>

**Notes:**
- (MP, M, B) refers to the initial state.
- ε₁, ε₂, ε₃, ε₄, ε₅, ε₆, ε₇, ε₈, ε₉, ε₁₀ are variables representing different actions or states.
- B, b are binary states.
- α, β are variables representing transitions.
The crises states are \{(0 0 1) (0 1 1) (1 0 1) (2 0 1)\}. Here, a state is said to be in crisis if full business may be there but there is shortage of manpower or money.

Using the above infinitesimal matrix, the steady state probability vector can be derived by using

\[ \prod Q = 0 \quad \text{and} \quad \prod e = 1 \quad (8.1.3) \]

where \( e = [1, 1, 1, \ldots, 1]^t \) is a vector of the type \( 12 \times 1 \) and \( \prod \) is given by,

\[ \prod = [\prod_{000} \prod_{001} \prod_{010} \prod_{011} \prod_{100} \prod_{101} \prod_{110} \prod_{111} \prod_{200} \prod_{201} \prod_{210} \prod_{211}] \quad (8.1.4) \]

Which is a \( 12 \times 1 \) vector.

The transitions taking place in all the three characteristics are assumed to be independent that the infinitesimal matrices of individual characteristic are as follows:

1. Money (Funds)
\[
F = \begin{bmatrix}
-\beta & \beta \\
\alpha & -\alpha
\end{bmatrix}
\]

Since the changes from full availability to zero and zero to full availability are also independent, the steady probability in money is given by

\[
\Pi_{F0} = \frac{\alpha}{\alpha + \beta} \quad \text{and} \quad \Pi_{F1} = \frac{\beta}{\alpha + \beta}
\quad (8.1.5)
\]

2. Business

\[
B = \begin{bmatrix}
-b & b \\
a & -a
\end{bmatrix}
\]

Since the changes take independent of each other, the steady state probabilities are given by

\[
\Pi_{B0} = \frac{a}{a + b} \quad \text{and} \quad \Pi_{B1} = \frac{b}{a + b}
\quad (8.1.6)
\]

3. Manpower

\[
M = \begin{bmatrix}
-(\mu_{01} + \mu_{02}) & \mu_{01} & \mu_{02} \\
\lambda_{10} & -(\mu_{12} + \lambda_{10}) & \mu_{12} \\
\lambda_{20} & \lambda_{21} & -(\lambda_{20} + \lambda_{21})
\end{bmatrix}
\]

The matrix \( M \) is partitioned as \( M' = \begin{bmatrix} M'' & \zeta \\ \mathbf{r} & -(\lambda_{21} + \lambda_{21}) \end{bmatrix} \) where \( \mathbf{r} = (\lambda_{20} \quad \lambda_{21}) \) and \( \zeta = (\mu_{02} \quad \mu_{12}) \) and \( M' = \begin{bmatrix}
-(\mu_{01} + \mu_{02}) & \mu_{01} \\
\lambda_{10} & -(\mu_{12} + \lambda_{10})
\end{bmatrix}. \)

Now using (8.3)

\[
\Pi = \frac{1}{1 + r (-(M'')^{-1})^{-1} \mathbf{r}^{-1}} (-(M'')^{-1} \mathbf{1})
\quad (8.1.7)
\]
Where $\Pi = [\pi_{M0}, \pi_{M1}, \pi_{M2}]$ whose values are obtained by solving (8.1.7) and are given by

$$\pi_{M0} = \frac{d_1}{X}, \quad \pi_{M1} = \frac{d_2}{X}, \quad \text{and} \quad \pi_{M2} = \frac{d_2}{X} \quad (8.1.8)$$

where

$$x = d_0 + d_1 + d_2 \quad \text{and} \quad d_0 = \lambda_{20} \mu_{12} + \lambda_{20} \mu_{10} + \lambda_{21} \mu_{10},$$

$$d_1 = \lambda_{20} \mu_{01} + \lambda_{21} \mu_{01} + \lambda_{21} \mu_{02} \quad \text{and} \quad d_2 = \mu_{01} \mu_{12} + \mu_{02} \mu_{12} + \lambda_{10} \mu_{02}$$

