CHAPTER 5
SYSTEM WITH STOCHASTIC MANPOWER HARDWARE AND SOFTWARE INTERACTION

5.1 Introduction

A business concern depends on so many characteristics to run smoothly. Apart from being dependent on money, manpower also depends on computer systems, which has become indispensable and a dominating criterion for survival and prosperity of business. One may classify the system failures as failure of software and software induced hardware failure. In the former, a substitute can be used to run the show without interruption, whereas hardware failure leads to catastrophe, necessitating a restart as a whole. A model is analyzed where crisis states occur on account of money and manpower becoming lean under situations of software failure and software induced hardware failure. Expressions for steady state probabilities and rate of crises are derived and numerical examples are discussed and cost analysis is carried out.

The objective of this model is to make an analysis of manpower, money and business with computers as additional characteristic considering hardware failure, software failure and software induced hardware failure.

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5.2 Hardware and Software Interaction Model

5.2.1 Assumptions

1. Busy and lean periods occur successively.
2. The time $T_1$ for which the staff strength remains full is exponentially distributed with parameter $\lambda$ and the time $R_1$ required to complete recruitment for filling up of vacancies is exponentially distributed with parameter $\mu$. $T_1$ and $R_1$ are independently distributed random variables.
3. The busy and lean periods of the business are exponentially distributed with parameter $\alpha$ and $\beta$ respectively.
4. The time $T_2$ for which the fund is fully available is taken to be exponentially distributed with parameter $a$ and the time $R_2$ required for raising of funds for meeting shortage is exponentially distributed with parameter $b$. $T_2$ and $R_2$ are independent random variables.
5. $a_1$ is hardware failure rate, $c_1$ is software failure rate, $c_2$ is failed software induced hardware failure rate, $d_1$ is software replacement rate and $d_2$ is both software and hardware replacement rate.
6. When the hardware fails all the characteristics instantly fail and entire system requires a restart.
7. When hardware is replaced the software is also replaced and the system is given a fresh start. All the characteristics money, manpower and business are started from lower strata.
5.2.2 System analysis

The system has 17 states and are given by the state space
\[ S = \{ (1, \eta, i, j, k) : \eta = 0 \text{ or } 1; i = 0 \text{ or } 1; j = 0 \text{ or } 1; k = 0 \text{ or } 1 \} \cup \{ \emptyset \}, \]

Where \(\emptyset\) is the singleton state of hardware failure indicating catastrophe and the system needs to start afresh. 1 refers to the state of full availability of resources software and hardware functioning well and 0 refers to the state of non availability or shortage of resources and software. The infinitesimal generator of the continuous Markov chain is a matrix of order 17 and is given by.

\[
\begin{array}{cccc}
Q = & Q_1 & \Delta c_1 & a_1 \\
\Delta d_1 & Q_2 & a_1 + c_2 \\
d_2 & 0 & -d_2 \\
\end{array}
\]  

(5.2.1)

S precisely means status of the states when hardware functions, software takes values 0 or 1, I stands for fund status which may be 0 or 1; j stands for manpower status which may be one or zero and k stands for business status which may be 0 or 1. Every state space is in the order Hardware / Software/ Money / Manpower / Business.
Here

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\[ Q_1 = \]

here

\[
\begin{align*}
\epsilon_1 &= -(\beta + \mu + b) - (a_1 + c_1), \\
\epsilon_2 &= -(\alpha + \mu + b) - (a_1 + c_1), \\
\epsilon_3 &= -(\lambda + \beta + b) - (a_1 + c_1), \\
\epsilon_4 &= -(\lambda + \alpha + b) - (a_1 + c_1), \\
\epsilon_5 &= -(a + \beta + \mu) - (a_1 + c_1), \\
\epsilon_6 &= -(a + \alpha + \mu) - (a_1 + c_1), \\
\epsilon_7 &= -(a + \lambda + \beta) - (a_1 + c_1), \\
\epsilon_8 &= -(a + \lambda + \alpha) - (a_1 + c_1)
\end{align*}
\]

\[ \Delta c_1 \text{ is } 8 \times 8 \text{ diagonal matrix with } c_1 \text{ along the principal diagonal.} \]

\[ \Delta d_1 \text{ is } 8 \times 8 \text{ diagonal matrix with } d_1 \text{ along the principal diagonal.} \]

\[ a_1 \text{ is a column vector of order } 8 \times 1, \text{ given by } a_1 = (a_1, a_1, \ldots, a_1)^t \]

\[ a_1 + c_2 \text{ is a column vector of order } 8 \times 1, \text{ given by } \]

\[ a_1 + c_2 = (a_1 + c_2, a_1 + c_2, \ldots, a_1 + c_2)^t \]

\[ Q_2 = \]

