In recent years there has been considerable interest in studying various properties of the materials in quantum domain and the development of the corresponding technologies. It is well-known that the concept of the effective electron mass in quantum materials is one of the basic pillars in the realm of modern low dimensional quantum science and technology [1]. Among the various definitions of the effective mass (e.g. effective acceleration mass, density-of-state effective mass, concentration effective mass, conductivity effective mass, Faraday rotation effective mass, etc) [2], it is the effective momentum mass that should be regarded as the basic quantity [3], since it is this mass which appears in the description of transport phenomena and all other properties of the carriers in materials having arbitrary band structures [3]. It can be shown that it is the effective momentum mass which enters in various transport coefficients and plays the most dominant role in explaining the experimental results of different scattering mechanisms through Boltzmann’s transport equation [4, 5].

The carrier degeneracy in materials influences the effective mass when it is energy dependent. Under degenerate conditions, only the electrons at the Fermi surface of n-type materials participate in the conduction process and hence, the effective momentum mass of the electrons (EMME) corresponding to the Fermi level would be of interest in electron transport under such conditions. The Fermi energy is again determined by the carrier energy spectrum and the electron statistics and therefore, these two features would determine the dependence of the EMME in degenerate n-type materials under the degree of carrier degeneracy. In recent years, various energy wave vector dispersion relations have been proposed [3-16] which have created the interest in studying the EMME in such materials under external conditions. Besides the physical properties of various quantum materials have been studied intensively [17]. The nature of variations of the EMME has widely been studied in the literature [18-33]. Some of the significant features, which have emerged from these investigations, are:

(a) The EMME increases monotonically with increasing carrier concentration.
(b) The EMME increases with doping in heavily doped materials in the presence of band tails.

(c) The nature of variations is significantly influenced by the energy band constants of various materials having different band structures.

(d) The EMME oscillates with inverse quantizing magnetic field due to SdH effect. The EMME in Bismuth under magnetic quantization depends both on the Fermi energy and the magnetic quantum number due to the presence of band nonparabolicity only.

(e) The EMME increases with the magnitude of the quantizing electric field in n-channel inversion layers of III-V materials and depends on the sub-band index for both weak and strong electric field limits.

(f) The EMME in ultrathin films of nonlinear optical materials depends on the Fermi energy and size quantum numbers due to the specific dispersion relations.

(g) The EMME has significantly different values in superlattices and also in the presence of quantum confined superlattices of small gap materials with graded interfaces together with effective mass of lattices.

The EMME in Ga\(_{x}\)As\(_{1-x}\)/GaAs quantum wells (QWs) has been investigated, and detected by cyclotron resonance technique by P. N. Hai et. al [34]. The values of EMME are 0.12\(m_0\) and 0.19\(m_0\) which are directly determined for the 70-Å-thick QWs with N composition of 1.2% and 2.0%, respectively. This sizable increase in the EMME is consistent with the earlier theoretical predictions based on the strong interaction of the lowest conduction band states with the upper lying band states or impurity band induced by the incorporation of N. DiVincenzo et.al. [35] formulated the self consistent effective mass theory for intra layer screening in graphite intercalation compounds. The effective mass approximation differential equations appropriate for impurities in a graphite host are constructed and are used to solve self-consistently for the screening response surrounding a single intercalant atom. The screening cloud is found to have a very slow algebraic decay with a characteristic length of 0.38 nm which is due to both the semi metallic and two dimensional character of graphite. The transferred charge in alkali-metal-graphite intercalation compounds is distributed nearly homogeneously on a carbon plane.

Perlin et al. [36] performed infrared reflectivity and Hall effect measurements on highly conducting \(n\)-type GaN \((n=6\times10^{19} \text{ cm}^{-3})\) bulk crystals grown by the high-pressure high-temperature method for the purpose of the experimental determination of the EMME of
GaN. Values of electron-plasma frequency and free-electron concentration were determined for each sample of the set of seven crystals. It enabled them to calculate the perpendicular EMME in the wurtzite structure of GaN as \( m^* = 0.22 \pm 0.02 \ m_0 \) and the effects of non-parabolicity together with the difference between parallel and perpendicular components of the effective mass are small and do not exceed the experimental error. The EMME has been determined by magneto-photoluminescence in as-grown and hydrogenated GaAs\(_{1-x}\)N\(_x\) samples for a wide range of nitrogen concentrations (from \( x < 0.01\% \) to \( x = 1.78\% \)) by the group of Masia et al. [37]. A modified \( k \cdot p \) model, which takes into account hybridization effects between N cluster states and the conduction band edge, reproduces quantitatively the experimental values up to \( x \leq 0.6\% \). Experimental and theoretical evidence is provided for the N complexes responsible for the non-monotonic and initially puzzling compositional dependence of the EMME. Sewall et al. [38] investigated the experimental tests of EMME and atomistic approaches to quantum dot electronic structure. The overall symmetry of the envelope functions for the four lowest energy excitonic states in colloidal CdSe quantum dots are assigned using excitonic state-resolved pump/probe spectroscopy.

G. E. Smith [39] performed the experimental determination of the EMME's in Bismuth-Antimony alloy. It was found that the EMME's in Bi\(_{95}\)Sb\(_5\) are smaller by about a factor of two than that of pure Bi and the hole masses are essentially unchanged. SdH investigations on n-InP are presented by the group of Schneider et al. [40] and EMME as a function of carrier concentration has been determined. The experiments were carried out with bulk and liquid phase epitaxially grown material and carrier concentrations between \( n_0 = 10^{20} \text{ m}^{-3} \) and \( 10^{22} \text{ m}^{-3} \) within the ranges of temperature between 2 K to 77 K and magnetic field \( B = 22 \) Tesla. The experimental result agrees very well with the theoretical relations. The values of the EMME in Cd\(_3\)As\(_2\) were obtained from low temperature SdH, magneto-Seebeck and Hall measurements by Caron et. al. [41]. The theoretical estimation of the variation of the energy gap at \( \Gamma \) as a function of temperature and pressure has been obtained. There is a band reversal in Cd\(_3\)Zn\(_x\)As\(_2\) and Cd\(_3\)As\(_x\)P\(_2\) alloys. Bhattacharya et al. [42] evaluated the EMME in compound semiconductor films of CdS\(_x\)Te\(_{1-x}\) and CdS\(_x\)Se\(_{1-x}\) which showed bowing phenomena similar to those for optical band gaps for the above alloy films.

B. Slomski et al. [43] has investigated the in-plane EMME of quantum well states in thin Pb films on a Bi reconstructed Si (111) surface by angle-resolved photoemission spectroscopy. It is found that this EMME is a factor of 3 lower than the unusually high
values reported for Pb films grown on a Pb reconstructed Si (111) surface. Through a quantitative low-energy electron diffraction analysis the change in EMME as a function of coverage and for the different interfaces is linked to a change of about 2% in the in-plane lattice constant. To corroborate this correlation, density functional theory calculations are performed on freestanding Pb slabs with different in-plane lattice constants. These calculations show an anomalous dependence of the EMME on the lattice constant including a change of sign for values close to the lattice constant of Si (111). This unexpected relation is due to a combination of reduced orbital overlap of the 6pz states and altered hybridization between the 6pz and the 6pxy derived quantum well states. Furthermore, it is shown by core-level spectroscopy that the Pb films are structurally and temporally stable at temperatures below 100 K.

The EMME’s for spin-up and spin-down electrons of a partially spin-polarized Fermi liquid are theoretically different as proposed by L.M. Wei et al [44]. They extracted the spin-up and spin-down EMME’s from magnetotransport measurements at different temperatures for a 2D electron gas in an In\(_{0.65}\)Ga\(_{0.35}\)As/In\(_{0.52}\)Al\(_{0.48}\) As quantum well exhibiting zero-field spin splitting. Two analytical methods are used, one involving the simultaneous fitting of fast Fourier transform (FFT) spectra and the other involving inverse FFT analysis. Both methods confirm that the EMME’s for spin-up and spin-down are different, consistent with theoretical expectations. The group of Karra et al [45] performed Cyclotron-resonance measurements for wide (100-300 nm) modulation-doped Al\(_x\)Ga\(_{1-x}\)As graded parabolic quantum wells for electron areal densities 10^9/cm^2–2.5x10^11/cm^2. A clear dependence of the cyclotron frequency on N\(_s\) is observed in the extreme quantum limit which is understood in terms of alloy effects. Self-consistent calculations that include the x dependence of the local effective mass and exchange-correlation effects in a local density approximation are in quantitative agreement with the measurements for high densities. At low densities a pinning of the cyclotron frequency is observed that is not predicted by the model.

Rožner et.al [46] reported the dependence of the effective masses on hole density in remotely doped strained Ge layers on relaxed Si\(_{0.3}\)Ge\(_{0.7}\) buffers with sheet densities from 2.9x10^{11} cm\(^{-2}\) to 1.9x10^{12} cm\(^{-2}\). The masses have been determined using temperature dependent Shubnikov–de Haas oscillations. No noticeable dependence of the mass on the magnetic field has been found. The extrapolated G point effective mass has been found to be 0.080 times the free electron mass. From the measured data the variation of the mass with kinetic energy and the shape of the topmost heavy hole sub-band have been
calculated. The results are in good agreement with theoretical predictions. The determination of the EMME of the two-dimensional electron gas (2DEG) and nonparabolicity effects in modulation-doped In$_{0.65}$Ga$_{0.35}$As/In$_{0.52}$Al$_{0.48}$As single quantum well were investigated by T. M. Kim et al [47] by performing temperature-dependent Shubnikov-de Haas (SdH) measurements and fast Fourier transformation (FFT) and the inverse FFT (IFFT) analyses. The result of the angular dependent SdH measurements clearly demonstrated the occupation of two sub-bands in the quantum wells by the 2DEGs. The EMME's determined from temperature-dependent S±dH measurements and the FFT and IFFT analyses were 0.05869 and 0.05385$m_e$ for the first and zeroth sub-bands, respectively. The EMME's obtained from the S-dH measurements and the FFT and IFFT analyses measurements qualitatively satisfy the nonparabolicity behavior in the In$_{0.65}$Ga$_{0.35}$As single quantum well.

The non-parabolic EMME's in InGaAs quantum wells (QWs), sandwiched by thick InAlAs barriers of 0.52-eV band offset, were studied by N. Kotera et al [48] in normal and parallel directions to the QW plane. The normal mass was experimentally obtained by observing interband photocurrent spectra of undoped InGaAs multi-QW structures. The mass increased by more than 50% from the bulk band edge mass, 0.041$m_0$. Electron eigenenergies were calculated in QWs based on Kane’s three-level band theory. The calculated ‘apparent’ normal mass as a function of kinetic energy up to 0.5 eV agreed well with experiments. The parallel mass in n-type modulation-doped InGaAs QWs was experimentally obtained by pulse cyclotron resonance up to 100 T. The analysis in quantizing magnetic fields, modified for two-dimensional QWs, fits well with cyclotron energy. The ‘apparent’ parallel mass as a function of energy was obtained consistently. Interband optical transitions of In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As multi-quantum wells have been observed Tanaka et al [49] in photocurrent spectra. Interband transitions were assigned from the spectral structures. Eigen-energies of conduction band were not proportional to the square of quantum numbers. An EMME normal to the quantum well plane was 50’%-heavier than the bulk band-edge mass of InGaAs.

The electronic structures of Bi$_2$Te$_3$ and Sb$_2$Te$_3$ were computed and related to the thermoelectric properties of Bi$_2$Te$_3$ and Sb$_2$Te$_3$ superlattices by Wang and Cagin [50]. They found that the similarity of the electronic structure of the two materials permits the Bi$_2$Te$_3$ and Sb$_2$Te$_3$ superlattices inherit high band edge degeneracy, and thus have high electrical conductivity. From the calculated EMME along the superlattice growth direction, they infer that presence of more Sb$_2$Te$_3$ than Bi$_2$Te$_3$ in the superlattice leads to...
a smaller EMME and enhanced carrier mobility. Furthermore, their results suggest that external tensile strain parallel to the interface may further improve the thermoelectric performance of the Bi$_2$Te$_3$ and Sb$_2$Te$_3$ superlattices. Engineered energy-wave-vector dispersion relations of either electrons or holes hold great promise for realizing fundamental oscillators at terahertz frequencies if they contain sections with a negative EMME at appropriate energy levels as suggested by Gribnikov et al [51], although, neither bulk semiconductor materials nor quantum wells or quantum wires exhibit such negative EMME sections in the dispersion relations at favorable energy levels.

Therefore, the novel use of a nanostructure is proposed to create an NEM section of electrons at suitable energy levels. This structure utilizes a heterojunction with a QW channel grown perpendicular to a superlattice. At small values of the wave-vector $k$; the electron wave function $\psi$ resides mostly in the QW channel and, as $k$ increases, $\psi$ extends further into the superlattice. This spread of $\psi$ induces a negative EMME section in the energy dispersion relation and several combinations of suitable material systems are considered by them.

Chen and Bajaj [52] have shown that the nonparabolicity and spin splitting enhance the EMME appreciably in a quantum wire. In bulk materials, these effects are usually small since electrons are near the conduction band edges. In nano wires, strong confinement puts the electrons far from the conduction-band edge, giving rise to large nonparabolicity effects. They derive a simple expression for the EMME parallel to the transport direction in the nano wire, taking into account the band non-parabolicity, anisotropy, and spin splitting. They apply their formalism to GaAs/Al$_{0.3}$Ga$_{0.7}$As nano wires with the conclusion that the parallel mass could be 50% more than the bulk value for a wire width of 50 Å. The space dependence of the EMME in nano wires results in the appearance of an additional momentum dependent potential has been considered by Borovitskaya and Shur [53]. If the EMME is anisotropic (as in silicon or germanium), this effect strongly depends on the transverse mass for a given sub-band and they consider Si-Ge p-type nano wires, where the impact ionization by holes should be determined by the impact ionization rates in silicon and not in SiGe. Dacal et al [54] investigated the conductance of three-dimensional semiconductor nano wires considering different EMME’s in the contacts and in the channel. They have shown that, with respect to the case with equal masses in the channel and in the contacts, the amplitude of the conductance oscillations increases if the EMME in the channel is larger and decreases if it is smaller than in the contacts. Effects on the density of probability are also considered.
and these effects of the EMME discontinuity are explained in terms of kinetic confinement and transmission coefficient modulation. Candidate materials for strained-layer EMME superlattices are investigated by Sasaki [55], and sixteen combinations of III–V semiconductor layers are presented. Among these In$_{0.69}$Ga$_{0.31}$As/InP possesses the smallest lattice mismatch, 1.1%. The electronic sub-band of In$_{0.69}$Ga$_{0.31}$As/InP layers is calculated through the Kronig-Penny approach. The energy gap obtained for the conduction band of composite materials is, for example, 52 meV for one period of alternating layer thickness of 40 Å/27 Å. Maan et al. [56] investigated the far infrared radiation transmission of a highly doped InAs-GaSb superlattice as a function of the magnetic field, exhibiting helicon wave propagation. The EMME and the carrier density are determined from an analysis of the results as a function of frequency to be 0.082 ± 0.005$m_0$ and $3.4 \times 10^{18}$ cm$^{-3}$. The carrier density is equal to that obtained from Hall measurements. The EMME is significantly higher than the value expected from the InAs conduction band nonparabolicity (0.063$m_0$).

Synthesizing single-walled carbon nanotubes (SWCNTs) with accurate structural control has been widely acknowledged as an exceedingly complex task culminating in the realization of CNT devices with uncertain electronic behavior. El Shabrawy et al. [57] applied a statistical approach in predicting the SWCNT band-gap and EMME variation for typical uncertainties associated with the geometrical structure. They carried out the same by proposing a simulation-efficient analytical model which evaluates the bandgap of an isolated SWCNT with a specified diameter and chirality. They developed an SWCNT EMME model, which is applicable to CNTs of any chirality and diameters >1 nm. A Monte Carlo method has been adopted to simulate the bandgap and effective mass variation for a selection of structural parameter distributions. They established analytical expressions that separately specify the bandgap and EMME variability with respect to the CNT mean diameter and standard deviation which offer insight from a theoretical perspective on the optimization of diameter-related process parameters with the aim of suppressing bandgap and effective mass variation.

In this chapter in Section 1, the EMME under magnetic quantization for microstructures of non-parabolic materials having different band structures has been investigated in details. It is well known that the band structure of materials can be dramatically changed by applying the external fields [58–60]. The effects of the quantizing magnetic field on the band structure of compound materials are more striking and can be observed easily in experiments. Under magnetic quantization, the motion of the electron parallel to the
magnetic field remains unaltered while the area of the wave vector space perpendicular to the direction of the magnetic field gets quantized in accordance with the Landau’s rule of area quantization in the wave-vector space. The energy levels of the carriers in a magnetic field (with the component of the wave-vector parallel to the direction of magnetic field be equated with zero) are termed as the Landau levels and the quantized energies are known as the Landau sub-bands. It is important to note that the same conclusion may be arrived either by solving the single-particle time independent Schrödinger differential equation in the presence of a quantizing magnetic field or by using the operator method. The quantizing magnetic field tends to remove the degeneracy and increases the band gap. A semiconductor, placed in a magnetic field $B$, can absorb radiative energy with the frequency $\omega = (|e| B/m_e)$ where $|e|$ is the magnitude of the carrier charge and $m_e$ is the effective carrier mass at the band edge respectively. This phenomenon is known as cyclotron or diamagnetic resonance. The effect of energy quantization is experimentally noticeable when the separation between any two consecutive Landau levels is greater than $k_BT$ ($k_B$ is the Boltzmann constant and $T$ is the temperature). A number of interesting transport phenomena originate from the change in the basic band structure of the semiconductor in the presence of quantizing magnetic field. These have been widely been investigated and also served as diagnostic tools for characterizing the different materials having various band structures.

At first we shall study the EMME in non-linear optical compounds, in the presence of an arbitrarily oriented quantizing magnetic field, which are being used in non-linear optics and light emitting diodes [61]. The quasi-cubic model can be used to investigate the symmetric properties of both the bands at the zone center of wave vector space of the same compound. Including the anisotropic crystal potential in the Hamiltonian, and special features of the non-linear optical compounds, Kildal [62] formulated the electron dispersion law under the assumptions of isotropic momentum matrix and the isotropic spin orbit splitting constant, respectively, although the anisotropies in the two aforementioned band constants are the significant physical features of the said materials [63]. In Section 1.2.1 of theoretical background, the magneto-EMME in non-linear optical materials has been investigated by considering the combined influence of the anisotropies of the said energy band constants together with the inclusion of the crystal field splitting respectively within the frame-work of $k.p$ formalism. The III-V compounds find applications in infrared detectors [64], quantum dot light emitting diodes [65], quantum cascade lasers [66], quantum well wires [67], optoelectronic
sensors [68], high electron mobility transistors [69] etc. The electron energy spectrum of III-V materials can be described by the three and two-band models of Kane [70], together with the models of Stillman et.al. [71], Newson and Kurobe [72] and Palik et.al. [73] respectively. In this context it may be noted that the ternary and quaternary compounds enjoy the singular position in the entire spectrum of optoelectronic materials. The ternary alloy Hg$_{1-x}$Cd$_x$Te is a classic narrow gap compound. The band gap of this ternary alloy can be varied to cover the spectral range from 0.8 to over 30μm [74] by adjusting the alloy composition. Hg$_{1-x}$Cd$_x$Te finds extensive applications in infrared detector materials and photovoltaic detector arrays in the 8-12μm wave bands [75]. The above uses have generated the Hg$_{1-x}$Cd$_x$Te technology for the experimental realization of high mobility single crystal with specially prepared surfaces. The same compound has emerged to be the optimum choice for illuminating the narrow sub-band physics because the relevant material constants can easily be experimentally measured [76]. Besides, the quaternary alloy In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP, also finds wide use in the fabrication of avalanche photo-detectors [77], hetero-junction lasers [78], light emitting diodes [79] and avalanche photodiodes [80], field effect transistors, detectors, switches, modulators, solar cells, filters and new types of integrated optical devices are made from the quaternary systems [81]. It may be noted that all types of band models as discussed for III-V materials are also applicable for ternary and quaternary compounds. In Section 1.2.2, the magneto-EMME in III-V, ternary and quaternary materials has been studied in accordance with the said band models and the simplified results for wide gap materials having parabolic energy bands under certain limiting conditions have further been demonstrated as a special case and thus confirming the compatibility test.

The II-VI materials are being used in nanoribbons, blue green diode lasers, photosensitive thin films, infrared detectors, ultra high-speed bipolar transistors, fiber optic communications, microwave devices, solar cells, semiconductor gamma-ray detector arrays, semiconductor detector gamma camera and allow for a greater density of data storage on optically addressed compact discs [82-84]. The carrier energy spectra in II-VI compounds are defined by the Hopfield model [85] where the splitting of the two-spin states by the spin-orbit coupling and the crystalline field has been taken into account. The Section 1.2.3 contains the investigation of the magneto-EMME in II-VI compounds.

In recent years, Bismuth (Bi) nano-lines have been fabricated and Bi also finds use in array of antennas which leads to the interaction of electromagnetic waves
with such Bi-nano-wires [86]. Several dispersion relations of the carriers have been proposed for Bi. Shoenberg [87] experimentally verified that the de Haas-Van Alphen and cyclotron resonance experiments supported the ellipsoidal parabolic model of Bi, although, the magnetic field dependence of many physical properties of Bi supports the two-band model [88]. The experimental investigations on the magneto-optical and the ultrasonic quantum oscillations support the Lax ellipsoidal non-parabolic model [88]. Kao [89], Dinger and Lawson [90] and Koch and Jensen [91] demonstrated that the Cohen model [92] is in conformity with the experimental results in a better way. Besides, the hybrid model of bismuth, as developed by Takoka et al. also finds use in the literature [93]. McClure and Choi [94] derived a new model of Bi and they showed that it can explain the data for a large number of magneto-oscillatory and resonance experiments. In section 1.2.4, the magneto-EMME in Bi has been formulated in accordance with the aforementioned energy band models for the purpose of relative assessment. Besides, under certain limiting conditions all the results for all the models in this case are reduced to the well-known result of the magneto-EMME in isotopic parabolic energy bands. This above statement exhibits the compatibility test of our theoretical analysis.

Lead chalcogenides (PbTe, PbSe, and PbS) are IV-VI non-parabolic materials whose studies over several decades have been motivated by their importance in infrared IR detectors, lasers, light-emitting devices, photovoltaics and high temperature thermoelectrics [95]. PbTe, in particular, is the end compound of several ternary and quaternary high performance high temperature thermoelectric materials [96]. It has been used not only as bulk but also as films [97], quantum wells [98], superlattices [99], nanowires [100], and colloidal and embedded nanocrystals [101] and PbTe films doped with various impurities have also been investigated [102]. These studies revealed some of the interesting features that had been seen in bulk PbTe, such as Fermi level pinning and, in the case of superconductivity [103]. In Section 1.2.5, the magneto-EMME in IV-VI materials has been studied taking PbTe, PbSe, and PbS as examples.

The stressed materials are being investigated for strained silicon transistors, quantum cascade lasers, semiconductor strain gages, thermal detectors and strained-layer structures [104]. The magneto-EMME in stressed compounds (taking stressed n-InSb as an example) has been investigated in Section 1.2.6. The vacuum deposited Tellurium (Te) has been used as the semiconductor layer in thin-film transistors (TFT) [105] which is being used in CO2 laser detectors [106], electronic imaging, strain sensitive devices
The n-Gallium Phoshide (n-GaP) is being used in quantum dot light emitting diode [109], high efficiency yellow solid state lamps, light sources, high peak current pulse for high gain tubes. The green and yellow light emitting diodes made of nitrogen-doped n-GaP possess a longer device life at high drive currents [110]. In Section 1.2.8, the magneto-EMME in n-GaP has been studied. The Platinum Antimonide (PtSb$_2$) finds application in device miniaturization, colloidal nanoparticle synthesis, sensors and detector materials and thermo-photovoltaic devices [111]. The Section 1.2.9 explores the magneto-EMME in PtSb$_2$. Bismuth telluride (Bi$_2$Te$_3$) was first identified as a material for thermoelectric refrigeration in 1954 [112] and its physical properties were later improved by the addition of bismuth selenide and antimony telluride to form solid solutions [113]. The alloys of Bi$_2$Te$_3$ are useful compounds for the thermoelectric industry and have been investigated in the literature [113]. In Section 1.2.10, the magneto-EMME in Bi$_2$Te$_3$ has been considered.

The usefulness of elemental semiconductor Germanium is already well known since the inception of transistor technology and, it is also being used in memory circuits, single photon detectors, single photon avalanche diode, ultrafast optical switch, THz lasers and THz spectrometers [114]. In Section 1.2.11, the magneto-EMME has been studied in Ge. Gallium Antimonide (GaSb) finds applications in the fiber optic transmission window, heterojunctions and quantum wells. A complementary heterojunction field effect transistor in which the channels for the p-FET device and the n-FET device forming the complementary FET are formed from GaSb. The band gap energy of GaSb makes it suitable for low power operation [115]. In Section 1.2.12, the magneto-EMME in GaSb has been studied. The section 1.2.13 investigates the magneto-EMME in II-V materials. The 1.3 contains the result and discussions pertaining to section 1.

1.2. Theoretical Background

1.2.1 The EMME under magnetic quantization in non-linear optical materials

The form of $k.p$ matrix for nonlinear optical compounds can be expressed extending Bodnar [63] as

$$H = \begin{bmatrix} H_1 & H_2 \\ H_2^* & H_1 \end{bmatrix}$$

where,
\[
H_1 = \begin{bmatrix}
E_{t_0} & 0 & P_0 k_z & 0 \\
0 & (-2\Delta_{t}/3) & (\sqrt{2}\Delta_{t}/3) & 0 \\
P_0 k_z & (\sqrt{2}\Delta_{t}/3) & -(\delta + 1/3) & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad H_2 = \begin{bmatrix}
0 & -f_+ & 0 & f_- \\
f_+ & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
f_- & 0 & 0 & 0 \\
\end{bmatrix},
\]

in which \(E_{t_0}\) is the band gap in the absence of any field, and \(P_0\) are \(P_\perp\) the momentum matrix elements parallel and perpendicular to the direction of crystal axis respectively, \(\delta\) is the crystal field splitting constant, \(\Delta_0\) and \(\Delta_\perp\) are the spin-orbit splitting constants parallel and perpendicular to the C-axis respectively, \(f_\pm \equiv \left(P_\perp \sqrt{2}\right)(k_z \pm ik_y)\) and \(i = \sqrt{-1}\). Thus, neglecting the contribution of the higher bands and the free electron term, the diagonalization of the above matrix leads to the dispersion relation of the conduction electrons in bulk specimens of nonlinear optical compounds as

\[
\gamma(E) = f_1(E)k_z^2 + f_2(E)k_z^2
\]

(2)

where,

\[
\gamma(E) \equiv E(E + E_{t_0}) \left[ \left( E + E_{t_0} \right) \left( E + E_{t_0} + \Delta_0 \right) + \delta \left( E + E_{t_0} + \frac{2}{3} \Delta_0 \right) + \frac{2}{9} \left( \Delta_0^2 - \Delta_\perp^2 \right) \right], \quad E \text{ is the total energy of the electron as measured from the edge of the conduction band in the vertically upward direction in the absence of any quantization, } k_z^2 = k_x^2 + k_y^2,
\]

\[
f_1(E) = \frac{\hbar^2 E_{t_0} \left( E_{t_0} + \Delta_\perp \right)}{2m^*_l \left( E_{t_0} + \frac{2}{3} \Delta_\perp \right)} \left[ \delta \left( E + E_{t_0} + \frac{1}{3} \Delta_0 \right) + \left( E + E_{t_0} \right) \left( E + E_{t_0} + \frac{2}{3} \Delta_0 \right) + \frac{1}{9} \left( \Delta_0^2 - \Delta_\perp^2 \right) \right],
\]

\[
f_2(E) = \frac{\hbar^2 E_{t_0} \left( E_{t_0} + \Delta_0 \right)}{2m^*_l \left( E_{t_0} + \frac{2}{3} \Delta_\parallel \right)} \left( E + E_{t_0} \right) \left( E + E_{t_0} + \frac{2}{3} \Delta_\parallel \right)
\]

\(\hbar = \frac{h}{2\pi}\), \(h\) is Planck’s constant and \(m^*_l\) and \(m^*_t\) are the longitudinal and transverse effective electron masses at the edge of the conduction band respectively.

In the presence of an arbitrarily oriented quantizing magnetic field \(B\) along \(k_z\) direction which makes an angle \(\theta\) with \(k_x\) axis and lies in the \(k_x - k_z\) plane, the magneto-dispersion law of the conduction electrons in non-linear optical materials can be expressed as

\[
\psi_1(E) = \bar{\Delta}_l (n, E, \theta) + a_x(E, \theta) k_x^2
\]

(3)

where, \(\psi_1(E) = \gamma(E), \psi_2(E) = f_1(E), \psi_3(E) = f_2(E)\).
\[ \bar{A}_k(n,E,\theta) = \frac{2|e|B}{\hbar} \left( n + \frac{1}{2} \right) \left\{ \psi_2(E)\cos^2 \theta + \psi_2(E)\sin^2 \theta \right\}^{1/2} \]

\[ + \left( \frac{|e|B\hbar E_k}{6} \right) \left[ \frac{(E_g + \Delta_1)}{m^*_{\parallel}(E_g + \frac{2}{3}\Delta_1)} \right]^{1/2} \]

\[ \left[ (E + E_g + \delta + \frac{2\Delta_1^2}{3\Delta_1}) \left\{ \frac{\Delta_1^2(E_g + \Delta_1)\cos^2 \theta}{m^*_{\parallel}(E_g + \frac{2}{3}\Delta_1)} \right\} + \left\{ \frac{(E + E_g)^2(E_g + \Delta_1)\sin^2 \theta}{m^*_{\parallel}(E_g + \frac{2}{3}\Delta_1)} \right\} \right]^{1/2} \]

\[ n(=0,1,2,3...) \] is the Landau quantum number and

\[ a_0(E,\theta) = \frac{\psi_2(E)\psi'_2(E)}{\psi_2(E)\cos^2 \theta + \psi_3(E)\sin^2 \theta} \]

The EMME for semiconductors with arbitrary band structures can be defined by the ratio of carrier momentum to the group velocity along the particular free direction. Under magnetic quantization, the electron motion along the direction of magnetic quantization is free. Therefore, the EMME at the Fermi level along the direction of the quantizing magnetic field, can be expressed as

\[ m_{\parallel}^*(E_{FB},n,\theta) = \hbar^2 k_{ztl} \left. \frac{\partial k_{ztl}}{\partial E} \right|_{E=E_{FB}} \]

(4)

where, \( E_{FB} \) is the Fermi energy in the presence of magnetic quantization as measured from the edge of the conduction band in the vertically upward direction in the absence of any field. Using (3) and (4) we can write

\[ m_{\parallel}^*(E_{FB},n,\theta) = \left( \frac{\hbar^2}{2} \right) \left( \frac{\psi_2(E_{FB})' - \bar{A}_k(n,E_{FB},\theta)}{a_0(E_{FB},\theta)} \right) \left( \frac{\psi_2(E_{FB})'}{a_0(E_{FB},\theta)} \right) \left[ \psi_2(E_{FB}) - \bar{A}_k(n,E_{FB},\theta) \right] \]

(5)

From (5), it appears that EMME is a function of the Fermi energy, the angle of orientation of the quantizing magnetic field, the magnetic quantum number and the electron spin for non linear optical materials due to the combined influence of the crystal field splitting and the anisotropic spin orbit splitting constant. The dependence of the oscillatory mobility on the spin dependent EMME in addition to Fermi energy is an important physical feature of such compounds.
To investigate the dependence of EMME on the electron concentration we have to determine the density-of-states function in the present case \( D_s(E) \) which can be written, including spin as
\[
D_s(E) = \frac{g_s |e| B}{2 \pi^2 \hbar} \left( \sum_{n=0}^{\infty} \frac{\partial k_z}{\partial E} H(E - E_s) \right)
\]
where, \( g_s \) is valley degeneracy, \( E_s \) is the Landau energy and \( H \) is the Heaviside step function.

Using (3) and (6), one obtains,
\[
D_s(E) = \left( \frac{g_s |e| B}{4 \pi^2 \hbar} \right) \left( \sum_{n=0}^{\infty} \left[ \frac{\psi_i(E) - \bar{A}_i(n,E,\theta)}{a_0(E,\theta)} \right] \right)^2
\]
\[
\left[ a_0(E,\theta) \left\{ \psi_i(E) - \bar{A}_i(n,E,\theta) \right\} - \psi_i(E) \right] \left( \frac{\psi_i(E) - \bar{A}_i(n,E,\theta)}{a_0(E,\theta)} \right) H(E - E_s)
\]
(7)

Thus, combining (7) with the Fermi-Dirac occupation probability factor \( f_0 \) and using the generalized Sommerfeld’s lemma [116], the electron concentration assumes the form
\[
n_0 = \frac{g_s |e| B}{\pi^2 \hbar} \sum_{n=0}^{\infty} \left[ T_{33}(n,E_{FB}) + T_{34}(n,E_{FB}) \right]
\]
(8)

where,
\[
T_{33}(n,E_{FB}) = \left\{ \frac{\psi_i(E_{FB}) - \left( \frac{2 |e| B}{\hbar} \left( n + \frac{1}{2} \right) \right) \left[ \psi_2(E_{FB}) \{ \psi_2(E_{FB}) \cos^2 \theta + \psi_3(E_{FB}) \sin^2 \theta \} \right]^{1/2}}{a_0(E_{FB},\theta)} \right\}^{1/2}
\]
and
\[
T_{34}(n,E_{FB}) = \sum_{r=1}^{\infty} L_8(r) \left[ T_{33}(n,E_{FB}) \right], \quad L_8(r) = 2 \left( k_B T \right)^{2r} \left( 1 - 2^{-2r} \right) \xi(2r) \frac{\partial^{2r}}{\partial E_{FB}^{2r}},
\]
r is the set of real positive integers whose upper limit is and \( \xi(2r) \) is the Zeta function of order \( 2r \).

1.2.2 The EMME in Kane type III-V materials under magnetic quantization

(a) Three band model of Kane:

Under the conditions \( \delta = 0, \Delta_0 = \Delta = \Delta \) (the isotropic spin orbit splitting constant) and \( m_u^* = m_\perp^* = m_e \) (the isotropic effective electron mass at the edge of the conduction band), (3) assumes the form
\[
I_{11}(E) = \left( n + \frac{1}{2} \right) \hbar \omega_b + \frac{\hbar^2 k^2}{2m_c} \pm \frac{e|B\hbar\Delta}{6m_c \left( E + E_g + \frac{2}{3} \Delta \right)} \left[ 6m_c \left( E + E_g + \frac{2}{3} \Delta \right) \right]^{-1}
\]

where, \(I_{11}(E)\) is given by
\[
\frac{E(E + E_{\sigma}) (E + E_{\delta} + \Delta) (E_{\delta} + \frac{2}{3} \Delta)}{E_{\sigma} (E + \Delta) (E + E_{\delta} + \frac{2}{3} \Delta)}
\]

The (9) is the dispersion relation of the conduction electrons of III-V, ternary and quaternary materials in the presence of a quantizing magnetic field \(B\) along \(z\)-direction [70].

From (9), the EMME along the direction of magnetic quantization can be written as
\[
m_{\perp}^*(E_{FB}) = m_c \left( \left[ I_{11}(E_{FB}) \right] \pm \frac{|e|B\hbar\Delta}{6m_c \left[ (E_{FB} + E_g + \frac{2}{3} \Delta) \right]^2} \right)
\]

Thus, the EMME is a function of the Fermi energy and the electron spin under magnetic quantization. The dependence of the EMME on the electron spin is due to the presence of the spin orbit splitting constant, excluding the dependence on \(\{I_{11}(E_{FB})\}'\), is a special property of the three band model of Kane.

Using (6) and (9), the density-of-states function in this case can be expressed as
\[
D_g(E) = \frac{g_s |e| B \sqrt{2m_c}}{4\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ I_{11}(E) - \left( n + \frac{1}{2} \right) \hbar \omega_b + \frac{|e|B\hbar\Delta}{6m_c \left( E + E_g + \frac{2}{3} \Delta \right)} \right] \left[ \left( I_{11}(E_{FB}) \right) \right] \left[ H(E - E_{ni}) \right] \left[ E_{ni}^2 \right] \left[ \left( E_{FB} + \frac{2}{3} \Delta \right) \right]
\]

where \(E_{ni}\) is the Landau energy in this case.

Thus, the electron concentration assumes the form
\[
n_0 = \frac{g_s |e| B \sqrt{2m_c}}{2\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ T_{35}(n, E_{FB}) + T_{36}(n, E_{FB}) \right]
\]

where, \(T_{35}(n, E_{FB})\) is given by
\[
\left[ I_{11}(E_{FB}) - \left( n + \frac{1}{2} \right) \hbar \omega_b + \frac{|e|B\hbar\Delta}{6m_c \left( E_{FB} + E_g + \frac{2}{3} \Delta \right)} \right] \left[ H(E - E_{ni}) \right] \left[ E_{ni}^2 \right] \left[ \left( E_{FB} + \frac{2}{3} \Delta \right) \right]
\]
and $T_{36}(n, E_{FB}) \equiv \sum_{r=1}^{r} L_{b}(r) T_{35}(n, E_{FB})$.

In the absence of spin, the electron concentration assume the form

$$n_0 = \frac{g e |e| B \sqrt{2m_e}}{\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ T_{37}(n, E_{FB}) + T_{38}(n, E_{FB}) \right]$$

where, $T_{37}(n, E_{FB}) \equiv \left[ \frac{1}{E_{FB}} \left( n + \frac{1}{2} \right) \hbar \omega_b \right]^{1/2}$ and $T_{38}(n, E_{FB}) \equiv \sum_{r=1}^{r} L_{b}(r) T_{37}(n, E_{FB})$.

(b) Two band model of Kane:

Under the condition $\Delta \gg E_s$, (9) can be expressed as

$$E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega_b + \left( \hbar^2 k^2 / 2m_e \right) \pm \frac{1}{2} \mu_0 g^* B$$

$$\alpha = \frac{1}{\mu_0}$$

where, $\mu_0 = \left( |e| \hbar / 2m_0 \right)$ is known as the Bohr magnetron, $g^*$ is the magnitude of the band edge g-factor and is equal to $\left( m_0 / m_e \right)$ in accordance with the two band model of Kane.

From (14), the EMME along the direction of magnetic quantization can be expressed as

$$m_{\parallel}^* (E_{FB}) = m_e \left[ 1 + 2\alpha E_{FB} \right]$$

Thus, the EMME is a function of Fermi energy only due to the presence of band nonparabolicity factor $\alpha$ and is independent of the electron spin under magnetic quantization.