The steady state probabilities can be derived easily taking into consideration the independent nature of the states and are given by,

$$\Pi_{000} = \frac{d_0 \alpha \alpha}{XYZ}, \quad \Pi_{001} = \frac{d_0 \beta \beta}{XYZ}, \quad \Pi_{010} = \frac{d_0 \beta \beta}{XYZ}, \quad \Pi_{011} = \frac{d_0 \beta \beta}{XYZ},$$

$$\Pi_{100} = \frac{d_1 \alpha \alpha}{XYZ}, \quad \Pi_{101} = \frac{d_1 \alpha \beta}{XYZ}, \quad \Pi_{110} = \frac{d_1 \beta \alpha}{XYZ}, \quad \Pi_{111} = \frac{d_1 \beta \beta}{XYZ},$$

$$\Pi_{200} = \frac{d_2 \alpha \alpha}{XYZ}, \quad \Pi_{201} = \frac{d_2 \alpha \beta}{XYZ}, \quad \Pi_{110} = \frac{d_1 \beta \alpha}{XYZ} \quad \text{and} \quad \Pi_{111} = \frac{d_1 \beta \beta}{XYZ}$$

Where $X = (d_0 + d_1 + d_2), \quad Y = (\alpha + \beta) \quad \text{and} \quad Z = (a + b)$.

**8.1.3 Rate of crisis**

Now the rate of crisis in steady state $(\mathbf{c}_\infty)$ is obtained as follows.

$$P(\text{crisis in } [t (t+\Delta t)] = P[X(t+\Delta t) = (001) / X(t) = (000)] \times P [(X(t) = (000)]$$

$$+ P[X(t+\Delta t) = (011) / X(t) = (111)] \times P [(X(t) = (111)]$$

$$+ P[x(t+\Delta t) = (201) / X(t) = (211)] \times P [(X(t) = (211)]$$
\[ + P[X(t+\Delta t) = (101) / X(t) = (100)] xP [(X(t) = (100)] \]
\[ +P[X(t+\Delta t) = (101) / X(t) = (111)] xP [(X(t) = (111)] \]
\[ + P[X(t+\Delta t) = (011) / X(t) = (010)] xP [(X(t) = (010)] \]
\[ + P[X(t+\Delta t) = (201) / X(t) = (200)] xP [(X(t) = (200)] \]
\[ + P[X(t+\Delta t) = (011) / X(t) = (211)] xP [(X(t) = (211)] \]
\[ + O\Delta(t). \]

Taking limit as \( \Delta t \to 0 \),

\[ C_t = bP_{000}(t)+\lambda_{10}P_{111}(t) +bP_{010}(t)+\beta P_{100}(t)+\alpha P_{111}(t)+\beta P_{200}(t)+\alpha P_{211}(t)+\lambda_{20}P_{211}(t) \]

Taking \( t \to \infty \),

\[ C_\infty = b\pi_{000}+\lambda_{10}\pi_{111}+b\pi_{010}+\beta\pi_{100}+\alpha\pi_{111}+\beta\pi_{200}+\alpha \pi_{211}+\lambda_{20}\pi_{211} \]

Using the steady state probabilities, gives

\[ C_\infty = \frac{b}{\alpha} [bd_0a + \beta(\lambda_{10}d_1b+bad_0+b\alpha d_1+a\alpha d_2+b\alpha d_2+bd_2\lambda_{20}] \quad (8.1.9) \]

8.1.4 Numerical illustrations

The various parametric values are assigned as given below to verify the nature of rate of crises by substitution of values in the equation of \( C_\infty \).
\[ \lambda_{20} = 2, \lambda_{21} = 5, \lambda_{10} = 4, \mu_{11} = 8, \mu_{01} = 6, \mu_{02} = 7, \alpha = 4, \beta = 6; a = 8, b = 9, \] and \( \beta = 8, 10, 12, 15 \) and \( b = 12, 15, 18, 22 \) and 25.

The table below gives the value of \( C_\infty \) for various values of \( b \) and \( \beta \), keeping rest of the variables as constants and the graph is drawn with \( b \) and \( \beta \) against \( C_\infty \). It is observed to have increasing crises rate (\( C_\infty \)) as they increase.
<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$C_\infty$</th>
<th>$b$</th>
<th>$C_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3.4373</td>
<td>12</td>
<td>3.1299</td>
</tr>
<tr>
<td>8</td>
<td>3.7374</td>
<td>15</td>
<td>3.3381</td>
</tr>
<tr>
<td>10</td>
<td>3.9518</td>
<td>18</td>
<td>3.4991</td>
</tr>
<tr>
<td>12</td>
<td>4.1125</td>
<td>22</td>
<td>3.6623</td>
</tr>
<tr>
<td>15</td>
<td>4.2902</td>
<td>25</td>
<td>3.7583</td>
</tr>
</tbody>
</table>