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€_1 = (β + µ + b) – (a_1 + c_2 + d_1),  €_2 = – (α + µ + β) – (a_1 + c_2 + d_1),
€_3 = (λ + β + b) – (a_1 + c_2 + d_1),  €_4 = (λ + α + b) – (a_1 + c_2 + d_1),
€_5 = (a + µ + β) – (a_1 + c_2 + d_1),  €_6 = (a + α + µ) – (a_1 + c_2 + d_1),
€_7 = (a + λ + β) – (a_1 + c_2 + d_1),  €_8 = (a + α + λ) – (a_1 + c_2 + d_1),

\( d^2 = (d_2, 0, 0, ..., 0) \) is a 1×8 row vector,

\( O = (0, 0, ...) \) is 1×8 row vector

\( Q \) is a matrix of order 17 × 17. The steady state probability vector \( \Pi \) of the matrix \( Q_1 \) satisfies the following equations.

### 5.2.3 Steady state probabilities

\[ \Pi Q = 0 \quad \text{and} \quad \Pi e = 1 \]  \hspace{1cm} (5.2.2)

Here,

\[ \Pi = \{ \Pi_{10000}, \Pi_{11001}, \Pi_{11010}, \Pi_{11101}, \Pi_{11110}, \Pi_{11111}, \Pi_{10000}, \Pi_{10001}, \]
\[ \Pi_{10010}, \Pi_{10011}, \Pi_{10100}, \Pi_{10101}, \Pi_{10110}, \Pi_{10111}, \Pi^* \} \]  \hspace{1cm} (5.2.3)

And \( \Pi_{11ijk} \) is the steady state probability that the system is in state (1, η, i, j, k) and \( \Pi^* \) is the steady state probability that the system is in the state 0 and \( e = (1, 1, ..., 1)^t \) is a vector of order 17 × 1.
The matrix $Q$ is portioned as now
\[ Q = \begin{pmatrix} Q_1' & c \\ r & -d_2 \end{pmatrix} \]  
(5.2.4)

Where $Q_1 = ((Q)_{i,j})$, $1 \leq i, j \leq 16$ is a sub matrix of $Q$ without the last row and last column of $Q$.

$r = (d_2 \ 0 \ 0 \ \ldots \ \ldots \ 0)$ is the vector of order $1 \times 16$.

$c = (a_1, a_1, \ldots, a_1 \ a_1+c_2, a_1+c_2 \ \ldots \ \ldots \ a_1+c_2)^t$ is the vector of order $16 \times 1$.

Equations (5.2.2) becomes:
\[ \Pi^t Q_1' + \Pi^* r = 0 \]  
(5.2.5)

Where $\Pi = (\Pi^t, \Pi^*)$, where $\Pi^t$ and $\Pi^*$ partition of $\Pi$ and $\Pi^t$ is a vector of order $1 \times 16$ using (5.2.5) gives
\[ \Pi^t = \Pi^* r (-Q_1')^{-1} \]  
(5.2.6)

As $\Pi e = 1$ we get
\[ \Pi^* r (-Q_1')^{-1} e + \Pi^* \Pi^* = 1 \]
\[ \Pi^* = [1 + r (-Q_1')^{-1} e]^t \]

Substituting in (5.2.6) we get
\[ \Pi^t \left[ \frac{1}{1 + r (-Q_1')^{-1} e} \right] \times (r (-Q_1')^{-1}) \]

Now $\Pi = (\Pi^t, \Pi^*)$
\[ \Pi = \left[ \frac{1}{1 + r (-Q_1')^{-1} e} \right] \times [r (-Q_1')^{-1} e, 1] \]  
(5.2.7)

Equation (5.2.7) presents all the steady probabilities as mentioned in (5.2.3).