In accordance with the two-band model of Kane, the density-of-states function assumes the form

$$D_{\parallel}(E) = \frac{g_s |e| B \sqrt{2m_e}}{4\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ 1 + 2\alpha E \right] \left[ E(1 + \alpha E) - \left( n + \frac{1}{2} \right) \hbar \omega_b \pm \frac{1}{2} g^* B \right]^{1/2} H(E-E_\alpha)$$

where $E_{\alpha_2}$ is the Landau energy in this case.

Thus, the electron concentration can be written as

$$n_0 = \frac{g_s |e| B \sqrt{2m_e}}{2\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ T_{39}(n, E_{s_\alpha}) + T_{310}(n, E_{s_\alpha}) \right]$$

where, $T_{39}(E_{FB}) \equiv \left[ E_{FB}(1 + \alpha E_{FB}) - \left( n + \frac{1}{2} \right) \hbar \omega_b \pm \frac{1}{2} g^* B \right]^{1/2}$.
and $T_{310}(n, E_{FB}) \equiv \sum_{r=1}^{r} I_{r}(r) T_{3s}(n, E_{FB})$.

In the absence of spin, the electron concentration assume the form

$$n_{0} = \frac{g_{e} |e| B \sqrt{2m_{e}}}{\pi^{2} \hbar^{2}} \sum_{n=0}^{\infty} \left[ T_{311}(n, E_{a}) + T_{312}(n, E_{a}) \right]$$

(17b)

where $T_{311}(n, E_{FB}) \equiv \left[ E_{FB}(1 + \alpha E_{FB}) - \left( n + \frac{1}{2} \right) \hbar \omega_{b} \right]^{1/2}$ and

$T_{312}(n, E_{FB}) \equiv \sum_{r=1}^{r} I_{r}(r) T_{311}(n, E_{FB})$.

From (16), under the condition $\alpha E << 1$, the density-of-states function can be written as

$$D_{s}(E) = \frac{g_{e} |e| B \sqrt{2m_{e}}}{4\pi^{2} \hbar^{2}} \sum_{n=0}^{\infty} \left[ E - \left( n + \frac{1}{2} \right) \hbar \omega_{b} + \frac{1}{2} g^{*} \mu_{0} B \right]^{1/2}$$

$$\times \left[ 1 + \alpha E \right] \frac{1}{f_{s} dE}$$

(18)

Therefore, the electron concentration is given by

$$n_{e} = \frac{g_{e} |e| B \sqrt{2m_{e}}}{4\pi^{2} \hbar^{2}} \sum_{n=0}^{\infty} \left[ E - \left( n + \frac{1}{2} \right) \hbar \omega_{b} + \frac{1}{2} g^{*} \mu_{0} B \right]^{1/2}$$

$$\times \left[ 1 + \alpha E \right] f_{s} dE$$

(19)

Let us substitute, $y = E - \left( n + \frac{1}{2} \right) \hbar \omega_{b} + \frac{1}{2} g^{*} \mu_{0} B (1 + \alpha E_{FB})^{-1}$

(20)

where, $y$ is a new variable

Therefore, from (20), after binomial expansion and neglecting the terms in the order of $(\alpha E)^{2}$, we can write

$$E = \frac{y}{a_{01}} + b_{01}$$

(21)

where, $a_{01} = \left[ 1 + \alpha \left( n + \frac{1}{2} \right) \hbar \omega_{b} \pm \frac{1}{2} g^{*} \mu_{0} B \right]$ and $b_{01} = (a_{01})^{-1} \left[ \left( n + \frac{1}{2} \right) \hbar \omega_{b} \pm \frac{1}{2} g^{*} \mu_{0} B \right]$

Therefore, combining (20) and (21) we get,

$$n_{0} = \frac{g_{e} |e| B \sqrt{2m_{e}}}{4\pi^{2} \hbar^{2}} \sum_{n=0}^{\infty} \frac{1}{\sqrt{a_{01} b_{01}}} \left[ y \left( y + a_{01} \right) + b_{01} \right]^{1/2}$$

$$\times \left[ 1 + \frac{3}{2} \alpha \left( y + a_{01} \right) + b_{01} \right]^{1/2} a_{01}^{1/4} b_{01}^{1/4} dy$$

(22)
Let us substitute, \( \beta_{01} = \frac{y}{a_{01}k_BT} \) and \( \eta_{B} = \frac{E_{FB} - b_{01}}{k_BT} \).

Thus, using (22) and the Fermi-Dirac integrals, the electron concentration in this case assumes the form

\[
n_0 = g_eN_c\beta_{B1} \left[ \sum_{n=0}^{\infty} \frac{1}{\alpha_{01}^*} \left[ \left( 1 + \frac{3}{2} n + \frac{1}{2} \right) \eta_{B1} \right] \right] \left[ \left( 1 + \frac{3}{2} \alpha_{01}^* \right) F_{-\frac{1}{2}}(\eta_{B1}) + \frac{3}{4} \alpha_{01}^* k_BT F_{-\frac{1}{2}}(\eta_{B1}) \right]^{\frac{3}{2}}
\]

where, \( N_c = 2 \left( \frac{2\pi m_c \theta_B}{\hbar^2} \right)^{3/2} \), \( \theta_{B1} = \frac{\hbar \alpha_{01}^*}{k_BT} \) and \( F_j(\eta_B) \) is the well known one parameter Fermi-Dirac integral of order \( j \) [59].

In the absence of spin, (23) assume the form [70],

\[
n_0 = g_eN_c\beta_{B1} \left[ \sum_{n=0}^{\infty} \frac{1}{\alpha_{01}^*} \left[ \left( 1 + \frac{3}{2} n + \frac{1}{2} \right) \eta_{B1} \right] \right] \left[ \left( 1 + \frac{3}{2} \alpha_{01}^* \right) F_{-\frac{1}{2}}(\eta_{B1}) + \frac{3}{4} \alpha_{01}^* k_BT F_{-\frac{1}{2}}(\eta_{B1}) \right]^{\frac{3}{2}}
\]

where, \( \alpha_{01}^* = 1 + \alpha \left( n + \frac{1}{2} \right) \hbar \alpha_b, \eta_{B1} = \eta_{B1}^{-1}, \) and \( \eta_{B1} = \frac{E_{FB} - b_{01}^*}{k_BT} \).

(c) **Parabolic energy bands:**

Under the condition \( \alpha \rightarrow 0 \) \( (14) \) becomes

\[
E = \left( n + \frac{1}{2} \right) \hbar \omega_b + \left( \hbar^2 k_e^2 / 2m_e \right) \pm \frac{1}{2} g^* \mu_B B
\]

From (25), the EMME along the direction of quantizing magnetic field can be expressed as

\[
m^*_e (E_{FB}) = m_e
\]

The electron concentration in this case assumes the form

\[
n_0 = g_eN_c\theta_{B1} \sum_{n=0}^{\infty} \left[ \left( n + \frac{1}{2} \right) \hbar \omega_b \right] \eta_{B1}^{-1}
\]

where, \( \eta_{B} = (k_BT)^{-1} \left[ E_{FB} - \left( n + \frac{1}{2} \right) \hbar \omega_b \right] \)

Thus, the quantizing magnetic field cannot influence the EMMEin relatively wide gap materials having parabolic energy bands.

(d) **The model of Stillman, et. al.**

In accordance with the model of Stillman et. al. [71], the electron dispersion law of III-V materials assumes the form
\[ E = \tilde{t}_{11} k^2 - \tilde{t}_{12} k^4 \]  \\
(28)

where,  \\
\( \tilde{t}_{11} = \frac{\hbar^2}{2m_e} \) ;  \\
\( \tilde{t}_{12} = \left( \frac{1 - m_e}{m_v} \right)^2 \left( \frac{\hbar^2}{2m_v} \right)^2 \left[ 3E_{g_0} + 4\Delta + \frac{2\Delta^2}{E_{g_0}} \right] \left( \frac{E_{g_0} + \Delta}{2E_{g_0} + 3E_{g_0}} \right)^{1/2} \)

and  \\
\( m_v \) is the free electron mass.

The (28) can be expressed as

\[ \frac{\hbar^2 k^2}{2m_e} = I_{12}(E) \]  \\
(29)

where,  \\
\( I_{12}(E) = a_{11} \left[ 1 - \left( 1 - a_{12} E \right)^{1/2} \right] \),  \\
a_{11} = \left( \frac{\hbar^2 \tilde{t}_{11}}{4m_v \tilde{t}_{12}} \right) \) and  \\
a_{12} = \frac{4\tilde{t}_{12}}{\tilde{t}_{11}} .

In accordance with model, the electron energy spectrum in III-V materials in the presence of the quantizing magnetic field \( \vec{B} \) along z-direction can be written following (29) as

\[ k_z^2 = \frac{2m_e}{\hbar^2} \left[ I_{12}(E) - \left( n + \frac{1}{2} \right) \hbar \omega_b \right] \]  \\
(30)

Therefore,  \\
k_z^2 = \frac{2m_e}{\hbar^2} A_{33}(E, n)

(31)

where,  \\
\( A_{33}(E, n) = I_{12}(E) - \left( n + \frac{1}{2} \right) \hbar \omega_b \)

The EMME for this model can be expressed as

\[ m_{e*}(E_{FB}) = m_e [I_{11'}(E_{FB})] \]  \\
(32)

The electron concentration is given by

\[ n_0 = \frac{2m_e e B_g}{\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ Y_{33}(E_{FB}, n) + Z_{33}(E_{FB}, n) \right] \]  \\
(33)

where,  \\
\( Y_{33}(E_{FB}, n) = \left[ \sqrt{A_{33}(E_{FB}, n)} \right] \)  \\
and  \\
\( Z_{33}(E_{FB}, n) = \sum_{r=1}^{N} L_g(r) [Y_{33}(E_{FB}, n)] \)

(c) The model of Palik et al.

The energy spectrum of the conduction electrons in III-V materials up to the fourth order in effective mass theory, taking into account the interactions of heavy hole, light hole and the split-off holes can be expressed in accordance with the model of Palik et al. [73] as

\[ E = \frac{\hbar^2 k^2}{2m_e} - B_{11} k^4 \]  \\
(34)

\[ n_0 = \frac{2m_e e B_g}{\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ Y_{33}(E_{FB}, n) + Z_{33}(E_{FB}, n) \right] \]  \\
(33)
where, \[ \bar{B}_{11} = \left[ \frac{\hbar^4}{4E_0(m_c)^2} \left( 1 + \frac{x_{11}^2}{2} \right) \right] (1 - y_{11})^2 \]
\[ x_{11} = \left[ 1 + \left( \frac{\Delta}{E_0} \right) \right] \] and \[ y_{11} = \frac{m_e}{m_0} \]

To the fourth order in effective mass theory and taking into account the interactions of the conduction, light hole, heavy-hole and split-off hole bands, the electron energy spectrum in III-V materials in the presence of a quantizing magnetic field \( \bar{B} \) can be written in accordance with the present model extending (34) as

\[ E = J_{31} + \left( n + \frac{1}{2} \right) \hbar \omega_b + \frac{\hbar^2 k_z^2}{2m_e} \pm \frac{1}{4} \left( \frac{m_e}{m_0} \right) \hbar \omega_b g_0 \cdot k_{30} \alpha (n + \frac{1}{2}) (\hbar \omega_b)^2 \pm k_{31} \alpha \hbar \omega_b \left( \frac{\hbar^2 k_z^2}{2m_e} \right) \]

\[ k_{32} \left( \hbar \omega_b \left( \frac{1}{2} + \frac{1}{2} \right) + \frac{\hbar^2 k_z^2}{2m_e} \right)^2 \]

where, \( J_{31} = -\frac{1}{2} \alpha \hbar \omega_b \left[ (1 - y_{11}) / (2 + x_{11})^2 \right] J_{32} \)

\[ J_{32} = \left\{ \left[ \frac{1}{3} (1 - x_{11})^2 - (2 + x_{11})^2 \right] (2 + x_{11}), \left( y_{11} + \frac{1}{2} (1 - x_{11})^2 (1 + x_{11}) (1 + y_{11}) \right) \right\} \]

\[ g_0 \cdot \left[ 2 \left[ \frac{1 - \frac{1}{2} (1 - x_{11})}{2 + x_{11}} \right] \left[ \frac{(1 - y_{11})}{y_{11}} \right] \right] \]

\[ k_{30} = (1 - y_{11})(1 - x_{11}) \left[ \left( 2 + \frac{3}{2} x_{11} + x_{11}^2 \right), \left( \frac{1 - y_{11}}{2 + x_{11}} \right) \right] \frac{2}{3} \left( 1 - x_{11} \right) y_{11} \]

\[ k_{31} = (1 - y_{11}) \left[ \frac{(1 - x_{11})}{(2 + x_{11})} \right] \left[ \left( 2 + \frac{3}{2} x_{11} + x_{11}^2 \right), \left( \frac{1 - y_{11}}{2 + x_{11}} \right) \right] \frac{2}{3} \left( 1 - x_{11} \right) y_{11} \]

and

\[ k_{32} = -\left[ \left( 1 + \frac{1}{2} x_{11} \right), \left( 1 + \frac{1}{2} x_{11} \right) \right] (1 - y_{11})^2 \]

The (35) assumes the form

\[ J_{34} k_z^4 + J_{35,\pm} (n) k_z^2 + J_{36,\pm} (n) - E = 0 \]

where \( J_{34} = \alpha k_{32} \left( \frac{\hbar^2}{2m_e} \right)^2 \), \( J_{35,\pm} (n) = \left[ \frac{\hbar^2}{2m_e} \pm \alpha k_{31} \hbar \omega_b \cdot \frac{\hbar^2}{2m_e} + \alpha k_{32} \hbar \omega_b \cdot \frac{\hbar^2}{2m_e} (n + \frac{1}{2}) \right] \),

\[ J_{36,\pm} (n) = \left[ J_{31} \pm \frac{1}{4} \left( \frac{m_e}{m_0} \right) \hbar \omega_b g_0 \cdot k_{30} \alpha (\hbar \omega_b)^2 (n + \frac{1}{2}) + k_{32} \alpha [(\hbar \omega_b) (n + \frac{1}{2})]^2 \right] \]
2J_{34}k_x^2 = -J_{35,z} + \sqrt{(J_{35,z})^2 - 4J_{34}J_{36,z} - E}

k_x^2 = A_{35,z}(E,n)

(37)

where, \(A_{35,z}(E,n) = (2J_{34})^{-1} \left[ -J_{35,z}(n) + \sqrt{(J_{35,z}(n))^2 - 4J_{34}J_{36,z}(n) - E} \right] \)

(38a)

The EMME for this model can be expressed as

\[ m_{k_x}^* (E_{FB}, n) = \frac{\hbar^2}{2} (A_{35,z}(E_{FB}, n)) \]

(38b)

Thus EMME in accordance with this model is a function of Fermi energy, Landau quantum number and the electron spin due to the presence of band non-parabolicity only.

The electron concentration is given by

\[ n_0 = \frac{eB_{g_x}}{2\pi^2 \hbar} \sum_{n=0}^{\text{sum}} \left[ Y_{34}(E_{FB}, n) + Z_{34}(E_{FB}, n) \right] \]

(39)

where, \(Y_{34}(E_{FB}, n) = [\sqrt{A_{35,z}(E_{FB}, n)} + \sqrt{A_{35,z}(E_{FB}, n)}] \) and

\[ Z_{34}(E_{FB}, n) = \sum_{r=3}^{5} L_g(r) [Y_{34}(E_{FB}, n)] \]

1.2.3 II-VI Materials

The carrier energy spectra in bulk specimens of II-VI compounds in accordance with Hopfield model [85] can be written as

\[ E = a' k_x^2 + b' k_z^2 \pm \hbar_0 k_z \]

(40)

where, \(a' = \hbar^2/2m^*_z, b' = \hbar^2/2m^*_0 \) and \(\hbar_0 \) represents the splitting of the two-spin states by the spin orbit coupling and the crystalline field.

The Hamiltonian of the conduction electron of II-VI materials in the presence of a quantizing magnetic field \(B\) along \(z\) direction assumes the form

\[ \hat{H}_g = - \hbar \left[ \begin{array}{c} \hat{p}_z \\ \frac{1}{2m^*_z} (\hat{p}_x - e|B|\hat{x})^2 + \frac{\hbar_0}{\hbar} \left[ (\hat{p}_z)^2 + (\hat{p}_y - e|B|\hat{y})^2 \right]^{1/2} \end{array} \right] + \frac{\hbar_0}{2m^*_0} \]

(41)

where, the "hats" denote the respective operators.

The application of the operator method leads to the magneto-dispersion relation of the carriers of II-VI materials, including spin, as

\[ E = \frac{\hbar e|B|}{m^*_z} \left( n + \frac{1}{2} \right) + \frac{\hbar^2 k_z^2}{2m^*_0} \pm \hbar_0 \left[ \frac{2e|B|}{\hbar} \left( n + \frac{1}{2} \right) \right]^{1/2} \pm \frac{1}{2} e^* \mu_0 |B| \]

(42)
From (42), the EMME along the direction of the magnetic quantization can be expressed as

$$m_i^* (E_{FB}) = m_i^1$$

(43)

Thus, the EMME in this case is a constants quantity and is not affected by magnetic field.

1.2.4 The EMME in Bismuth Under Magnetic Quantization

(a) The McClure and Choi Model

The dispersion relation of the carriers in Bi can be written, following the McClure and Choi [94], as

$$E(1 + \alpha E) = \frac{p_x^2}{2m_1} + \frac{p_y^2}{2m_2} + \frac{p_z^2}{2m_2} - \alpha E \left\{ 1 - \left( \frac{m_2}{m_2} \right) \right\} + \frac{p_x^4 \alpha}{4m_1 m_2} - \frac{\alpha p_y^2 p_z^2}{4m_2 m_3}$$

(44)

where, $p_i = \hbar k_i, i = x, y, z$, $m_1, m_2$ and $m_3$ are the effective carrier masses at the band-edge along x, y and z directions respectively and $m'$ is the effective-mass tensor component at the top of the valence band (for electrons) or at the bottom of the conduction band (for holes).

The Hamiltonian in the presence of a quantizing magnetic field $B$ along the z-direction in accordance with this model can be written as

$$\hat{H}_B = \frac{\hbar}{2m_1} \left( \hat{p}_x^2 + \hat{p}_y^2 - |e| B \hat{x} \right)^2 + \frac{\hbar}{4m_1 m_2} \left[ \alpha \left( \hat{p}_y - |e| B \hat{x} \right)^4 + \alpha \left( \hat{p}_z \right)^2 - \alpha \left( \hat{p}_y - |e| B \hat{x} \right)^2 \right]$$

(45)

Thus, the modified carrier energy spectrum in accordance with McClure and Choi model up to the first order by including spin effects can be expressed as [94]

$$E(1 + \alpha E) = \frac{\hbar \alpha}{2} \frac{\hat{x} \hat{y} (E)}{4} + \frac{\hbar^2 k^2}{2m} \left[ 1 - \frac{(n+1/2) \hbar \alpha (E)}{2} \right] + \frac{1}{2} g^* |\mu_B|$$

(46)

where, $\alpha(E) = \frac{|e| B}{\sqrt{m_1 m_2}} \left[ 1 + \alpha E \left( 1 - \frac{m_2}{m_2} \right) \right]^{1/2}$. 
From (46), the EMME along the direction of magnetic quantization assumes the form as

\[
m^*_{\parallel}(n, E_{FB}) = m_3 \left[ \frac{1}{2} \frac{\alpha}{n+\frac{1}{2}} \hbar \alpha(E_{FB}) \right]^{1/2} \left[ 1 + 2\alpha E_{FB} - \left( n + \frac{1}{2} \right) \hbar \alpha(E_{FB}) \right]^{1/2} \frac{1}{2} \left( n^2 + n + \frac{1}{2} \right) \alpha^2 \hbar^2 \alpha(E_{FB}) \hbar \alpha(E_{FB})
\]

\[
\frac{\alpha}{2} \frac{\alpha}{n+\frac{1}{2}} \hbar \alpha(E_{FB}) \left[ 1 - \frac{1}{2} \frac{\alpha}{n+\frac{1}{2}} \hbar \alpha(E_{FB}) \right]^{1/2}
\]

(47)

In absence of band nonparabolicity, from (47) we get

\[
m^*_{\parallel}(n, E_{FB}) = m_3
\]

(48)

It is interesting to note that for the two band model of Kane, the band nonparabolicity alone explains the dependence of the EMME on Fermi energy, and the EMME is independent of magnetic quantum number and the electron spin. In the case of McClure and Choi model of Bi under magnetic quantization, the same band nonparabolcity again alone explains the dependence of the EMME on the magnetic quantum number, electron spin and the Fermi energy respectively.

The density-of-states function for this model under magnetic quantization is given by

\[
D_b(E) = \frac{g_v e^{-2m_3}}{4\pi^2 \hbar^2} \sum_{\nu=0}^{\infty} \left[ \left[ 1 - \frac{\alpha}{2} \frac{\alpha}{n+\frac{1}{2}} \hbar \alpha(E) \right]^{1/2} \left( \frac{1}{2} \right) \alpha \left( n + \frac{1}{2} \right) \hbar \alpha(E) \right]^{1/2} \frac{\alpha \hbar^2 \alpha^2(E)}{4} \frac{1}{2} g \cdot \mu_e B
\]

\[
+ \left[ E(1 + \alpha E) - \left( n + \frac{1}{2} \right) \hbar \alpha(E) - \left( n^2 + 1 + n \right) \frac{\alpha \hbar^2 \alpha^2(E)}{4} \frac{1}{2} g \cdot \mu_e B \right]^{1/2}
\]

\[
\left[ 1 + 2\alpha E - \left( n + \frac{1}{2} \right) \hbar \alpha(E) \right]^{1/2} \left[ \left( n^2 + n + \frac{1}{2} \right) \alpha \hbar^2 \alpha^2(E) \hbar \alpha(E) \right]^{1/2} \frac{1}{2} \left( \frac{n+\frac{1}{2}}{2} \right) \alpha \hbar \alpha(E)
\]

(49)

Combining (49) with the Fermi-Dirac occupation probability and using the generalized Sommerfeld’s lemma [116], the electron concentration in this case assumes the form

\[
n_0 = \frac{g_v e^{-2m_3}}{2\pi^2 \hbar^2} \sum_{\nu=0}^{\infty} \left[ T_{313}(n, E_{FB}) + T_{314}(n, E_{FB}) \right]
\]

(50)

where
Under the condition $\alpha \to 0$, (50) get simplified as

$$n_0 = \frac{g_s N c \theta_3}{2} \sum_{n=0}^{\infty} \frac{F_3}{2} (\eta_n)$$

(51)

where, $N_s = 2 \left( \frac{2 \pi m_0}{h^2} k^2 E_p \right)^{\frac{3}{2}}$, $m_{n+1} = (m_1 m_2 m_3)^{\frac{1}{3}}$, $\theta_3 = \frac{\hbar a_0}{k^2 T}$, $a_0 = (|E| / \sqrt{m_1 m_2})$ and

$$\eta_n = (k^2 T)^{-1} \left[ E_p - \left( n + \frac{1}{2} \right) \hbar \omega_0 + \frac{1}{2} \hbar \mu B \right].$$

**b) The Cohen Model**

In accordance with the Cohen model [92], the dispersion law of the carriers in Bi is given by

$$E(1 + \alpha E) = \frac{P_x^2}{2m_1} + \frac{P_y^2}{2m_2} + \frac{P_z^2}{2m_3} + \frac{\alpha E p_x^2}{2m_2} + \frac{\alpha P_z^4}{4m_2 m_3'}$$

(52)

The application of the above method in Cohen model leads to the electron energy spectrum in Bi in the presence of quantizing magnetic field $B$ along $z$-direction as

$$E(1 + \alpha E) = \left( n + \frac{1}{2} \right) \hbar \omega(E) \pm \frac{1}{2} g \cdot \mu B + \frac{3}{8} \alpha \left( n^2 + n + \frac{1}{2} \right) \hbar^2 \omega^2(E) + \frac{\hbar^2 k^2}{2m_3}$$

(53)

From (53), the EMME along the direction of the quantizing magnetic field can be expressed as

$$m_s^*(n, E_p) = m_3 \left[ 2 \alpha E_p + 1 - \left( n + \frac{1}{2} \right) \hbar \omega(E_p) \right] - \frac{3}{4} \alpha \hbar^2 \omega(E_p) \omega'(E_p) \left( n^2 + n + \frac{1}{2} \right)$$

(54)

In absence of band nonparabolicity, the (54) gets transformed into the well known form of parabolic energy bands and the mass becomes independent of Fermi energy and magnetic quantum number.

It is important to note that the band nonparabolicity has been introduced between the McClure and Choi model and the Cohen model in two different ways so that in the first case, the band nonparabolicity alone explains the dependence of the EMME on the Fermi energy, magnetic quantum number and the electron spin whereas for the Cohen model, the same band nonparabolicity alone explains the independence of the EMME on the
electron spin excluding the other two dependences. In the absence of band nonparabolicity for both the models of Bi, the mass along the direction of the magnetic field is not perturbed by the magnetic quantization.

The density-of-states function under magnetic quantization in accordance with the Cohen model is given by

\[ D_B(E) = \frac{g_e |e| B \sqrt{2m_3}}{4\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ E(1+\alpha E) - \left(n + \frac{1}{2}\right) \hbar \omega(E) - \frac{3\alpha \hbar^2 \omega(E) \pm 1}{2} \frac{1}{2} \frac{1}{2} \mu_B B \right]^2 \]

\[ + \left[1 + 2\alpha E\left(n + \frac{1}{2}\right) \hbar \omega(E) - \frac{3\alpha \hbar^2 \omega(E)}{4} \right] \left[E-E_n\right] \]  

(55)

Thus, the electron concentration assumes the form

\[ n_0 = \frac{g_e |e| B \sqrt{2m_3}}{2\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ T_{319}(n, E_{FB}) + T_{320}(n, E_{FB}) \right] \]

(56)

where,

\[ T_{319}(n, E_{FB}) = \left[E_{FB}(1+\alpha E_{FB}) - \left(n + \frac{1}{2}\right) \hbar \omega(E_{FB}) + \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \mu_B B - \frac{3}{8} \alpha \left(n^2 + \frac{1}{2}\right) \hbar^2 \omega(E_{FB}) \right]^2 \]

\[ \omega(E_{FB}) = \sqrt{m_1 m_2} \left[1 + \alpha E_{FB} \left(1 - \frac{m_2^3}{m_1^3}\right) \right]^2 \]

and

\[ T_{320}(n, E_{FB}) = \sum_{r=1}^{N} L_r(r, 0) \left[T_{319}(n, E_{FB}) \right] \]

c) The Lax Model

In accordance with this model, the magneto dispersion relation assumes the form

\[ E(1+\alpha E) = (n + \frac{1}{2}) \hbar \omega_0(E) + \frac{\hbar^2 k^2}{2m_3} + \frac{1}{2} \mu_B g^* B \]

(57)

where, \( \omega(E) = \frac{|e| B}{\sqrt{m_1 m_2}} \)

Therefore \( k^2 = \frac{2m_3}{\hbar^2} \left[A_{40,1}(E, n)\right] \)

(58)

where, \( A_{40,1}(E, n) = E(1+\alpha E) - (n + \frac{1}{2}) \hbar \omega_0 + \frac{1}{2} \mu_B g^* B \)

The EMME assumes the form
\( m^*(E_{FB}) = m_0(1 + 2\alpha E_{FB}) \) \hspace{1cm} (59)

The electron concentration is given by

\[
n_0 = \frac{eB_g}{\sqrt{2m_0}} \sum_{n=0}^{\infty} \left[ Y_{40}(E_{FB}, n) + Z_{40}(E_{FB}, n) \right]
\]

where, \( Y_{40}(E_{FB}, n) = \sqrt{A_{40, +}(E_{FB}, n) + \sqrt{A_{40, -}(E_{FB}, n)}} \) and

\[
Z_{40}(E_{FB}, n) = \sum_{r=0}^{n} L_g(r)[Y_{40}(E_{FB}, n)]
\]

\( d) \) Ellipsoidal parabolic energy bands

For this model, the magneto-dispersion relation can be written as

\[
E = \left( n + \frac{1}{2} \right) \hbar \omega_{ba} + \left( \hbar^2 k^2 / 2m \right) \pm \frac{1}{2} \hbar \mu_0 B
\] \hspace{1cm} (61)

The expression of the electron concentration for this model is the special case of the models of the McClure and Choi, the Cohen and the Lax respectively.

1.2.5 The EMME From IV-VI Materials Under Magnetic Quantization

\( a) \) The Dimmock Model

The dispersion relation of the conduction electrons in IV-VI materials can be expressed in accordance with Dimmock [117] as

\[
\left[ \underbrace{\frac{E_{\xi_0}}{2} - \frac{\hbar^2 k^2}{2m_\xi}}_{\bar{\varepsilon}} \right] \left[ \frac{E_{\xi_0}}{2} + \frac{\hbar^2 k^2}{2m_\xi} \right] = P_{\xi_+}^2 k_{\xi_+}^2 + P_{\xi_-}^2 k_{\xi_-}^2
\]

where \( \bar{\varepsilon} \) is the energy as measured from the center of the band gap \( E_{\xi_0} \), \( m_\xi^+ \) and \( m_\xi^- \) represent the contributions to the transverse and longitudinal effective masses of the external \( L_\xi^+ \) and \( L_\xi^- \) bands arising from the \( \vec{k}, \vec{p} \) perturbations with the other bands taken to the second order.

Using \( E = E + \left( E_{\xi_0} / 2 \right) \), \( P_{\xi_+}^2 = \hbar^2 E_{\xi_0} \), and \( m_\xi^+ \) are the transverse and longitudinal effective electron masses at \( k=0 \) in (62), we can write

\[
\left[ \frac{\hbar^2 k_{\xi_+}^2}{2m_\xi^+} \right] \left[ \frac{\hbar^2 k_{\xi_-}^2}{2m_\xi^-} \right] = \frac{\hbar^2 k_{\xi_+}^2}{2m_\xi^+} + \frac{\hbar^2 k_{\xi_-}^2}{2m_\xi^-}
\]

In accordance with Dimmock model, the electron energy spectrum in IV-VI materials in presence of a quantizing magnetic field \( \vec{B} \) along \( z \)-direction can be written following (63) as
Thus (64) assumes the form

$$ k_z^2 = A_{41}(E, n) $$

where $A_{41}(E, n) = [2C_{31}]^{-1} \left[ -C_{32}(E, n) + \left[ C_{32}^2(E, n) - 4C_{31}\left[C_{33}(E, n) - E(1 + \alpha E)\right] \right]^{1/2} \right]$

$$ C_{31} = \frac{\alpha \hbar^4}{4m_i^*m_i^-}, $$

$$ C_{32}(E, n) = \left[ -\frac{\alpha E \hbar^2}{2m_i^*} + \frac{\alpha \hbar^3 eB}{2m_i^*m_i^-} \right] \left( n + \frac{1}{2} \right) + \frac{1 + \alpha E}{2m_i^-} \left( n + \frac{1}{2} \right) + \frac{\hbar^2}{2m_i^-}, $$

$$ C_{33}(E, n) = \left[ \frac{\hbar eB}{m_i^-} \left( n + \frac{1}{2} \right) - \frac{\alpha E \hbar eB}{m_i^*m_i^-} \left( n + \frac{1}{2} \right) + \frac{1 + \alpha E}{m_i^-} \left( n + \frac{1}{2} \right) + \frac{\alpha (\hbar eB)^2}{m_i^*m_i^-} \left( n + \frac{1}{2} \right)^2 \right] $$

The EMME for this model can be expressed as

$$ m^*(E_{FB}, n) = \frac{\hbar^2}{2} A_{41}(E_{FB}, n)^2 $$

Therefore, the EMME is a function of Fermi energy and Landau quantum number due to the presence of band nonparabolicity only.

The electron concentration can be written as

$$ n_0 = \left( \frac{eB_{gy}}{\pi^* \hbar} \right) \sum_{m=0}^{2m} \left[ Y_{41}(E_{FB}, n) + Z_{41}(E_{FB}, n) \right] $$

where, $Y_{41}(E_{FB}, n) = [\sqrt{A_{42}(E_{FB}, n)}]$ and, $Z_{41}(E_{FB}, n) = \sum_{r=1}^{2m} L_{gr}(r) [Y_{41}(E_{FB}, n)]$

b) The Model of Bangert and Kastner

The electron energy spectrum of IV-VI materials in accordance with the model of Bangert and Kastner can be written as [118]

$$ \Gamma(E) = \overline{F}_1(E) k_z^2 + \overline{F}_2(E) k_z^2 $$

where, $\Gamma(E) = 2E, \overline{F}_1(E) = \left[ \frac{(\overline{R})^2}{E + E_{\delta_0}} + \frac{(\overline{S})^2}{E + \Delta_1} + \frac{(\overline{Q})^2}{E + \Delta_1'} \right], \overline{F}_2(E) = \left[ \frac{2A^2}{E + E_{\delta_0}} + \frac{(\overline{S} + \overline{Q})^2}{E + \Delta_1^*} \right]$

and $\overline{R}, \overline{S}, \overline{Q}, \overline{A}, \Delta_1, \Delta_1', \Delta_1^*$ are the electron energy spectrum constants.

In presence of a quantizing magnetic field $\vec{B}$ along z-direction, (68) assumes the form

$$ \Gamma(E) = \overline{F}_1(E) \frac{2eB}{\hbar} (n + \frac{1}{2}) + \overline{F}_2(E) k_z^2 $$
Therefore, \( k_z^2 = A_{44}(E, n) \) \( \quad (70) \)

\[
\Gamma(E) - F_1(E) 2eB(n + \frac{1}{2}) \\
where A_{44}(E, n) = \frac{F_2(E)}{F_2(E)}
\]

The EMME for this model can be expressed as

\[
m^*(E_{FB}, n) = \frac{\hbar^2}{2} A'_{44}(E_{FB}, n) \quad (71)
\]

Therefore, the EMME is a function of Fermi energy and Landau quantum number which is the characteristics feature of this model.

The electron concentration can be expressed as

\[
n_0 = \left( \frac{eB_{eq}}{\pi^2 \hbar} \right) \sum_{n=0}^{\infty} \left[ Y_{42}(E_{FB}, n) + Z_{42}(E_{FB}, n) \right] \\
\]

where \( Y_{42}(E_{FB}, n) = \sqrt{A_{44}(E_{FB}, n)} \) and, \( Z_{42}(E_{FB}, n) = \sum_{r=1}^{3} L_g(r)[Y_{42}(E_{FB}, n)] \)

**c) The Model of Foley and Landenberg**

In accordance with the model of Foley and Landenberg, the electron energy spectrum in IV-VI materials assumes the form \([119]\)

\[
E + E_{eg} = \frac{\hbar^2 k_x^2}{2m^*_x} + \frac{\hbar^2 k_y^2}{2m^*_y} + \left[ \frac{\hbar^2 k_z^2}{2m^*_z} + \frac{E_{eg}}{2} \right]^2 + \frac{p^2 k_z^2 + p^2 k_y^2}{2m^*_z} \quad (73)
\]

Where, \( \frac{1}{m^*_x} = \frac{1}{m^*_c} \left[ \frac{1}{m^*_c} + \frac{1}{m^*_l} \right] \), \( \frac{1}{m^*_y} = \frac{1}{m^*_c} \left[ \frac{1}{m^*_c} + \frac{1}{m^*_l} \right] \), \( m^*_c \) and \( m^*_l \) are the transverse and longitudinal effective electron masses of the conduction electrons at the edge of the conduction band and \( m^*_z \) and \( m^*_l \) are the transverse and longitudinal effective hole masses at the edge of the valence band. In the presence of magnetic quantization \( \vec{B} \) along z-direction (73) assumes the form

\( \therefore k_z^2 = A_{46}(E, n) \) \( \quad (74) \)

where \( A_{46}(E, n) = (2D_{31})^{-1} \left[ -D_{32}(E, n) + D_{32}^2(E, n) + 4 \left[ E(E + E_{eg}) - D_{33}(E, n) \right] D_{31} \right]^{1/2} \)

\[
D_{31} = \left[ \frac{\hbar^4}{4(m^*_c)^2} - \frac{\hbar^4}{4(m^*_l)^2} \right]
\]

\[
D_{32}(E, n) = \left[ \frac{\hbar^2}{2m^*_l} \left( E_{eg} + \frac{2\hbar eB}{m^*_l} \left( n + \frac{1}{2} \right) \right) + \frac{p^2}{m^*_l} - \frac{\hbar^2 eB}{m^*_l m^*_c} \left( n + \frac{1}{2} \right) + \left( E_{eg} + 2E \right) \right] \frac{\hbar^2}{2m^*_l}
\]

\[
D_{33}(E, n) = \left[ \frac{\hbar^2}{2m^*_l} \left( E_{eg} + \frac{2\hbar eB}{m^*_l} \left( n + \frac{1}{2} \right) \right) + \frac{p^2}{m^*_l} - \frac{\hbar^2 eB}{m^*_l m^*_c} \left( n + \frac{1}{2} \right) + \left( E_{eg} + 2E \right) \right] \frac{\hbar^2}{2m^*_l}
\]
The EMME for this model can be expressed as

\[ m'(E_{FB}, n) = \frac{\hbar^2}{2} A'_o (E_{FB}, n) \]  

(75)

Therefore, the EMME is a function of Fermi energy and Landau quantum number which is the characteristic feature of this model.