$\beta - C_\infty$ graph
The steady state cost in different situations are determined by using the following formula:

$$\text{STEADY STATE COST} = \prod_{i,j,k} [C_M^i + C_F^j + C_B^k]$$

where $M$ refers to manpower, $F$ refers to money and $B$ refers to business. The cost table is prepared by assuming different values for the three characteristics under different situations

$C_O^F = 25$, $C_1^F = 15$, $C_0^M = 10$, $C_1^M = 8$, $C_2^M = 5$, $C_0^B = 15$, $C_1^B = 8$.

Now applying these values in the steady state cost formula at $(i, j, k)$, the whole set of steady state costs are obtained and are given in the table.
<table>
<thead>
<tr>
<th>S. No</th>
<th>Steady State Probability</th>
<th>Cost of state</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\Pi_{000}$</td>
<td>1.5360</td>
</tr>
<tr>
<td>2</td>
<td>$\Pi_{001}$</td>
<td>1.4760</td>
</tr>
<tr>
<td>3</td>
<td>$\Pi_{010}$</td>
<td>2.1600</td>
</tr>
<tr>
<td>4</td>
<td>$\Pi_{011}$</td>
<td>2.7360</td>
</tr>
<tr>
<td>5</td>
<td>$\Pi_{100}$</td>
<td>2.7650</td>
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<tr>
<td>6</td>
<td>$\Pi_{101}$</td>
<td>2.6570</td>
</tr>
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<td>7</td>
<td>$\Pi_{110}$</td>
<td>3.8880</td>
</tr>
<tr>
<td>8</td>
<td>$\Pi_{111}$</td>
<td>4.9250</td>
</tr>
<tr>
<td>9</td>
<td>$\Pi_{200}$</td>
<td>3.1680</td>
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<tr>
<td>10</td>
<td>$\Pi_{201}$</td>
<td>2.8080</td>
</tr>
<tr>
<td>11</td>
<td>$\Pi_{210}$</td>
<td>4.3200</td>
</tr>
<tr>
<td>12</td>
<td>$\Pi_{211}$</td>
<td>4.9680</td>
</tr>
<tr>
<td></td>
<td>Total expected steady state cost</td>
<td>37.4070</td>
</tr>
</tbody>
</table>

### 8.1.5 Observation

Observe that the crises rate increases as the values $\beta$ and $b$ increases and the steady cost probability is high when all characteristics are full which signifies that the business is doing very fine and the lowest steady state probability cost makes it clear that despite nil business we have to incur non-productive expenditures.
REASONS FOR SELECTION OF MODELS AND THEIR APPLICATION TO INDUSTRIES / ORGANIZATIONS

There are three characteristics, but manpower only fluctuates among three levels, others between two levels only. Transitions are considered in manpower in any state to any state and operating characteristics are determined. This type of situation is common in any institution, organization and educational institutions because of stay of employees for a long time is probabilistic because of job opportunities and management approach that many a time they run with insufficient staff so business would suffer. The model can help to prepare beforehand how to tackle such situations.
8.2 MODEL – II
A PROBABILITIC ANALYSIS OF THREE LEVELS OF MANPOWER AFFECTING BUSINESS – CONTINUOUS TIME MARKOV CHAIN MODEL

8.2.1 Introduction

Nowadays it is observed that there is no guarantee for labor to stay for along time unlike some years back in any business concern. The reason may be that plenty of jobs are available or attitude of the employer. This trend may be advantageous and disadvantageous for both employer and employee. But a business concern should try to keep its skilled and specialist employees intact otherwise the abusiveness would suffer on their leaving. A company may be forced to face worst situation if the employees carry along with them the goodwill of the company which they have earned because of their reputation as producers and ambassadors of the company.

The results obtained for the second model has been published as a paper with the title “A Probabilistic Analysis of Three Levels of Manpower Affecting Business - Continuous Time Markov Chain Model”, in Applied Mathematical Sciences, Vol. 7, 2013, no. 3, 127 - 134.
In this model is considered a business organization under varying conditions which are restricted to depend on manpower, money and business. Considered are three phases of manpower viz manpower is fully available, moderately or insufficiently available and manpower not at all available or scarcely available and the transitions take place from full availability to nil availability and from nil availability to full availability in case of money and business but in the case of manpower it is assumed that the transitions can be from nil availability to moderate availability and vice versa and from moderate availability to full availability and vice versa. No transitions take place between the lowest and the highest.