### 5.2.4 Rate of crisis

The crisis states are given by
\[ C = \{(11001), (11011), (11101), (10001), (10011), (10101)\} \]  
(5.2.8)

Now the rate of crisis in steady state is given by,
\[ C_t = P[X(t+\Delta t) = 11001 \mid X(t)=11000] \times P[X(t) = 11000] \\
+ P[X(t+\Delta t) = 11101 \mid X(t) = 11100] \times P[X(t) = 11100] \\
+ P[X(t+\Delta t) = 11011 \mid X(t) = 11010] \times P[X(t) = 11010] \\
+ P[X(t+\Delta t) = 11111 \mid X(t) = 11111] \times P[X(t) = 11111] \\
+ P[X(t+\Delta t) = 11011 \mid X(t) = 11010] \times P[X(t) = 11010] \\
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+ P[X(t+\Delta t) = 10101 \mid X(t) = 10100] \times P[X(t) = 10100] \\
+ P[X(t+\Delta t) = 10011 \mid X(t) = 10010] \times P[X(t) = 10010] \\
+ P[X(t+\Delta t) = 10111 \mid X(t) = 10111] \times P[X(t) = 10111] \\
+ P[x(t+\Delta t) = 10101 \mid X(t) = 10111] \times P[x(t) = 10111] + O \Delta t \text{ (5.2.9)} \]

Taking the minimal as \( \Delta t \to 0 \),

\[ C_t = \beta p_{11001}(t) + \beta p_{11100}(t) + \beta p_{11010}(t) + a p_{11111}(t) + \lambda p_{11111}(t) \]
\[ + \beta p_{10000}(t) + \beta p_{10100}(t) + \beta p_{10010}(t) + a p_{10111}(t) + \lambda p_{10111}(t) \]

Taking limit as \( t \to \infty \),

\[ C_\infty = \beta \Pi_{11000} + \beta \Pi_{11100} + \beta \Pi_{11010} + a \Pi_{11111} + \lambda \Pi_{11111} + \beta \Pi_{10000} + \beta \Pi_{10100} \]
\[ + \beta \Pi_{10010} + a \Pi_{10111} + \lambda \Pi_{10111} \text{ (5.2.10 )} \]

**5.2.5 Cost analysis**

The steady state cost of the system in different situations are determined by taking,

\( C^1_{F(11)} \) as cost of funds when fund is fully available and both hardware and software function;

\( C^0_{F(11)} \) – cost of funds when there is shortage of funds and both hardware and software function;

\( C^1_{F(10)} \) - cost of fund when fund is fully available and there is software failure;
\( C^0_{\text{F(10)}} \) – cost of fund when there is shortage of funds and there is software failure;

\( C^1_{\text{M(11)}} \) is the cost of manpower when manpower is fully available and both hardware and software function;

\( C^0_{\text{M(11)}} \) is cost of manpower when there is shortage of manpower and both hardware and software function;

\( C^1_{\text{M(10)}} \) is cost of manpower when manpower is fully available and there is software failure;

\( C^0_{\text{M(10)}} \) is cost of manpower when there is shortage of manpower and there is software failure;

\( C^1_{\text{B(11)}} \) is cost of business when it is fully available and hardware and software function;

\( C^0_{\text{B(11)}} \) is cost of business when there shortage in business and both hardare and software function;

\( C^1_{\text{B(10)}} \) is cost of business when it is full and there is software failure;

\( C^0_{\text{B(10)}} \) is cost of business when there is shortage in business and there is software failure;

The cost with respect to the states \( (\epsilon, \eta, i, j, k) \) is given by

\[
C_{\epsilon\eta\epsilon\eta\epsilon\eta} = \Pi_{\epsilon\eta\epsilon\eta\epsilon\eta} \{ C^i_{\text{F(}\epsilon\eta)} + C^j_{\text{M(}\epsilon\eta)} + C^k_{\text{B(}\epsilon\eta)} \} \quad (5.2.11)
\]

\( \epsilon, \eta, i, j \) and \( k \) take the values 0 or 1.