The electron concentration can be expressed as

\[ n_o = \left( \frac{e B g_e}{\pi^2 \hbar} \right) \sum_{n=0}^{\infty} \left[ Y_{43}(E_{FB}, n) + Z_{43}(E_{FB}, n) \right] \]

(76)

where, \( Y_{43}(E_{FB}, n) = \sqrt{A_4(E_{FB}, n)} \) and, \( Z_{43}(E_{FB}, n) = \sum_{r=1}^{n} I_y(r) [Y_{43}(E_{FB}, n)] \)

1.2.6 The EMME From Stressed Materials Under Magnetic Quantization

The electron energy spectrum in stressed Kane type materials can be written as [120]

\[ \left( \frac{\mathbf{k}_x}{a_0(E)} \right)^2 + \left( \frac{\mathbf{k}_y}{b_0(E)} \right)^2 + \left( \frac{\mathbf{k}_z}{c_0(E)} \right)^2 = 1 \]

(77)

where, \( \left[ a_0(E) \right]^2 = \frac{\bar{K}_0(E)}{A_0(E) + \frac{1}{2} \bar{D}_0(E)} \), \( \bar{K}_0(E) \equiv \left[ E - C_1 \mathbf{\varepsilon} - \frac{2 C_2^2 \mathbf{\varepsilon}^2}{3E'_s} \right] \left( \frac{3E'_s}{2B_2^2} \right) \), \( C_1 \) is the conduction band deformation potential, \( \mathbf{\varepsilon} \) is the trace of the strain tensor \( \hat{\mathbf{\varepsilon}} \) which can be written as \( \hat{\mathbf{\varepsilon}} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{xy} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \), \( C_2 \) is a constant which describes the strain interaction between the conduction and valance bands, \( E'_s \equiv E_s + E - C_1 \mathbf{\varepsilon}, B_2 \) is the momentum matrix element,

\[ \bar{A}_0(E) \equiv 1 - \frac{\bar{a}_0 + C_1}{E'_s} + \frac{3 \bar{b}_0 \mathbf{\varepsilon}_{xx} - \bar{b}_0 \mathbf{\varepsilon}}{2E'_s} - \frac{\bar{b}_0 \mathbf{\varepsilon}_{yy}}{2E'_s} \]

\[ \bar{a}_0 = -\frac{1}{3} \left( \bar{b}_0 + 2 \bar{m} \right), \quad \bar{b}_0 = \frac{1}{3} \left( -\bar{m} - \bar{m} \right), \quad \bar{d}_0 = \frac{2 \bar{m}}{\sqrt{3}}, \]

\( \bar{l}, \bar{m}, \bar{n} \) are the matrix elements of the strain perturbation operator, \( \bar{D}_0(E) \equiv \left( \bar{d}_0 \sqrt{3} \right) \mathbf{\varepsilon}_{xx} \)

\[ \left[ \bar{E}_0(E) \right]^2 = \frac{\bar{K}_0(E)}{\bar{A}_0(E) - \frac{1}{2} \bar{D}_0(E)}, \quad \left[ \bar{E}_0(E) \right]^2 = \frac{\bar{K}_0(E)}{\bar{A}_0(E)}, \quad \text{and} \]
The simplified expression of the electron energy spectrum in stressed Kane type materials in the presence of an arbitrarily oriented quantizing magnetic field \( B \), which makes angles \( \alpha, \beta, \) and \( \gamma \) with \( k_x, k_y, \) and \( k_z \) axes respectively can be written following (77) as

\[
1 - [k']^2 [I_2 (E)]^{-1} = I_3 (n, E)
\]  

(78)

where, \( I_2 (E) \equiv \left[ \hat{a}_0 (E) \right]^2 \cos^2 \alpha + \left[ \hat{b}_0 (E) \right]^2 \cos^2 \beta + \left[ \hat{c}_0 (E) \right]^2 \cos^2 \gamma \) and

\[
I_3 (n, E) = \left( \frac{2 |d B|}{h} \right) \left[ (n + \frac{1}{2}) [\hat{a}_0 (E)] [\hat{b}_0 (E)] [\hat{c}_0 (E)]^{-1} [I_2 (E)]^{-1} \right] \]

The use of (78) leads to the expression of the EMME as

\[
m_{n}^* (n, E_{FB}) = \frac{\hbar^2}{2} \left[ -I_3 (n, E_{FB}) + I_2 (n, E_{FB}) \right] \]

(79)

In the absence of stress, together with the substitution \( B_z^2 = \frac{3\hbar^2 E_F}{4m_e} \), the (78) gets simplified into (15). By comparing (79) and (15), one can observe that the stress makes the EMME quantum number dependent in stressed Kane type compounds under magnetic quantization, in addition to Fermi energy.

The density-of-states function in this case is given by

\[
D_{\theta} (E) = \frac{g_e |e| B_{\text{max}}}{2\pi^2 \hbar} \sum_{\Delta =0} \left[ I_2 (E) \right] \left[ 1 - I_3 (n, E) \right]^{-1/2} \left[ 1 - I_3 (n, E) \right]^{-1/2} \left[ I_2 (E) \right] \sqrt{E - E_F}
\]

(80)

The use of (80) leads to the expression of electron concentration as

\[
n_e = \frac{g_e |e| B_{\text{max}}}{\pi^2 \hbar} \sum_{\Delta =0} \left[ T_{327} (n, E_{FB}) + T_{328} (n, E_{FB}) \right]
\]

(81)

where, \( T_{327} (n, E_{FB}) = \sqrt{I_2 (E_{FB})} \left[ \sqrt{1 - I_3 (n, E_{FB})} \right] \) and \( T_{328} (n, E_{FB}) = \sum_{r=1}^{\infty} L (r) T_{327} (n, E_{FB}) \).

1.2.7 The EMME From Tellurium Under Magnetic Quantization

The dispersion relation of the conduction electrons in Te can be expressed as \[121\]

\[
E = \psi_1 k_x^2 + \psi_2 k_y^2 \pm \sqrt{[\psi_1 k_x^2 + \psi_2 k_y^2]^2}
\]

(82)

where,

\[
\psi_1 = 6.7 \times 10^{-14} \text{mev.m}^2, \quad \psi_2 = 4.2 \times 10^{-14} \text{mev.m}^2, \quad \psi_3 = 6 \times 10^{-8} \text{mev.m}
\]
and $V_4 = 3.6 \times 10^{-8} \text{mev.m}$

The dispersion under magnetic quantization can be written following (82) as

$$E = \Psi_{4} k_z^2 + \Psi_2 \frac{2eB}{\hbar} (n + \frac{1}{2}) \pm \left[ \Psi_{5}^2 k_z^2 + \Psi_4 \frac{2eB}{\hbar} (n + \frac{1}{2}) \right]^{1/2}$$

(83)

Therefore, $k_z^2 = A_{0,2}(E,n)$

(84)

where $A_{0,2}(E,n) = \left(2\Psi_{4}^2\right)^{-1} \left[ \Psi_5(E,n) \pm \left[ \Psi_5^2(E,n) - 4\Psi_2^2 \Psi_6(E,n) \right]^{1/2} \right],$

$$\Psi_5(E,n) = \left[ 2\Psi_1 \left[ E - \Psi_2 \frac{2eB}{\hbar} (n + \frac{1}{2}) \right] + \Psi_3^2 \right],$$

$$\Psi_6(E,n) = \left[ E - \Psi_2 \frac{2eB}{\hbar} (n + \frac{1}{2}) \right] - \Psi_4 \frac{2eB}{\hbar} (n + \frac{1}{2})$$

The expression for EMME can be written as

$$m^*_{(E_{FB},n)} = \frac{\hbar^2}{2} A'_{0,2}(E_{FB},n)$$

(85)

The presence of the term $\Psi_3$ in (85) makes the mass both quantum number and the Fermi energy dependent in this case.

The electron concentration can be expressed as

$$n_0 = \left(\frac{eB_{G}}{2\pi^2 \hbar}\right) \sum_{n=0}^{\infty} \left[ Y_{45}(E_{FB},n) + Z_{45}(E_{FB},n) \right]$$

(86)

where, \(Y_{45}(E_{FB},n) = \left[ \sqrt{A_{0,2}^+, \left( E_{FB},n \right) + \sqrt{A_{0,2}^-, \left( E_{FB},n \right) } \right] \)

and $Z_{45}(E_{FB},n) = \sum_{i=1}^{\infty} \left[ L_i(\mathbf{r}) \left[ Y_{45}(E_{FB},n) \right] \right]$

1.2.8 The EMME From N-Gallium Phosphide Under Magnetic Quantization

The energy spectrum of the conduction electrons in n-GaP can be written as [122]

$$E = \frac{\hbar^2 k_z^2}{2m_i} + \frac{\hbar^2}{2m_i} \left[ \frac{A' k_z^2 + k_z^2}{2m_i^2} \right] - \left[ \frac{\hbar^2 k_0^2}{2m_i^2} \left( k_0^2 + k_z^2 \right) + |V_G| \right]^{1/2} + |V_G|$$

(87)

where $K_0$, and $|V_G|$ are constants of the energy spectrum and $A' = 1$.

The magneto electron energy spectrum can be written following (87) as

$$E = a_0 \frac{2eB}{\hbar} \left( n + \frac{1}{2} \right) + b_0 k_z^2 - \left[ C \frac{2eB}{\hbar} \left( n + \frac{1}{2} \right) + |V_G| \right]^{1/2} + |V_G|$$

(88)
where \( \alpha_0 = \frac{\hbar^2}{2m_e}, \frac{\Delta \hbar^2}{2m_e}, b_0 = \frac{\hbar^2}{2m_0}, C = \frac{\hbar^4 k_0^2}{(m_1)^2} \).

Therefore \( k^2 = A_{22,1}(E,n) \) (89)

\[ m^*(E_{FB}, n) = \frac{\hbar^2}{2} A'_{22,1}(E_{FB}, n) \] (90)

The presence of the term \( |V_0| \) in (90) makes the mass both quantum number and the Fermi energy dependent in this case.

The electron concentration can be expressed as

\[ n_0 = \frac{(eB g_e)^{\frac{2m}{4}}}{2 \pi^2 \hbar} \sum_{n=0}^{2m} \left[ Y_{46}(E_{FB}, n) + Z_{46}(E_{FB}, n) \right] \] (91)

where \( y_{46}(E_{FB}, n) = \sqrt{A_{22,1}(E_{FB}, n)} + \sqrt{A_{52,2}(E_{FB}, n)} \)

and, \( y_{46}(E_{FB}, n) = \sum_{r=1}^{N} I_{B}(r)Y_{46}(E_{FB}, n) \)

1.2.9 The EMME From Platinum Antimonide Under Magnetic Quantization

The dispersion relation for the n-type PtSb can be written as [123]

\[
\begin{bmatrix}
E + \lambda_0 \left( \frac{a^2}{4} \right) - lk_x^2 \left( \frac{a^2}{4} \right) \\
E + \delta_0 - \nu \left( \frac{a^2}{4} \right) - n \left( \frac{a^2}{4} \right) k_x^2
\end{bmatrix}
= I \left( \frac{a^4}{16} \right)
\]

(92)

where \( \omega_1 = \lambda_0 \left( \frac{a^2}{4} \right), \omega_2 = \lambda_0 \left( \frac{a^2}{4} \right), \omega_3 = \nu \left( \frac{a^2}{4} \right), \omega_4 = \nu \left( \frac{a^2}{4} \right), \omega_5 = \nu \left( \frac{a^2}{4} \right), \omega_6 = \nu \left( \frac{a^2}{4} \right), \omega_7 = \nu \left( \frac{a^2}{4} \right). \)

\( L = \left( \frac{a^2}{4} \right), \lambda_0, I, \delta_0, \nu \) and \( n \) are the band constants and \( a \) is the lattice constant.
The magneto dispersion relation can be written following (92) as

\[
E + \frac{\bar{\alpha}(\bar{a})^2 eB}{2\hbar} (n+\frac{1}{2}) + \frac{\bar{\alpha}(\bar{a})^2}{4} k_z^2 - \frac{\bar{l}(\bar{a})^2 eB}{2\hbar} (n+\frac{1}{2}) \left[ E + \bar{\alpha}_0 - \frac{\bar{v}(\bar{a})^2 eB}{2\hbar} (n+\frac{1}{2}) \right] \]

Therefore, \( K_x^2 = A_{55,\pm} (E, n) \) (93)

where \( A_{55} (E, n) = (2\Psi_{17})^{-1} \left[ -\Psi_{18}(E, n) + \left( \Psi_{18}^2(E, n) - 4\Psi_{17,19}(E, n) \right)^{1/2} \right] \),

\[
\Psi_{17} = \left[ \frac{\bar{l}(\bar{a})^4}{16} + \frac{\bar{\alpha}_0 \bar{v}(\bar{a})^4}{16} \right],
\]

\[
\Psi_{18}(E, n) = \left[ \frac{\bar{l}(\bar{a})^4 eB}{4\hbar} (n+\frac{1}{2}) + \Psi_{15}(E, n) \frac{\bar{v}(\bar{a})^2}{4} - \Psi_{16}(E, n) \bar{\alpha}_0 (\bar{a})^2 \right],
\]

\[
\Psi_{20} = \frac{\bar{v}(\bar{a})^2}{4}, \quad \Psi_{21}(E, n) = [\Psi_{16}(E, n) + \Psi_{15}(E, n)]
\]

\[
\Psi_{19}(E, n) = (\Psi_{13}(E, n) \Psi_{16}(E, n) - \frac{I(\bar{a})^4 e^2B^2(n+\frac{1}{2})^2}{4\hbar^2})
\]

\[
\Psi_{15}(E, n) = \left[ E + \frac{\bar{\alpha}_0 (\bar{a})^2 eB}{2\hbar} (n+\frac{1}{2}) - \frac{\bar{l}(\bar{a})^2 eB}{2\hbar} (n+\frac{1}{2}) \right]
\]

and \( \Psi_{16}(E, n) = \left[ E + \bar{\alpha}_0 - \frac{\bar{v}(\bar{a})^2 eB}{2\hbar} (n+\frac{1}{2}) \right] \)

The EMME for this case can be written as

\[
m^*(E_{FB}, n) = \frac{\hbar^2}{2} A_{55,\pm}'(E_{FB}, n) \quad (95)
\]

The electron concentration can be expressed as

\[
n_0 = \frac{(eB_{crx})}{2\pi^2 \hbar} \sum_{n=0}^{\infty} \left[ Y_{47}(E_{FB}, n) + Z_{47}(E_{FB}, n) \right] \quad (96)
\]

where \( Y_{47}(E_{FB}, n) = \sqrt{A_{55,+,}^2(E_{FB}, n) + A_{55,-}^2(E_{FB}, n)} \)

and \( Z_{47}(E_{FB}, n) = \sum_{r=1}^{N_l} L_r (r) [Y_{47}(E_{FB}, n)] \)
1.2.10 The EMME From Bismuth Telluride Under Magnetic Quantization

The dispersion relation of the conduction electron in Bi$_2$Te$_3$ can be written as [124]

\[ E(1 + \alpha E) = \tilde{\omega}_1 k_x^2 + \tilde{\omega}_2 k_y^2 + \tilde{\omega}_3 k_z^2 + 2\tilde{\omega}_4 k_x k_y \]  

(97)

where \( \tilde{\omega}_1 = \frac{\hbar^2}{2m_0} \alpha_{11}, \tilde{\omega}_2 = \frac{\hbar^2}{2m_0} \alpha_{22}, \tilde{\omega}_3 = \frac{\hbar^2}{2m_0} \alpha_{33} \) in which \( \alpha_{11}, \alpha_{22}, \alpha_{33} \) and \( \alpha_{23} \) are system constants.

In the presence of a quantizing magnetic field \( \vec{B} \) along \( k_x \) direction, the magneto dispersion relation of the carriers in Bi$_2$Te$_3$ can be written following (97) as

\[ E(1 + \alpha E) = \tilde{\omega}_1 k_x^2 + \hbar \omega_3 \left( n + \frac{1}{2} \right) \]  

(98)

where,

\[ \omega_3 = \frac{eB}{M_3}, M_3 = \frac{m_0}{\sqrt{\alpha_{22} \alpha_{23} - \frac{\alpha_{23}^2}{4}}} \]

Therefore

\[ k_x^2 = \frac{E(1 + \alpha E) - \hbar \omega_3 \left( n + \frac{1}{2} \right)}{\tilde{\omega}_1} \]  

(99)

The EMME can be expressed as

\[ m^* (E_{FB}) = \frac{\hbar^2}{2} \left[ \frac{1 + 2\alpha E_{FB}}{\omega_h} \right] \]  

(100)

The electron concentration can be expressed as

\[ n_0 = \left( \frac{eB_{sat}}{\pi^2 \hbar} \right) \sum_{n=0}^{\infty} \left[ Y_{48}(E_{FB}, n) + Z_{48}(E_{FB}, n) \right] \]  

(101)

where

\[ Y_{48}(E_{FB}, n) = \left\{ \frac{1}{\omega_h} \left[ E_{FB} (1 + \alpha E_{FB}) - \left( n + \frac{1}{2} \right) \hbar \omega_3 \right] \right\}^{1/2} \]

and

\[ Z_{48}(E_{FB}, n) = \sum_{r=1}^{b} Z_b(r) \left[ Y_{48}(E_{FB}, n) \right] \]

1.2.11 The EMME From Germanium Under Magnetic Quantization

(a) The model of Cardona et al

It is well known that the conduction electrons of n-Ge obey two different types of dispersion laws since band non-parabolicity has been included in two different ways as given in the literature.

a) The energy spectrum of the conduction electrons in bulk specimens of n-Ge can be
expressed in accordance with Cardona et. al. as [125]

\[ E = \frac{-E_{80}}{2} + \frac{\hbar^2 k_x^2}{2m^*_y} + \left[ \frac{E^2_{80}}{4} + E_{80} k_x^2 \left( \frac{\hbar^2}{2m^*_y} \right)^2 \right]^{1/2} \]

(102)

where in this case \( m_0^* \) and \( m^*_y \) are the longitudinal and transverse effective masses along <111> direction at the edge of the conduction band respectively.

The dispersion relation of the conduction electrons in \( n-Ge \) in accordance with the model of Cardona et al in presence of quantizing magnetic field \( B \) along z-direction can be written as

\[ E(1+\alpha E) = \hbar \omega_{\pm} \left( n + \frac{1}{2} \right) + \frac{\hbar^2 k^2}{2m^*_y} + 2\alpha E \left( \frac{\hbar^2 k^2}{2m^*_y} \right) - \alpha \left( \frac{\hbar^2 k^2}{2m^*_y} \right)^2 \]

(103)

where, \( \omega_{\pm} = \frac{eB}{m^*_y} \), \( m^*_y \), and \( m^*_z \) are the longitudinal and transverse effective masses along <111> direction at the edge of the conduction band respectively.

Therefore, \( k^2 = \frac{2m^*_y}{\hbar^2} A_{E_0} (E, n) \)

(104)

where, \( A_{E_0} (E, n) = (2\alpha)^{-1} \left[ 1 + 2\alpha E - \left[ 1 + 4\alpha \left( n + \frac{1}{2} \right) \hbar \omega_{\pm} \right]^{1/2} \right] \)

(105)

The EMME can be written as

\[ m^*(E_{FB}, n) = m^*_y A_{E_0} (E_{FB}, n) \]

(106)

The electron concentration can be expressed as

\[ n_0 = \left( \frac{eB_s}{\pi^2 \hbar} \right)^{3/2} \sum_{m=0}^{m_{max}} \left[ Y_{E_0} (E_{FB}, n) + Z_{E_0} (E_{FB}, n) \right] \]

(107)

where, \( Y_{E_0} (E_{FB}, n) = \frac{\sqrt{2m^*_y}}{\hbar} \left[ \sqrt{A_{E_0} (E_{FB}, n)} \right] \) and \( Z_{E_0} (E_{FB}, n) = \sum_{r=1}^{r_{max}} I_{E_0} (r) \left[ Y_{E_0} (E_{FB}, n) \right] \)

(b) The model of Wang and Ressler

The dispersion relation of the conduction electron in bulk specimens of n-Ge can be expressed in accordance with the model of Wang and Ressler [126] can be written as

\[ E = \frac{\hbar^2 k_x^2}{2m^*_y} + \frac{\hbar^2 k_x^2}{2m^*_z} - \tilde{c}_1 \left( \frac{\hbar^2 k_x^2}{2m^*_y} \right)^2 - \tilde{d}_1 \left( \frac{\hbar^2 k_x^2}{2m^*_z} \right)^2 - \tilde{e}_1 \left( \frac{\hbar^2 k_x^2}{2m^*_y} \right)^2 \]

(108)

where \( \tilde{c}_1 = \tilde{c} \left( 2m^*_y / \hbar^2 \right)^2, \tilde{c} = 1.4A, \tilde{A} = \frac{1}{4} \left( \frac{\hbar^4}{E_{FB} m^*_y} \right) \left( 1 - \frac{m^*_y}{m_0^*} \right), \tilde{d}_1 = \tilde{d} \left( \frac{4m^*_y m^*_z}{\hbar^4} \right), \tilde{d} = 0.8A, \tilde{e}_1 = \tilde{e}_0 \left( 2m^*_y / \hbar^2 \right)^2 \) and \( \tilde{e}_0 = 0.005\tilde{A} \).
The magneto dispersion law in $n-$Ge in accordance with the model of Wang and Ressler can be written as

$$k^2 = \frac{2m^*}{\hbar^2} [\mathcal{A}_n(E,n)]$$  \hspace{1cm} (109)$$

where

$$\mathcal{A}_n(E,n) = \left[ \Psi_{24}(n) - \frac{1}{2e_i} \left( \Psi_{25}(n) - 4e_iE \right) \right]^{1/2}, \Psi_{24}(n) = (2\pi_i)^{-1} \left[ 1 - d_1 \left( n + \frac{1}{2} \right) \hbar \omega \right]$$

and

$$\Psi_{25}(n) = \left\{ \left( 1 - d_1(n + \frac{1}{2}) \hbar \omega \right) \right\}^2 + e_i \left( \left( n + \frac{1}{2} \right) \hbar \omega - c_i \right) \left( \left( n + \frac{1}{2} \right) \hbar \omega \right)^2$$

The EMME is given by

$$m^0(E_{FB},n) = m^0 \mathcal{A}_n(E_{FB},n)$$  \hspace{1cm} (110)$$

The electron concentration can be expressed as

$$n_0 = \left( \frac{eB_{\tau}}{\pi^2 \hbar} \right) \sum_{n=0}^{\infty} \left[ Y_{50}(E_{FB},n) + Z_{50}(E_{FB},n) \right]$$  \hspace{1cm} (111)$$

where $Y_{50}(E_{FB},n) = \sum_{n=0}^{\infty} L_{\tau}(r) [Y_{50}(E_{FB},n)]$ and $Z_{50}(E_{FB},n) = \sum_{n=0}^{\infty} L_{\tau}(r) [Z_{50}(E_{FB},n)]$

1.2.12 The EMME From Gallium Antimonide Under Magnetic Quantization

The dispersion relation of the conduction electrons in n-GaSb can be written as \cite{127}

$$E = \frac{\hbar^2 k^2}{2m_0} - \frac{E_{g0}}{2} + \frac{E_{g0}'}{2} \left[ 1 + \frac{2\hbar^2 k^2}{E_{g0}'} \left( \frac{1}{m_c} - \frac{1}{m_0} \right) \right]^{1/2}$$  \hspace{1cm} (112)$$

where $E_{g0}' = \left[ E_{g0} + \frac{5 \times 10^{-5} T^2}{2(112+T)} \right] \text{ eV}$

The (112) can be expressed as

$$\frac{\hbar^2 k^2}{2m_c} = I_{16}(E)$$  \hspace{1cm} (113)$$

where

$$I_{16}(E) = \left[ E + E_{g0}' - (m_c / m_0)(E_{g0}' / 2) - [(E_{g0}' / 2)^2 + [(E_{g0}')^2 / 2(1 - (m_c / m_0))] + [(E_{g0}')^2 / 2(1 - (m_c / m_0))] \right]^{1/2}$$

The magneto dispersion relation is given by

$$k^2 = \frac{2m_c}{\hbar^2} \left[ I_{16}(E) - (n + \frac{1}{2}) \hbar \omega \right]$$  \hspace{1cm} (114)$$
(114) can be written as
\[ k_x^2 = \frac{2m_e}{\hbar^2} [A_n(E, n)] \] (115)

where \[ [A_n(E_{FB}, n)] = \left[ I_{36}(E) - \left( n + \frac{1}{2} \right) \hbar \omega_e \right] \]

The EMME can be expressed as
\[ m'(E_{FB}) = m_e I_{16}(E_{FB}) \] (116)

The electron concentration can be expressed as
\[ n_0 = \left( \frac{eB_g}{\pi^2 \hbar} \right) \sum_{n=0}^{\infty} [Y_{501}(E_{FB}, n) + Z_{501}(E_{FB}, n)] \] (117)

where \[ Y_{501}(E_{FB}, n) = \sqrt{\frac{2m_e}{\hbar}} \left[ \sqrt{A_n(E_{FB}, n)} \right] \] and \[ Z_{501}(E_{FB}, n) = \sum_{n=1}^{\infty} L_n(r) [Y_{501}(E_{FB}, n)] \]

1.2.13 The EMME From II-V Materials Under Magnetic Quantization

The dispersion relation of the holes in this case is given by [128]
\[ E = \theta(k_x^2 + \theta_1 k_y^2 + \theta_2 k_z^2 + \delta u_k + \theta_4 k_z^2 + \theta_5 k_z^2 + \delta_3 k_z^2 + G_3 k_y^2 + \Delta_3^2) \pm \Delta_3 \] (118 a)

where, \( k_x, k_y, \) and \( k_z \) are expressed in the units of \( 10^{10} \) m\(^{-1}\),
\[ \theta_1 = \frac{1}{2} (a_1 + b_1), \theta_2 = \frac{1}{2} (a_2 + b_2), \theta_3 = \frac{1}{2} (a_3 + b_3), \delta = \frac{1}{2} (A + B), \]
\[ \theta_4 = \frac{1}{2} (a_4 - b_4), \theta_5 = \frac{1}{2} (a_5 - b_5), \delta_3 = \frac{1}{2} (A - B), \]
\[ a_i (i = 1, 2, 3, 4), b_i, A, B, G_3 \text{ and } \Delta_3 \text{ are system constants} \]

The magneto dispersion law in II-V materials in the presence of a magnetic field \( B \) along \( k_y \) direction can be written as
\[ k_y^2 = A_{35,\pm}(E, n) \] (118 b)

where \[ A_{35,\pm}(E, n) = \left[ I_{35} E + I_{36,\pm}(n) \pm \sqrt{E^2 + EI_{38,\pm}(n) + I_{39,\pm}(n)} \right], I_{35} = \frac{\theta_2}{(\theta_3^2 - \theta_5^2)} \]
\[ I_{36,\pm}(n) = \frac{I_{33,\pm}(n)}{2(\theta_3^2 - \theta_5^2)}, I_{38,\pm}(n) = (4\theta_3^2)^{-1} \left[ 4\theta_3^2 I_{33,\pm}(n) + 8\theta_3^2 I_{31,\pm}(n) - \theta_3^2 I_{31,\pm}(n) \right], \]
\[ I_{39,\pm}(n) = (4\theta_3^2)^{-1} \left[ I_{33,\pm}(n) + 4\theta_3^2 I_{34,\pm}(n) - 4\theta_3^2 I_{34,\pm}(n) \right], \]
\[ I_{33,\pm}(n) = \left[ G_3^2 + 2\theta_3 I_{32}(n) - 2\theta_3 I_{31,\pm}(n) \right]. \]
\[ I_{34,\pm}(n) = \left[ I_{32}^2(n) + \Delta_3^2 - I_{31,\pm}(n) \right], \quad I_{31,\pm}(n) = \left[ \left( n + \frac{1}{2} \right) \hbar \omega_{31} \frac{\delta_4^2 \Delta_3}{4\Theta} \right] \]

\[ M_{32} = \frac{\hbar^2}{2\Theta}, \quad M_{33} = \frac{\hbar^2}{2\Theta_3}, \quad \text{and} \quad M_{34} = \frac{\hbar^2}{2\Theta} \]

The EMME is given by

\[ m^*(E_{FB}, n) = \frac{\hbar^2}{2} A_{35,\pm}^e(E_{FB}, n) \]  

(119)

The electron concentration can be expressed as

\[ n_0 = \left( \frac{eB_{EF,n}}{2\pi^2\hbar} \right) \sum_{n=0}^{\infty} \left[ Y_{31}(E_{FB}, n) + Z_{31}(E_{FB}, n) \right] \]

(120)

where \( Y_{31}(E_{FB}, n) = \left[ \sqrt{A_{35,\pm}(E_{FB}, n)} + \sqrt{A_{35,-}(E_{FB}, n)} \right] \) and

\[ Z_{31}(E_{FB}, n) = \sum_{r=1}^{5} L_{r}(r) \left[ Y_{31}(E_{FB}, n) \right] \]

1.3 Results and Discussions:

Using the appropriate equations together with the energy band constants as given in Table-1, we have plotted the EMME in \( n-Cd_3As_2 \) and \( CdGeAs_2 \) as functions of inverse magnetic field for the first two magnetic sub-bands in Figs. 1.1 and 1.2 respectively. For the purpose of self-assessment, in the same figures, we have also plotted the effect of absence of the crystal field splitting together with the equivalent simplified three and the two band models of Kane.

From these figs., it appears that the EMME is an oscillatory function of the inverse quantizing magnetic field. The magnetic field has been tilted to an angle of 45° to the \( k_x \) direction in both the figures. The oscillatory dependence is due to the crossing over of the Fermi level by the Landau sub-bands in steps resulting in successive reduction of the number of occupied Landau levels as the magnetic field is increased. For each coincidence of a Landau level, with the Fermi level, there would be a discontinuity in the density-of-states function resulting in a peak of oscillation. Thus the peaks should occur whenever the Fermi energy is a multiple of energy separation between the two consecutive Landau levels and it may be noted that the origin of oscillations in the EMME is the same as that of the Subhnikov-de Hass oscillations. With increase in magnetic field, the amplitude of the oscillation increases and, ultimately, at very large
values of the magnetic field, the conditions for the quantum limit is reached when the EMME is found to decrease monotonically with increase in magnetic field. Further, in this case we see that the EMME is a strong function of the sub-band quantum number \( n \). For this reason, we have also plotted the EMME for the next higher sub-band \( n=1 \). It thus appears that the increasing the index decreases the EMME for lower values of the field. However at higher field, the difference between them diminishes and all the respective band models tend to coincide with each other which stand out to be a remarkable mathematical simplicity in deriving the analytical expressions of the EMME.

Table 1

The Numerical Values of the Energy Band Constants of Few Materials [129]

<table>
<thead>
<tr>
<th>Materials</th>
<th>Numerical Values of the Energy Band Constants</th>
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</table>
| a) The conduction electrons of n-Cadmium Germanium Arsenide can be described by three types of band models | 1. The values of the energy band constants in accordance with the generalized electron dispersion relation of nonlinear optical materials are as follows

\[
E_{t_0} = 0.57\text{eV}, \Delta_0 = 0.30\text{eV}, \Delta_\perp = 0.36\text{eV}, m_0^* = 0.034m_0, \\
m_\perp = 0.039m_0, T = 4K, \delta = -0.21eV, g_v = 1
\]

2. In accordance with the three band model of Kane the spectrum constants are given by

\[
\Delta = \frac{\Delta_0 + \Delta_\perp}{2} = 0.33\text{eV}, E_{t_0} = 0.57\text{eV}, m_\perp = \frac{(m_0^* + m_\perp^*)}{2} = 0.0365m_0, \text{ and } \delta = 0\text{eV}.
\]

3. In accordance with two band model of Kane, the spectrum constants are given by

\[
E_{t_0} = 0.57\text{eV} \text{ and } m_\perp = \left(\frac{m_0^* + m_\perp^*}{2}\right) = 0.0365m_0.
\]

b) The conduction electrons of n-Cadmium Arsenide can be described by three types of band models | 1. The values of the energy band constants in accordance with the generalized electron dispersion relation of nonlinear optical materials are as follows

\[
|E_{t_0}| = 0.095eV, \Delta_0 = 0.27eV, \Delta_\perp = 0.25eV, m_0^* = 0.00697m_0, \\
m_\perp = 0.013933m_0, T = 4K, \delta = 0.085eV, g_v = 1
\]

2. In accordance with the three band model of Kane, the spectrum constants are given by

\[
\Delta = \frac{\Delta_0 + \Delta_\perp}{2} = 0.26eV, |E_{t_0}| = 0.095eV, m_\perp = \left(\frac{m_0^* + m_\perp^*}{2}\right) = 0.0105m_0 \text{ and } \delta = 0\text{eV}.
\]
3. In accordance with two band model of Kane, the spectrum constants are given by 
\[ |E_{\text{so}}| = 0.095 \text{eV}, \] 
and 
\[ m_t = \left( m_t^s + m_t^s \right) / 2 = 0.0105 m_0. \]

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<tbody>
<tr>
<td>2</td>
<td>n-Indium Arsenide</td>
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</tbody>
</table>
| 3 | n-Gallium Arsenide | The values \( E_{\text{so}} = 1.55 \text{eV}, \Delta = 0.35 \text{eV} \), \( m_t = 0.066 m_0 \), \( g_v = 1 \) are valid for three band model of Kane. The values 
\[ a_{13} = -1.97 \times 10^{-37} \text{eV} \text{m}^4 \] and 
\[ a_{15} = -2.3 \times 10^{-34} \text{eV} \text{m}^4 \] are valid for the Newson and Kurobe model |
| 4 | n-Gallium Aluminium Arsenide | \( E_{\text{so}} = \left( 1.424 + 1.266 x + 0.26 x^2 \right) \text{eV} \), \( \Delta = \left( 0.34 - 0.5 x \right) \text{eV} \), \( m_t = \left( 0.066 + 0.088 x \right) m_0 \), \( g_v = 1 \) |
| 5 | n-Mercury Cadmium Telluride | \( E_{\text{so}} = \left( -0.302 + 1.93 x + 5.35 \times 10^{-14} \left( 1 - 2 x \right) T - 0.810 x^2 + 0.832 x^3 \right) \text{eV} \), \( \Delta = \left( 0.63 + 0.24 x - 0.27 x^2 \right) \text{eV} \), \( m_t = 0.1 m_0 \), \( m_{\text{so}} \), \( g_v = 1 \) |
| 6 | n-Indium Gallium Arsenide Phosphide lattice matched to Indium Phosphide | \( E_{\text{so}} = \left( 1.337 - 0.73 y + 0.13 y^2 \right) \text{eV} \), \( \Delta = \left( 0.114 + 0.26 y - 0.02 y^2 \right) \text{eV} \), 
\[ y = \frac{(0.1896 - 0.4052 x) / (0.1896 - 0.0123 x), m_t = (0.08 - 0.039 y) m_0, g_v = 1 \] |
| 7 | n-Indium Antimonide | \( E_{\text{so}} = 0.2352 \text{eV}, \Delta = 0.81 \text{eV} \), \( m_t = 0.0135 m_0 \), \( g_v = 1 \) |
| 8 | n-Gallium Antimonide | The values of \( E_{\text{so}} = 0.81 \text{eV}, \Delta = 0.80 \text{eV}, P = 9.48 \times 10^{-10} \text{eV} \text{m}, \) 
\( \bar{\gamma}_0 = -2.1, \bar{\nu}_0 = -1.49, \bar{\delta}_0 = 0.42, g_v = 1 \) |
| 9 | n-Cadmium Sulphide | \( m_{t}^* = 0.7 m_0, m_{l}^* = 1.5 m_0, C_0 = 1.4 \times 10^{-8} \text{eV} \text{m}, g_v = 1 \) |
| 10 | n-Lead Telluride | The values \( m_t = 0.070 m_0, m_2 = 0.54 m_0, m_4 = 0.010 m_0, m_0^* = 1.4 m_0 \), 
\( P_{ij} = 141 \text{meV} \text{nm}, P_{ij} = 486 \text{meV} \text{nm}, E_{\text{so}} = 190 \text{meV}, g_v = 4 \) are valid for the Dimmock model. 
The values \( m_t = 0.0239 m_0, m_2 = 0.024 m_0, m_4 = 0.31 m_0, m_5 = 0.24 m_0 \) are valid for the Cohen model. |
<table>
<thead>
<tr>
<th>Page</th>
<th>Material</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Stressed n-Indium Antimonide</td>
<td>$m_c = 0.01359 m_0, E_{t_0} = 0.081 eV, B_2 = 9 \times 10^{-6} eV m$,</td>
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<td></td>
<td></td>
<td>$C_1 = 3 eV, C_2 = 2 eV, \bar{a}_0 = -10 eV, \bar{b}_0 = -1.7 eV, \bar{d}_0 = -4.4 eV$,</td>
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<tr>
<td></td>
<td></td>
<td>$S_{xx} = 0.6 \times 10^{-3} (kbar)^{-1}, S_{yy} = 0.42 \times 10^{-3} (kbar)^{-1}$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$S_{zz} = 0.39 \times 10^{-3} (kbar)^{-1}, S_{xy} = 0.5 \times 10^{-3} (kbar)^{-1}$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma = \sigma_{xx}, \sigma_{yy} = \sigma_{yy}$,</td>
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<tr>
<td></td>
<td></td>
<td>$\sigma_{zz} = \sigma_{zz}, \sigma$ is the stress in kilobar, $g_v = 1$ are valid for</td>
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<tr>
<td></td>
<td></td>
<td>the model of Seiler et. al.</td>
</tr>
<tr>
<td>12</td>
<td>Bismuth</td>
<td>$E_{t_0} = 0.0153 eV, m_1 = 0.00194 m_0, m_2 = 0.313 m_0, m_3 = 0.00246 m_0$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$m'_2 = 0.36 m_0, g_v = 3, M_2 = 1.25 m_0, M'_2 = 0.36 m_0$</td>
</tr>
<tr>
<td>13</td>
<td>Mercury Telluride</td>
<td>$m'_1 = 0.028 m_0, g_v = 1$</td>
</tr>
<tr>
<td>14</td>
<td>Platinum Antimonide</td>
<td>For valence bands, along &lt;100&gt; direction, $\bar{\sigma}_0 = (0.02/4) eV$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{\sigma} = (-0.32/4) eV, \bar{\nu}_0 = (0.39/4) eV$,</td>
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<tr>
<td></td>
<td></td>
<td>$\bar{\nu} = (-0.65/4) eV$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{a} = 0.643 a m, I = 0.30 (eV)^2, \bar{\sigma}_0 = 0.02 eV, g_v = 6$ For conduction</td>
</tr>
<tr>
<td></td>
<td></td>
<td>bands, along &lt;111&gt; direction, $g_v = 8, \bar{\sigma}_0 = (0.33/4) eV$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{\sigma} = (1.09/4) eV, \bar{\nu} = (0.17/4) eV$ and $\bar{\nu} = (0.22/4) eV$</td>
</tr>
<tr>
<td>15</td>
<td>n-Gallium Phosphide</td>
<td>$m'_1 = 0.92 m_0, m'_1 = 0.25 m_0, k_0 = 1.7 \times 10^{15} m^{-1},</td>
</tr>
<tr>
<td>16</td>
<td>Germanium</td>
<td>$E_{t_0} = 0.785 eV, m'_1 = 1.57 m_0, m'_3 = 0.0807 m_0, g_v = 4$</td>
</tr>
<tr>
<td>17</td>
<td>Tellurium</td>
<td>The values $\psi_1 = 6.7 \times 10^{-16} meV m^2, \psi_2 = 4.2 \times 10^{-16} meV m^2$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\psi_3 = 9.6 \times 10^{-8} meV m$ and $\psi_4 = \left(3.6 \times 10^{-8} meV m\right)$ are valid for the</td>
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<tr>
<td></td>
<td></td>
<td>model of Bouat et. al.</td>
</tr>
<tr>
<td>18</td>
<td>Bismuth Telluride</td>
<td>The values $E_{t_0} = 0.145 eV, \bar{a}<em>{11} = 4.9, \bar{a}</em>{22} = 5.92, \bar{a}<em>{33} = 9.5, \bar{a}</em>{12} = 4.22,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{a}<em>{23} = 4.6, \bar{a}</em>{31} = 4.6, g_v = 6$</td>
</tr>
<tr>
<td>19</td>
<td>Zinc Selenide</td>
<td>$m_{c2} = 0.16 m_0, \Delta_2 = 0.42 eV, E_{t_{22}} = 2.82 eV$</td>
</tr>
<tr>
<td>20</td>
<td>Lead Selenide</td>
<td>$m_{c2} = 0.23 m_0, m'_2 = 0.32 m_0, m'_3 = 0.115 m_0, m'_1 = 0.303 m_0$,</td>
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<td>$P_{</td>
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</table>
We also see that the effect of the crystal field considering a particular sub-band is insignificant. The presence and absence of the crystal field in case of Cd$_3$As$_2$ in both the sub-bands are indistinguishable. However, the presence of the isotropic spin orbit splitting constant in the three band model of Kane changes the value of the EMME as compared with the corresponding two band model. Figs. 1.3 and 1.4 exhibit the dependency of the EMME on the carrier degeneracy in both the aforementioned materials. Oscillatory dependences is exhibited in the case of the equivalent three and the two band model of Kane, the deviation among which, for both the materials, is almost zero. Further, we also see that there is almost no significant change in the variation of the sub-band index from $n=1$ to $n=2$ in both the cases. An exponential rise in the EMME can be observed beyond $10^{23}$ m$^{-3}$ for both the materials. In case of Cd$_3$As$_2$, we see that decreasing the carrier degeneracy converge the EMME from all the band models to a unique value. However, this is not arrested in case of CdGeAs$_2$. There is a crossing over of the EMME near to the concentration zone of $10^{23}$ m$^{-3}$ which overestimates the numerical result. In addition, the EMME exhibits different numerical values for both the materials, the rate of variations of which are different due to the influence of the energy band constants in accordance with all the types of the band models and follow the same trend as shown in Figs. 1.3 and 1.4. The dependency of the EMME on the angular orientation of the quantizing magnetic field has been exhibited in Figs. 1.5 and 1.6 in both n-Cd$_3$As$_2$ and n-CdGeAs$_2$. It appears that the EMME exhibits a periodic variation increasing $\theta$ from $0^\circ$ to $120^\circ$. The effect of the crystal field splitting parameter in both the cases has been exhibited for relative assessment. It appears that the influence of the crystal field parameter on the EMME for Cd$_3$As$_2$ is relatively insignificant, while there appears a cross-over regime in the EMME in CdGeAs$_2$ around $50^\circ$. In the later case, the field parameters tend to reduce the EMME beyond $50^\circ$ which exhibits the influence of $\delta$. For the three and the two band models of Kane, the EMME becomes independent of $\theta$, since the dispersion relation of the bulk materials in accordance with the said band models is spherical in constant energy wave-vector space, whereas, the generalized band model represents the ellipsoid of revolution in the same space. Figs. 1.7-1.11 exhibit the variation of the EMME on the quantizing magnetic field for n-InAs, n-GaAs, n-InSb, n-Hg$_{1-x}$Cd$_x$Te and In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP in accordance with the three and two band models of Kane respectively. The variations of the EMME are periodic and independent of the sub-band index number with the quantizing magnetic field and the
Fig. 1.1: Plot of the sub-band index dependent effective electron mass as function of inverse magnetic field for $n$-Cd$_3$As$_2$ both in the presence and absence of crystal field-splitting parameter. The magnetic field has been oriented at $45^\circ$ to the $k_z$ axis. The equivalent three and two band models of Kane in the absence of crystal field splitting parameter has also been exhibited.