The different states have been discussed under the assumption that changes from one state to another in any three characteristic occur in exponential times with different parameters. The steady state probabilities of the continuous time Markov chain describing the transitions in various states are derived and critical states are identified for presenting the cost analysis. An expression for “Rate of Crisis under steady state \( C_x^\infty \)” is arrived and steady state costs have also been worked by assuming different costs for the parameters under different conditions.

8.2.2 Assumptions

1. The random length of time the staff strength is assumed to remain moderate to become nil is exponentially distributed with parameter \( \lambda_s \), the length of time required for filling up of vacancies from nil to moderate level is exponentially distributed with parameter \( \mu_d \). The length of time the staff strength is assumed to remain full to become moderate is assumed to be exponentially distributed with parameter \( \lambda_c \). The length of time required for filling up of vacancies from
moderate to full is also exponentially distributed with parameter $\mu$. The above
periods of transitions are random variables and are independent.

2. The random length of time for which the fund is fully available is exponentially
distributed with parameter $\alpha$ and the length of time required for the funds to move
from shortage level to fully available level is also exponentially distributed with
parameter $\beta$ and the periods of transition are independent random variables.

3. The random length of time for which the business is busy is exponentially
distributed with parameter $a$ and the random length of time for which the business
remains lean is also exponentially distributed with parameter $b$. The periods of
transitions are independent random variables.

4. Busy and lean periods are assumed to occur in cycles.

8.2.3 System analysis

The stochastic process $x(t)$ describing the state of the system is
continuous Markov chain with 12 points state space and is given by,

$$S = \{ (0 0 0), (0 01), (0 10), (0 11), (1/2 0 0), (1/2 0 1),
(1/2 1 0), (1/2 1 1), (1 0 0), (1 0 1), (1 1 0), (1 1 1) \} \ (8.2.1)$$

0 refers to nil availability or shortages of resources, $1/2$ refers to semi
availability and manpower and 1 refers to full availability of resources. The
infinitesimal generator $Q$ of the continuous time Markov chain of the state
space is given below using the transitions.
\[
\begin{array}{cccccccc}
\text{MP, M, B)} & (0 & 0 & 0) & (0 & 1 & 0) & (0 & 1 & 1) & (0 & 1/2 & 0) & (0 & 1/2 & 1) & (0 & 1 & 0) & (0 & 1 & 1) \\
(0 0 0) & \varepsilon_1 & b & B & 0 & \mu_d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(0 0 1) & a & \varepsilon_2 & 0 & \beta & 0 & \mu_d & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(0 1 0) & \alpha & 0 & \varepsilon_3 & b & 0 & 0 & \mu_d & 0 & 0 & 0 & 0 & 0 & 0 \\
(0 1 1) & 0 & \alpha & \alpha & \varepsilon_4 & 0 & 0 & 0 & \mu_d & 0 & 0 & 0 & 0 & 0 \\
(1/2 0 0) & \lambda_s & 0 & 0 & 0 & \varepsilon_5 & b & \beta & 0 & \mu_s & 0 & 0 & 0 & 0 \\
(1/2 0 1) & 0 & \lambda_s & 0 & 0 & a & \varepsilon_6 & 0 & \beta & 0 & \mu_s & 0 & 0 & 0 \\
(1/2 1 0) & 0 & 0 & \lambda_s & 0 & \alpha & 0 & \varepsilon_7 & b & 0 & 0 & \mu_s & 0 & 0 \\
(1 0 0) & 0 & 0 & 0 & \lambda_c & 0 & 0 & 0 & \varepsilon_9 & b & \beta & 0 & 0 & 0 \\
(1 0 1) & 0 & 0 & 0 & 0 & \lambda_c & 0 & 0 & \alpha & \varepsilon_{10} & 0 & \beta & 0 & 0 \\
(1 1 0) & 0 & 0 & 0 & 0 & 0 & \lambda_c & 0 & \alpha & 0 & \varepsilon_{11} & b & 0 & 0 \\
(1 1 1) & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_c & 0 & \alpha & \varepsilon_{12} & 0 & 0 & 0 \\
\end{array}
\]