**5.3 Numerical Example**

We assume the values \( a = 1/75, b = 1/98, \lambda = 1/80, \mu = 1/50, \alpha = 1/60, \beta = 1/70, a_1 = 1/100, c_1 = 1/4, c_2 = 4/10, d_1 = 9/10, d_2 = 8/10. \)

The infinitesimal generator \( Q \) for the above set of values can be obtained using (5.2.1) Using the infinitesimal generator,
\( r(-Q_1^1)^{-1} = \{ 0.295536, 0.36942, 0.556032, 0.140744, 0.255768, 0.540232, \\
0.105904, 0.024072, 0.560808, 0.073528, 0.112088, 0.0284, 0.051656, 0.012216, \\
0.021288, 0.005072 \} \)

And \( r(-Q_1^1)^{-1} e = 3.01525316 \)

Now using (5.2.7) the steady state probabilities are:

\[
\begin{align*}
\Pi_{11000} &= 0.07127, \quad \Pi_{11010} = 0.13390, \quad \Pi_{11011} = 0.03389, \\
\Pi_{11100} &= 0.06159, \quad \Pi_{11101} = 0.13009, \quad \Pi_{11110} = 0.025990, \quad \Pi_{11111} = 0.00684, \\
\Pi_{10000} &= 0.1350, \quad \Pi_{10001} = 0.01771, \quad \Pi_{10010} = 0.02699, \quad \Pi_{10011} = 0.00512, \\
\Pi_{10100} &= 0.01244, \quad \Pi_{10101} = 0.00294, \quad \Pi_{10110} = 0.00512, \quad \Pi_{10111} = 0.00122, \\
\Pi_{\cdot e} &= 0.26875 \\
\end{align*}
\]

(5.2.12)

The crisis states in case of both hardware and software functioning and software failure is given by

\( \mathbf{C} = \{ 11001, 11011, 11101, 10001, 10011, 10101 \} \)

And using (5.2.10), the rate of crises in steady state when both hardware and software function and when there is software failure are obtained.

\[
\begin{align*}
C_{(11)}(\infty) &= 0.004 \text{ and } C_{(10)}(\infty) = 0.011 \\
\end{align*}
\]

(5.2.13)

Let us assume the cost under various situations (busy and lean) when both hardware and software function or software fail as below

\[
\begin{align*}
C^1_{F(11)} &= 5, \quad C^0_{F(11)} = 8, \quad C^1_{F(10)} = 10, \quad C^1_{M(11)} = 10, \quad C^0_{M(11)} = 15, \quad C^1_{M(10)} = 15, \\
C^0_{M(10)} &= 25, \quad C^1_{B(11)} = 25, \quad C^0_{B(11)} = 30, \quad C^1_{B(10)} = 30, \quad C^0_{B(10)} = 40, \quad C^0_{F(10)} = 18 \\
a_1 &= 1/100, \quad c_1 = 1/4, \quad d_1 = 9/10, \quad d_2 = 8/10. \\
\end{align*}
\]

Now using (5.2.11) the steady state costs are:

\[
\begin{align*}
C_{11000} &= 3.7773, \quad C_{11001} = 4.0036, \quad C_{11010} = 6.4272, \quad C_{11011} = 1.4573, \\
C_{11100} &= 3.0795, \quad C_{11101} = 5.8541, \quad C_{11110} = 1.16660, \quad C_{11111} = 0.2320, \\
C_{10000} &= 11.209, \quad C_{10001} = 1.2928, \quad C_{10010} = 1.9703, \quad C_{10011} = 0.4309, \\
\end{align*}
\]
\[ C_{10100} = 0.9330, \quad C_{10101} = 0.1617, \quad C_{10110} = 0.3328, \quad C_{10111} = 0.0671 \]  

(5.2.14)

The first eight steady state costs given above are when software functions and the next eight when software fails

### 5.4 Observation

It is being observed from (5.2.13) that the rate of crisis in steady state is more in the case of software failure than both hardware and software function. Also it is clear from (5.2.14) that cost of business is very high when software failure is there when compared to situation when both hardware and software function and that the cost is very less when all are available and hardware and software function. When hardware fails, the situation is said to be in catastrophic situation that everything must be strengthened to restart the business.

**REASONS FOR SELECTION OF MODELS AND THEIR APPLICATION TO INDUSTRIES / ORGANIZATIONS**

Computer usage finds a place from home to big industry. Its failure may be because of software or software induced hardware failure or hardware failure. A business getting affected because of the above is dealt in this model. Its effect on the cost and what rate of crisis it brings are discussed and expressions are derived for the same. Numerical examples are given to show the effect on cost when system works well and when there is system failure because of above reasons. Computers are used as teaching aid in schools and colleges, to get us connected to the entire world, so failure of system will drastically affect the institutions. Banking industry, railway reservations, insurance companies are some of the places where computers are widely used, so the consequence will be serious if computers fail.