Fig. 1.2: Plot of the sub-band index dependent effective electron mass as function of inverse magnetic field for $n$-CdGeAs$_2$ in the presence of crystal field-splitting parameter. The magnetic field has been oriented at $45^\circ$ to the $k_z$ axis. The equivalent three and two band models of Kane have also been exhibited.
Fig. 1.3: Plot of the sub-band index dependent effective electron mass as function of carrier degeneracy for $n$-$\text{Cd}_3\text{As}_2$ considering in the presence of crystal field-splitting parameter. The magnetic field has been oriented at $45^\circ$ to the $k_z$ axis. The equivalent three and two band models of Kane have also been exhibited.

Fig. 1.4: Plot of the sub-band index dependent effective electron mass as function of carrier degeneracy for $n$-$\text{CdGeAs}_2$ considering in the presence of crystal field-splitting parameter. The magnetic field has been oriented at $45^\circ$ to the $k_z$ axis. The equivalent three and two band models of Kane have also been exhibited.
Fig. 1.5: Plot of the lowest sub-band index dependent effective electron mass as function of angular dependency of the magnetic field in n-Cd$_3$As$_2$ considering the presence and absence of the crystal field-splitting parameter.

Fig. 1.6: Plot of the lowest sub-band index dependent effective electron mass as function of angular dependency of the magnetic field in n-CdGeAs$_2$ considering the presence and absence of the crystal field-splitting parameter.
Fig. 1.7: Plot of the effective electron mass as function of inverse magnetic field for n-InAs considering the three and two band models of Kane.

Fig. 1.8: Plot of the effective electron mass as function of inverse magnetic field for n-GaAs considering the three and two band models of Kane.
Fig. 1.9: Plot of the effective electron mass as function of inverse magnetic field for n-InSb considering the three and two band models of Kane.

Fig. 1.10: Plot of the effective electron mass as function of inverse magnetic field for n-Hg$_{0.3}$Cd$_{0.7}$Te considering the three and two band models of Kane.
Fig. 1.11: Plot of the effective electron mass as function of inverse magnetic field for $n$-In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP considering the three and two band models of Kane.

Fig. 1.12: Plot of the effective electron mass as function of carrier degeneracy for $n$-InAs considering the three and two band models of Kane.
Fig. 1.13: Plot of the effective electron mass as function of carrier degeneracy for n-GaAs considering the three and two band models of Kane.

Fig. 1.14: Plot of the effective electron mass as function of carrier degeneracy for n-InSb considering the three and two band models of Kane.
Fig. 1.15: Plot of the effective electron mass as function of carrier degeneracy for \( \text{n-Hg}_{0.3}\text{Cd}_{0.7}\text{Te} \) considering the three and two band models of Kane.

Fig. 1.16: Plot of the effective electron mass as function of carrier degeneracy for \( \text{n-In}_{1-x}\text{Ga}_{x}\text{As}_{y}\text{P}_{1-y} \) lattice matched to InP considering the three and two band models of Kane.
Fig. 1.17: Plot of the effective electron mass as function of alloy composition for n-HgxCd_{1-x}Te and n-In_{1-x}Ga_{x}As_{y}P_{1-y} lattice matched to InP considering the three and two band models of Kane at a quantizing magnetic field of 2 tesla and carrier degeneracy of 5 \times 10^{23} \text{m}^{-3}.

Fig. 1.18: Plot of the effective electron mass as function of inverse magnetic field for Bi considering the energy band models of McClure and Choi, Cohen and Lax respectively.
influence of the energy band constants on the EMME in accordance with all the band models is apparent from the said figures. The Figs. 1.12-1.16 exhibit the concentration dependence of the periodic EMME for all the respective aforementioned materials. It appears from Figs 1.12-1.16 that the periodic oscillatory numerical values of the EMME is greatest for the quaternary materials while the least for InSb for all types of variables in accordance with all types of band models of III-V, ternary and quaternary materials. In Fig. 1.17, we have plotted the variation of the EMME as function of alloy composition in HgCdTe and InGaAsP lattice matched to InP. It appears that the EMME increases with the alloy fraction in almost linearly. The result of the EMME arising due to the difference in the band structures also appears to be extremely less.

Using the appropriate equations for McClure-Choi model, the model of Cohen and the model of Lax, we have plotted the EMME for Bi as functions of inverse quantizing magnetic field and carrier degeneracy as shown in Figs 1.18 and 1.19 respectively considering the first two magnetic sub-bands for models of McClure-Choi. From Fig.

Fig. 1.19: Plot of the effective electron mass as function of carrier degeneracy for Bi considering the energy band models of McClure and Choi, Cohen and Lax respectively.
1.18, it appears that the effect of the energy band structure namely, due to the model of Cohen and the Lax on the EMME almost coincides with each other. However, the quadratic nonlinear energy dispersion relations of McClure and Choi tend to increase the EMME. It appears that the increase in the magnetic sub-band index increases the EMME in this case as compared with that of Fig. 1.20 exhibit the variation of the EMME against the quantizing magnetic field for IV-VI materials considering PbTe as an example using the dispersion relation provided by the Dimmock model at the Figs. 1.2 and 1.4 for nonlinear optical materials.

This increase of the EMME for the present case results due to the presence of the respective dominant energy spectrum parameters. As the magnetic field increases, we see that with increase in the sub-band index the EMME exhibits a sharp discontinuity and can become a negative quantity, thus questioning the validity of the McClure and Choi model in the beyond-10 tesla zone.

![Graph](image)

Fig. 1.20: Plot of the effective electron mass as function of quantizing magnetic field for PbTe and stressed InSb at the two lowest sub-bands.
Fig. 1.21: Plot of the effective electron mass as function of angular orientation of the magnetic field for stressed InSb at the two lowest sub-bands.

Fig. 1.22: Plot of the effective electron mass as function quantizing magnetic field for $\text{Bi}_2\text{Te}_3$, GaSb, Te, GaP and PtSb$_2$ at the lowest sub-band.
The variation of the EMME on the carrier degeneracy for Bi in Fig. 1.19 is rather slow over \(0.1-0.5 \times 10^{23} \text{ m}^{-3}\) zone for lowest magnetic sub-band. In the same figure we have demonstrated the variation of the EMME for stressed InSb for the first two lowest sub-bands. Large oscillations are exhibited for PbTe case as compared with that of the stressed case, where we have considered the stress to be composed of all the diagonal and off-diagonal strain components as given in Table 1. It appears that the deviation of the EMME from its ground state value is almost zero when the angular dependency is \(45^0\). To exhibit this difference, we have further plotted the EMME at the lowest two sub-bands as function of the angle of orientation of the field in Fig. 1.20. It appears that the EMME exhibits periodical variation over the entire angular range as shown in the same figure with the deviation between the sub-band values at the two minima and the mid angular zone.

Fig. 1.22 exhibits the EMME in Te, GaP, PtSb\(_2\), Bi\(_2\)Te\(_3\), GaSb and as function of quantizing magnetic field at the lowest sub-band level. The usual periodical oscillatory nature is exhibited for all the said materials with Bi\(_2\)Te\(_3\) to exhibit the highest EMME numerical values. In case of Ge, the Cardona et. al. model registers a constant EMME, although the Wang et. al. registers a sub-band index dependent model.

We wish to note that the effect of electron spin has not been considered in obtaining the oscillatory plots. The peaks in all the figures would increase in number with decrease in amplitude if spin splitting term is included in the respective numerical computations. Though, the effects of collisions are usually small at low temperatures, the sharpness of the amplitude of the oscillatory plots would somewhat be reduced by collision broadening. Nevertheless, the present analysis would remain valid since the effects of collision broadening can usually be taken into account by an effective increase in temperature. Although in a more rigorous treatment the many body effects should be considered along with the self-consistent procedure, this simplified analysis as presented exhibits the basic qualitative features of the EMME under the magnetic quantization with reasonable accuracy.
SECTION 2
Study of the EMME under Size Quantization

2.1 Introduction
With the advent of fine lithographical methods [130] molecular beam epitaxy [131], organometallic vapor-phase epitaxy [132] and other experimental techniques, the restriction of the motion of the carriers of bulk materials in one (ultrathin films, NIPI structures, inversion and accumulation layers), two (nanowires) and three (quantum dots, magnetosize quantized systems, magneto accumulation layers, magneto inversion layers, quantum dot superlattices, magneto ultrathin film superlattices and magneto NIPI structures) dimensions have in the last few years, attracted much attention not only for their potential in uncovering new phenomena in nanoscience but also for their interesting quantum device applications [133-135]. In ultrathin films (UFs), the restriction of the motion of the carriers in the direction normal to the body (say, the z direction) may be viewed as carrier confinement in an infinitely deep 1D rectangular potential well, leading to quantization [known as quantum size effect (QSE)] of the wave vector of the carrier along the direction of the potential well, allowing 2D carrier transport parallel to the surface of the body representing new physical features not exhibited in bulk materials [136]. The low-dimensional heterostructures based on various materials are widely investigated because of the enhancement of carrier mobility [137]. These properties make such structures suitable for applications in ultrathin film lasers [138], heterojunction FETs [139], high-speed digital networks [140], high-frequency microwave circuits [141], optical modulators [142], optical switching systems [143], and other devices. The constant energy 3D wave-vector space of bulk materials becomes 2D wave-vector surface in UFs due to dimensional quantization. Thus, the concept of reduction of symmetry of the wave-vector space and its consequence can unlock the physics of low dimensional structures. In theoretical background 2.2 of section 2, we study the EMME in UFs of nonlinear optical, III-V, II-VI, Bi, IV-VI, stressed, Tellurium, Gallium Phosphide, Platinum Antimonide, Bismuth Telluride, Germanium, Gallium Antimonide in 2.2.1, 2.2.2, 2.2.3, 2.2.4, 2.2.5, 2.2.6, 2.2.7, 2.2.8, 2.2.9, 2.2.10, 2.2.11 and 2.2.12 respectively. The 2.3 contains result and discussions pertinent to section 2.

2.2 Theoretical Background
2.2.1 Formulation of EMME in UFs of nonlinear optical materials
For dimensional quantization along z- direction, the dispersion relation of the 2D electrons in this case can be written following (2) as

\[ E = \frac{\hbar^2 k^2}{2m} \]

where \( E \) is the energy, \( k \) is the wave vector, \( \hbar \) is the reduced Planck constant, and \( m \) is the effective mass of the electron. This equation describes the quantization of the wave vector along the direction of the potential well, leading to a discrete set of energy levels that depend on the size of the potential well in the z direction.
\[\psi_l(E) = \psi_2(E)k_s^2 + \psi_3(E)(n_z\pi/d_z)^2\]  \hspace{1cm} (121)

where, \(n_z(=1,2,3,...)\) and \(d_z\) are the size quantum number and the nano-thickness along the z-direction respectively.

The EMME at the Fermi level in the xy-plane can be written as

\[m^*(E_F,n_z) = \hbar^2k_s\frac{\partial k_s}{\partial E}\bigg|_{E=E_F}\]  \hspace{1cm} (122)

where, \(E_F\) is the Fermi energy in the presence of size quantization as measured from the edge of the conduction band in the vertically upward direction in the absence of any quantization.

From (121) and (122), the EMME in this case can be written as

\[m^*(E_F,n_z) = \frac{\hbar^2}{2} \sum_{n_z} \left[ \psi_2(E_F)^* \left[ \psi_1(E_F)^* - \psi_2(E_F)^* \right] \left[ \frac{\partial \psi_2}{\partial E} \right]^* \right] \left[ \psi_2(E_F) - \psi_2(E_F) \right] \left[ \frac{\partial \psi_2}{\partial E} \right]^* \]  \hspace{1cm} (123)

where, the primes denote the differentiation of the differentiable functions with respect to Fermi energy. Thus, we observe that the EMME is the function of size quantum number and the Fermi energy due to the combined influence of the crystal field splitting constant and the anisotropic spin-orbit splitting constants respectively. To study the dependence of the EMME as a function of electron concentration per unit area we have to formulate the corresponding density-of-states function (DOS).

The general expression of the total 2D DOS \(N_{2DT}(E)\) in this case is given by

\[N_{2DT}(E) = \frac{2e\gamma}{(2\pi)^2} \sum_{n_z} \frac{\partial A(E,n_z)}{\partial E} H(E-E_{n_z})\]  \hspace{1cm} (124)

where \(A(E,n_z)\) is the area of the constant energy 2D wave vector space for UFs, \(H(E-E_{n_z})\) is the Heaviside step function and \(E_{n_z}\) is the corresponding sub-band energy.

Using (121) and (124), the expression of the \(N_{2DT}(E)\) for UFs of nonlinear optical materials can be written as

\[N_{2DT}(E) = \frac{2\gamma}{(2\pi)^2} \sum_{n_z} \left[ \psi_2(E)^* \left[ \psi_1(E)^* - \psi_2(E)^* \right] \left[ \frac{\partial \psi_2}{\partial E} \right]^* \right] \left[ \psi_2(E) - \psi_2(E) \right] \left[ \frac{\partial \psi_2}{\partial E} \right]^* H(E-E_{n_z})\]  \hspace{1cm} (125)

where, the sub band energies \(E_{n_z}\) in this case given by

\[\psi_1(E_{n_z}) = \psi_2(E_{n_z})(n_z\pi/d_z)^2\]  \hspace{1cm} (126)
Combining (125) with the Fermi-Dirac occupation probability factor, integrating between \( E_{F} \) to infinity and applying the generalized Somerfeld's lemma, the 2D carrier statistics in this case assumes the form

\[
n_{2D} = \frac{g_y}{2\pi} \sum_{n_z=1}^{n_{\text{max}}} \left[ T_{51}\left( E_{Fz}, n_z \right) + T_{52}\left( E_{Fz}, n_z \right) \right]
\]

where,

\[
T_{51}\left( E_{Fz}, n_z \right) = \frac{\psi_i(E_{Fz}) - \psi_j(E_{Fz}) (n_z \pi/d_z)^2}{\psi_i(E_{Fz})}
\]

and

\[
L(r) = 2(k_B T)^{2r}(1 - 2^{-2r})\zeta(2r) \frac{d^{2r}}{dE_F^{2r}}
\]

2.2.2 The EMME from UFs of III-V materials

(a) The three band model of Kane

Thus, under the conditions \( \delta = 0, \Delta_\parallel = \Delta_\perp = \Delta \) and \( m_0^* = m^*_\parallel = m_c^* \), (121) assumes the form

\[
\frac{\hbar^2 k_x^2}{2m_c} + \frac{\hbar^2}{2m_c} (n_z \pi/d_z)^2 = I_{11}(E)
\]

Using (128) and (122), the EMME in x-y plane for this case can be written as

\[
m^*\left( E_{Fz} \right) = m_c \left\{ I_{11}\left( E_{Fz} \right) \right\}
\]

It is worth noting that the EMME in this case is a function of Fermi energy alone and is independent of size quantum number.

The total 2D density-of-states function can be written as

\[
N_{2DF}(E) = \left( \frac{m_c g_y}{\pi \hbar^2} \right) \sum_{n_{\text{max}}}^{n_{\text{max}}} \left\{ \left[ I_{11}(E) \right] \ H\left( E - E_{n_2} \right) \right\}
\]

where, the sub band energies \( E_{n_2} \) can be expressed as

\[
I_{11}(E_{n_2}) = \frac{\hbar^2}{2m_c} (n_z \pi/d_z)^2
\]

The 2D carrier concentration assumes the form

\[
n_{2D} = \frac{m_c g_y}{\pi \hbar^2} \sum_{n_z=1}^{n_{\text{max}}} \left[ T_{53}\left( E_{Fz}, n_z \right) + T_{54}\left( E_{Fz}, n_z \right) \right]
\]
where \( T_{33}(E_{Fz}, n_z) \equiv \left[ I_{11}(E_{Fz}) = \frac{\hbar^2}{2m_c} \left( \frac{n \pi}{d_z} \right)^2 \right] \) and \( T_{54}(E_{Fz}, n_z) \equiv \sum_{i=1}^{i} L(r) T_{33}(E_{Fz}, n_z) \).

Under the inequalities \( \Delta \gg E_F \) or \( \Delta \ll E_F \), (128) can be expressed as

\[
E(1 + \alpha E) = \frac{\hbar^2 k_x^2}{2m_c} + \frac{\hbar^2}{2m_c} \left( \frac{n \pi}{d_z} \right)^2
\]

(133)

The EMME in this case can be written as

\[
m^*(E_{Fz}) = m_c \left( 1 + 2\alpha E_{Fz} \right)
\]

(134)

Thus, we observe that the EMME in the present case is a function of Fermi energy only due to the presence of band non-parabolicity.

The total 2D density-of-states function assumes the form

\[
N_{2D}(E) = \frac{m_c g_v}{2\pi \hbar^2} \sum_{n_z=1}^{n_{\text{max}}} (1 + 2\alpha E) H \left( E - E_{n_z} \right)
\]

(135)

where, the sub-band energy \( (E_{n_z}) \) can be expressed as

\[
\frac{\hbar^2}{2m_c} \left( \frac{n \pi}{d_z} \right)^2 = E_{n_z} \left( 1 + \alpha E_{n_z} \right)
\]

(136)

The 2D electron statistics can be written as

\[
n_{2D} = \frac{m_c g_v}{2\pi \hbar^2} \sum_{n_z=1}^{n_{\text{max}}} \frac{(1 + 2\alpha E) \, dE}{1 + \exp \left( \frac{E - E_{Fz}}{k_B T} \right)}
\]

\[
= \frac{m_c g_v}{2\pi \hbar^2} \sum_{n_z=1}^{n_{\text{max}}} \left[ (1 + 2\alpha E_{n_z}) F_0(\eta_{n_z}) + 2\alpha k_B T F_1(\eta_{n_z}) \right]
\]

(137)

where, \( \eta_{n_z} = (E_{Fz} - E_{n_z}) / k_B T \)

Under the condition \( \alpha \to 0 \), the expressions of total 2D density-of-states, for materials whose bulk electrons are defined by the isotropic parabolic energy bands can, be written as

\[
N_{2D}(E) = \frac{m_c g_v}{2\pi \hbar^2} \sum_{n_z=1}^{n_{\text{max}}} H \left( E - E_{n_z} \right)
\]

(138)

The sub-band energy \( (E_{n_z}) \), the EMME and the \( n_{2D} \) can respectively be expressed as
\[ E_{n_z} = \frac{\hbar^2}{2m_e} \left( \frac{n_z \pi}{d_z} \right)^2 \]  \hspace{1cm} (139)

\[ m^* (E_{F_z}) = m_e \]  \hspace{1cm} (140)

and \[ n_{2D} = \frac{m_e k_B T_{\text{G} \text{v}}}{\pi \hbar^2} \sum_{n_z=1}^{n_{\text{max}}} F_0 (\eta_{n_z}) \]  \hspace{1cm} (141)

where, \[ \eta_{n_z} = \frac{1}{k_B T} \left[ \frac{E_{F_z}}{\frac{\hbar^2}{2m_e} \left( \frac{n_z \pi}{d_z} \right)^2} \right]. \]

It may be noted that the results of this section are already well known in the literature [22].

(b) The model of Stillman et al.

The 2D electron dispersion relation in this case assumes the form following (29) as

\[ \frac{\hbar^2 k_z^2}{2m_e} + \frac{\hbar^2}{2m_e} \left( \frac{n_z \pi}{d_z} \right)^2 = I_{12} (E) \]  \hspace{1cm} (142)

Using (142) and (122), the EMME in x-y plane for this case can be written as

\[ m^* (E_{F_z}) = m_e \left\{ I_{12} (E_{F_z}) \right\}' \]  \hspace{1cm} (143)

It appears that the EMME in this case is a function of Fermi energy alone and is independent of size quantum number.

The total 2D density-of-states function can be written as

\[ N_{2D} (E) = \left( \frac{m_e}{\pi \hbar^2} \right) \sum_{n_z=1}^{n_{\text{max}}} \left\{ I_{12} (E) \right\}' H \left( E - E_{n_z} \right) \]  \hspace{1cm} (144)

where, the sub band energies \( E_{n_z} \) can be expressed as

\[ I_{12} (E_{n_z}) = \frac{\hbar^2}{2m_e} \left( \frac{n_z \pi}{d_z} \right)^2 \]  \hspace{1cm} (145)

The 2D carrier concentration assumes the form

\[ n_{2D} = \frac{m_e g_v}{\pi \hbar^2} \sum_{n_z=1}^{n_{\text{max}}} \left[ T_{55} (E_{F_z}, n_z) + T_{56} (E_{F_z}, n_z) \right] \]  \hspace{1cm} (146)

where \[ T_{55} (E_{F_z}, n_z) \equiv \left[ I_{12} (E_{F_z}) - \frac{\hbar^2}{2m_e} \left( \frac{n_z \pi}{d_z} \right)^2 \right] \] and \[ T_{56} (E_{F_z}, n_z) \equiv \sum_{r=1}^{r_{\text{max}}} L (r) T_{55} (E_{F_z}, n_z) \]

(c) Model of Palik et al.

The (34) can be expressed as
\[ \frac{\hbar^2 k^2}{2m_e} = I_{13}(E) \]  

where \( I_{13}(E) = \tilde{a}_{12} \left[ \bar{a}_{12} - \left( (\tilde{a}_{12})^2 - 4E\tilde{B}_{11} \right)^{1/2} \right] , \quad \bar{a}_{12} = \left( \frac{\hbar^2}{2m_e} \right) \) and \( \tilde{a}_{12} = \left[ \frac{\bar{a}_{12}}{2\tilde{B}_{11}} \right] \)

The 2D electron dispersion relation in this case assumes the form

\[ \frac{\hbar^2 k_x^2}{2m_e} + \frac{\hbar^2}{2m_e} \left( n_x \pi / d_z \right)^2 = I_{13}(E) \]  

(148)

Using (151) and (122), the EMME in x-y plane for this case can be written as

\[ m^*(E_{F_z}) = m_e \left\{ I_{13}(E_{F_z}) \right\}' \]  

(149)

It appears that the EMME in this case is a function of Fermi energy alone and is independent of size quantum number.

The total 2D density-of-states function can be written as

\[ N_{z\text{or}}(E) = \left( \frac{m_e g_v}{\pi \hbar^2} \right) \sum_{n_{1z}} \left[ I_{13}(E) \right]' H \left[ E - E_{n_{1z}} \right] \]  

(150)

where, the sub band energies \( E_{n_{1z}} \) can be expressed as

\[ I_{13}(E_{n_{1z}}) = \frac{\hbar^2}{2m_e} \left( n_x \pi / d_z \right)^2 \]  

(151)

The 2D carrier concentration assumes the form

\[ n_{2D} = \frac{m_e g_v}{\pi \hbar^2} \sum_{n_{1z}} \left[ T_{57}(E_{F_z}, n_{1z}) + T_{58}(E_{F_z}, n_{1z}) \right] \]  

(152)

where \( T_{57}(E_{F_z}, n_{1z}) = \left[ I_{13}(E_{F_z}) - \frac{\hbar^2}{2m_e} \left( n_x \pi / d_z \right)^2 \right] \) and \( T_{58}(E_{F_z}, n_{1z}) = \sum_{r=1}^{r} \left( L(r) T_{37}(E_{F_z}, n_{1z}) \right) \)

2.2.3 The EMME from UFs of II-VI materials

The dispersion relation of the conduction electrons of UFs of II-VI materials for dimensional quantization along z-direction can be written following (40) as

\[ E = \tilde{a}_0 k_x^2 + b_0 \left( \frac{n_x \pi}{d_z} \right)^2 \pm \tilde{\alpha}_0 k_z \]  

(153)

Using (122), the EMME in this case can be written as
Thus, we can infer that the EMME in the UFs of II-VI compounds is a function of both the size quantum number and the Fermi energy due to the presence of the term $\bar{\lambda}_0$.

The sub-band energy $E_{n_0}$ assumes the form

$$E_{n_0} = b'_0 \left( n \pi / d_z \right)^2$$  \hspace{1cm} (155)

The area of constant energy 2D quantized surface in this case is given by

$$A_x(E, n_z) = \left[ \frac{\pi}{2(\alpha_0)^2} \left( \frac{\bar{\lambda}_0}{2} \right)^2 + 2a_0 (E - E_{n_0}) \pm \bar{\lambda}_0 \left( \frac{\bar{\lambda}_0}{2} \right)^2 + 4a_0 (E - E_{n_0}) \right]^{1/2}$$  \hspace{1cm} (156)

The surface electron concentration under the condition of extreme carrier degeneracy can be expressed in this case as

$$n_{2D} = \frac{g_m^{*} \sum_{n_{\text{perm}}} A_x(E_{n_z}, n_z)}{2(2\pi)^2}$$  \hspace{1cm} (157a)

using (156) and (157a) we get

$$n_{2D} = \frac{g_m^{*} \sum_{n_{\text{perm}}} (E_{n_z} - E_{n_0} + \left( \frac{\bar{\lambda}_0}{2} \right)^2 m^*_h \hbar^{-2})}{4\alpha_0 \pi \hbar^2}$$  \hspace{1cm} (157b)

2.2.4 The EMME from UFs of Bismuth

(a) The McClure and Choi model

The dispersion relation of the conduction electrons in UFs of Bi for dimensional quantization along $k_z$ direction can be written following (44) for this model as

$$E(1 + \alpha E) = \frac{p_z^2}{2m_z} + \frac{p_y^2}{2m_y} + \hbar^2 \left( \frac{n \pi}{d_z} \right)^2 + \frac{p_z^2}{2m_z} \alpha E \left[ 1 - \left( \frac{m_z}{m_y} \right) \right] \frac{\alpha p_z}{4m_z m_y} + \frac{\alpha p_z^2}{4m_z m_y} + \frac{\alpha p_y^2}{4m_y m_z} \left( \frac{n \pi}{d_z} \right)^2$$  \hspace{1cm} (158)

The (158) can, approximately, be expressed as

$$\gamma_i(E, n_z) = p_i k_z^2 + q_i(E) k_y^2 + R_i(E, n_z) k_z^4$$  \hspace{1cm} (159)

where, $\gamma_i(E, n_z) = \left[ E(1 + \alpha E) \frac{\hbar^2}{2m_z} \left( \frac{n \pi}{d_z} \right)^2 \right]$, $p_i = \frac{\hbar^2}{2m_i}$, $q_i(E) = \frac{\hbar^2}{2m_y} \left[ 1 + \alpha E \left( 1 - \frac{m_z}{m_y} \right) - \alpha E \left( 1 + \alpha E \right) \right]$
and \( R_t(E,n_z) = \frac{aE}{4m_2} + \frac{\hbar^2}{2m_2} \left( 1 + \frac{\alpha E}{2m_2} \right) \left( 1 - \frac{m_2}{m_1} \right) \).

The area enclosed by (159) is defined by the following integral

\[
A(E,n_z) = \frac{4}{\pi \lambda_1} \left( \frac{R_t(E,n_z)}{\rho_1} \right)^{3/2} \cdot J_1(E,n_z)
\]

where,
\[
J_1(E,n_z) = \int_0^{E(n_z)} \left[ \frac{\gamma_1(E,n_z)}{R_t(E,n_z)} - \frac{q_1(E)k^2}{k}\right]^{1/2} dk,
\]

and
\[
u_0(E,n_z) = \sqrt{\frac{q_1(E)}{4R_t^n(E,n_z)} + \frac{\gamma_1(E,n_z)}{R_t(E,n_z)} - q_1(E)}.
\]

Thus, the area enclosed can be written as

\[
A(E,n_z) = \frac{4}{3} \left( \frac{R_t(E,n_z)}{\rho_1} \right)^{1/2} \left[ a^2(E,n_z) + b^2(E,n_z) \right]^{1/2}
\]

\[
a^2(E,n_z) = \frac{1}{2} \left[ \frac{q_1^2(E)}{R_t^2(E,n_z)} + \frac{4\gamma_1(E,n_z)}{R_t(E,n_z)} \right]^{1/2},
\]

\[
b^2(E,n_z) = \frac{1}{2} \left[ \frac{q_1^2(E)}{R_t^2(E,n_z)} + \frac{4\gamma_1(E,n_z)}{R_t(E,n_z)} \right]^{1/2} - \left( \frac{q_1(E)}{2R_t(E,n_z)} \right),
\]

\[
\ell(E,n_z) = \frac{b(E,n_z)}{\sqrt{a^2(E,n_z) + b^2(E,n_z)}} \cdot F \left[ \frac{\pi}{2}, \ell(E,n_z) \right]
\]

\[
E \left[ \frac{\pi}{2}, \ell(E,n_z) \right]
\]

are the complete elliptic integral of the first and second kinds respectively.

Using (160), the EMME can be written as

\[
m^*_{(E_x,n_z)} = \left( \frac{2\hbar^2}{3\pi \sqrt{\rho_1}} \right) \left[ R_3(E,n_z) \right]_{E=E_x}
\]

where,
\[
R_3(E_x,n_z) = \frac{1}{2} \left[ R_t(E_x,n_z) \right]^{1/2} \left[ R_t(E_x,n_z) \right] \left[ a^2(E_x,n_z) + p^2(E_x,n_z) \right]^{1/2}
\]
Thus, the EMME in this case is a function of both the Fermi energy and the size quantum number due to the presence of band non-parabolicity only.

The total 2D density-of-states function can be written following (160), as

\[ N_{2D}(E) = \left( \frac{2\gamma}{3\pi^2} \right) \sum_{n_i=1}^{\infty} R_{i}(E,n_{t}) H \left( E - E_{n_{t}} \right) \]  

(163)

where, the sub-band energies \( E_{n_{t}} \) assumes the form

\[ E_{n_{t}} = \frac{\hbar^2}{2m_{t}} \left( \frac{n_{t} \pi}{d_{t}} \right)^2 \]  

(164)

Combining (163) with the Fermi-Dirac occupation probability factor, the 2D electron statistics in UFs of Bi in accordance with the McClure and Choi model can be expressed as

\[ n_{2D} = \left( \frac{2\gamma}{3\pi^2} \right) \sum_{n_i=1}^{\infty} \left[ \theta_1(E_{F_{n_{t}}},n_{t}) + \theta_2(E_{F_{n_{t}}},n_{t}) \right] \]  

(165)

where, \( \theta_1(E_{F_{n_{t}}},n_{t}) \equiv \sqrt{R(E_{F_{n_{t}}},n_{t})} \left[ a^2(E_{F_{n_{t}}},n_{t}) + b^2(E_{F_{n_{t}}},n_{t}) \right]^{1/2} \)

\[ \left[ a^2(E_{F_{n_{t}}},n_{t}) F\left( \frac{\pi}{2}, \ell(E_{F_{n_{t}}},n_{t}) \right) - \left[ a^2(E_{F_{n_{t}}},n_{t}) - b^2(E_{F_{n_{t}}},n_{t}) \right] F\left( \frac{\pi}{2}, \ell(E_{F_{n_{t}}},n_{t}) \right) \right] \]

and \( \theta_2(E_{F_{n_{t}}},n_{t}) \equiv \sum_{r=1}^{5} L(r) \left[ \theta_1(E_{F_{n_{t}}},n_{t}) \right] \).
The Hybrid Model

The dispersion relation of the carriers in bulk specimens of Bi in accordance with the Hybrid model can be represented as

\[
E(1+\alpha E) = \frac{\theta_0(E)\left(\hbar k_y\right)^2}{2M_2} + \frac{\alpha\gamma_0\hbar^4k^4}{4M_2^2} + \frac{\hbar^2k_x^2}{2m_1} + \frac{\hbar^2k_z^2}{2m_3}
\]  

(166)

in which \(\theta_0(E) = \left[1 + \alpha E(1 - \gamma_0) + \delta_0\right]\), \(\gamma_0 = \frac{M_2}{m_2}\), \(\delta_0 = \frac{M_2}{M_2}\) and the other notations are defined in [93]. In the presence of size quantization along y-direction, the 2D electron dispersion relation can be written as

\[
\frac{\hbar^2k_x^2}{2m_1} + \frac{\hbar^2k_z^2}{2m_3} = E(1+\alpha E) - \frac{\theta_0(E)\hbar^2}{2M_2} \left(\frac{\pi n_y}{d_y}\right)^2 - \frac{\alpha\gamma_0\hbar^4}{4M_2^2} \left(\frac{\pi n_z}{d_z}\right)^4
\]  

(167 a)

The 2D area is given by

\[
A(E, n_y) = \frac{2\pi\sqrt{m_1m_3}}{\hbar^2} t_{29}(E, n_y)
\]  

(167 b)

where,

\[
t_{29}(E, n_y) = \left[ \left(1 + \alpha E\right) - \frac{\theta_0(E)\hbar^2}{2M_2} \left(\frac{\pi n_y}{d_y}\right)^2 - \frac{\alpha\gamma_0\hbar^4}{4M_2^2} \left(\frac{\pi n_z}{d_z}\right)^4 \right]
\]

The effective mass in the X-Z plane can be written as

\[
m^*(E_{Fy}, n_y) = [\sqrt{m_1m_3}]^\prime t_{29}(E, n_y)
\]  

(168)

Therefore, the effective mass in UFs of Bi in accordance with Hybrid model is a function of Fermi energy and the size quantum number due to the presence of band non-parabolicity only.

The sub-band energy are given as

\[
E_n(1+\alpha E_n) - \frac{\theta_0(E_n)\hbar^2}{2M_2} \left(\frac{\pi n_y}{d_y}\right)^2 - \frac{\alpha\gamma_0\hbar^4}{4M_2^2} \left(\frac{\pi n_z}{d_z}\right)^4 = 0
\]  

(169)

The total DOS function in this case can be written as

\[
N_{2DT}(E) = \frac{g_0\sqrt{m_1m_3}}{\pi\hbar^2} \sum_{n=1}^{\infty} \left[ t_{29}(E, n_y) \right]^{\prime} H\left(E - E_n\right)
\]  

(170)

The use of (170) leads to the 2D electron statistics in UFs of Bi in this case as
\[ n_{2D} = \frac{g_s \sqrt{m_1 m_2}}{\pi \hbar^2} \sum_{n_x, n_y} \left[ t_{30}(E_{Fs}, n_x) + t_{30}(E_{Fs}, n_y) \right] \]  

in which \( t_{30}(E_{Fs}, n_y) = \sum_{r=1}^{N_r} L(r) \left[ t_{30}(E_{Fs}, n_y) \right] \)

(c) The Cohen model

The 2D electron dispersion law in UF's of Bi in accordance with this model can be written following (52) as

\[ E(1 + \alpha E) = \frac{2}{m_1} \left[ \frac{n_x}{d_x} \right]^2 + \frac{n_y}{2m_2} \frac{\alpha E p_x^2}{2m_2} + \frac{\alpha p_y^4}{4m_2 m'_2} \frac{1}{2nh} \]  

The (172) can be written as

\[ \chi_1(E, n_x) = p_1 k_x^2 + q_2(E) k_y^2 + R_2 k_4 \]  

where, \( q_2(E) \equiv \left[ \frac{\hbar^2}{2m_2} 1 + \alpha E \right] - \frac{\alpha E h^2}{2m'_2} \) and \( R_2 \equiv \left( \frac{\alpha h^4}{4m_2 m'_2} \right) \).