Where

\[
\begin{align*}
\varepsilon_1 &= -(b+\beta+\mu_d), & \varepsilon_2 &= -(a+\mu_d+\beta), & \varepsilon_3 &= -(\mu_d+b+\alpha), \\
\varepsilon_4 &= -(\mu_d+\alpha+a), & \varepsilon_5 &= -(\lambda_s+b+\beta+\mu_s), & \varepsilon_6 &= -(\lambda_s+a+\beta+\mu_s), \\
\varepsilon_7 &= -(\lambda_s+a+b+\mu_s), & \varepsilon_8 &= -(\lambda_s+a+\alpha+\mu_s), & \varepsilon_9 &= -(\lambda_c+b+\beta), \\
\varepsilon_{10} &= -(\lambda_c+a+\beta), & \varepsilon_{11} &= -(\lambda_c+a+b), & \varepsilon_{12} &= -(\lambda_c+a+a)
\end{align*}
\]
The crises states are \{ (001), (011), (101), (½ 01) \}. Here, consider a state to be in crisis if full business may be there but there is shortage of manpower or money using the above infinitesimal matrix.

### 8.2.4 Steady state probabilities

The steady state probability vector can be derived by using,

\[ \Pi \mathbf{Q} = 0 \quad \text{and} \quad \Pi \mathbf{e} = 1 \]  \hspace{1cm} (8.2.3)

where \( \mathbf{e} = [1,1,1,\ldots,1]^T \) is a vector of the type 12 x 1. and \( \Pi \) is given by,

\[ \Pi = [\Pi_{000}, \Pi_{001}, \Pi_{010}, \Pi_{011}, \Pi_{1/200}, \Pi_{1/201}, \Pi_{1/210}, \Pi_{1/211}, \Pi_{100}, \Pi_{101}, \Pi_{110}, \Pi_{111}] \]

Which is a 12 x 1 vector.

The steady state probabilities can be derived easily taking into consideration the independent nature of the states and are given by,

\[ \Pi_{000} = \frac{\alpha \lambda_c \lambda_d}{XYZ}, \quad \Pi_{001} = \frac{a b \lambda_c \lambda_d}{XYZ}, \quad \Pi_{010} = \frac{a b \lambda_c \lambda_d}{XYZ}, \quad \Pi_{011} = \frac{a b \lambda_c \lambda_d}{XYZ}, \]

\[ \Pi_{1200} = \frac{b \beta \lambda_c \mu_d}{XYZ}, \quad \Pi_{1201} = \frac{b \beta \lambda_c \mu_d}{XYZ}, \quad \Pi_{1210} = \frac{b \beta \lambda_c \mu_d}{XYZ}, \quad \Pi_{1211} = \frac{b \beta \lambda_c \mu_d}{XYZ}, \quad \Pi_{100} = \frac{a \mu_c \mu_d}{XYZ}, \quad \Pi_{101} = \frac{a \mu_c \mu_d}{XYZ}, \quad \Pi_{110} = \frac{a \mu_c \mu_d}{XYZ}, \quad \Pi_{111} = \frac{a \mu_c \mu_d}{XYZ} \]

Where \( X = (\lambda_c \lambda_d + \lambda_c \mu_d + \mu_c \mu_d), \quad Y = (\alpha + \beta) \) and \( Z = (a + b) \) \hspace{1cm} (8.2.4)

### 8.2.5 Rate of crisis

Now the rate of crisis in steady state (\( C_{XZ} \)) is obtained as follows.

\[ P(\text{crisis in } [t(t+\Delta t)] \)
\[ P[X(t+\Delta t) = (0,1,1)/X(t) = (1/2, 1, 1)] \times P[(X(t) = (1/2, 1, 1)] + P[X(t+\Delta t) = (0,1,1)/X(t) = (1,1,1)] \times P[(X(t) = (1,1,1)] + P[X(t+\Delta t) = (1,0,1)/X(t) = (1,0,0)] \times P[(X(t) = (1,0,0)] + P[X(t+\Delta t) = (1,1,0)/X(t) = (1,1,0)] \times P[(X(t) = (1,1,0)] + O\Delta(t). \]