The EMME in this case can be written as

\[ m^*(E_{Fs}, n_x) = \left( \frac{2 \hbar^2}{3 \pi \sqrt{p_1}} \right) R_4(E_{Fs}, n_x) \]  

in which,

\[ R_4(E_{Fs}, n_x) = \sqrt{R_2} \left[ q_2^2(E_{Fs}, n_x) + R_2^2(E_{Fs}, n_x) \right]^{1/2} \left[ a_1(E_{Fs}, n_x) (a_1(E_{Fs}, n_x))' + b_1(E_{Fs}, n_x) (b_1(E_{Fs}, n_x))' \right] \]

\[ \left[ a_1^2(E_{Fs}, n_x) F \left( \frac{\pi}{2}, \ell_1(E_{Fs}, n_x) \right) - \left[ a_1^2(E_{Fs}, n_x) - b_1^2(E_{Fs}, n_x) \right] E \left( \frac{\pi}{2}, \ell_1(E_{Fs}, n_x) \right) \right] \]

\[ + \sqrt{R_2} \left[ q_2^2(E_{Fs}, n_x) + R_2^2(E_{Fs}, n_x) \right]^{1/2} \left[ 2a_1(E_{Fs}, n_x) (a_1(E_{Fs}, n_x))' F \left( \frac{\pi}{2}, \ell_1(E_{Fs}, n_x) \right) + q_2^2(E_{Fs}, n_x) F \left( \frac{\pi}{2}, \ell_1(E_{Fs}, n_x) \right) \right] \]

\[ \left[ 2a_1(E_{Fs}, n_x) (a_1(E_{Fs}, n_x))' - 2b_1(E_{Fs}, n_x) (b_1(E_{Fs}, n_x))' \right] E \left( \frac{\pi}{2}, \ell_1(E_{Fs}, n_x) \right) \left[ a_1^2(E_{Fs}, n_x) - b_1^2(E_{Fs}, n_x) \right] E \left( \frac{\pi}{2}, \ell_1(E_{Fs}, n_x) \right) \]

\[ \cdot a_1^2(E_{Fs}, n_x) = \frac{q_2(E_{Fs})}{2R_2} \frac{1}{\left[ \frac{q_2^2(E_{Fs})}{R_2} + \frac{4\chi(E_{Fs}, n_x)^2}{R_2} \right]^{1/2}} , \]

\[ b_1^2(E_{Fs}, n_x) = \left[ \frac{q_2^2(E_{Fs})}{R_2^2} + \frac{4\chi(E_{Fs}, n_x)^2}{R_2} \right]^{1/2} - \left( \frac{q_2(E_{Fs})}{2R_2} \right) \]  

70
and \( \epsilon_1(E_{F_3}, n_z) = \frac{b_1(E_{F_3}, n_z)}{\sqrt{a_1^2(E_{F_3}, n_z) + b_1^2(E_{F_3}, n_z)}}. \)

which shows that the EMME in this present case is again function of both the size quantum number and the Fermi energy due to the presence of the band nonparabolicity only.

The total density-of-states is given by

\[
N_{2DR}(E) = \left(\frac{2g}{3\sqrt{\pi}}\right) \sum_{n_z} R_4(E, n_z) H \left(E - E_{n_0}\right)
\]

where, \( E_{n_0} \) is the lowest positive root of the equation

\[
\chi_1(E_{n_0}, n_z) = 0
\]

Combining (175 a) with the Fermi-Dirac occupation probability factor, the 2D electron statistics in UFs of Bi in accordance with the Cohen model can be written as

\[
n_{2D} = \left(\frac{2g}{3\sqrt{\pi}}\right) \sum_{n_z} \left[ \theta_3(E_{F_3}, n_z) + \theta_4(E_{F_3}, n_z) \right]
\]

where, \( \theta_i(E_{F_3}, n_z) = \left[ \sqrt{R_2 \left[ a_1^2(E_{F_3}, n_z) + b_1^2(E_{F_3}, n_z) \right]} \right]^{1/2} \)

\[
\left[ a_1^2(E_{F_3}, n_z) F \left(\frac{\pi}{2}, \epsilon_1(E_{F_3}, n_z)\right) - \left[ a_1^2(E_{F_3}, n_z) - b_1^2(E_{F_3}, n_z) \right] F \left(\frac{\pi}{2}, \epsilon_1(E_{F_3}, n_z)\right) \right],
\]

and \( \theta_4(E_{F_3}, n_z) = \sum_{r=1}^{\infty} L(r) \left[ \theta_4(E_{F_3}, n_z) \right]. \)

\( \textbf{(d) The Lax model} \)

The 2D electron dispersion law in this case can be written following (57) as

\[
E(1 + \alpha E) = \frac{\hbar^2 k_x^2}{2m_1} + \frac{\hbar^2 k_y^2}{2m_2} + \frac{\hbar^2}{2m_2} \left( \frac{n_z \pi}{d_z} \right)^2
\]

The EMME in this case assumes the form

\[
m^*(E_{F_3}) = \sqrt{m_1 m_2} \left(1 + 2\alpha E_{F_3}\right)
\]

Thus, we see that the EMME for the Lax model is a function of the Fermi energy alone due to the band non-parabolicity.

The sub-band energy, the total density- of states function and the 2D electron statistics for this model can, respectively, be expressed as

\[
E_{n_0} \left(1 + \alpha E_{n_0}\right) = \frac{\hbar^2}{2m_2} \left( \frac{n_z \pi}{d_z} \right)^2
\]
\[ N_{2dr} (E) = \frac{g_s \sqrt{m_1 m_2}}{\pi \hbar^2} \sum_{n=1}^{n_{\text{max}}} \left( 1 + 2\alpha E \right) H (E - E_{n_s}) \] (180)

\[ n_{2D} = \frac{g_s \sqrt{m_1 m_2} k_B T}{\pi \hbar^2} \sum_{n=1}^{n_{\text{max}}} \left[ (1 + 2\alpha E_{n_s}) F_0 (\eta_{n_s}) 2\alpha k_B T F_0 (\eta_{n_s}) \right] \] (181)

where, \( \eta_{n_2} = \frac{E_{F_s} - E_{n_s}}{k_B T} \).

(e) The ellipsoidal parabolic model

The 2D dispersion relation, the EMME, the sub-band energy \( E_{n_s} \), the total density-of-states and the 2D electron statistics for this model can respectively be written as

\[ E = \left( \frac{\hbar^2 k_x^2}{2m_1} + \frac{\hbar^2 k_y^2}{2m_2} + \left( \frac{\hbar^2}{2m_3} \right) \left( \frac{n_s \pi}{d_z} \right)^2 \right) \] (182)

\[ m^* (E_{F_s}) = \left( \sqrt{m_1 m_2} \right) \] (183)

\[ N_{2dr} (E) = \frac{g_s \sqrt{m_1 m_2}}{\pi \hbar^2} \sum_{n=1}^{n_{\text{max}}} H (E - E_{n_s}) \] (184)

\[ E_{n_s} = \left( \frac{\hbar^2}{2m_3} \right) \left( \frac{n_s \pi}{d_z} \right)^2 \] (185)

\[ n_{2D} = \left[ \frac{k_B T g_s \sqrt{m_1 m_2}}{\pi \hbar^2} \right] \sum_{n=1}^{n_{\text{max}}} F_0 (\eta_{n_2}) \] (186)

where, \( \eta_{n_2} = (k_B T)^{-1} \left[ E_{F_s} - E_{n_s} \right] \).

2.2.5 The EMME from UFs of IV-VI materials

The 2D dispersion relation of the conduction electrons in IV-VI materials in UFs for the dimensional quantization along \( z \) direction can be expressed following (63) as

\[ E \left( 1 + \alpha E \right) + \alpha E \left( \frac{\hbar^2 k_x^2}{2x_4} + \frac{\hbar^2 k_y^2}{2x_5} \right) + \alpha E \left( \frac{\hbar^2}{2x_6} \left( \frac{n_s \pi}{d_z} \right)^2 \right) - (1 + \alpha E) \left( \frac{\hbar^2 k_x^2}{2x_1} + \frac{\hbar^2 k_y^2}{2x_2} \right) \]

\[-\alpha \left( \frac{\hbar^2 k_x^2}{2x_1} + \frac{k_B T g_s \sqrt{m_1 m_2}}{\pi \hbar^2} \right) \left( \frac{\hbar^2 k_x^2}{2x_1} + \frac{\hbar^2 k_y^2}{2x_2} \right) - \left( \frac{\hbar^2}{2x_6} \right) \left( \frac{n_s \pi}{d_z} \right)^2 \right) - (1 + \alpha E) \left( \frac{\hbar^2}{2x_5} \left( \frac{n_s \pi}{d_z} \right)^2 \right) \]

\[-\alpha \left( \frac{\hbar^2}{2x_3} \left( \frac{n_s \pi}{d_z} \right)^2 \right) \left( \frac{\hbar^2}{2x_3} \left( \frac{n_s \pi}{d_z} \right)^2 \right) - \alpha \left( \frac{\hbar^2}{2x_5} \left( \frac{n_s \pi}{d_z} \right)^2 \right) \left( \frac{\hbar^2}{2x_6} \left( \frac{n_s \pi}{d_z} \right)^2 \right) = \frac{\hbar^2}{2m_1} + \frac{\hbar^2}{2m_2} + \frac{\hbar^2}{2m_3} \left( \frac{n_s \pi}{d_z} \right)^2 \] (187)
where $x_4 = m_1^+$, $x_5 = m_1^* + 2m_1^*$, $x_6 = m_1^* + m_2^*$, $x_1 = m_2^*$, $x_2 = m_2^* + 2m_2^*$, $x_3 = m_3^*$, $x_3^* = m_3^* + m_4^*$, $x_3^+ = m_3^* + m_5^*$, $x_4^+ = m_3^* + m_6^*$, with $m_1 = m_1^*$, $m_2 = m_2^* + 2m_2^*$, and $m_3 = 3m_3^* m_4^* + m_3^* + 2m_3^* + m_3^*$.

Substituting $k_z = r \cos \theta$ and $k_y = r \sin \theta$ (where $r$ and $\theta$ are 2D polar coordinates in 2D wave vector space) in (187), we can write

$$ r^4 \left[ \alpha \frac{1}{4} \left( \frac{h^2 \cos^2 \theta}{x_1} + \frac{h^2 \sin^2 \theta}{x_2} \right) \left( \frac{h^2 \cos^2 \theta}{x_4} + \frac{h^2 \sin^2 \theta}{x_5} \right) \right] + r^2 \frac{1}{2} \left[ \left( \frac{h^2 \cos^2 \theta}{m_1} + \frac{h^2 \sin^2 \theta}{m_2} \right) \right] + \alpha \frac{h^2}{2x_3} \left( \frac{n_x \pi}{d_z} \right)^2 \left( \frac{h^2 \cos^2 \theta}{x_4} + \frac{h^2 \sin^2 \theta}{x_5} \right) + \alpha \left( \frac{h^2 \cos^2 \theta}{x_1} + \frac{h^2 \sin^2 \theta}{x_2} \right) \frac{h^2}{2x_6} \left( \frac{n_x \pi}{d_z} \right)^2 $$

$$ + h^2 (1 + \alpha E) \left( \frac{\cos^2 \theta}{x_1} + \frac{\sin^2 \theta}{x_2} \right) - h^2 \alpha E \left( \frac{\cos^2 \theta}{x_4} + \frac{\sin^2 \theta}{x_5} \right) - [E(1 + \alpha E)]^2 \left[ (1 + \alpha E) \frac{h^2}{2x_3} \left( \frac{n_x \pi}{d_z} \right)^2 - \alpha \left( \frac{h^4}{4x_3^2 x_6} \left( \frac{n_x \pi}{d_z} \right)^4 \right) \right] = 0 \quad (188) $$

The area $A(E, n_z)$ of the 2D wave vector space can be expressed as

$$ A(E, n_z) = \bar{J}_1 - \bar{J}_2 \quad (189) $$

where

$$ \bar{J}_1 = 2 \int_0^{\pi/2} \frac{c}{b} d \theta \quad (190) $$

and

$$ \bar{J}_2 = 2 \int_0^{\pi/2} \frac{ac^2}{b^3} d \theta \quad (191) $$

in which

$$ a = \left[ \alpha \left( \frac{h^4}{4} \right) \left( \frac{\cos^2 \theta}{x_1} + \frac{\sin^2 \theta}{x_2} \right) \left( \frac{\cos^2 \theta}{x_4} + \frac{\sin^2 \theta}{x_5} \right) \right], $$

$$ b = \left( \frac{h^2}{2} \right) \left( \frac{\cos^2 \theta + \sin^2 \theta}{m_1} + \frac{\cos^2 \theta}{m_2} \right) + \alpha \left( \frac{h^2}{2x_3} \right) \left( \frac{n_x \pi}{d_z} \right)^2 \left( \frac{\cos^2 \theta}{x_4} + \frac{\sin^2 \theta}{x_5} \right) $$

$$ + \alpha \left( \frac{h^2}{2x_3} \right) \left( \frac{n_x \pi}{d_z} \right)^2 \left( \frac{\cos^2 \theta + \sin^2 \theta}{m_1} \right) + (1 + \alpha E) \left( \frac{\cos^2 \theta + \sin^2 \theta}{x_1} + \frac{\sin^2 \theta}{x_2} \right) - \alpha E \left( \frac{\cos^2 \theta}{x_4} + \frac{\sin^2 \theta}{x_5} \right) $$

and

$$ c = \left[ E(1 + \alpha E) + \alpha E \left( \frac{h^2}{2x_3} \right) \left( \frac{n_x \pi}{d_z} \right)^2 - (1 + \alpha E) \left( \frac{h^2}{2x_3} \right) \left( \frac{n_x \pi}{d_z} \right)^2 - \alpha \left( \frac{h^4}{4x_3^2 x_6} \left( \frac{n_x \pi}{d_z} \right)^4 \right) \right]. $$
(190) can be expressed as

\[ J_1 = 2 \int_0^{\pi/2} \frac{t_3(E, n_z)}{A_1(E, n_z) \cos^2 \theta + B_1(E, n_z) \sin^2 \theta} d\theta \]

where,

\[ t_3(E, n_z) = c, A_1(E, n_z) = \frac{\hbar^2}{2m_i} t_1(E, n_z), \]

\[ t_1(E, n_z) = \left[ 1 + m_t \left( \frac{\alpha \hbar^2 (n, \pi) \alpha \hbar^2 (n, \pi)}{2x_2x_5 d_z} + \frac{\alpha \hbar^2 (n, \pi) \alpha \hbar^2 (n, \pi)}{2x_2x_5 d_z} + \frac{1 + \alpha \hbar^2}{x_1 - x_4} \right) \right] \]

\[ B_1(E, n_z) = \frac{\hbar^2}{2m_2} t_2(E, n_z) \text{ and} \]

\[ t_2(E, n_z) = \left[ 1 + m_t \left( \frac{\alpha \hbar^2 (n, \pi) \alpha \hbar^2 (n, \pi)}{2x_2x_5 d_z} + \frac{\alpha \hbar^2 (n, \pi) \alpha \hbar^2 (n, \pi)}{2x_2x_5 d_z} + \frac{1 + \alpha \hbar^2}{x_2 - x_5} \right) \right]. \]

Performing the integration, we get

\[ J_1 = \pi t_2(E, n_z) [A_1(E, n_z) B_1(E, n_z)]^{1/2} \quad (192) \]

From (191) we can write

\[ J_2 = \frac{\alpha t_1(E, n_z) \hbar^4}{2B^2(E, n_z)} \quad (193) \]

where

\[ I = \int_0^\infty \left\{ \frac{(a_1 + a_2 z^2)(a_3 + a_4 z^2)}{[(\bar{a})^2 + z^2]^3} \right\} dz \quad (194) \]

in which \( a_1 \equiv 1 \), \( a_2 \equiv 1 \), \( z = \tan \theta \), \( \theta \) is a new variable, \( a_3 \equiv 1 \), \( a_4 \equiv 1 \) and

\[ (\bar{a})^2 = \frac{A_1(E, n_z)}{B_1(E, n_z)}. \]

The use of the Residue theorem leads to the evaluation of the integral in (194) as

\[ I = \frac{\pi}{4 \bar{a}} [a_1 a_4 + 3 a_2 a_4] \quad (195) \]

Therefore, the 2D area of the 2D wave vector space can be written as

\[ A(E, n_z) = \frac{\pi t_1(E, n_z)}{\sqrt{A_1(E, n_z) B_1(E, n_z)}} \left[ 1 - \frac{1}{x_5} \left( \frac{1 + 3}{x_1 - x_4} \right) A_1(E, n_z) \hbar^4 \right] \quad (196) \]

The EMME for the UFs of IV-VI materials can thus be written as

\[ m^*(E, n_z) = \frac{\hbar^2}{2} [\theta_3(E, n_z)]^2 \quad (197) \]

74
where, $\theta_3(E, n_z) = \left[ 1 - \frac{1}{x_1} \left( \frac{1}{x_1} + \frac{3}{x_2} \right) \frac{J_s}{8 \left( B_1(E, n_z) \right)^2} \right] \left( A_i(E, n_z) B_i(E, n_z) \right)^{1/4}$$$
\left[ \sqrt{A(E, n_i) B(E, n_i)} \left\{ t_5(E, n_i) \right\} - t_5(E, n_i) \right\} \left\{ 1 + \frac{1}{2} \left( A(E, n_i) / \frac{B(E, n_i)}{A(E, n_i)} \right)^{1/2} \right\} + \frac{1}{2} \right] \left\{ B(E, n_i) \right\}^{1/2} \left\{ A(E, n_i) / B(E, n_i) \right\}^{1/2} \right\} \]$$

Thus, the EMME is a function of Fermi energy and the quantum number due to the band non-parabolicity.

The total density-of-states function can be written as

$$N_{2D} (E) = \left( \frac{g_v}{2\pi} \right) \sum_{n_z} \theta_3(E, n_z) H\left( E - E_{n_z} \right)$$

(198)

where the sub-band energy $\left( E_{n_z} \right)$ in this case can be written as

$$E_{n_z} \left( 1 + \alpha E_{n_z} \right) + \alpha E_{n_z} \left( \frac{\hbar^2}{2x_1} \left( \frac{n_z \pi}{d_z} \right)^2 - \left( 1 + \alpha E_{n_z} \right) \left( \frac{\hbar^2}{2x_3} \left( \frac{n_z \pi}{d_z} \right)^2 \right) \right)$$

$$- \alpha \left( \frac{\hbar^2}{2x_1} \left( \frac{n_z \pi}{d_z} \right)^2 \right) \left( \frac{\hbar^2}{2x_6} \left( \frac{n_z \pi}{d_z} \right)^2 \right) = 0$$

(199)

The use (198) leads to the expression of 2D electron statistics as

$$n_{2D} = \frac{g_v}{2\pi} \sum_{n_z} \left[ T_{39} (E_{Fz}, n_z) + T_{60} (E_{Fz}, n_z) \right]$$

(200)

where $T_{39} (E_{Fz}, n_z) = \frac{A(E_{Fz}, n_z)}{\pi}$ and $T_{60} (E_{Fz}, n_z) = \frac{\sum L(r) T_{50} (E_{Fz}, n_z)}{\pi}$.

2.2.6 The EMME from UFs of Stressed Materials

The 2D electron energy spectrum in UFs of stressed materials assumes the form following (77) as

$$\frac{k^2}{[\tilde{a}_0(E)]^2} + \frac{k^2}{[\tilde{b}_0(E)]^2} + \frac{1}{[\tilde{c}_0(E)]^2} (n_z \pi / d_z)^2 = 1$$

(201)

The area of 2D wave vector space enclosed by (201) can be written as

$$A(E, n_z) = \pi P^2 (E, n_z) \tilde{a}_0(E) \tilde{b}_0(E)$$

(202)
where \( P_2(E, n_z) = \left[ 1 - \left( \frac{n_z \pi}{d_z \bar{c}_0(E)} \right)^2 \right] \).

The expression of the surface EMME in this case can be written as

\[ m^* (E_{n_z}, n_z) = \frac{\hbar^2}{2} \left[ \frac{\theta_q (E, n_z)}{E - \bar{c}_0 (E)} \right] \]

in which,

\[ \theta_q (E, n_z) = \left[ 2P(E_{n_z}) \left[ P(E_{n_z}) \right] \bar{a}_0(E) \bar{b}_0(E) + \left[ P(E_{n_z}) \right]^2 \left[ \bar{a}_0(E) \right] b_0(E) + \left[ P(E_{n_z}) \right]^2 \left[ b_0(E) \right]^2 \bar{a}_0(E) \right] \]

The EMME in this case is the function of Fermi energy and the size quantization number due to the presence of stress only.

Thus, the total 2D density-of-states function can be expressed as

\[ N_{2DT} (E) = \left( \frac{g_v}{2\pi} \right) \sum_{n_z=1}^{N_{\text{lat}}} \theta_0 (E, n_z) H(E - E_{n_{z1}}) \]

The sub band energies \( (E_{n_{z1}}) \) are given by

\[ \bar{c}_0 (E_{n_{z1}}) = n_z \pi / d_z \]

The 2D surface electron concentration per unit area for UF of stressed Kane type compounds can be written as

\[ n_{z2D} = \frac{g_v}{2\pi} \sum_{n_z=1}^{N_{\text{lat}}} \left[ T_{61} (E_{n_z}, n_z) + T_{62} (E_{n_z}, n_z) \right] \]

where \( T_{61} (E_{n_z}, n_z) \equiv \left[ P^2 (E_{n_z}, n_z) \bar{a}_0 (E_{n_z}) b_0 (E_{n_z}) \right] \) and \( T_{62} (E_{n_z}, n_z) \equiv \sum_{r=1}^{s} L(r) T_{61} (E_{n_z}, n_z) \).

### 2.2.7 The EMME from UF of Tellurium

The 2D electron energy spectrum in ultrathin films of Te assumes the form following (82) as

\[ k_z^2 = \psi_5 (E) - \psi_6 \left( \frac{\pi n_z}{d_z} \right)^2 \pm \psi_7 \left[ \psi_8^2 (E) - \left( \frac{\pi n_z}{d_z} \right)^2 \right]^{1/2} \]

where, \( \psi_5 (E) = \left[ \frac{E}{\psi_2} + \frac{\psi_4^2}{2 \psi_2^2} \right], \psi_6 = \frac{\psi_1}{\psi_2}, \psi_7 = \frac{\psi_4 \sqrt{\psi_1}}{\psi_2^{3/2}} \),

\[ \psi_8^2 (E) = \frac{\psi_4^4 + 4E \psi_4 \psi_2^4 + 4 \psi_2^2 \psi_3^2}{4 \psi_2^4 \psi_4^2} \]

The EMME in this case is given by
where, \( t_{40}(E, n_z) = |\psi_5(E) - \psi_6(\frac{\pi n_z}{d_z})^2 \pm \psi_7(\frac{\pi n_z}{d_z})^2| \)

It appears that the EMME in UFs of Te is a function of Fermi energy and size quantum number which is the characteristics of such systems.

Thus, the total 2D density-of-states function can be expressed as

\[
N_{2DF}(E) = \left( \frac{g_x}{\pi} \right) \sum_{n_z=1}^{n_{max}} t_{40}(E, n_z) H(E - E_{n_z})
\]

The sub band energies \( E_{n_z} \) are given by

\[
E_{n_z} = \psi_1(n_z \pi/d_z)^2 \pm \psi_5(n_z \pi/d_z)
\]

The 2D surface electron concentration per unit area for UFs of Te can be written as

\[
n_{2D} = \frac{g_x}{\pi} \sum_{n_z=1}^{n_{max}} [t_{40}(E_{n_z}, n_z) + t_{41}(E_{n_z}, n_z)]
\]

where \( t_{41}(E_{n_z}, n_z) \equiv \sum_{r = 1}^r L(r) t_{40}(E_{n_z}, n_z) \).

2.2.8 The EMME from UFs of Gallium Phosphide

The 2D electron dispersion relation in size quantized n-GaP can be expressed following (87) as

\[
E = ak_1^2 + C(n_z \pi/d_z)^2 + |V_G| - \left[ Dk_1^2 + |V_G|^2 + D(n_z \pi/d_z)^2 \right]^{1/2}
\]

in which, \( a = \frac{\hbar^2}{2m_1} + \frac{\hbar^2}{2m_\perp} \), \( C = \frac{\hbar^2}{2m_\perp} \) and \( D \equiv \left( \frac{\hbar^2 k_0^2}{m_0^*} \right)^2 \).

The sub-band energy \( E_{n_z} \) are given by

\[
E_{n_z} = C(n_z \pi/d_z)^2 + |V_G| - \left[ |V_G|^2 + D(n_z \pi/d_z)^2 \right]^{1/2}
\]

(212) can be expressed as

\[
k_1^2 = t_{42}(E, n_z)
\]

in which, \( t_{42}(E, n_z) \equiv \left[ 2a(E - t_i) + D \right] - \left[ \left[ 2a(E - t_i) + D \right]^2 - 4a^2 \left( E - t_i \right)^2 - t_2 \right]^{1/2} \),

\[
t_i \equiv |V_G| + C(n_z \pi/d_z)^2 \quad \text{and} \quad t_2 \equiv |V_G|^2 + D(n_z \pi/d_z)^2.
\]

The EMME can be expressed from (214) as
\[ m^*(E_{Fz}, n_z) = \frac{\hbar^2}{2} t_{44}(E_{Fz}, n_z) \]  

It appears that the EMME in UFs of GaP is a function of Fermi energy and size quantum number due to the presence of the system constant \( k_0 \).

The total DOS function is given by

\[ N_{2D}(E) = \frac{2\pi}{4\pi d^2} \sum_{n_z} \left[ t_{42}(E, n_z) \right] H(E - E_{n_z}) \]  

The electron statistics in UFs in n-GaP assumes the form

\[ n_{2D} = \frac{2\pi}{4\pi d^2} \sum_{n_z} \left[ t_{42}(E_{F2D}, n_z) + t_{43}(E_{F2D}, n_z) \right] \]  

where, \( t_{43}(E_{F2D}, n_z) \equiv \sum_{r=1}^{\frac{s}{42}} L(r) \left[ t_{42}(E_{F2D}, n_z) \right] \)

### 2.2.9 The EMME from UFs of Platinum Antimonide

(93) can be written as

\[ \left[ E + \omega_4 k_z^2 \right] \left[ E + \delta_0 - \omega_4 k_z^2 \right] = I_1 \left( k_z^2 + k_x^2 \right)^2 \]  

The use of (218) leads to the expression of the 2D dispersion law in UFs of n-PtSb as

\[ k_z^2 = t_{44}(E, n_z) \]  

where,

\[ t_{44}(E, n_z) = \left[ 2A_y \right]^{-1} \left[ -A_{10}(E, n_z) + \sqrt{A_{10}^2(E, n_z) + 4A_y A_{11}(E, n_z)} \right] \]  

\[ A_y = [I_1 + \omega_4 \omega_1], \]

\[ A_{10}(E, n_z) = \omega_1 E + \omega_1 \left( E + \delta_0 - \omega_4 \left( \frac{\pi n_z}{d_z} \right)^2 \right) + \omega_4 \left( \frac{\pi n_z}{d_z} \right)^2 + 2I_1 \left( \frac{\pi n_z}{d_z} \right)^2 \]

and

\[ A_{11}(E, n_z) = \left[ E + \delta_0 - \omega_4 \left( \frac{\pi n_z}{d_z} \right)^2 + \omega_4 \left( \frac{\pi n_z}{d_z} \right)^2 \right] \left[ E + \delta_0 - \omega_4 \left( \frac{\pi n_z}{d_z} \right)^2 \right] - I_1 \left( \frac{\pi n_z}{d_z} \right)^4 \]

The area of \( k_z \) space can be expressed as

\[ A(E, n_z) = \frac{\pi}{2A_y} t_{44}(E, n_z) \]  

The EMME can be written as

\[ m^*(E_{Fz}, n_z) = \frac{\hbar^2}{4A_y} t_{44}(E, n_z) \]
It appears that the EMME in UFs of PtSb$_2$ is a function of Fermi energy and size quantum number which is the characteristic features of such systems.

The total DOS function assumes the form

$$N_{2DT}(E) = \frac{g_v}{4\pi A_y} \sum_{n=1}^{\infty} \left[ t_{44}^{(n)}(E, n_z) \right] H \left( E - E_{n_{st}} \right)$$

(223)

where the quantized levels $E_{n_{st}}$ can be expressed through the equation

$$E_{n_{st}} = \left[ \frac{\alpha}{d_y} \left( \frac{n_y\pi}{d_y} \right)^2 + \frac{\alpha}{d_z} \left( \frac{n_z\pi}{d_z} \right)^2 \right] + \frac{4}{c} \left[ t \left( \frac{n_x\pi}{d_x} \right)^2 + \alpha q \left( \frac{n_x\pi}{d_x} \right)^2 - \alpha \delta \left( \frac{n_x\pi}{d_x} \right)^2 \right]$$

(224)

The electron statistics can be written as

$$n_{2D} = \frac{2g_v}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial}{\partial E} \left[ t_{44}(E, n_z) \right] f(E) dE$$

$$n_{2D} = \frac{g_v}{4\pi A_y} \sum_{n=1}^{\infty} \left[ t_{44}(E_{F2D}, n_z) + t_{45}(E_{F2D}, n_z) \right]$$

(225)

where $t_{46}(E_{F2D}, n_z) = \sum_{r=1}^{5} L(r) [t_{44}(E_{F2D}, n_z)]$

### 2.2.10 The EMME from UFs of Bismuth Telluride

The 2D electron dispersion law in UFs of Bi$_2$Te$_3$ assumes the form following (98) as

$$E(1 + \alpha E) = \bar{\omega}_1 \left( \frac{n_x\pi}{d_x} \right)^2 + \bar{\omega}_2 k_y^2 + \bar{\omega}_3 k_z^2 + 2\bar{\omega}_4 k_x k_y$$

(226)

The area of the ellipse is given by

$$A_n(E, n_z) = \frac{\pi}{\sqrt{\bar{\omega}_2 \bar{\omega}_3} - 4\bar{\omega}_2} \left[ \frac{2m_y(E(1 + \alpha E))}{\hbar^2} - \bar{\omega}_1 \left( \frac{n_x\pi}{d_x} \right)^2 \right]$$

(227)

The EMME can be expressed as

$$m^*(E_{n_z}) = \frac{m_y(1 + 2\alpha E)}{\sqrt{\bar{\omega}_2 \bar{\omega}_3} - 4\bar{\omega}_2}$$

(228)

It appears that the EMME in UFs of Bi$_2$Te$_3$ is a function of Fermi energy due to the presence of the band non-parabolicity.

The total DOS function assumes the form

$$N_{2DT}(E) = \frac{g_v m_y}{\pi \hbar^2} \sum_{n=1}^{\infty} (1 + 2\alpha E) H \left( E - E_{n_{st}} \right)$$

(229)

where, $(E_{n_{st}})$ can be expressed through the equation
The electron concentration can be written as

\[ n_{2D} = \frac{k_B T}{\pi \hbar^2} \left( \frac{m_0}{\sqrt{\alpha_{22} \alpha_{33} - 4 \alpha_{23}^2}} \right) \sum_{n_z=1}^{\infty} \left( 1 + 2 \alpha E_{n_z} \right) F_0(\eta_{n_z}) + 2 \alpha k_B T F_1(\eta_{n_z}) \]

(231)

where, \( \eta_{n_z} = \frac{E_{F_z} - E_{n_z}}{k_B T} \)

(232)

2.2.11 The EMME from UFs of Germanium

(a) In the presence of size quantization along \( k_z \) direction, the 2D dispersion relation of the conduction relations in UFs of \( n \)-Ge can be written by extending the method as given in [144] as

\[ \frac{\hbar^2 k_x^2}{2m_x^*} + \frac{\hbar^2 k_y^2}{2m_y^*} = \gamma(E, n_z) \]

(233)

where, \( m_x^* = m_{1x}^* + m_1^* \),

\[ m_y^* = \frac{3m_{1y}^*}{2m_1^* + m_x^*}. \]

The area of ellipse of the 2D surface as given by (233) can be written as

\[ A(E, n_z) = \frac{2\pi \sqrt{m_x^* m_y^*}}{\hbar^2} \gamma(E, n_z) \]

(234)

The EMME can be expressed using (234) as

\[ m^*(E_{F_z}, n_z) = \sqrt{m_x^* m_y^*} \left[ (1 + 2 \alpha E_{F_z}) - (2 \alpha) \frac{\hbar^2}{2m_3^*} \left( \frac{n \pi}{d_z} \right)^2 \right] \]

(235)

Therefore, the EMME is a function of Fermi energy and size quantum number due to the presence of band non-parabolicity only.

The DOS function per sub band can be expressed as

\[ N_{2D}(E) = \frac{4}{\pi \hbar^2} \left[ 1 + 2 \alpha E - 2 \alpha \left( \frac{\hbar^2}{2m_3^*} \left( \frac{\pi n \pi}{d_z} \right)^2 \right) \right] \]

(236)

The total DOS function is given by
\[ N_{2D}(E) = \frac{4}{\pi \hbar^2} \left[ \frac{m^*_{1} m^*_{2}}{\hbar^2} \sum_{n_{1},n_{2}} \left[ 1 + 2\alpha E - 2\alpha \left( \frac{\hbar^2}{2m^*_1} \left( \frac{\pi n_{1,t}}{d_z} \right)^2 \right) \right] H \left( E - E_{n_{16}} \right) \]  \tag{237}

where, \( E_{n_{16}} \) is the positive root of the following equation

\[ E_{n_{16}} \left( 1 + \alpha E_{n_{16}} \right) - \left( 1 + 2\alpha E_{n_{16}} \right) \left( \frac{\hbar^2}{2m^*_1} \left( \frac{\pi n_{1,t}}{d_z} \right)^2 \right) + \alpha \left( \frac{\hbar^2}{2m^*_1} \left( \frac{\pi n_{1,t}}{d_z} \right)^2 \right)^2 = 0 \]  \tag{238}

Thus combining (237) with the Fermi Dirac occupation probability factor), the electron statistics in this case can be written as

\[ n_{2D} = \frac{4\sqrt{m^*_1 m^*_2 k_B T}}{\hbar^2} \sum_{n_{1}} \left[ \left( A_{1}(n_{z}) + 2\alpha E_{n_{16}} \right) F_{0} \left( E_{n_{16}} \right) + 2\alpha k_B T F_{1} \left( E_{n_{16}} \right) \right] \]  \tag{239}

where \( A_{1}(n_{z}) = \left[ 1 + 2\alpha \left( \frac{\hbar^2}{2m^*_1} \right) \left( \frac{\pi n_{z}/d_z}{d_z} \right)^2 \right] \) and \( n_{n_{16}} = \frac{1}{k_B T} \left[ E_{F_{2D}} - E_{n_{16}} \right] \).

(b) Therefore the 2D dispersion law can be expressed following (108) as

\[ E = A_{5}(n_{z}) + A_{6}(n_{z})\beta - \bar{c}_{2}\beta^2 \]  \tag{240}

where \( A_{5}(n_{z}) = \frac{\hbar^2}{2m^*_1} \left( \frac{\pi n_{z}}{d_z} \right)^2 \) and \( A_{6}(n_{z}) = \left[ 1 - \bar{c}_{2} \right] \left( \frac{\hbar^2}{2m^*_1} \left( \frac{n_{z}/d_z}{d_z} \right)^2 \right] \).

The (240) can be written as

\[ \frac{\hbar^2 k_x^2}{2m^*_1} + \frac{\hbar^2 k_y^2}{2m^*_2} = I_{1}(E,n_{z}) \]  \tag{241}

where \( \bar{c}_{2} \equiv \left( 2\bar{c}_{1} \right)^{-1} \left[ A_{5}(n_{z}) - A_{6}(n_{z}) + 4\bar{c}_{1}E + 4\bar{c}_{1}A_{5}(n_{z}) \right]^{1/2} \).

From (241), the area of the 2D \( k_z \)-space is given by

\[ A(E,n_{z}) = \frac{2\pi \sqrt{m^*_1 m^*_2}}{\hbar^2} I_{1}(E,n_{z}) \]  \tag{242}

The EMME can be expressed using (242) as

\[ m^*(E_F,n_{z}) = \sqrt{m^*_1 m^*_2} \left[ I_{1}(E,n_{z}) \right] \]  \tag{243}

where \( \left[ I_{1}(E,n_{z}) \right]' = \frac{\partial}{\partial E} \left[ I_{1}(E,n_{z}) \right] \).

Therefore, the EMME according to this model is a function of Fermi energy and size quantum number due to the presence of band non-parabolicity only.
The DOS function per sub band can be written as

\[ N_{2D}(E) = \frac{4}{\pi} \sqrt{\frac{m_i m_s}{\hbar^2}} \left\{ I_1(E, n_z) \right\}' \]  \hspace{1cm} (244)

The total DOS function assumes the form

\[ N_{2DT}(E) = \frac{4}{\pi} \sqrt{\frac{m_i m_s}{\hbar^2}} \sum_{n_{sz}} \left\{ I_1(E, n_z) \right\}' H(E - E_{n_{sz}}) \]  \hspace{1cm} (245)

where, the sub band energy \( E_{n_{sz}} \) are given by

\[ E_{n_{sz}} = \frac{\hbar^2}{2m_s} \left( \frac{\pi n_z}{d_z} \right)^2 \left[ 1 - e^{\left( \frac{\hbar^2}{2m_s} \right) \left( \frac{m_i}{d_z} \right)^2} \right] \]  \hspace{1cm} (246)

The electron statistics can be written as

\[ n_{2D} = \frac{4}{\pi} \sqrt{\frac{m_i m_s}{\hbar^2}} \sum_{n_{sz}} \left[ t_{46}(E_{F_s}, n_z) + t_{47}(E_{F_s}, n_z) \right] \]  \hspace{1cm} (247)

where \( t_{46}(E_{F_s}, n_z) \equiv I_1(E_{F_s}, n_z) \), \( t_{47}(E_{F_s}, n_z) \equiv \sum R \left( t_{46}(E_{F_s}, n_z) \right) \)

### 2.2.12 The EMME from UFs of Gallium Antimonide

The 2D electron dispersion relation in this case assumes the form

\[ \frac{\hbar^2 k_x^2}{2m_s} + \frac{\hbar^2}{2m_c} \left( n_x \pi / d_z \right)^2 = I_{36}(E) \]  \hspace{1cm} (248)

Using (248), the EMME in x-y plane for this case can be written as

\[ m^*(E_{F_s}) = m_c \left\{ I_{36}(E_{F_s}) \right\}' \]  \hspace{1cm} (249)

It appears that the EMME in this case is a function of Fermi energy alone and is independent of size quantum number.

The total 2D density-of-states function can be written as

\[ N_{2DS}(E) = \left( \frac{m_g e_s}{\pi \hbar^2} \right) \sum_{n_{sz}} \left\{ I_{36}(E) \right\}' H(E - E_{n_{sz}}) \]  \hspace{1cm} (250)

where, the sub band energies \( E_{n_{sz}} \) can be expressed as

\[ I_{36}(E_{n_{sz}}) = \frac{\hbar^2}{2m_c} \left( n_x \pi / d_z \right)^2 \]  \hspace{1cm} (251)

The 2D carrier concentration assumes the form
\[ n_{2D} = \frac{m^* g_F}{\pi \hbar^2} \sum_{n_i=1}^{n_{\text{max}}} \left[ t_{55}(E_{Fz}, n_z) + t_{56}(E_{Fz}, n_z) \right] \]

where \( t_{55}(E_{Fz}, n_z) \equiv I_{36}(E_{Fz}) - \frac{\hbar^2}{2m^*} \left( \frac{n_z \pi}{d_z} \right)^2 \) and \( t_{56}(E_{Fz}, n_z) \equiv \sum_{r=1}^{N} L(r) t_{55}(E_{Fz}, n_z) \)

2.3 Results and Discussions:

Using the appropriate equations and taking the energy band constants as given in the table 1, we have plotted the EMME in UFs of Cd$_3$As$_2$ as a function of film thickness and have been shown in Fig. 2.1. For comparison, we have also plotted the EMME in the absence of the crystal field splitting for the three and the two band models of Kane. Fig. 2.1 exhibits the effect of size quantization on the EMME in general, and bears a good amount of discussion. It appears that the effect of van Hove singularity makes the EMME to suffer severe discontinuities. Assuming a carrier degeneracy of $10^{15} \text{ m}^{-2}$, fig. 2.1a shows that the EMME can reach up to about 10% of its free mass at a film thickness of 5 nm, which is quite high from its bulk value and may degrade the carrier mobility to a great extent. In the same figure, we have also demonstrated the effect of assuming only the lowest level sub-band. It appears that with this approximation, the EMME approaches to the bulk value $m_i^* = 0.0139 m_0$ more quickly than that of by considering the sub-bands. With this, it is now more obvious to note that the assumption of a single sub-band occupancy throughout leads to the practical approach to the determination of EMME. All the models of the single sub-band occupied curves tend to merge with the bulk value near 50 nm thickness. The increase in the EMME with the reduction of film thickness is due to the increased Fermi energy of the material. It must be noted that with such a highly doped system, the Fermi energy is determined by the carrier statistics equation. It is this Fermi energy which should be used in the determination of the EMME. This is not with the case in an intrinsic material. In such a case, the Fermi energy coincides with the intrinsic energy level, which is very near to the energy band gap of the material and thus the variation of the energy band gap with the film thickness needs a great concern. The variation of the energy band gap however is significant at extremely narrow film thickness more, in the region below sub-4 nm. Thus all the curves below such thickness are expected to suffer deviation with our existing theoretical model, if plotted. In all the subsequent geometry dependent curves, we have restricted ourselves above sub-4 nm regime. Since Cd$_3$As$_2$ crystals are usually grown as degenerate n-type specimens, the Fermi level mass will be the effective mass of consideration for
Fig. 2.1a: Plot of the EMME as function of film thickness for UFs of n-Cd$_3$As$_2$. The plots for three and two band models of Kane have also been exhibited in which, $m^*_{1} = 0.0139m_0$ and $m^*_{c} = \frac{1}{2}(m^*_{11} + m^*_{12})m_0 = 0.0105m_0$ are the corresponding bulk values.

Fig. 2.1b: Plot of the EMME as function of film thickness for UFs of n-Cd$_3$As$_2$ for all cases of Fig. 2.1a at different three sub-band levels.
transport in Cd$_3$As$_2$. Hence in the quantum limit, the effective mass at the Fermi level corresponding to the lowest electric sub-band will be the effective conductivity mass for electron transport in Cd$_3$As$_2$.

It appears from these figures that the Fermi level mass is significantly influenced by the effects of size quantization particularly in tetragonal materials like n- Cd$_3$As$_2$ having crystalline field-effects and energydependent anisotropy of the effective mass. It has been found that the effective mass at the Fermi level depends on the size quantum number due to the combined influence of crystal-field splitting and the anisotropic spin orbit splitting constants, resulting in different effective masses at the Fermi level corresponding to different electric sub-bands (the different effective masses being the same in the absence of field splitting as can be seen from the Figs. 2.1a and 2.1b). It has further been observed that the different effective masses corresponding to different electric sub-bands closely approach each other, for a given film thickness, with increasing electron concentration and, for a given electron concentration with increasing film thickness.

![Graph](image.png)

Fig. 2.2: Plot of the EMME as function of film thickness for UFs of n-CdGeAs$_2$ for all the cases of Fig. 2.1 in which, $m_z^* = 0.039m_0$ and $m_e = \frac{1}{2}(m_{1,2}^* + m_{3}^*)m_0 = 0.0105m_0$ are the corresponding bulk values.
These are in conformity with expectations since both with increasing electron concentration at a given film thickness and with increasing film thickness for a given electron concentration, the effects of size quantization gradually become less and less significant. As in bulk specimens, the Fermi level mass increases with increasing carrier concentration at a given value of the film thickness. Besides, for particular values of the film thickness and electron concentration, the combined effect of $\delta \neq 0$ and $\Delta_{11} \neq \Delta_{\perp}$ effect of crystal-field splitting is to reduce the effective mass corresponding to any particular sub-band. It may further be noted that if the direction normal to the film is taken as one of the transverse directions of the single ellipsoid at the zone centre and not as the longitudinal direction as assumed in the present section, the effective mass at the Fermi level corresponding to any given sub-band would be somewhat different.

![Graph](image)

Fig. 2.3: Plot of the EMME as function of film thickness for UFs of n-InAs in accordance with the three and the two band models of Kane.

Nevertheless, since the mass anisotropy in Cd$_3$As$_2$ is indeed small as can be seen from the values of $P_{11}$ and $P_{\perp}$ which are very close to each other, the arbitrary choice of the direction normal to the film with respect to the major axis of the ellipsoid would not
result in a significant change in the effective mass at the Fermi level corresponding to a particular sub-band. The Fermi level mass should gradually become closer to that of bulk specimens with increasing film thickness since, for such thicknesses, the effects of size quantization are greatly diminished. This has also been confirmed in our present work. Furthermore, the general features of the effects of size quantization on the effective mass as discussed here would also be valid with the only exception that the effective mass at the Fermi level will be independent of the size quantum number in the absence of crystal-field splitting and anisotropic spin orbit splitting constant for the III-V small-gap materials since these materials have non-parabolic energy bands obeying Kane's dispersion relation and the present chapter is based on the generalized Kane's model.

Fig. 2.2 exhibits the plot EMME in UF of n-CdGeAs$_2$ as a function of film thickness in accordance with the three and two band models of Kane together with the incorporation of the crystal field parameter. It appears that the effect of the crystal field splitting increases the EMME sharply below sub-20 nm. The EMME also increases about 7% at 5 nm and converges to its bulk value beyond 20 nm at the same value of electron degeneracy. The effect of film thickness on the EMME of III-V materials, most important with respect to extremely low field high mobility of which are n-InAs, n-InSb and n-GaAs has been exhibited in figs. 2.3, 2.4 and 2.5 respectively.

The effect of non-linearity of the energy band structure on the respective EMMEs has been clearly indicated. It appears that in the determination of the EMME, it is sufficient to take the two band model of Kane to explain the variation of the EMME over a wide range of thickness. The deviation from the three band model of Kane is much less; indicating that the complexity in the energy band model can be reduced to large extent by considering only two band models of Kane. This is extremely important with respect to the numerical computation in device analyses performance where sufficient longer computation time affects the efficiency in characterizing the compact model with respect to the said materials. In all the Figs. 2.3-2.5, we have demonstrated the effect of two widely known models viz. the three and the two band models. Figs. 2.6 and 2.7 exhibit the variation of the EMME with respect to the film thickness for the ternary and quaternary materials at same carrier degeneracy level. It appears that at an alloy composition x=0.3, the EMME in both the cases tends to about 0.1 times the rest mass at film thickness of 5 nm. Th effect of variation of EMME on the alloy composition for these two materials has been exhibited in Fig. 2.8.
Fig. 2.4: Plot of EMME as function of film thickness for UFs of n-InSb for all the cases of Fig. 2.3.

Fig. 2.5: Plot of the EMME as function of film thickness for UFs of n-GaAs for all the cases of Fig. 2.3.
Fig. 2.6: Plot of the EMME as function of film thickness for UFs of n-Hg$_{0.3}$Cd$_{0.7}$Te for all the cases of Fig. 2.3.

Fig. 2.7: Plot of the EMME as function of film thickness for UFs of n-In$_{1-x}$Ga$_x$As$_{1-y}$P$_{1-y}$ for all the cases of Fig. 2.3.
The effect of increasing the alloy composition increases the EMME for the said two materials. For the purpose of comparison, we have also plotted the the variation of the bulk effective mass with the alloy composition. For the quaternary material, the difference between the two energy band model is not much, as also can be seen from Figs. 2.6 and 2.7. The increment in EMME is rather linear in case of InGaAsP than that of HgCdTe. This also exhibits the variation of the electron mobility in these systems as the alloy composition changes. It appears that with increase in $x$, the mobility falls down assuming a constant relaxation rate. The effect of carrier degeneracy on the EMME in nonlinear optical, III-V, ternary and quaternary materials have been exhibited in the Figs. 2.9-2.15. It appears that the EMME for all the aforementioned materials at 10 nm film thickness are almost invariant below sub $10^{15}$ m$^{-2}$. The effect of inclusion of both the higher order sub-bands and lowest sub-band has been exhibited. From all the curves, it appears that the EMME bears almost exponential relation with the carrier degeneracy.

The variation of the EMME in II-VI materials like p-CdS has been exhibited in Figs. 2.16 and 2.17 as functions of film thickness and Fermi energy respectively. In these two figures, instead of obtaining the Fermi energy from the corresponding carrier statistics, we have followed the opposite route, i.e., what values of the Fermi energy makes the
Fig. 2.9a: Plot of the EMME as function of surface electron concentration in UFs of n-Cd$_3$As$_2$. The plots for three and two band models of Kane have also been exhibited in which, $m_1^* = 0.0139m_0$ and $m_z = \frac{1}{2}(m_1^* + m_2^*)m_0 = 0.0105m_0$ are the corresponding bulk values.

Fig. 2.9b: Plot of the EMME as function of surface electron concentration in UFs of n-Cd$_3$As$_2$ at different sub-band levels for all cases of Fig. 2.9a.
Fig. 2.10: Plot of the EMME as function of surface electron concentration in UFs of n-CdGeAs$_2$ for all the cases of Fig. 2.9.

Fig. 2.11: Plot of the EMME as function of surface electron concentration in UFs of n-InAs.
Fig. 2.12: Plot of the EMME as function of surface electron concentration in UFs of n-InSb.

Fig. 2.13: Plot of the EMME as function of surface electron concentration in UFs of n-GaAs.
Fig. 2.14: Plot of the EMME as function of surface electron concentration in UFs of n-HgCdTe.

Fig. 2.15: Plot of the EMME as function of surface electron concentration in UFs of n-InGaAsP.
EMME to be very low or very high. A corresponding concentration of that order can then be evaluated. A decision of this kind aids a good amount of estimation in the optimization. Using this approach, we estimate that the EMME can soar up to 0.77 times rest mass in the higher valley, while for the lower valley, it may plunge up to about 0.55 times rest mass.

The effect of valley degeneracy as we see from this two curves expresses much in the understanding the electron transport direction. It appears from the two curves that the channel oriented along the lower valley direction will most probably result in an increased value of current due to the low EMME. Figs. 2.18 and 2.19 exhibit the effect of film thickness and the carrier concentration on the EMME of UFs of Bismuth. The effect of increasing the carrier degeneracy has also been exhibited in Fig. 2.18. It appears that the EMME increases from its corresponding bulk value sharply at the 5 nm film thickness implying a tremendous decrease in the carrier mobility. Fig. 2.19 exhibits the effect of different energy band model of Bi on EMME for a varying surface electron concentration. It appears that at the lowest sub-band energy level, there is almost no difference between the Mc-Clure and Cohen model extracted EMME, however there is a significant change in the Hybrid and Lax ellipsoidal model.

Figs. 2.20, 2.21 and 2.22 exhibit the variation of the EMME at the lowest sub-band level for QWs of IV-VI, strained InSb and Ge. The effective mass in IV-VI materials exhibits strong variation for PbTe, an excellent thermoelectric material, whereas least for PbSnSe. It also appears that the EMME of PbTe is higher than that of PbSnSe and PbSnTe. With the advent of strained quantum effect devices, the analysis in EMME in strained quantum wells becomes very much important. It appears that the compressive and tensile strain does not tend to modify the respective magnitude of the EMME in strained quantum wells of InSb. It should be noted that the EMME in Fig. 2.21 has been evaluated by considering the momentum matrix element $B_2 = 0.9 \text{ eVnm}$. This is a bulk value. However, an arbitrary increase in this geometry dependent parameter sufficiently reduces the EMME and finds thus extensive use in strained film transistors. The variation of the EMME in Ge has been exhibited in Fig. 2.22 as function of film thickness for the model of Cardona et. al. The general trend of increase in the EMME has also been exhibited here at least 6 times the bulk value $\sqrt{m_1 m_2}$ for three different carrier concentration levels.
Fig. 2.16: Plot of the EMME as function of film thickness in UFs of p-CdS in two different conduction band valleys.

Fig. 2.17: Plot of the EMME as function of Fermi energy for all the cases of Fig. 2.16.
Fig. 2.18: Plot of the EMME as function of film thickness in UFsl of Bismuth for different carrier concentration values using the Hybrid model.

Fig. 2.19: Plot of the EMME at the lowest sub-band as function of surface electron concentration in UFsl of Bismuth using the Mc Clure, Cohen, Hybrid and Lax energy band models.
Fig. 2.20: Plot of the EMME at the lowest sub-band as function of film thickness in UFs of IV-VI materials.

Fig. 2.21: Plot of the EMME at the lowest sub-band as function of film thickness in UFs of uniaxial strained InSb
We observe that considering the various sub-band levels, the EMME exhibits a step-functional decreasing dependence with increase in film thickness for UFs of all the single valley materials. The combined influence of the anisotropies of the energy band constants and the crystal field splitting is to enhance the EMME as compared with the corresponding which is based on two band model of Kane in the whole range of thicknesses as considered in Fig. 2.1. The periodicity with respect to the film thickness is same in both the cases and is invariant of the energy band constants.

The influence of quantum confinement is immediately apparent from Figs. 2.1-2.7 and 2.9-2.15 since the EMME depends strongly on the thickness of the quantum-confined materials in contrast with the corresponding bulk specimens. The EMME changes with increasing carrier concentration suffering discontinuities with different numerical magnitudes. It appears from the aforementioned figures that the EMME exhibits spikes for particular values of film thickness which, in turn, depends on the particular band structure of the specific semiconductor. Moreover, the EMME from QWs of different compounds can be smaller than of bulk specimens of the same materials having multi valley conduction band like in case of p-CdS, which is also a direct signature of quantum confinement. This effect of the discontinuity on the EMME will be less and less prominent with increasing film thickness. For bulk specimens of the same material, the EMME will be found to increase continuously with increasing electron
degeneracy in a non-oscillatory manner. The appearance of the discrete jumps in the respective figures is due to the redistribution of the electrons among the quantized energy levels when the size quantum number corresponding to the highest occupied level changes from one fixed value to the others.

With varying electron degeneracy, a change is reflected in the EMME through the redistribution of the electrons among the size-quantized levels. It may be noted that at the transition zone from one sub band to another, the height of the peaks between any two sub-bands decreases with the increasing in the degree of quantum confinement and is clearly shown in the respective figures. It should be noted that although, the EMME changes in various manners with all the variables as evident from all the figures, the rates of variations are totally band-structure dependent. Our theoretical formulation comprises the dispersion relation dependent properties of various technologically important quantum-confined materials having different band-structures. The nature of variations of the EMME as shown here would be similar for the other types of materials and the simplified analysis of this chapter exhibits the basic qualitative features of the EMME.
3.1 Introduction

It is well known that the electrons in bulk materials in general, have three dimensional freedom of motion. When, these electrons are confined in a one dimensional potential well whose width is of the order of the carrier wavelength, the motion in that particular direction gets quantized while that along the other two directions remains as free. Thus, the energy spectrum appears in the shape of discrete levels for the one dimensional quantization, each of which has a continuum for the two dimensional free motion. The transport phenomena of such one dimensional confined carriers have recently studied [145-164] with great interest. For the metal-oxide-semiconductor (MOS) structures, the work functions of the metal and the semiconductor substrate are different and the application of an external voltage at the metal-gate causes the change in the charge density at the oxide semiconductor interface leading to a bending of the energy bands of the semiconductor near the surface. As a result, a one dimensional potential well is formed at the semiconductor interface. The spatial variation of the potential profile is so sharp that for considerable large values of the electric field, the width of the potential well becomes of the order of the de Broglie wavelength of the carriers. The Fermi energy, which is near the edge of the conduction band in the bulk, becomes nearer to the edge of the valance band at the surface creating inversion layers. The energy levels of the carriers bound with in the potential well get quantized and form electric sub-bands. Each of the sub-bands corresponds to a quantized level in a plane perpendicular to the surface leading to a quasi two dimensional electron gas. Thus, the extreme band bending at low temperature allows us to observe the quantum effects at the surface.

In section 3.2.1 of the theoretical background, the EMME in n-channel inversion layers of tetragonal materials has been investigated for both weak and strong electric field limits. The section 3.2.2 contains the results for n-channel inversion layers of III-V, ternary and quaternary compounds for both the electric field limits whose bulk electrons obey the three and the two band models of Kane together with parabolic energy bands and they form the special cases of section 3.2.1. The section 3.2.3 contains the study of the EMME for n-channel inversion layers of II-VI materials. The sections 3.2.4 and 3.2.5 contains the study of the EMME in n-channel inversion layers of IV-VI and stressed materials for both the limits respectively. The sections 3.2.6 and 3.2.7 contains the study
of the EMME in n-channel inversion layers of Ge and GaSb for both the limits respectively. The 3.3 contains the results and discussion of this section.

3.2. Theoretical background

3.2.1 Formulation of the EMME in n-channel inversion layers of Non-linear optical materials

In the presence of a surface electric field $F_s$ along z direction and perpendicular to the surface, (2) assumes the form

$$
\psi_1(E) = \psi_2(E) + \psi_3(E) F_s z k_z^2,
$$

where, for this chapter, $E$ represents the electron energy as measured from the edge of the conduction band at the surface in the vertically upward direction.

The quantization rule for inversion layers is given by [149]

$$
\int_0^\infty k_z dz = \frac{2}{3} (S_i)^{3/2}
$$

where, $S_i$ is the classical turning point and $S_i$ is the zeros of the Airy function $\text{Ai}(-S_i) = 0$.

Using (253) and (254), under the weak electric field limit, one can write,

$$
\int_0^\infty \sqrt{A_s(E) - |e| F_s z D_s(E)} dz = \frac{2}{3} (S_i)^{3/2}
$$

in which, $z_i \equiv \frac{A_s(E)}{|e| F_s D_s(E)}$, $A_s(E) \equiv \frac{\psi_1(E) - \psi_2(E) k_z^2}{\psi_3(E)}$, $D_s(E) \equiv [B_s(E) - A_s(E) C_s(E)]$, $B_s(E) \equiv \frac{\psi_3(E)'}{\psi_3(E)}$, $C_s(E) \equiv \left[\frac{\psi_5(E)}{\psi_3(E)}\right]'$ and

$$
C_s(E) = \left[\frac{\psi_5'(E)}{\psi_3(E)}\right].
$$
Thus, the 2D electron dispersion law in n-channel inversion layers of tetragonal materials under the weak electric field limit can approximately be written as

\[ \psi_1(E) = P_1(E,i) k^2_x + Q_1(E,i) \]  

(256)

where,

\[ P_1(E,i) = \left[ \psi_2(E) - \left( \frac{2 \psi(E)}{3 \psi(E)} \right) \psi_3(E) S_i \right] \]

and

\[ Q_1(E,i) = S_i \psi_4(E) \left| e | F_r \right| \]  

\[ \left( E_E \right)^{2/3} \]

The EMME in the x-y plane can be expressed as

\[ m^*(E_{Fw}, i) = \left( \frac{\hbar^2}{2} \right) G_1(E,i) \]  

(257)

where,

\[ G_1(E,i) = \left[ P_1(E,i) \right]^2 \left( \psi_1(E) - \left( Q_1(E,i) \right)' - \left( Q_1(E,i) \right) \right) \]

and \( E_{Fw} \) is the Fermi energy under the weak electric field limit as measured from the edge of the conduction band at the surface in the vertically upward direction.

Thus, we observe that the EMME is the function of sub-band index, the Fermi energy and other band constants due to the combined influence of the crystal filed splitting constant and the anisotropic spin-orbit splitting constants respectively.

The sub-band energy \( E_{n+1} \) in this case can be obtained from (256) as

\[ \psi_1(E_{n+1}) = Q_1 \left( E_{n+1}, i \right) \]  

(258)

The general expression of the 2D total density-of-states function in this case can be written as

\[ N_{2D}(E) = \frac{2g_s}{(2\pi)^2} \sum_{n=0}^{\infty} \frac{\partial}{\partial E} \left[ A(E,i) H \left( E - E_n \right) \right] \]  

(259)
where, \( A(E, i) \) is the area of the constant energy 2D wave vector space for inversion layers and \( E_n \) is the corresponding sub-band energy.

Using (256) and (259), the total 2D density-of-states function under the weak electric field limit can be expressed as

\[
N_{2D}(E) = \frac{g_s}{(2\pi)^2} \sum_{r=0}^{\infty} \left[ G_r(E, i) H(E - E_{n_r}) \right]
\]

(260)

where, \( E_{n_r} \) is the sub-band energy for the weak electric field limit in this case.

Using (260) and the Fermi-Dirac occupation probability factor, the 2D surface electron concentration in n-channel inversion of tetragonal materials under the weak electric field limit \((n_{2DW})\) can be written as

\[
n_{2DW} = g_s \frac{(2\pi)^2}{(2\pi)^2} \sum_{r=0}^{\infty} \left[ P_{rw}(E_{Fm}, i) + Q_{rw}(E_{Fm}, i) \right]
\]

(261)

where, \( P_{rw}(E_{Fm}, i) = \left[ \psi_i(E_{Fm}, i) - Q_i(E_{Fm}, i) \right] \left[ P_i(E_{Fm}, i) \right]^{-1} \) and

\[
Q_{rw}(E_{Fm}, i) = \sum_{r=0}^{\infty} \left[ L(r) \left[ P_i(E_{Fm}, i) \right] \right].
\]

\[
F_s = \frac{|e| n_{2DW}}{\varepsilon_{sc}} \quad \text{and} \quad \varepsilon_{sc} \quad \text{is the semiconductor permittivity.}
\]

Using (253) and (254), the 2D electron dispersion law in n-channel inversion layers of tetragonal materials under the strong electric field limit can be written as

\[
k_s^2 = P_2(E, i)
\]

(262)

\[
P_2(E, i) = \left[ F_7(E) \right]^{-1} \left[ F_6(E) - F_6(E, i) \right],
\]

\[
F_6(E) = \left[ \psi_i(E) \right] \left[ \psi_i(E) \right]^T \left( \psi_i(E) \right)^T + \frac{\psi_i(E) \left[ \psi_i(E) \right]^T}{\psi_i(E) \psi_i(E)^T + 2 \left[ \psi_i(E) \right]^T \left[ \psi_i(E) \right]}
\]

104
$$F_1(E) = \left[ \frac{\psi_3(E)}{\psi_2(E)} \right] - \left( \frac{\psi_1(E)}{2[\psi_1(E)]^* \psi_3(E)} \right) + \left( \frac{\psi_2(E)}{\psi_5(E)} \right) \left[ \frac{\psi_3(E)}{\psi_3(E)} + \frac{\psi_1(E)}{2[\psi_1(E)]^* \psi_3(E)} \right] + \left( \frac{\psi_1(E)}{\psi_3(E)} \right) \left[ \frac{\psi_2(E)}{\psi_5(E)} \right] \left[ \frac{\psi_3(E)}{\psi_3(E)} + \frac{\psi_1(E)}{2[\psi_1(E)]^* \psi_3(E)} \right]$$

and $F_s(E)$ is defined as

$$F_s(E) = \frac{2\sqrt{2}}{3}(S)^{\frac{1}{2}} |\mu F_s| \sqrt{\psi_1(E)}$$

The EMME in the x-y plane can be written in this case as

$$m^*(E_{fu}, i) = \left( \frac{k^2}{2} \right) \left[ P_i(E_{fu}, i) \right]$$

where, $E_{fu}$ is the Fermi energy under the strong electric field limit as measured from the edge of the conduction band at the surface. Thus, we note that the EMME is the function of sub-band index and the Fermi energy due to the combined influence of the crystal field splitting constant and the anisotropic spin-orbit splitting constants respectively.

The sub-band energy $E_{n_{su}}$ in this case can be obtained from the (262) as

$$P_2(E_{n_{su}}, i) = 0 \quad (264)$$

The total 2D density-of-states function under the strong electric field limit can be written as

$$N_{2D}(E) = \frac{g_s}{(2\pi)} \sum_{n_{su}} \left[ P_i(E_{n_{su}}, i) \right] H(E - E_{n_{su}})$$

Using (265) and the Fermi-Dirac occupation probability factor, the 2D surface electron concentration $n_{2D}$ in this case can be expressed as

$$n_{2D} = g_s \left( \frac{2\pi}{4} \right) \sum_{n_{su}} \left[ P_i(E_{n_{su}}, i) + \Omega_{2s}(E_{n_{su}}, i) \right]$$

105
where, \( Q_2(E_{\nu}, i) = \sum_{r=1}^{L} L(r) [P_3(E_{\nu}, i)] \).

### 3.2.2 Formulation of the EMME in n-channel inversion layers of III-V, ternary and quaternary materials

Using the substitutions \( \delta = 0, \Delta_0 = \Delta = \Delta_0 \) and \( m_0^* = m_1^* = m_c \), (266) under the condition of weak electric field limit, assumes the form

\[
I_{11}(E) = \frac{\hbar^2 k_z^2}{2m_c} + \sum_i \left[ \frac{\hbar |e| F_z [I_{11}(E)]}{\sqrt{2m_c}} \right]^{2/3}
\]  

(267) represents the dispersion relation of the 2D electrons in n-channel inversion layers of III-V, ternary and quaternary materials under the weak electric field limit whose bulk electrons obey the three band model of Kane.

The EMME can be expressed as

\[
m^*(E_{\nu}, i) = m_c [P_3(E, i)]_{E=E_{\nu}}
\]

where,

\[
P_3(E, i) = \left[ I_{11}(E) \right]' - \frac{2}{3} S_i \left[ \frac{\hbar |e| F_z}{\sqrt{2m_c}} \right]^{2/3} \left[ I_{11}(E) \right]^{1/3} \left[ I_{11}(E) \right]^{1/3}
\]

Thus, one can observe that the EMME is a function of the sub-band index, surface electric field, the Fermi energy and the other spectrum constants due to the combined influence of \( E_g \) and \( \Delta \). The sub-band energy \( E_{n_{\nu1}} \) in this case can be obtained from (267) as

\[
I_{11}(E_{n_{\nu1}}) = \sum_i \left[ \frac{\hbar |e| F_z [I_{11}(E_{n_{\nu1}})]}{\sqrt{2m_c}} \right]^{2/3}
\]

(269)

Using (259) and (267), the 2D total density-of-states function in weak electric field limit can be expressed as

\[
N_{2D}(E) = \frac{m_c g_v}{\pi \hbar^2} \sum_{\nu=0}^{\infty} P_3(E, i) H(E - E_{n_{\nu1}})
\]

(270)
Using (270) and the occupation probability, the $n_{Dw}$ in the present case can be written

\[ n_{Dw} = \frac{g_e m_e}{\pi \hbar^2} \sum_{i=0}^{\infty} \left[ P_{4w} (E_{Fw}, i) + Q_{4w} (E_{Fw}, i) \right] \]  

(271)

where, \( P_{4w} (E_{Fw}, i) \equiv I_{11} (E_{Fw}) - S_i \left[ \frac{\hbar e F_w}{\sqrt{2m_e}} \right] \left[ I_{11} (E_{Fw}) \right]^2 \) and \( Q_{4w} (E_{Fw}, i) \equiv \sum_{i=1}^{t} \left[ L (r) \left[ P_{4} (E_{Fw}, i) \right] \right]\).

Using the substitutions \( \delta = 0, \Delta = - \Delta = \Delta \) and \( m_{\perp}^* = m_{\parallel}^* = m_e \), (262) under the condition of strong electric field limit, assumes the form

\[ I_{11} (E) - \frac{\left[ I_{11} (E) \right]^2}{\sqrt{2m_e}} \left[ I_{11} (E) \right]^2 = \frac{\hbar^2 k_i^2}{2m_e} \]  

(272)

(272) represents the dispersion relation of the 2D electrons in n-channel inversion layers of III-V, ternary and quaternary materials under the strong electric field limit whose bulk conduction electrons are defined by the three band model of Kane.

The EMME can be expressed as

\[ m^* (E_{Fw}, i) = m_e \left[ P_5 (E, i) \right]_{E_{Fw}} \]  

(273)

where, \( P_5 (E, i) \equiv \left\{ I_{11} (E) \right\}^2 - \frac{\left[ I_{11} (E) \right]^2}{\sqrt{2m_e}} \left[ I_{11} (E) \right]^2 \left[ I_{11} (E) \right]^2 \left[ I_{11} (E) \right]^2 \) \].

Thus, one can observe that the EMME is a function of the sub-band index, surface electric field, the Fermi energy and the other spectrum constants due to the combined influence of \( E_g \) and \( \Delta \).

The sub-band energy \( E_{n_{11}} \) in this case can be obtained from (272) as

\[ I_{11} (E_{n_{11}}) = \left[ \frac{\left[ I_{11} (E_{n_{11}}) \right]^2 \left[ I_{11} (E_{n_{11}}) \right]^2 \left[ I_{11} (E_{n_{11}}) \right]^2 \right]}{2m_e} \right] = 0 \]  

(274)
Using (259) and (272), the total 2D density-of-states function under the strong electric field limit can be expressed as

\[ N_{2D}(E) = \frac{g_m g_v}{\pi \hbar^2} \sum_{i=0}^{\infty} \left[ P_5(E, i) H(E - E_{\text{li}}) \right] \]  \hspace{1cm} (275)

Using (275) and the Fermi-Dirac occupation probability factor, the \( n_{2D} \) in the present case under the strong electric field can be written as

\[ n_{2D} = \frac{g_m g_v}{\pi \hbar^2} \sum_{i=0}^{\infty} \left[ P_6(E_{\text{li}}, i) + Q_{6a}(E_{\text{li}}, i) \right] \]  \hspace{1cm} (276)

where, \( P_6(E_{\text{li}}, i) = \left[ I_{11}(E_{\text{li}}) - \frac{2\sqrt{2}}{3} S_i \frac{\hbar|e| F_i}{\sqrt{2m_c}} \right] \) and

\[ Q_{6a}(E_{\text{li}}, i) = \sum_{r=4}^{\infty} \left[ L(r) \left[ P_6(E_{\text{li}}, i) \right] \right] \]

Using the constraints \( \Delta \ll E_\delta \) or \( \Delta \gg E_\delta \), (267) under the low electric field limit assumes the form

\[ E(1 + \alpha E) = \frac{\hbar^2 k_i^2}{2m_c} + S_i \left[ \frac{\hbar|e| F_i (1 + 2\alpha E)}{\sqrt{2m_c}} \right] \]  \hspace{1cm} (277)

For large values of \( i, S_i \rightarrow \left[ \frac{3\pi}{2} \left( i + \frac{3}{4} \right) \right] ^{2/3} \) \hspace{1cm} [149] and (277) gets simplified as

\[ E(1 + \alpha E) = \frac{\hbar^2 k_i^2}{2m_c} + \left[ \frac{3\pi \hbar|e| F_i}{2} \left( i + \frac{3}{4} \right) \left( 1 + 2\alpha E \right) \right] ^{2/3} \]  \hspace{1cm} (278)

(278) was derived for the first time by Antcliffe et. al. [147].

The EMME in this case is given by

\[ m^*(E_{\text{li}}, i) = m_c \left[ P_6(E, i) \right]_{E=E_{\text{li}}} \]  \hspace{1cm} (279)

where, \( P_6(E, i) = \left\{ (1 + 2\alpha E) - \frac{4\alpha}{3} S_i \left[ \frac{\hbar|e| F_i}{\sqrt{2m_c}} \right] ^{2/3} \right\} ^{2/3} \).
Thus, one can observe that the EMME is a function of the sub-band index, surface electric field and the Fermi energy due to the presence of band nonparabolicity only.

The sub-band energies $E_{n_3}$ are given by

$$E_{n_3} = S_i \left[ \frac{\hbar |e| F_i}{\sqrt{2m_c}} (1 + 2\alpha E_{n_3}) \right]^{2/3}$$  \hspace{1cm} (280)

The total 2D density-of-states function can be written as

$$N_{2D}(E) = \frac{m g_s}{\pi \hbar^2} \sum_{i=0}^{\infty} \left[ 1 + 2\alpha E - \frac{4\alpha}{3} S_i \left[ \frac{\hbar |e| F_i}{\sqrt{2m_c}} \right]^{2/3} (1 + 2\alpha E)^{-1/3} \right] H(E - E_{n_3})$$  \hspace{1cm} (281)

Under the condition $\alpha E \ll 1$, the use of (281) and the Fermi-Dirac integral leads to the expression of $n_{2Dw}$ as

$$n_{2Dw} = \left( \frac{g_s m_c k_B}{\pi \hbar^2} \right) \sum_{i=0}^{\infty} \left[ 1 + D_i + 2\alpha E_{n_3} \right] F_0(\eta_w) + 2\alpha k_B T \left[ \eta_w \right]$$  \hspace{1cm} (282)

where, 

$$D_i = \frac{4\alpha S_i}{3} \left( \frac{\hbar |e| F_i}{\sqrt{2m_c}} \right)^{2/3}$$

$$\eta_w = \left[ \frac{E_{n_3} - E^w}{k_B T} \right]$$

For all values of $\alpha E_{F^w}$, the $n_{2Dw}$ can be written as

$$n_{2Dw} = \left( \frac{g_s m_c k_B}{\pi \hbar^2} \right) \sum_{i=0}^{\infty} \left[ P_{2w}(E_{F^w}, i) + Q_{5w}(E_{F^w}, i) \right]$$  \hspace{1cm} (283)

where,

$$P_{2w}(E_{F^w}, i) = \left[ E_{F^w} (1 + \alpha E_{F^w}) - S_i \left( \frac{h |e| F_i}{\sqrt{2m_c}} (1 + 2\alpha E_{F^w}) \right)^{2/3} \right]$$

and

$$Q_{5w}(E_{F^w}, i) = \sum_{r=1}^{5} L(r) P_{2w}(E_{F^w}, i)$$

For $\alpha \to 0$, as for inversion layers, whose bulk electrons are defined by the parabolic energy bands, we can write,
\[ E = \frac{\hbar^2 k_x^2}{2m_c} + S_i \left[ \frac{\hbar |e| F_s}{\sqrt{2m_c}} \right]^{2/3} \]  

The (284) is valid for all values of the surface electric field [145].

The electric sub-band energy \( E_{n_s} \) assumes the form, from (284) as

\[ E_{n_s} = S_i \left[ \frac{\hbar |e| F_s}{\sqrt{2m_c}} \right]^{2/3} \]  

(285)

The total density-of-states function can be written as

\[ N_{2D}(E) = \frac{m_e g_e}{\pi \hbar^2} \sum_{i=0}^{i_{\text{max}}} H(E - E_{n_s}) \]  

(286)

The use of (286) leads to the expression of \( n_{2D} \) as [145]

\[ n_{2D} = \frac{g_e m_e k_F T}{\pi \hbar^2} \sum_{i=0}^{i_{\text{max}}} \tau_{_1} \eta_i \]  

(287)

where, \( \eta_i = (k_F T)^{-1} \left[ E_{F_i} - S_i \left[ \frac{\hbar |e| F_s}{\sqrt{2m_c}} \right]^{2/3} \right] \), \( E_{F_i} \) is the Fermi energy as measured from the edge of the conduction band at the surface.

Using the constraints \( \Delta \ll E_g \) or \( \Delta \gg E_g \), the (272) under the strong electric field limit assumes the form

\[ E(1 + \alpha E) = \frac{\hbar^2 k_s^2}{2m_c} + \frac{2\sqrt{2}}{3} \left( S_i \right)^{3/2} \left[ \frac{\hbar |e| F_s}{\sqrt{2m_c E_s}} \right] \]  

(288)

For large values of \( i \), \( S_i \to \left[ \frac{3\pi}{2} \left( i + \frac{3}{4} \right) \right]^{2/3} \) [149] and (288) gets simplified as

\[ E(1 + \alpha E) = \frac{\hbar^2 k_s^2}{2m_c} + \frac{\pi \hbar |e| F_s \sqrt{2}}{\sqrt{m_e E_s}} \left( i + \frac{3}{4} \right) \]  

(289)

The (289) was derived for the first time by Antcliffe et. al. [147].
From (289), we observe that under the condition $E_z \to 0$, one cannot obtain the corresponding parabolic case, since under high electric field limit, the band becomes permanently nonparabolic.

The EMME is given by

$$m^*(E_{Fu}, i) = m_c \left(1 + 2\alpha E_{Fu}\right)|_{i=0}$$

(290)

Thus, in the high electric field limit, the EMME is a function of Fermi energy due to the presence of band nonparabolicity only and is independent of the sub-band index.

The electric sub-band energy $E_{_{\text{em}i}}$ in the high electric field limit is given by

$$E_{_{\text{em}i}}(1 + \alpha E_{_{\text{em}i}}) = \frac{\pi \hbar e^2 F_i \sqrt{2}}{m_c E_z} \left(i + \frac{3}{4}\right)$$

(291)

The 2D total density-of-states function in this case can be written as

$$N_{2D}(E) = \frac{m_c \epsilon_r}{\hbar^2} \sum_{i=0}^{\infty} \left[1 + 2\alpha E \right] H \left(E - E_{\text{em}i}\right)$$

(292)

The surface electron concentration for all values of $\alpha E_{Fu}$ in this case assumes the form

$$n_{2De} = \left(\frac{\epsilon_r m_c k_BT}{\pi \hbar^2}\right) \left[1 + 2\alpha k_BT \right] F_0(\eta_{s}) + 2\alpha k_BT E_{_{\text{em}i}} F_1(\eta_{s})$$

(293)

where, $\eta_{s} = \left[\frac{E_{Fu} - E_{_{\text{em}i}}}{k_BT}\right]$.

### 3.2.3 Formulation of the EMME in n-channel inversion layers of II-VI materials

The use of (254) and (40) leads to the quantization integral as

$$\frac{\sqrt{2m^*_i}}{\hbar} \int_{z_i} \left[E - |e| F_z - \alpha_0 k_z^2 \mp (\lambda_0) k_z\right]^{1/2} dz = \frac{2}{3} (S_i)^{1/2}$$

(294)

where, $z_i = (|e| F_z)^{-1} \left[E - \alpha_0 k_z^2 \mp (\lambda_0) k_z\right]$. 