Taking limit as \( \Delta t \to 0 \), we get,

\[ C_t = b P_{000} (t) + \lambda_s P_{1/211} (t) + \alpha P_{111} (t) + b P_{1/200} (t) + b P_{100} (t) + b P_{010} (t) \]

\[ C_{\infty} = \lim_{t \to \infty} [b P_{000} (t) + \lambda_s P_{1/211} (t) + \alpha P_{111} (t) + b P_{1/200} (t) + \alpha P_{1/211} (t) + b P_{100} (t) + b P_{010} (t)] \]

that is

\[ C_{\infty} = \lim_{t \to \infty} [b \prod_{000} (t) + \lambda_s \prod_{1/211} (t) + \alpha \prod_{111} (t) + b \prod_{1/200} (t) + \alpha \prod_{1/211} (t) + b \prod_{100} (t) + b \prod_{010} (t)] \]

Using the steady state probabilities,

\[ C_{\infty} = \frac{b}{\lambda c \lambda s (a \alpha + a \beta + \beta \mu_d) + \mu_s \mu_d (a \beta + a \alpha) + \lambda c \mu_d (a \beta + a \alpha)} \]

8.2.6 Cost analysis

The various parametric values are assigned as given to verify the nature of rate of crises by substitution of values in the equation of \( C_{\infty} \).

\[ \lambda_c = 2, \ \lambda_s = 3, \ \mu_s = 9, \ \mu_d = 5; \ \alpha = 4, \ \beta = 6; \ a = 8, \ b = 9, 11, 13, 15, 17. \]

The table below gives the value of \( c_{\infty} \) for various values of \( b \) keeping rest of the other variables as constants and the graph is drawn with \( b \) against \( C_{\infty} \) and it is observed to have increasing crises rate \( c_{\infty} \) as \( b \) increases.
The steady state cost in different situations are determined by assuming the following values:

\[ C_M^0 = 25 \quad C_M^{1/2} = 15 \quad C_M^1 = 10 \quad C_F^0 = 8 \quad C_F^1 = 5 \quad C_B^0 = 15 \quad C_B^1 = 25. \]

Now applying these values in the steady state cost formula at \((i j k)\), the whole set of steady state costs are as given in the table:

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Steady State Probability</th>
<th>Cost of state</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[\prod_{000}]</td>
<td>0.8400</td>
</tr>
<tr>
<td>2</td>
<td>[\prod_{001}]</td>
<td>0.8938</td>
</tr>
<tr>
<td>3</td>
<td>[\prod_{010}]</td>
<td>1.1790</td>
</tr>
<tr>
<td>#</td>
<td>( \Pi )</td>
<td>Value</td>
</tr>
<tr>
<td>----</td>
<td>---------------------</td>
<td>---------</td>
</tr>
<tr>
<td>4</td>
<td>( \Pi_{011} )</td>
<td>1.2464</td>
</tr>
<tr>
<td>5</td>
<td>( \Pi_{1/200} )</td>
<td>1.6878</td>
</tr>
<tr>
<td>6</td>
<td>( \Pi_{1/201} )</td>
<td>1.8615</td>
</tr>
<tr>
<td>7</td>
<td>( \Pi_{1/210} )</td>
<td>2.4035</td>
</tr>
<tr>
<td>8</td>
<td>( \Pi_{1/211} )</td>
<td>2.6208</td>
</tr>
<tr>
<td>9</td>
<td>( \Pi_{100} )</td>
<td>4.3263</td>
</tr>
<tr>
<td>10</td>
<td>( \Pi_{101} )</td>
<td>42614</td>
</tr>
<tr>
<td>11</td>
<td>( \Pi_{110} )</td>
<td>5.9010</td>
</tr>
<tr>
<td>12</td>
<td>( \Pi_{111} )</td>
<td>5.6557</td>
</tr>
<tr>
<td></td>
<td>Total expected steady state cost</td>
<td>32.8715</td>
</tr>
</tbody>
</table>

### 8.2.7 Observation

It is observed that (i) rate of crisis increases as the value of \( b \) increases (ii) the steady state cost when both money and manpower are full and the business is zero is highest because the business has to be got by paying premium (iii) the steady cost when all are full is also highest which means that the business is doing well. The cost is the least when all the characteristics are zero is the least.

#### REASONS FOR SELECTION OF MODELS AND THEIR APPLICATION TO INDUSTRIES / ORGANIZATIONS

This is same as the previous model with the assumption that no transitions take place between lowest and highest level in manpower. There are industries where manpower level becomes full only from moderate level and it becomes nil only from moderate level. The transitions take place strictly from 0 to \( \frac{1}{2} \) and \( \frac{1}{2} \) to 1 or vice versa.