111
Therefore, the 2D electron dispersion law for n-channel inversion layers of II-VI materials can be expressed for all values of $F_i$ as

$$E = \alpha_0 k_z^2 \pm (\lambda_0) k_x + S_i \left( \frac{\hbar |e| F_i}{\sqrt{2m_i^*}} \right)^{2/3}$$  \hspace{1cm} (295)$$

The area of the 2D surface as enclosed by (295) can be expressed as

$$A(E) = \frac{\pi (m_i^*)^2}{h^2} \left[ 2(\lambda_0)^{2/3} \frac{2\hbar^2}{m_i^*} S \left( \frac{\hbar |e| F_i}{\sqrt{2m_i^*}} \right)^{2/3} \right]$$

The EMME is given by

$$m^* (E_n, i) = m_i^* \left[ \frac{1}{2} \left( \frac{\rho_n}{E_n + \rho_{\tau_2}} \right) \right]$$

where, $E_n$ is the Fermi energy in this case, $\rho_n = \frac{\lambda_0}{2 \alpha_0^2}$ and

$$\rho_{\tau_2} = \left[ (\rho_n)^{2/3} - \frac{\hbar^2}{2m_i^*} \left( \frac{|e| F_i}{2m_i^*} \right)^{2/3} \right].$$

Thus, the EMME depends on both the Fermi energy and the sub-band index due to the presence of the term $\lambda_0$.

The sub-band energy $E_{n_i}$ can be written as

$$E_{n_i} = S_i \left( \frac{\hbar |e| F_i}{\sqrt{2m_i^*}} \right)^{2/3}$$

The total 2D density-of-states function can be written as

$$N_{2D}(E) = \frac{m_i^* g_v}{\pi \hbar^2} \sum_{i=0}^{\infty} \left[ 1 - \frac{\rho_n}{\sqrt{E + \rho_{\tau_2}}} \right] H \left( E - E_{n_i} \right)$$

The surface electron concentration assumes the form
3.2.4 Formulation of the EMME in n-channel inversion layers of IV-VI materials

In the low electric field limit (63) assumes the form

\[ E(1+\alpha E) - |E| F(x)(1+2\alpha E) = \frac{p_x^2}{2M_1} + \frac{p_z^2}{2M_3} + \left( \frac{\alpha p_x^4}{4M_2M'_2} \right) + \frac{p_y^2}{2M_2} - \alpha E |E| F(x) \left( \frac{M_2}{M'_2} \right) \]  

(301)

where, \( M_1 = m_{1c} \), \( m_{1c} \) is the transverse effective electron mass at the edge of the conduction band at \( k=0 \), \( M_2 = \left( \frac{m_{1c} + 2m_e}{3} \right) \), \( m_e \) is the longitudinal effective electron mass at the edge of the conduction band at \( k=0 \), \( M_3 = \left( \frac{3m_{1c}m_e}{2m_e + m_{1c}} \right) \), \( M'_2 = \left( \frac{m_{1\nu} + 2m_{\nu}}{3} \right) \), \( m_{1\nu} \) and \( m_{\nu} \) are the effective transverse and longitudinal hole masses at the edge of the valence band at \( k=0 \).

The use of (254) and (301) leads to the simplified expression of the 2D electron dispersion law in n-channel inversion layers of IV-VI materials under the weak electric field limit as

\[ \gamma_{11}(E,i) = p_{11}k_x^2 + q_{11}(E,i)k_y^2 + r_{11}k_y^4 \]  

(302)

where, \( \gamma_{11}(E,i) \equiv \left[ E(1+\alpha E) - S_i \left( \frac{\hbar |E| F(x)}{\sqrt{2M_3}} \right)^2 \right] \left( 1 + \frac{4}{3} \alpha E \right) \), \( p_{11} \equiv \frac{\hbar^2}{2M_1} \),

\[ q_{11}(E,i) \equiv \left( \frac{\hbar^2}{2M_2} \right) \left[ 1 + \alpha E \left( 1 - \frac{M_2}{M'_2} \right) - 2\alpha S_i \left( \frac{\hbar |F(x)}{\sqrt{2M_3}} \right)^2 \left( 1 - \frac{M_2}{M'_2} \right) \right] \]  

and \( r_{11} \equiv \left( \frac{\alpha \hbar^4}{4M_2M'_2} \right) \).

The area enclosed by (302) is given by

\[ A(E,i) = \frac{4}{3} \left( \frac{r_{11}}{p_{11}} \right)^{1/2} \left( \{a_{1w}(E,i)\}^2 + \{b_{1w}(E,i)\}^2 \right)^{1/2} \]

\[ \left[ \{a_{1w}(E,i)\}^2 F \left[ \frac{\pi}{2}, \ell_{1w}(E,i) \right] - \left( \{a_{1w}(E,i)\}^2 - \{b_{1w}(E,i)\}^2 \right) E \left[ \frac{\pi}{2}, \ell_{1w}(E,i) \right] \right] \]  

(303)
in which, \( a_{1w}(E,i) \) and \( b_{1w}(E,i) \) are defined by

\[
\begin{align*}
\{a_{1w}(E,i)\}^2 &= \left[ \frac{q_{11}(E,i)}{2r_{11}} + \frac{1}{2} \left[ \frac{\{q_{11}(E,i)\}^2}{r_{11}} + \frac{4\gamma_{11}(E,i)}{r_{11}} \right]^{1/2} \right], \\
\{b_{1w}(E,i)\}^2 &= \left[ \frac{1}{2} \left[ \frac{\{q_{11}(E,i)\}^2}{(r_{11})^2} + \frac{4\gamma_{11}(E,i)}{r_{11}} \right] - \left( \frac{q_{11}(E,i)}{2r_{11}} \right) \right],
\end{align*}
\]

\( \ell_{1w}(E,i) \) is the complete elliptic integral of the first and second kinds respectively. Using (303), the EMME in this case can be expressed as

\[
m^*(E_{1w}, i) = \frac{\hbar^2}{2\pi} R_{11}(E, i)_{E = E_{1w}}
\]

where,

\[
\begin{align*}
R_{11}(E, i) &\equiv \left[ \frac{4}{3} \left( \frac{r_{11}}{p_{11}} \right)^{1/2} \right] \left[ \left\{ a_{1w}(E,i) \right\}^2 + \left\{ b_{1w}(E,i) \right\}^2 \right]^{-1/2} \\
&\quad \times \left[ \{a_{1w}(E,i)\}^2 F\left[ \frac{\pi}{2}, \ell_{1w}(E,i) \right] - \{a_{1w}(E,i)\}^2 F\left[ \frac{\pi}{2}, \ell_{1w}(E,i) \right] \right] E\left[ \frac{\pi}{2}, \ell_{1w}(E,i) \right] \\
&\quad + \left( \frac{r_{11}}{3} \right)^{1/2} \left[ \left\{ a_{1w}(E,i) \right\}^2 + \left\{ b_{1w}(E,i) \right\}^2 \right]^{1/2} \\
&\quad \times \left[ 2a_{1w}(E,i)\{a_{1w}(E,i)\}^\prime F\left[ \frac{\pi}{2}, \ell_{1w}(E,i) \right] + \{a_{1w}(E,i)\}^2 F\left[ \frac{\pi}{2}, \ell_{1w}(E,i) \right] \right] \left[ 2a_{1w}(E,i)\{a_{1w}(E,i)\}^\prime F\left[ \frac{\pi}{2}, \ell_{1w}(E,i) \right] - 2b_{1w}(E,i)\{a_{1w}(E,i)\}^\prime F\left[ \frac{\pi}{2}, \ell_{1w}(E,i) \right] \right]
\end{align*}
\]

114
Thus, the EMME is a function of the sub-band index number and the Fermi energy due to the presence of band nonparabolicity only.

The sub-band energies \( E_{n_{i}} \) are given by

\[
E_{n_{i}} \left(1 + \alpha E_{n_{i}} \right) - S \left( \frac{\hbar c F_{z}}{2M} \right)^{2/3} \left(1 + \frac{4}{3} \alpha E_{n_{i}} \right) = 0
\]  

(305)

The total 2D density-of-states function can be written as

\[
N_{2D} (E) = \frac{g_{v}}{2\pi^{2}} \sum_{i=0}^{\infty} \left\{ R_{n_{i}} (E, i) \right\} H \left( E - E_{n_{i}} \right) \}
\]

(306)

The surface electron concentration assumes the form

\[
n_{2D} (E) = \frac{2g_{v}}{3\pi^{2}} \left( \frac{r_{1}}{p_{11}} \right)^{1/2} \sum_{i=0}^{\infty} \left[ P_{n_{i}} (E_{F_{n_{i}}}, i) + Q_{n_{i}} (E_{F_{n_{i}}}, i) \right]
\]

(307)

where,

\[
P_{n_{i}} (E_{F_{n_{i}}}, i) \equiv \left[ \left( a_{n_{i}} (E_{F_{n_{i}}}, i) \right)^{2} + \left( b_{n_{i}} (E_{F_{n_{i}}}, i) \right)^{2} \right]^{1/2}
\]

\[
\left[ \left( a_{n_{i}} (E_{i}) \right)^{2} \right] F \left[ \pi \frac{r_{i}}{2} E_{F_{n_{i}}}, i \right] - \left[ \left( a_{n_{i}} (E_{F_{n_{i}}}, i) \right)^{2} - \left( b_{n_{i}} (E_{F_{n_{i}}}, i) \right)^{2} \right] E \left[ \frac{\pi}{2} r_{i} E_{F_{n_{i}}}, i \right]
\]

and

\[
Q_{n_{i}} (E_{F_{n_{i}}}, i) \equiv \sum_{r=1}^{r_{n_{i}}} L (r) P_{n_{i}} (E_{F_{n_{i}}}, i)
\]

Under the strong electric field limit, the dispersion relation assumes the form

\[
\gamma_{22} (E, i) = p_{22} p_{x}^{2} + q_{22} (E) k_{x}^{2} + r_{22} k_{y}^{4}
\]

(308)

where,

\[
\gamma_{22} (E, i) \equiv \left[ E \left( 1 + \alpha E \right) - \frac{2\sqrt{2} \alpha}{3} \frac{\hbar c F_{z}}{\sqrt{2M} \left( S_{i} \right)^{1/2}} \right], \quad P_{22} \equiv \left( \frac{\hbar^{2}}{2M} \right),
\]

\[
q_{22} (E) \equiv \left( \frac{\hbar^{2}}{2M_{2}} \right) \left[ 1 + \alpha E \left( 1 - \frac{M_{2}}{M_{2}'} \right) \right] \quad \text{and} \quad r_{22} \equiv \left( \frac{\alpha \hbar^{4}}{4M_{2}M_{2}'} \right).
\]

Comparing (302) with (308), we observe that the forms of the (302) and (308) remain unchanged provided, \( \gamma_{22} (E, i), p_{22}, q_{22} (E, i) \) and \( r_{22} \) are being replaced by the corresponding quantities \( \gamma_{22} (E, i), p_{22}, q_{22} (E) \) and \( r_{22} \) respectively.
3.2.5 Formulation of the EMME in n-channel inversion layers of stressed materials

The use of (254) and (77) leads to the expression of the dispersion relation of the 2D electrons in n-channel inversion layers of stressed III-V materials under the low electric field limit as

\[
\begin{bmatrix}
T_{57} (E,i) \\
T_{67} (E,i)
\end{bmatrix}
\begin{bmatrix}
k_x^2 \\
k_y^2
\end{bmatrix}
= T_{77} (E,i)
\]  

(309)

where, 

\[
T_{57} (E,i) = \left[ E - \alpha_1 + \frac{2}{3} S \left( \frac{|\epsilon|^2}{e^*} \right)^{2/3} \right] (n_{2i\omega})^{2/3} L_{57} (E)
\]

\[
L_{57} (E) = \left[ \frac{(E - \alpha_3)}{(E - \alpha_3)^{2/3} \left[ \bar{T}_{47} (E) \right]^{1/3}} \right] - \left( E - \alpha_1 \right)^{1/3} \left[ \bar{T}_{47} (E) \right]^{-1/3}
\]

\[
\bar{T}_{47} (E) = \left[ \left\{ \rho_5 (E) \right\}' - \left( \frac{\rho_5 (E)}{E - \alpha_3} \right) \right]
\]

\[
T_{67} (E,i) = \left[ E - \alpha_2 + \frac{2}{3} S \left( \frac{|\epsilon|^2}{e^*} \right)^{2/3} \right] (n_{2i\omega})^{2/3} L_{67} (E)
\]

\[
L_{67} (E) = \left[ \frac{(E - \alpha_3)}{(E - \alpha_3)^{2/3} \left[ \bar{T}_{47} (E) \right]^{1/3}} \right] - \left( E - \alpha_2 \right)^{1/3} \left[ \bar{T}_{47} (E) \right]^{-1/3}
\]

\[
\bar{T}_{27} (E) = \left[ \rho_3 (E) - S \left( \frac{|\epsilon|^2}{e^*} \right)^{2/3} \right] (n_{2i\omega})^{2/3} L_{27} (E)
\]

and 

\[
L_{27} (E) = (E - \alpha_3)^{1/3} \left[ \bar{T}_{47} (E) \right]^{2/3}
\]

The area of the 2D surface under the weak electric field limit can be written as

\[
A (E,i) = \frac{\pi T_{77} (E,i)}{\sqrt{T_{57} (E,i) T_{67} (E,i)}}
\]

(310)

The sub-band energies \( E_{n\omega} \) in this case are defined by
The expression of the EMME in this case can be written as

\[
m^* (E_{Fw}, i) = \frac{\hbar^2}{2} L_{47} (E, i) |_{E = E_{Fw}}
\]  

(312)

where, \( L_{47} (E, i) = \left[ \frac{1}{T_{57} (E, i) T_{67} (E, i)} \right] [T_{57} (E, i)]^{-1/2} \left( \frac{T_{77} (E, i)}{2} \right) \) 

\[
\left\{ \left[ T_{57} (E, i) \right]' \left( \frac{T_{67} (E, i)}{T_{57} (E, i)} \right)^{-1/2} + \left[ T_{67} (E, i) \right]' \left( \frac{T_{77} (E, i)}{T_{67} (E, i)} \right)^{-1/2} \right\}.
\]

The total 2D density-of-states function can be expressed as

\[
N_{2D} (E) = \frac{g_s}{2\pi} \sum_{\eta=0}^{\infty} \left\{ \rho_{2D} (E, i) H \left( E - E_{\eta\kappa} \right) \right\}
\]  

(313)

The surface electron concentration under the weak electric field limit assumes the form

\[
n_{2Dw} = \frac{g_s}{(2\pi)} \left[ \sum_{\eta=0}^{\infty} \left( \rho_{2D} (E_{Fw}, i) + Q_{8w} (E_{Fw}, i) \right) \right]
\]  

(314)

where, \( \rho_{2D} (E_{Fw}, i) = \frac{T_{77} (E_{Fw}, i)}{\sqrt{T_{57} (E_{Fw}, i) T_{67} (E_{Fw}, i)}} \) and \( Q_{8w} (E_{Fw}, i) = \sum_{r=1}^{r} L(r) P_{8w} (E_{Fw}, i) \).

The simplified dispersion relation of the 2D electrons in n-channel inversion layers of stressed III-V materials under the high electric field limit can be expressed as

\[
\left[ T_{117} (E, i) \right] k_x^2 + \left[ T_{127} (E, i) \right] k_y^2 = T_{137} (E, i)
\]  

(315)

where, \( T_{117} (E, i) = \left[ E - \alpha_i + \frac{4}{3} \left| F_i (S_i) \right|^{2/3} a_\eta \right] \),

\[
a_\eta (E) = \frac{1}{2} \left[ \frac{1}{\sqrt{\left( E - \alpha_i \right)} T_\eta (E)} \left\{ \frac{(E - \alpha_i)^2}{\sqrt{T_\eta (E)}} \right\} \right].
\]
\[
[T_{m}(E)] = \left[ \frac{\rho_3(E)}{2} \left( \frac{(\rho_3(E))'}{E-\alpha_3} \right) + \left( \frac{\rho_3(E)}{(E-\alpha_3)^2} \right) \right],
\]

\[
T_{127}(E,i) = \left[ E - \alpha_2 + \frac{4}{3} |e| F_t(S_t)^{3/2} a_{\gamma 7}(E) \right],
\]

\[
a_{\gamma 7}(E) = \frac{1}{2} \left[ \frac{1}{\sqrt{(E-\alpha_3)T_{177}(E)V}} \left( \frac{(E-\alpha_3)^{1/3}}{\sqrt{T_{177}(E)(E-T_3)^{3/2}}} \right) \right],
\]

\[
T_{137}(E,i) = \left[ \rho_3(E) - (S_t)^{3/2} F_t a_{\gamma 7}(E) \right]
\]

and \( a_{\gamma 7}(E) = \frac{4}{3} \sqrt{T_{177}(E)(E-\alpha_3)}. \)

The area of the 2D surface in this case is given by

\[
A(E,i) = \frac{\pi T_{137}(E,i)}{\sqrt{T_{117}(E,i)T_{127}(E,i)}}
\]

The sub-band energies \( \{E_{n\alpha}^i\} \) in this case can be written as

\[
T_{137}(E_{n\alpha},i) = 0
\]

The EMME in this case assumes the form

\[
m^*(E_{n\alpha},i) = \frac{\hbar^2}{2} T_{147}(E,i) \bigg|_{E=E_{n\alpha}}
\]

where,

\[
T_{147}(E,i) = \left[ \frac{1}{T_{117}(E,i)T_{127}(E,i)} \right] \left[ \{T_{137}(E,i)^\prime \} \left[ T_{117}(E,i)T_{127}(E,i) \right]^{1/2} - \left( \frac{T_{137}(E,i)}{2} \right) \right]
\]

\[
\left\{ \left[ T_{177}(E,i)^\prime \right] \left[ T_{127}(E,i) \right]^{1/2} + \left[ T_{137}(E,i)^\prime \right] \left[ T_{127}(E,i) \right]^{1/2} \right\}.
\]

The expression of the total 2D density-of-states function is given by

\[
N_{2D}(E) = \frac{g_v}{2\pi} \sum_{i=0}^{\infty} \left\{ T_{147}(E,i) H (E - E_{n\alpha}) \right\}
\]

(319)
The surface electron concentration in the strong electric field limit can be expressed as

\[ n_{2D} = \frac{g_s}{(2\pi)} \left\{ \sum_{i=0}^{\infty} \left[ P_{9i} \left( E_{F,i} \right) + Q_{9i} \left( E_{F,i} \right) \right] \right\} \]  \hspace{1cm} (320)

where,

\[ P_{9i} \left( E_{F,i} \right) \equiv \frac{T_{117} \left( E_{F,i} \right)}{\sqrt{T_{117} \left( E_{F,i} \right) T_{127} \left( E_{F,i} \right)}} \]  \hspace{1cm} and \hspace{1cm} \[ Q_{9i} \left( E_{F,i} \right) \equiv \sum_{r=1}^{i} L(r) P_{9r} \left( E_{F,i} \right). \]

### 3.2.6 Formulation of the EMME in n-channel inversion layers of Germanium

The 2D electron dispersion law in n-channel inversion layers of Ge can be expressed as

\[ \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} = \left[ E(1 + \alpha E) + \alpha E_{12}^2 - E_{12}(1 + 2\alpha E) \right] \]  \hspace{1cm} (321)

where,\[ E_{12} = S \left( \frac{hE}{2m_x} \right)^{2/3} \]

The area of 2D space is

\[ A = \frac{2\pi \sqrt{m_x m_y}}{\hbar^2} \left[ E(1 + \alpha E) + \alpha E_{12}^2 - E_{12}(1 + 2\alpha E) \right] \]  \hspace{1cm} (322)

The EMME assumes the form

\[ m^* \left( E_{Fw}, i \right) = \sqrt{m_x m_y} \left[ 1 + 2\alpha E_{Fw} - E_{12} 2\alpha \right] \]  \hspace{1cm} (323)

Thus the EMME is the function of both Fermi energy and quantum number due to band nonparabolicity.

The density-of-states function is given by

\[ N_{2D}(E) = \frac{2g_v}{(2\pi)^2} \frac{2\pi \sqrt{m_x m_y}}{\hbar^2} \sum_{i=0}^{\infty} \left[ 1 + 2\alpha E - 2\alpha E_{12} \right] H(E - E_{12}) \]  \hspace{1cm} (324)

The surface electron concentration is given by

\[ n_{sF} = \frac{g_e k_B T}{\pi} \frac{\sqrt{m_x m_y}}{\hbar^2} \sum_{i=0}^{\infty} \left[ F_0(\eta_i) + 2\alpha k_B T F_1(\eta_i) \right] \]  \hspace{1cm} (325)

where,\[ \eta_i = (k_B T)^{-1} \left( E_{Fw} - E_{12} \right) \]
3.2.7 Formulation of the EMME in n-channel inversion layers of GaSb

The 2D electron dispersion law in n-channel inversion layers of GaSb under weak electric field limit can be expressed as

\[ I_{36}(E) = \frac{\hbar^2 k_x^2}{2m_e} + S_i \left( \frac{\hbar|e| F_s I_{36}(E)}{\sqrt{2m_e}} \right)^{2/3} \]  

(326)

(326) represents the dispersion relation of the 2D electrons in n-channel inversion layers of III-V, ternary and quaternary materials under the weak electric field limit whose bulk electrons obey the three band model of Kane.

The EMME can be expressed as

\[ m^*(E_{F_{sw}}, i) = m_e \left[ \frac{P_{36}(E, i)}{E_{F_{sw}}} \right] \]  

(327)

where, \[ P_{36}(E, i) \equiv \left\{ \left[ I_{36}(E) \right]^{2/3} - \frac{2}{3} S_i \left[ \frac{\hbar|e| F_s}{\sqrt{2m^*}} \right]^{2/3} \left\{ \left[ I_{36}(E) \right]^{2/3} \left[ I_{36}(E) \right]^{1/3} \right\} \right\} \]

Thus, one can observe that the EMME is a function of the sub-band index, surface electric field, the Fermi energy and the other spectrum constants due to the combined influence of \( E_g \) and \( \Delta \).

The sub-band energy \( (E_{n_{sw}}) \) in this case can be obtained as

\[ I_{36}(E_{n_{sw}}) = S_i \left( \frac{\hbar|e| F_s I_{36}(E_{n_{sw}})}{\sqrt{2m_e}} \right)^{2/3} \]  

(328)

The 2D total density-of-states function in weak electric field limit can be expressed as

\[ N_{2D}(E) = \frac{m_e g_k}{\pi \hbar^2} \sum_{i=0}^{i_m} \left[ P_{36}(E, i) H \left( E_{36} - E_{n_{sw}} \right) \right] \]  

(329)

Using (329) and the occupation probability, the \( n_{2Dw} \) in the present case can be written as
\[ n_{2Dw} = \frac{g_e m_e}{\pi \hbar^2} \sum_{i=0}^{\infty} \left[ P_{46w}(E_{Fw}, i) + Q_{46w}(E_{Fw}, i) \right] \] (330)

where,

\[ P_{46w}(E_{Fw}, i) = \left[ I_{36}(E_{Fw}) - S_i \left( \frac{\hbar e F_s}{\sqrt{2m_e}} \right)^{-2/3} \right] \] and

\[ Q_{46w}(E_{Fw}, i) = \sum_{r=1}^{r=0} \left[ L(r) \left[ P_{46w}(E_{Fw}, i) \right] \right]. \]

Under the condition of strong electric field limit, assumes the form

\[ I_{36}(E) \left[ I_{36}(E) \right]^{3/2} = \frac{\hbar^2 k_i^2}{2m_e} \] (331)

(331) represents the dispersion relation of the 2D electrons in n-channel inversion layers of III-V, ternary and quaternary materials under the strong electric field limit whose bulk conduction electrons are defined by the three band model of Kane.

The EMME can be expressed as

\[ m^*(E_{Fz}, i) = m_e \left[ P_{56}(E, i) \right]_{E=E_{Fz}} \] (332 a)

where,

\[ P_{56}(E, i) = \left[ I_{36}(E) \right]^{3/2} - \left( \frac{\epsilon F_s \hbar}{\sqrt{2m_e}} \right) \left[ \sqrt{2} (S_i)^{3/2} \left( I_{36}(E) \right)^{3/2} \right]^{1/2} = 0 \] (332 b)

Thus, one can observe that the EMME is a function of the sub-band index, surface electric field, the Fermi energy and the other spectrum constants due to the combined influence of \( E_s \) and \( \Lambda \).

The sub-band energy \( E_{\nu_2} \) in this case can be obtained as

\[ I_{36}(E_{\nu_2}) = \left[ \frac{\epsilon F_s \hbar}{\sqrt{2m_e}} \right] \left[ \sqrt{2} (S_i)^{3/2} \right] = 0 \] (332 b)

The total 2D density-of-states function under the strong electric field limit can be expressed as
\[ N_{2D}(E) = \frac{m_e g_v}{\pi \hbar^2} \sum_{i=0}^{\infty} \left[ P_{65}(E, i) H(E - E_{\text{m}}) \right] \]  
(332 c)

The \( n_{2D} \) in the present case under the strong electric field can be written as

\[ n_{2D} = \frac{g_e m_e}{\pi \hbar^2} \sum_{i=0}^{\infty} \left[ P_{65}(E_{\text{Fr}}, i) + Q_{55}(E_{\text{Fr}}, i) \right] \]  
(332 d)

where, \( P_{65}(E_{\text{Fr}}, i) = \left[ I_{65}(E_{\text{Fr}}) - \frac{2\sqrt{2}}{3}(S_i)^2 \frac{h |E_i|}{\sqrt{2m_e}} \frac{L_{65}(E_{\text{Fr}})}{2m_e} \right] \) and

\[ Q_{55}(E_{\text{Fr}}, i) = \sum_{n=1}^{r} \left[ L(r) \left[ P_{55}(E_{\text{Fr}}, i) \right] \right] \]

### 3.3 Results and Discussions:

The effect of surface electric field on the EMME at the quantum limit in n-channel inversion layers of Cd$_3$As$_2$ and CdGeAs$_2$ has been exhibited in figs. 3.1-3.4. In figs. 3.1 and 3.2, we have demonstrated the variation of the EMME with electric field in the weak inversion regime which was extended up to \( 10^5 \text{ Vm}^{-1} \). It appears that with the increase in the electric field, the EMME in fig. 3.1 increases considering the generalized energy band model and the three and the two band models of Kane which are the special cases of our generalized analysis. It appears that in the weak field regime, the deviation between the three and two band models of Kane is less however significant difference is with the consideration of the crystal field. It should be noted that it is these two models which tends to the isotropic bulk effective mass value 0.0105\( m_0 \), rather than the generalized model. In the high field regime, Fig. 3.2, the difference in the three and two band models of Kane appears which marks a significant variation in the value of the EMME. The effect of crystal field splitting tends to decrease the EMME considering the generalized energy band model.

A closer look at the two figures reveal more interesting features of the continuity of the weak inversion energy band model in the strong field and strong inversion energy band model in the weak field. It is due to this non-convergence there is a slight mismatch of the EMME at the boundary of \( 10^5 \text{ Vm}^{-1} \) in both the figs. 3.1 and 3.2 and needs more attention towards the development of the generalized theory valid for all values of electric field is still a formidable problem in this case. Figs. 3.3 and 3.4 exhibit the EMME in n-CdGeAs$_2$ for all the cases of figs. 3.1 and 3.2 respectively.

122
Fig. 3.1: Plot of the EMME at weak inversion as the function of surface electric field for n-channel inversion layers of Cd$_3$As$_2$ in accordance with the generalized theory. The simplified results for three and two band models of Kane have also been exhibited in which, $m_{i}^* = 0.0139 m_0$ and $m_z = \frac{1}{2}(m_{i}^* + m_{c}^*) = 0.0105 m_0$ are the corresponding bulk values.

Fig. 3.2: Plot of the EMME at strong inversion as function of surface electric field for n-channel inversion layers of Cd$_3$As$_2$ for all cases of Fig. 3.1.
Fig. 3.3: Plot of the EMME at weak inversion as function of surface electric field for n-channel inversion layers of CdGeAs$_2$ in accordance with the generalized theory. The simplified results for three and two band models of Kane have also been exhibited in which, $m_1^* = 0.039m_0$ and $m_c = \frac{1}{2}(m_1^* + m_0^*) = 0.03365m_0$ are the corresponding bulk values.

Fig. 3.4: Plot of the EMME at strong inversion as function of surface electric field for n-channel inversion layers of CdGeAs$_2$ for all cases of Fig. 3.3.
Fig. 3.5: Plot of the EMME at low electric field limit as function of surface electric field for n-InAs.

Fig. 3.6: Plot of the EMME at high electric field limit as function of surface electric field for n-InAs.
Fig. 3.7: Plot of the EMME at low and high electric field limits as function of surface electric field for n-channel inversion layers of GaAs and InSb respectively.

Fig. 3.8: Plot of the EMME as function of surface electric field for p-channel inversion layers of CdS.
It appears that in the weak inversion regime the EMME is almost invariant of the electric field; however with the increase in the field, the EMM sharply decreases and tends to take negative values which challenge the applicability of quantization condition (254) at strong electric field for n-channel CdGeAs$_2$. The effect of electric field on the EMME of n-channel InAs has been exhibited in figs. 3.5 and 3.6. Almost no variation of the EMME in weak field appears for n-InAs channel while for higher fields, the EMME tends to decrease. The effect of surface electric field on n-channel GaAs and InSb at weak and strong electric field has been exhibited in fig. 3.7. Same trend as InAs in weak field again follows for GaAs, where the difference in the energy band model in determining the EMME is vanishing small. With the increase in the electric field at high value, the EMME in n-channel InSb tends to fall down. This is not with the case of CdS in fig. 3.8 where the effect of increasing the electric field increases the EMME monotonically presenting a significant change.

Finally, in fig. 3.9, we present the variation of the EMME in n-channel inversion layers of Ge following Cardona model under weak inversion regime and n-channel inversion layers of GaSb under strong inversion regime. It appears that the EMME in case of GaSb increases within the regime of $10^8$ Vm$^{-1}$, while the EMME in Ge stays almost constant. Thus we infer that as the dispersion relation changes, all the physical properties of all low dimensional systems will change accordingly although the mathematical formulations and the complex computer algorithms and the physical interpretations differ widely.
SECTION 4

Study of the EMME in NIPI Structures of Non-parabolic Materials

4.1 Introduction

The concept of doping semiconductor super-lattices (SLs) was introduced by Esaki and Tsu [165] and extensive work in this subject was initiated by Dohler [166]. In the compositional SL the periodic potential is due to a change in the band gap of two materials. In doping SLs, the periodicity is space-charge induced and in addition a homogeneous material is used. With the advent of modern experimental techniques of fabricating nanomaterials, it is possible to grow semiconductor super-lattices (SLs) composed of alternative layers of two different degenerate layers with controlled thickness. These structures have found wide applications in many new devices such as photodiodes, photoresistors [167], transistors [168], light emitters [169], tunneling devices [170], etc [171-182]. The investigations of the physical properties of narrow gap SLs have increased extensively, since they are important for optoelectronic devices and also since the quality of hetero-structures involving narrow gap materials has been greatly improved. It may be noted that the nipi structures, also called the doping super-lattices as mentioned above, are crystals with a periodic sequence of ultrathin film layers [183, 184] of the same semiconductor with the intrinsic layer in between together with the opposite sign of doping. All the donors will be positively charged and all the acceptors negatively. This periodic space charge causes a periodic space charge potential which quantizes the motions of the carriers in the z-direction together with the formation of the sub-band energies. Therefore it appears that the electronic structures of the nipi’s differ radically from the corresponding bulk materials as stated below:

a. Each band is split into mini-bands
b. The magnitude and the spacing of these mini bands may be designed by the choice of the super lattices parameters and
c. The electron energy spectrum of the nipi crystal becomes two-dimensional leading to the step functional dependence of the density-of-states function.

In section 4.2.1, of the theoretical background, the EMME in nipi structures of non-linear optical materials has been investigated. The section 4.2.2 contains the results for nipi structures of III-V, ternary and quaternary compounds in accordance with the three and the two band models of Kane together with parabolic energy bands and they form the
special cases of section 4.2.1. The sections 4.2.3, 4.2.4 and 4.2.5 contain the study of the EMME for nipi’s of II-VI, IV-VI and stressed Kane type materials respectively. The 4.3 contains the results and discussion of this section.

4.2. Theoretical background

4.2.1 Formulation of the EMME in nipi structures of non-linear optical materials

The dispersion relation of the conduction electrons in nipi structures of nonlinear optical materials can be expressed by using (2) and following the method as given in [183, 184] as

$$\psi_i(E) = \psi_2(E)k_x^2 + \psi_3(E)\left(n_i + \frac{1}{2}\right)\frac{2m^*_{ni}}{\hbar} \omega_k(E)$$  \hspace{1cm} (333)

where \(\omega_k(E) = \left(\frac{n_0 |e|^2}{\varepsilon_{ec} \left(\frac{\theta(E)}{\epsilon\nu}\right)}\right)^{1/2}\) and

$$\theta(E) = \frac{\hbar^2}{2} \left\{ \frac{\psi_3(E) \left[\psi_1(E)\right]' - \psi_1(E) \left[\psi_3(E)\right]'}{\left[\psi_3(E)\right]^2} \right\}$$

and \(n_i (=0,1,2...)\) is the miniband index for nipi structures.

The EMME in this case assumes the form

$$m^*(E_{Fn}, n_i) = \left(\frac{\hbar^2}{2}\right) R_{ni}(E, n_i) \left|_{E=E_{Fn}} \right. \hspace{1cm} (334)$$

where,

$$R_{ni}(E, n_i) = \left[\psi_2(E)\right]^2 \left[\psi_1(E)\right]' - \left(\frac{2m^*_{ni}}{\hbar}\right) \left[\psi_3(E)\right] \left(n_i + \frac{1}{2}\right) \left[\omega_k(E)\right]' - \left(\frac{2m^*_{ni}}{\hbar}\right) \left[\psi_3(E)\right] \left(n_i + \frac{1}{2}\right) \left[\omega_k(E)\right]$$

$$- \left[\psi_1(E)\right]' - \left(\frac{2m^*_{ni}}{\hbar}\right) \left[\psi_3(E)\right] \left(n_i + \frac{1}{2}\right) \left[\omega_k(E)\right][\psi_2(E)]'$$

and \(E_{Fn}\) is the Fermi energy in the present case as measured from the edge of the conduction band in vertically upward direction in the absence of any quantization.
From (334), we observe that the EMME is a function of the Fermi energy, nipi sub-band index and the other material constants which is the characteristic feature of nipi structures of non-linear optical materials.

The sub-band energy \( E_{i\nu} \) can be written as

\[
\psi_i(E_{i\nu}) = \psi_3(E_{i\nu}) \left( n_i + \frac{1}{2} \right) \frac{2m^*}{\hbar} \partial_\nu (E_{i\nu})
\]  
(335)

The density-of-states function for nipi structures of non-linear optical materials can be expressed as

\[
N_{npi}(E) = \frac{g_v}{2\pi d_0} \sum_{\eta=0}^{n_{\text{max}}} R_{g1}(E, n_i) H(E - E_{i\nu})
\]  
(336)

in which \( d_0 \) is the superlattice period.

The electron concentration, can be written as

\[
r_0 = \frac{g_v}{2\pi d_0} \sum_{\eta=0}^{n_{\text{max}}} \left[ T_{g1}\left( E_{\nu}, n_i \right) + T_{g2}\left( E_{\nu}, n_i \right) \right]
\]  
(337)

where, \( T_{g1}(E_\nu, n_i) = \left[ \psi_1(E_\nu) - \psi_3(E_\nu) \left( n_i + \frac{1}{2} \right) \frac{2m^*}{\hbar} \partial_\nu (E_\nu) \right] \psi_2(E_\nu) \right]^2 \) and

\[
T_{g2}(E_\nu, n_i) = \sum_{r=1}^{5} L(r) T_{g1}(E_\nu, n_i).
\]

4.2.2 EMME in the nipi structures of III-V, ternary and quaternary materials

(a) The electron energy spectrum in nipi structures of III-V, ternary and quaternary materials can be expressed from (333) under the conditions \( \Delta_\perp = \Delta_\parallel = \Delta, \delta = 0 \) and \( m^*_\parallel = m^*_\perp = m_0 \), as

\[
I_1(E) = \left( n_i + \frac{1}{2} \right) \hbar \partial_\nu (E) + \frac{\hbar^2 k_\parallel^2}{2m^*_\parallel}
\]  
(338)

where \( \partial_\nu (E) = \left( \frac{n_0 |\psi|^2}{E_n I'(E) m_0} \right)^{1/2} \).

The EMME in this case can be written as

\[
m^*(E_{\nu}, n_i) = m_n R_{g2}(E, n_i) \bigg|_{E=E_\nu}
\]  
(339)
in which, \( R_{42}(E, n_i) \equiv \left\{ I_{11}(E) - \left( n_i + \frac{1}{2} \right) \hbar \alpha_k(E) \right\} \).

From (339) we observe that the EMME in this case is a function of the Fermi energy, nipisub-band index and the other material constants which is the characteristic feature of nipi structures of III-V, ternary and quaternary compounds whose bulk dispersion relations is defined by the three band model of Kane.

The sub-band energies (\( E_{2nl} \)) can be written as

\[
I_{11}(E_{2nl}) = \left( n_i + \frac{1}{2} \right) \hbar \alpha_k(E_{2nl})
\]  

The density-of-states function in this case can be expressed as

\[
N_{nipi}(E) = \frac{m_c \varepsilon_v}{\pi \hbar^2 d_0} \sum_{n_i=0}^{\text{max}} R_{42}(E, n_i) H(E - E_{2nl})
\]  

The use of (341) leads to the expression of the electron concentration as

\[
n_0 = \frac{m_c \varepsilon_v}{\pi \hbar^2 d_0} \sum_{n_i=0}^{\text{max}} \left[ T_{83}(\bar{E}_{Fn}, n_i) + T_{84}(\bar{E}_{Fn}, n_i) \right]
\]  

where

\[
T_{83}(\bar{E}_{Fn}, n_i) = \left\{ I_{11}(\bar{E}_{Fn}) - \left( n_i + \frac{1}{2} \right) \hbar \alpha_k(\bar{E}_{Fn}) \right\}
\]

and

\[
T_{84}(\bar{E}_{Fn}, n_i) = \sum_{r=1}^{s} L(r) T_{83}(\bar{E}_{Fn}, n_i).
\]

(b) For the two band model of Kane, the expressions of the dispersion relation, the EMME, the sub-band energies, the density-of-states function and \( n_0 \) remain same where

\[
i_{11}(E) = E(1 + \alpha E), \quad \{ I_{11}(E) \}' = (1 + 2\alpha E) \quad \text{and} \quad \{ I_{11}(E) \}'' = 2\alpha.
\]

The EMME in this case can be written as

\[
m'(E_{Fn}, n_i) = m_c \left\{ (1 + 2\alpha E_{Fn}) + \left( n_i + \frac{1}{2} \right) \hbar \alpha_k(E_{Fn}) \right\} \frac{\alpha}{(1 + 2\alpha E_{Fn})}
\]  

From (343) we observe that the EMME in this case is a function of the Fermi energy, nipisub-band index and the other material constants due to the band non-parabolicity only.
(c) For parabolic energy bands, the forms of the expressions of dispersion relation, the EMME, the sub-band energies, the density-of-states function and \( n_0 \) remain same, where \( I_{11}(E) = E \).

\[ \{I_{11}(E)\}' = 1 \text{ and } \{I_{11}(E)\}'' = 0. \]

The EMME can be written as

\[ m^*(E_{fn}, n_i) = m_e \]  \hspace{1cm} (344)

From (344) we observe that the EMME in this case is a constant quantity.

4.2.3 EMME in the nipi structures of II-VI materials

The carrier dispersion law in nipi structures of II-VI compounds can be expressed as

\[ E = \alpha_0 k_z^2 + \left( n_i + \frac{1}{2} \right) \hbar \omega_0 \pm \lambda_0 k_z, \]  \hspace{1cm} (345)

Using (345), the EMME in this case can be written as

\[ m^*(E_{fn}, n_i) = m_i^* \left\{ 1 - \lambda_0^2 \left[ \frac{\lambda_0}{\lambda_0} \right]^2 + 4 \alpha_0^2 E_{fn} - 4 \alpha_0^2 \left( n_i + \frac{1}{2} \right) \hbar \omega_0 \right\}^{-1/2} \]  \hspace{1cm} (346)

Thus, the EMME in this case is a function of the Fermi energy, the nipi-sub-band index number and the energy spectrum constants due to the only presence of \( \lambda_0 \).

The sub-band energies \( (E_{3n}) \) can be written as

\[ E_{3n} = \left( n_i + \frac{1}{2} \right) \hbar \omega_0 \]  \hspace{1cm} (347)

The density-of-states function in this case can be expressed as

\[ N_{\text{npi}}(E) = \frac{m_i^* g_e}{\pi \hbar^2} \sum_{n=0}^{\infty} \left[ 1 - \frac{a_{3n}}{\sqrt{E + b_{3n}(n_i)}} \right] H(E - E_{3n}) \]  \hspace{1cm} (348)

in which, \( a_{3n} = \frac{\lambda_0}{2 \sqrt{\alpha_0}} \) and \( b_{3n}(n_i) = \left[ \frac{1}{4 \alpha_0^2} \left( \frac{\lambda_0}{\lambda_0} \right)^2 - 4 \alpha_0^2 \left( n_i + \frac{1}{2} \right) \hbar \omega_0 \right] \).

The use of the (348) leads to the electron concentration under the condition of extreme degeneracy as
The carrier energy spectrum in nipi structures of IV-VI compounds can be written as

$$k_x^2 = (\hbar^2 S_{19})^{-1} \left[ -S_{20}(E, n_i) + \sqrt{S_{20}^2(E, n_i) + 4S_{19}S_{21}(E, n_i)} \right]$$  \hspace{1cm} (350)

in which, $S_{19} = \left( \frac{e}{m_i^*} \right)$, $S_{20}(E, n_i) = \left( \frac{1}{m_i^*} \right) \left( \frac{\alpha E}{m_i^*} \right) + 1 + \frac{\hbar^2}{2m_i^* m_i^*} \left( n_i + \frac{1}{2} \right) T(E) + \frac{\hbar^2}{2m_i^* m_i^*} \left( n_i + \frac{1}{2} \right) T(E)$,

$$T(E) = \frac{-2m_i^*(0)}{\hbar} \alpha_{11}(E), m_i^*(0) = \left( \frac{m_i^* m_i^*}{m_i^* + m_i^*} \right), \alpha_{11}(E) = \left( \frac{n_i |e|^2}{\epsilon n_i m_i^*(E)} \right)^{1/2},$$

$$m_i^*(E) = \frac{1}{4t_1} \left[ (t_2(E))' + \frac{t_2(E)(t_2(E))'}{\sqrt{t_2(E) + 4E \alpha (1 + \alpha E)}} \right],$$

$$t_1 = \left( \frac{\alpha}{4m_i^* m_i^*} \right), t_2(E) = \frac{1}{2} \left[ \left( \frac{1}{m_i^*} \right) - \frac{\alpha E}{m_i^*} + \frac{1 + \alpha E}{m_i^*} \right], (t_2(E))' = \frac{\alpha}{2} \left( \frac{1}{m_i^*} - \frac{1}{m_i^*} \right)$$

and

$$S_{21}(E, n_i) = \left[ E(1 + \alpha E) + \frac{\alpha E \hbar^2}{2m_i^*} \left( n_i + \frac{1}{2} \right) T(E) + \frac{\hbar^2}{2m_i^*} \left( n_i + \frac{1}{2} \right) T(E)(1 + \alpha E) \right] + \frac{\hbar^4}{4m_i^* m_i^*} \left( n_i + \frac{1}{2} \right) T(E) - \frac{\hbar^2}{2m_i^*} T(E) \left( n_i + \frac{1}{2} \right).$$

Using (350), the EMME in this case can be written as

$$m_i^*(E_{Fn}, n_i) = R_{94}(E, n_i) \bigg|_{E=E_{Fn}} \hspace{1cm} (351)$$

where, $R_{94}(E, n_i) = (2S_{19})^{-1} \left[ -S_{20}(E, n_i) + S_{20}(E, n_i) \left[ S_{20}(E, n_i) \right]' + 2S_{19} \left[ S_{21}(E, n_i) \right]' \right]^{1/2} + 4S_{19}S_{21}(E, n_i)$. 

Using (350), the EMME in this case can be written as

$$m_i^*(E_{Fn}, n_i) = R_{94}(E, n_i) \bigg|_{E=E_{Fn}} \hspace{1cm} (351)$$

where, $R_{94}(E, n_i) = (2S_{19})^{-1} \left[ -S_{20}(E, n_i) + S_{20}(E, n_i) \left[ S_{20}(E, n_i) \right]' + 2S_{19} \left[ S_{21}(E, n_i) \right]' \right]^{1/2} + 4S_{19}S_{21}(E, n_i)$. 

133
Thus, one can observe that the EMME in this case is a function of both the Fermi energy and the nipisub-band index number together with the spectrum constants of the system due to the presence of band nonparabolicity.

The sub-band energies \( E_{4n} \) can be written as

\[
E_{4n} = \frac{\hbar^2}{2m^*_n} T(E_{4n}) \left( n + \frac{1}{2} \right) \left[ 1 + \alpha E_{4n} + \alpha^2 \frac{\hbar^2}{2m^*_n} T(E_{4n}) \left( n + \frac{1}{2} \right) \right] - \frac{\hbar^2}{2m^*_n} T(E_{4n}) \left( n + \frac{1}{2} \right)
\]  

(352)

The density-of-states function in this case assumes the form as

\[
N_{\text{ext}}(E) = \frac{8\pi}{\pi \hbar^2 d_0} \sum_{n=0}^{\infty} R_{4n}(E, n_1) H(E - E_{4n})
\]  

(353)

The use of (353) leads to the expression of the electron concentration as

\[
n_0 = \frac{8\pi}{2\pi \hbar^2 s_0 d_0} \sum_{n=0}^{\infty} \left[ T_{85}(E_{4n}, n_1) + T_{86}(E_{4n}, n_1) \right]
\]  

(354)

where,

\[
T_{85}(E_{4n}, n_1) = \left[ -S_{20}(E_{4n}, n_1) + \sqrt{S_{20}(E_{4n}, n_1)^2 + 4S_{20}S_{21}(E_{4n}, n_1)} \right]
\]

and \( T_{86}(E_{4n}, n_1) = \sum_{r=1}^{4} L(r) T_{85}(E_{4n}, n_1) \).

4.2.5 EMME in the nipi structures of stressed Kane type materials

The electron dispersion law in the nipi structures of stressed materials can be written as

\[
\frac{k_x^2}{[a_0(E)]^2} + \frac{k_y^2}{[b_0(E)]^2} + \frac{1}{[c_0(E)]^2} \left( \frac{2m^*_n(0)}{\hbar} \right) \left( n + \frac{1}{2} \right) \omega_{12}(E) = 1
\]  

(355)

where \( \omega_{12}(E) \equiv \left( \frac{n_0 |\sigma|^2}{e_n m^*_n(E)} \right)^{1/2} \) and \( m^*_n(E) \equiv \hbar^2 c_0(E) \frac{\partial}{\partial E} \left[ c_0(E) \right] \).

The use of (355) leads to the expression of the EMME as

\[
m^*_n(E_{4n}, n_1) = \left. \left( \frac{\hbar^2}{2} \right) R_{85}(E, n_1) \right|_{E=E_{4n}}
\]  

(356)
where,

\[ R_{55}(E, n_i) = \left[ (\bar{a}_b(E))' b_0(E) + (\bar{b}_0(E)') \bar{a}_0(E) \right] \left[ 1 - \frac{1}{(\bar{c}_0(E))^2} \frac{2m^*_i(0)}{\hbar} \left( n_i + \frac{1}{2} \right) \omega_{12}(E) \right] \]

Thus, the EMME is a function of the Fermi energy and the nipisub-band index due to the presence of stress and band non-parabolicity only.

The sub-band energies \( E_{5m} \) can be written as

\[ E_{5m} = \frac{1}{(\bar{c}_0(E_{4m}))^2} \frac{2m^*_i(0)}{\hbar} \left( n_i + \frac{1}{2} \right) \omega_{12}(E_{4m}) = 1 \]  

(357)

The density-of-states function can be expressed as

\[ N_{nip}_{s}(E) = \frac{g_v}{\pi \hbar^2 d_0} \sum_{n=0}^{n_{nm}} R_{55}(E, n_i) H(E - E_{5m}) \]  

(358)

Thus, using (358), the electron concentration in nipi structures of stressed compounds can be written as

\[ n_0 = \frac{g_v}{2\pi d_0} \sum_{n=0}^{n_{nm}} \left[ C_3(\bar{E}_{5m}, n_i) + C_4(\bar{E}_{5m}, n_i) \right] \]  

(359)

where \( C_3(\bar{E}_{5m}, n_i) \equiv \bar{a}_0(\bar{E}_{5m}) \bar{b}_0(\bar{E}_{5m}) \left[ 1 - \frac{2m^*_i(0)}{\hbar} \left( n_i + \frac{1}{2} \right) \frac{\omega_{12}(\bar{E}_{5m})}{(\bar{c}_0(\bar{E}_{5m}))^2} \right] \)

and \( C_4(\bar{E}_{5m}, n_i) \equiv \sum_{r=1}^{L} L(r) C_3(\bar{E}_{5m}, n_i) \)

4.3 Results and Discussions:

The effect of nipi super-lattice period on the EMME has been exhibited in figs. 4.1 to 4.9 for different materials. Using the appropriate equations together with the energy band constants as given in Table 1, we have plotted the EMME in nipi structures of non-linear optical materials taking Cd_{3}As_{2} and CdGeAs_{2} as examples in Figs. 4.1 and 4.2. From both the figs. 4.1 and 4.2, it appears that the effect of increment of the super-lattice period increases the EMME in the presence of extreme carrier degeneracy of the order of
10^{23} \text{ m}^3. For comparison with the bulk anisotropic effective masses, we have also exhibited the same in the said figures. It appears that the EMME can be much less than that of the corresponding bulk values below 10 nm period for Cd$_3$As$_2$. Thus in such condition, one can expect an increase in the carrier mobility to a great extent, in fact almost double. The effect of crystal field splitting has also been exhibited in the same figs. 4.1 and 4.2. It appears that the effect of $\delta$ on the EMME is largest in case of Cd$_3$As$_2$. The approximation in the energy band structure also makes a significant deviation of the EMME in case of Cd$_3$As$_2$. However for CdGeAs$_2$, the EMME exhibits a slow variation over the super-lattice period as compare with fig. 4.1. At this point it should be noted that with the increase with the super-lattice period, the EMME in Cd$_3$As$_2$ by considering the energy dispersion relation with the absence of the crystal field splitting and the three band model of Kane actually tends to the anisotropic bulk value 0.01393$m_0$. This is not with the case when the effect of crystal field splitting and the two band equivalent model is considered.

Fig. 4.1: Plot of the EMME as function of super-lattice period for n-Cd$_3$As$_2$. The plots for three and two band models of Kane have also been exhibited with their corresponding bulk values.
Fig. 4.2: Plot of the EMME as function of superlattice period for n-CdGeAs$_2$ for all cases.

Fig. 4.3: Plot of the EMME as function of superlattice period for n-InAs considering the three and two band model of Kane with the corresponding bulk value.
Fig. 4.4: Plot of the EMME as function of super-lattice period for n-InSb considering the three and two band models of Kane with the corresponding bulk value.

Fig. 4.5: Plot of the EMME as function of super-lattice period for n-GaAs considering the three and two band model of Kane with the corresponding bulk value.
Fig. 4.6: Plot of the EMME as function of superlattice period for \( n-\text{Hg}_{1-x}\text{Cd}_x\text{Te} \) considering the three and two band model of Kane with the corresponding bulk value at \( x = 0.3 \).

Fig. 4.7: Plot of the EMME as function of superlattice period for \( n-\text{In}_{1-x}\text{Ga}_x\text{As}_{1-y}\text{P}_y \) considering the three and two band model of Kane with the corresponding bulk value at \( x = 0.3 \).
Fig. 4.8: Plot of the EMME as function of super-lattice period for p-CdS.

Fig. 4.9: Plot of the EMME as function of super-lattice period for PbTe.
In these two cases, the EMME is overestimated against the bulk value. This is not with the case of Fig. 4.2 of CdGeAs$_2$, where the EMME converges to the bulk anisotropic value at larger super-lattice period.

The effect of super-lattice period on the EMME in the ground state sub-band in III-V materials has been evaluated using the three and the two band models of Kane in figs. 4.3-4.5 for InAs, InSb and GaAs respectively. It appears from these figures that the effect of the variation of the energy dispersion relation model on EMME is almost insignificant for GaAs nipi structures, whereas for InSb, the EMME exhibits a significant deviation. In almost all the cases of about 1 nm period, the EMME approximately becomes half of the respective isotropic effective bulk masses indicating the mobility rise of up to 200%. In figs. 4.6 and 4.7, the EMME as function of the periods has been further evaluated for the ternary and quaternary materials like Hg$_{1-x}$Cd$_x$Te and In$_{1-x}$Ga$_x$As$_{1-y}$P$_y$, where the energy band gap in these materials can be modulated by changing the alloy fraction $x$. We see that the EMME in case of In$_{1-x}$Ga$_x$As$_{1-y}$P$_y$ almost exhibits no significant variation as it approaches quickly its bulk normalized value 0.0287 at $x = 0.3$ as compared to Hg$_{1-x}$Cd$_x$Te. Figs. 4.8 and 4.9 exhibit the EMME at the lowest sub-band in II-VI and IV-VI nipi structures of CdS and PbTe respectively.

The effect of increasing the doping concentration from $4 \times 10^{25}$ to $6 \times 10^{25}$ m$^{-3}$ on EMME in CdS has been also exhibited in Fig. 4.8 for a period bandwidth of 50-100 μm. It appears that with the increase in the doping concentration, the EMME in CdS increases and approaches the bulk longitudinal normalized value 1.5. However in case of PbTe, we see that the EMME saturates above super-lattice period of about 20 μm. The effect of doping concentration on the EMME in the lowest sub-band level in all the aforementioned materials has been exhibited in Figs. 4.10-4.17. It appears that the EMME increases with the increases in carrier degeneracy for all the cases and may becomes even large than that of their corresponding bulk value along the proper transport direction. From Fig. 4.11 we see that the EMME is almost constant below the degeneracy of about $10^{23}$ m$^{-3}$. The effect of different models of energy band structures has been exhibited to present the dependency of the EMME on the same. It appears from Figs. 4.14 and 4.16 that the second and third order Kane model almost exhibits no differences of the EMME from the two. The respective saturation of the EMME with the decrease in the degeneracy is different for the materials as this depends on the Fermi energy which is a function of the energy band parameters.
Fig. 4.10: Plot of the EMME as function of doping concentration for n-Cd$_3$As$_2$ considering all cases of Fig. 4.2.

Fig. 4.11: Plot of the EMME as function of doping concentration for n-CdGeAs$_2$ considering all cases of Fig. 4.3.
Fig. 4.12: Plot of the EMME as function of doping concentration for n-InAs considering all cases of Fig. 4.3.

Fig. 4.13: Plot of the EMME as function of doping concentration for n-InSb considering all cases of Fig. 4.4.
Fig. 4.14: Plot of the EMME as function of doping concentration for n-GaAs considering all cases of Fig. 4.5.

Fig. 4.15: Plot of the EMME as function of doping concentration for n-Hg$_{1-x}$Cd$_x$Te considering all cases of Fig. 4.6.
Fig. 4.16: Plot of the effective electron mass as function of doping concentration for n-\(\text{In}_{1-x}\text{Ga}_{x}\text{As}_{1-y}\text{P}_{y}\).

Fig. 4.17: Plot of the effective electron mass as function of doping concentration for PbTe.
Fig. 4.18: Plot of the EMME as function of alloy composition for \( \text{Hg}_{1-x}\text{Cd}_x\text{Te} \) and \( \text{n-In}_{1-x}\text{Ga}_x\text{As}_{1-y}\text{P}_y \) considering the three and two band model of Kane with the corresponding bulk value at different \( x \).

Fig. 4.17 exhibits the variation of the EMME with increasing degeneracy for \( \text{PbTe nipi} \). Anomalous behavior in the variation of the EMME has been exhibited as one increases the degeneracy. It appears that above \( 2 \times 10^{22} \text{ m}^{-3} \), the EMME decreases. This should not be in general confused with other plots since an increase in the degeneracy increases the Fermi energy which increases the EMME. However, in this case, the effect of the different spectrum constants defines the variation of the EMME.

The variation of the EMME with alloy composition for the ternary and quaternary materials has been exhibited in Fig. 4.18 for the three and the two energy band models of Kane. Almost no difference in the two energy band model in this case is exhibited. The variation of the EMME for the quaternary is rather slow than that of the ternary which is due to the variation of the energy band gap through the alloy composition.
SECTION 5

Study of the EMME in Nanowires of Non-Parabolic Materials

5.1. Introduction

It is well-known that in nanowires (NWs), the restriction of the motion of the carriers along two directions may be viewed as carrier confinement by two infinitely deep 1D rectangular potential wells, along any two orthogonal directions leading to quantization of the wave vectors along the said directions, allowing 1D carrier transport [185]. With the help of modern fabrication techniques, such one dimensional quantized structures have been experimentally realized and enjoy an enormous range of important applications in the realm of nanoscience. They have generated much interest in the analysis of nanostructured devices for investigating their electronic, optical and allied properties [186-192]. Examples of such new applications are based on the different transport properties of ballistic charge carriers which include nanoresistors [193-195], resonant tunneling diodes and band filters [196,197], nanoswitches [198], nanosensors [199-200], nanologic gates [201-202], nanotransistors and sub tuners [203-204], heterojunction FETs [205], high-speed digital networks [206], high-frequency microwave circuits [207], optical modulators [208], optical switching systems [209], and other nanoscale devices. In this chapter, we shall study the EMME in NWs of non-parabolic materials having different band structures.

In Section 5.2.1, the EMME in NWs of non-linear optical materials has been investigated. In Section 5.2.2, the EMME in NWs of III-V, ternary and quaternary materials has been studied in accordance with the said band models and the simplified results for wide gap materials having parabolic energy bands under certain limiting conditions have further been demonstrated as a special case and thus confirming the compatibility test. The Section 5.2.3 contains the investigation of the EMME in NWs of II-VI compounds. In section 5.2.4, the EMME in NWs of Bi has been formulated in accordance with the aforementioned energy band models for the purpose of relative assessment. Besides, under certain limiting conditions all the results for all the models of 1D systems are reduced to the well-known result of the EMME in NWs of wide gap materials. This above statement exhibits the compatibility test of our theoretical analysis. In Section 5.2.5, the EMME in NWs of IV-VI materials has been studied taking PbTe, PbSe, and PbS as examples. The EMME in NWs of stressed compounds (taking stressed n-InSb as an example) has been investigated in Section 5.2.6. The Section 5.2.7 contains
the investigation of EMME in NWs of Tellurium. In Section 5.2.8, the EMME in NWs of n-GaP has been studied. The Section 5.2.9 explores the EMME in NWs of PtSb₂. In Section 5.2.10, the EMME in NWs of Bi₂Te₃ has been considered. In Section 5.2.11, the EMME has been studied in NWs of Ge. In Section 5.2.12, the EMME in NWs of GaSb has been studied. In Section 5.2.13, we shall study the EMME in NWs of II-V materials. The 5.3 contains the result and discussions pertaining to this section.

5.2 Theoretical Background

5.2.1 The EMME in nanowires of non-linear optical materials

For two dimensional quantizations along x and y directions, (121) assumes the form

\[ k_x^2 = A_1(E, n_x, n_y) \]  

(360)

where \( A_1(E, n_x, n_y) = \left[ f_2(E) \right]^{-1} \{ \gamma(E) - \phi_1(n_x, n_y) f_1(E) \}, \) \( \phi_1(n_x, n_y) = \left( \frac{n_x \pi}{d_x} \right)^2 + \left( \frac{n_y \pi}{d_y} \right)^2, \)

\( n_x = (1, 2, 3, \ldots), n_y = (1, 2, 3, \ldots) \) are the size quantum numbers along x and y directions respectively and \( d_x \) and \( d_y \) are the nano-thickness along x and y directions respectively.

The quantized sub-band energy \( E_{11} \) is given by

\[ \gamma(E_{11}) = f_1(E_{11}) \phi_1(n_x, n_y) \]  

(361)

The EMME can be expressed as

\[ m^*(E_{FID}, n_x, n_y) = \frac{\hbar^2}{2} A'_1(E_{FID}, n_x, n_y) \]  

(362)

where \( E_{FID} \) is the Fermi energy in the presence of 2D quantization as measured from the edge of the conduction band in the vertically upward direction in the absence of any quantization.

\[ A'_1(E_{FID}, n_x, n_y) = \frac{-2A_1(E_{FID}, n_x, n_y) f'_1(E_{FID})}{f_2(E_{FID})} \left[ f_2(E_{FID}) \right]^{-1} \{ \gamma(E_{FID}) - f_1(E_{FID}) \phi_1(n_x, n_y) \} \]

\[ f'_1(E_{FID}) = \left[ \left( \hbar^2 E g_0 (E g_0 + \Delta_0) \right) \right]^{-1} \left[ \left( \hbar^2 E g_0 + \frac{2}{3} \Delta_0 \right) \right]^{-1} \left[ 2E_{FID} + 2E g_0 + \frac{2}{3} \Delta_0 \right], \]

\[ f'(E_{FID}) = \left[ \left( \hbar^2 E g_0 (E g_0 + \Delta_0) \right) \right]^{-1} \left[ \left( \hbar^2 E g_0 + \frac{2}{3} \Delta_0 \right) \right]^{-1} \left[ 2E_{FID} + 2E g_0 + \frac{2}{3} \Delta_0 + \delta \right] \]
and \( \gamma'(F_{FD}) = \left[ \frac{\gamma(E_{FD})(2E_{FD} + E_{g})}{E_{FD}(E_{FD} + E_{g})} \right] + (E_{FD})(E_{FD} + E_{g})[2E_{FD} + 2E_{g} + \Delta_{0} + \delta] \)

Thus, we observe that the EMME is the function of both the size quantum numbers \((n_x, n_y)\) and the Fermi energy due to the combined influence of the crystal field splitting constant and the anisotropic spin-orbit splitting constants respectively.

The density-of-states function per sub-band \( (N_{ID}(E)) \) is given by,

\[
N_{ID}(E) = \frac{g_s}{\pi} \left[ \left\{ \psi_1(E) - \psi_2(E) \phi_(n_x, n_y) \right\} \right]^{-1/2} \psi_3(E) \\
\left[ \psi_3(E) \right]^{-1} \left\{ \psi_3(E) \right\}^{-1} \left\{ \psi_3(E) \right\}^{-1} \left\{ \psi_3(E) \right\}
\]

The electron concentration per unit length can be expressed as

\[
n_0 = \frac{2g_s}{\pi} \sum_{n_x} \sum_{n_y} [B_{11}(E_{FD}, n_x, n_y) + B_{12}(E_{FD}, n_x, n_y)]
\]

where \( B_{11}(E_{FD}, n_x, n_y) = [A_{11}(E_{FD}, n_x, n_y)]^{1/2} \), \( B_{12}(E_{FD}, n_x, n_y) = \sum_{n_x} Z_{10}(r) \sum_{n_x} [B_{11}(E_{FD}, n_x, n_y)]\)

and \( Z_{10}(r) = 2(k_B T)^{2r}(1 - 2^{-2r}) \xi(2r) \frac{\partial^{2r}}{\partial E_{FD}^{2r}} \)

5.2.2 The EMME in nanowires of III-V materials

For the purpose of complete and coherent presentation, the EMME in NWs of III-V materials have also been investigated in accordance with the aforementioned different dispersion relations for III-V materials for the purpose of relative comparison as follows:

(a) Under the substitutions \( \delta = 0, \Delta_0 = \Delta_\perp = \Delta \) and \( m_\parallel = m_\perp = m_c \)

(360) assumes the form

\[
\frac{\hbar^2 k^2}{2m_c} = I_{11}(E) - \frac{\hbar^2}{2m_c} \phi(n_x, n_y)
\]

Using (365), the EMME along \( k_x \) direction for this case can be written as

\[
m^\ast(E_{FD}) = m_c \left[ I_{11}(E_{FD}) \right]^{1/2}
\]
where,

$$I_{11}(E_{FID}) = \left[ \frac{1}{E_{FID}} + \frac{1}{E_{FID} + E_{\theta}} + \frac{1}{E_{FID} + E_{\theta} + \Delta} + \frac{1}{E_{FID} + E_{\theta} + (2/3)\Delta} \right]$$

It is worth noting that the EMME in this case is a function of Fermi energy alone and is independent of size quantum number.

The sub band energy $E_{n_{\alpha}}$ can be written as

$$I_{11}(E_{n_{\alpha}}) = \frac{\hbar^2}{2m_c} \phi(n_x, n_y)$$

(367)

The 1D carrier concentration can thus be written as

$$n_{1D} = \frac{2g_v}{\pi \hbar^2} \left[ \sum_{n_{\alpha}=1}^{N_{\alpha}} \sum_{n_{\beta}=1}^{N_{\beta}} T_{63} \left( E_{FID}, n_x, n_y \right) + T_{64} \left( E_{FID}, n_x, n_y \right) \right]$$

(368)

where, $T_{63} \left( E_{FID}, n_x, n_y \right) = \left[ I_{11}(E_{FID}) - \frac{\hbar^2}{2m_c} \phi(n_x, n_y) \right]^{1/2}$ and

$$T_{64} \left( E_{FID}, n_x, n_y \right) = \sum_{r=1}^{L_{ID}} Z_{1D}(r) T_{63} \left( E_{FID}, n_x, n_y \right)$$

(b) Under the inequalities $\Delta \gg E_s$ or $\Delta \ll E_s$, (365) assumes the form

$$E(1 + \alpha E) = \frac{\hbar^2}{2m_c} \phi(n_x, n_y) + \frac{\hbar^2 k^2}{2m_c}$$

(369)

The EMME along $k_z$ direction can be written as

$$m^* \left( E_{FID} \right) = m_c \left( 1 + 2\alpha E_{FID} \right)$$

(370)

Thus, we observe that the EMME in the present case is a function of Fermi energy only due to the presence of band non-parabolicity.

For NWs, whose energy band structures for the corresponding bulk materials obey the two-band model of Kane, the density-of-states function per sub band assumes the form

$$N_{1D}(E) = \frac{g_v}{\pi \hbar^2} \left( \frac{2m_c}{\hbar^2} \right)^{1/2} \left[ \frac{(1 + 2\alpha E)}{E \left( 1 + \alpha E \right) - \frac{\hbar^2}{2m_c} \phi(n_x, n_y)} \right]^{1/2}$$

(371)

In this case the sub-band energy $E_{n_{\alpha}}$ can be expressed as
\[
\frac{\hbar^2}{2m_e} \phi(n_x, n_y) = E_{n_{m0}} \left( 1 + \alpha E_{n_{m0}} \right)
\]

(372)

The use of (371) leads to the expression of the 1D electron statistics as

\[
n_{1D} = \frac{2g_s}{\pi} \left( \frac{2m_e}{\hbar^2} \right)^{1/2} \sum_{n_{x=1}}^{n_{max}} n_{y=1} n_{y=1} \left[ T_{65} \left( E_{F1D}, n_x, n_y \right) + T_{66} \left( E_{F1D}, n_x, n_y \right) \right]
\]

(373)

where, \( T_{66} \left( E_{F1D}, n_x, n_y \right) = \left[ E_{F1D} \left( 1 + \alpha E_{F1D} \right) \right] \frac{\hbar^2}{2m_e} \phi(n_x, n_y) \) and

\[
T_{66} \left( E_{F1D}, n_x, n_y \right) = \sum_{r=1}^t Z_{1D} (r) T_{66} \left( E_{F1D}, n_x, n_y \right).
\]

Under the condition, \( \alpha E_{F1D} \ll 1 \), the expressions of the 1D electron statistics can be written as

\[
n_{1D} = \frac{2g_s}{\pi} \sqrt{2m_e \pi k_b T} \sum_{n_{x=1}}^{n_{max}} n_{y=1} \sum_{i=1}^{n_{y=1}} \frac{1}{\sqrt{i}} \left[ \left( \frac{3}{4} \alpha i_2 \right) F_{u2/1} (\xi_6) + \frac{3}{4} \alpha k_b T F_{u2/1} (\xi_6) \right]
\]

(374)

where, \( i_i = \left[ 1 + \alpha \frac{\hbar^2}{2m_e} \phi (n_x, n_y) \right] \), \( i_2 = \left( \frac{\hbar^2}{2m_e} \right) \phi (n_x, n_y) (i_2)^{-1} \) and

\[\eta_6 = (E_F - i_2) / k_b T .\]

Under the condition \( \alpha \rightarrow 0 \), the expression of \( n_{1D} \) for NWs of isotropic parabolic energy bands can be written from (374) as

\[
n_{1D} = \frac{2g_s}{\pi} \sqrt{2m_e \pi k_b T} \sum_{n_{x=1}}^{n_{max}} n_{y=1} \sum_{i=1}^{n_{y=1}} \left[ F_{u2/1} (\eta_i) \right], \eta_i = \left( \frac{1}{k_b T} \right) \left[ E_{F1D} \left( \frac{\hbar^2}{2m_e} \phi (n_x, n_y) \right) \right]
\]

(375)

(c) The model of Stillman et al.

In accordance with the model of Stillman et al. [71], the electron dispersion law of NWs of 1D III-V materials assumes the form

\[
k_i^2 = A_{l_2}(E, n_x, n_y)
\]

(376)

where \( A_{l_2}(E, n_x, n_y) = \frac{2m_e}{\hbar^2} (I_{l_2}(E) - \phi (n_x, n_y)) \)

The EMME in this case assume the from

\[
m^* (E_{F1D}) = m_e I_{l_2}^* (E_{F1D})
\]

(377)

where, \( I_{l_2}^* (E_{F1D}) = \left( \frac{a_1 a_2}{2} \right) (1 - a_2 E_{F1D})^{-1/2} \)
The quantized sub-band energy \((E_{l4})\) is given by

\[
I_{l4}(E_{l4}) = \frac{\hbar^2}{2m_c} A(n_x, n_y) \tag{378}
\]

The electron concentration per unit length can be expressed as

\[
n_0 = \frac{2g_x}{\pi} \sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \left[ B_{l7}(E_{F1D}, n_x, n_y) + B_{l8}(E_{F1D}, n_x, n_y) \right] \tag{379}
\]

where \(B_{l7}(E_{F1D}, n_x, n_y) = [A_{l4}(E_{F1D}, n_x, n_y)]^{l/2} \) and

\[
B_{l8}(E_{F1D}, n_x, n_y) = \sum_{r=1}^{\infty} Z_{1D}(r) B_{l7}(E_{F1D}, n_x, n_y) \]

(d) The model of Newson and Kurobe

In accordance with the model of Newson and Kurobe [72], the electron dispersion law in this case assumes the form

\[
E = a_{l3} k_z^4 + \left[ \frac{\hbar^2}{2m_c} + a_{l4} k_z^2 \right] k_z^2 + \frac{\hbar^2}{2m_c} k_z^2 + a_{l5} k_z^2 k_z^2 + a_{l3} \left(k_z^4 + k_z^4\right) \tag{380}
\]

where \(a_{l3}\) is the non-parabolicity constant, \(a_{l4} (= 2a_{l3} + a_{l5})\) and \(a_{l3}\) is known as the warping constant.

The 1-D \(E-k_z\) relation can be expressed as

\[
k_z^2 = A_{l5}(E, n_x, n_y) \tag{381}
\]

where \(A_{l5}(E, n_x, n_y) = (2a_{l3})^{l/2} \left[-\overline{L}_1(n_x, n_y) + [\overline{L}_1(n_x, n_y)]^2 - 4a_{l3}[\overline{L}_2(n_x, n_y) - E]^{l/2} \right], \)

\[
\overline{L}_1(n_x, n_y) = \frac{\hbar^2}{2m_c} + a_{l4} \left(\frac{n_x \pi}{d_x} \right)^2 + \left(\frac{n_y \pi}{d_y} \right)^2 \quad \text{and}
\]

\[
\overline{L}_2(n_x, n_y) = \left[\frac{\hbar^2}{2m_c} \phi(n_x, n_y) + a_{l4} \left(\frac{n_x \pi}{d_x} \right)^2 + a_{l3} \left(\frac{n_y \pi}{d_y} \right)^2 \right]^{l/2}
\]

The EMME can be written from (381) as

\[
m^\ast(E_{F1D}, n_x, n_y) = \frac{\hbar^2}{2} A_{l5}'(E_{F1D}, n_x, n_y) \tag{382}
\]

where, \(A_{l5}'(E_{F1D}, n_x, n_y) = [\overline{L}_1(n_x, n_y)]^2 - 4a_{l3}[\overline{L}_2(n_x, n_y) - E_{F1D}]^{l/2} \)
The mass is a function of quantum numbers in addition to Fermi energy due to band non-parabolicity.

The quantized sub-band energy \( E_{16} \) is given by

\[
E_{16} = L_2(n_x, n_y)
\]

The electron concentration per unit length can be written as

\[
n_0 = \frac{2 g_e}{\pi} \sum_{n_x=1}^{N_x} \sum_{n_y=1}^{N_y} \left[ B_{19}(E_{F1D}, n_x, n_y) + B_{20}(E_{F1D}, n_x, n_y) \right]
\]

where \( B_{19}(E_{F1D}, n_x, n_y) = [A_{15}(E_{F1D}, n_x, n_y)]^{1/2} \)

and \( B_{20}(E_{F1D}, n_x, n_y) = \sum_{r=1}^{N_{1D}} Z_{1D}(r)[B_{19}(E_{F1D}, n_x, n_y)] \)

(e) Model of Palik et al.

The energy spectrum of the conduction electrons in NWs of III-V materials up to the fourth order in effective mass theory, taking into account the interactions of heavy hole, light hole and the split-off holes can be expressed in accordance with the model of Palik et al.[73] can be written as

\[
k^2_x = A_{16}(E, n_x, n_y)
\]

where, \( A_{16}(E, n_x, n_y) = \left[ \frac{2m}{\hbar^2} \left( I_{13}(E) \right) - \phi(n_x, n_y) \right] \)

The EMME can be written from (385) as

\[
m^*(E_{F1D}) = m_e I_{13}^*(E_{F1D})
\]

where, \( I_{13}^*(E_{F1D}) = 2b_{12}B_{11} \left[ \left( \bar{a}_{12} \right)^2 - 4E_{F1D} \bar{B}_{11} \right]^{-1/2} \)

The electron concentration per unit length can be expressed as

\[
n_0 = \frac{2 g_e}{\pi} \sum_{n_x=1}^{N_x} \sum_{n_y=1}^{N_y} \left[ B_{21}(E_{F1D}, n_x, n_y) + B_{22}(E_{F1D}, n_x, n_y) \right]
\]

where \( B_{21}(E_{F1D}, n_x, n_y) = [A_{16}(E_{F1D}, n_x, n_y)]^{1/2} \)

and \( B_{22}(E_{F1D}, n_x, n_y) = \sum_{r=1}^{N_{1D}} Z_{1D}(r)[B_{21}(E_{F1D}, n_x, n_y)] \)

5.2.3 The EMME in nanowires of II-VI materials

The 1D dispersion relation for nanowires of II-VI materials can be expressed as
\[ E = \hbar^2 k_z^2 + G_{\text{zz}} \left( n_x, n_y \right) \]  
where \( G_{\text{zz}}(n_x, n_y) = \frac{\hbar^2}{d_y} \left( \frac{\pi n_x}{d_y} \right)^2 + \frac{\hbar^2}{d_z} \left( \frac{\pi n_y}{d_z} \right)^2 \left[ 1 + \frac{\hbar^2}{2m^*_t} \right] \]

The EMME can be written from (388) as

\[ m^*(E_{\text{FID}}) = m_t^* \]  
From (389), it appears that the EMME is constant in this case.

The 1D electron statistics can be written as

\[ n_{1\text{D}} = \frac{g_v}{\pi \sqrt{\hbar^2 \varepsilon_0}} \sum_{n_x} \sum_{n_y} \left[ n_x \left( E_{\text{FID}}, n_x, n_y \right) + n_y \left( E_{\text{FID}}, n_x, n_y \right) \right] \]

where\[ t_r \left( E_{\text{FID}}, n_x, n_y \right) = \left( E_{\text{FID}} - \left[ G_{\text{zz}} \left( n_x, n_y \right) \right] \right)^{1/2} \]

and

\[ t_s \left( E_{\text{FID}}, n_x, n_y \right) = \sum_{r=1}^{R} Z_{\text{ID}}(r) \left[ t_r \left( E_{\text{FID}}, n_x, n_y \right) \right] \]

5.2.4 The EMME in nanowires of Bismuth

(a) The McClure and Choi model

The dispersion relation of the carriers in NWs of Bi can be written in accordance with the model of McClure and Choi as

\[ E(1 + \alpha E) = \left[ \frac{\hbar^2 k_z^2}{2m_t} - \frac{\hbar^2}{2m_2} \left( \frac{\pi n_y}{d_y} \right)^2 \right] + G_{12} + \frac{\hbar^2}{2m_2} \alpha E \left[ 1 - \left( \frac{m_t}{m_2} \right) \left( \frac{\pi n_y}{d_y} \right)^2 \right] \]

where \( G_{12} = \left[ \frac{\hbar^2}{2m_2} \left( \frac{\pi n_x}{d_x} \right)^2 + \frac{\hbar^2}{2m_3} \left( \frac{\pi n_z}{d_z} \right)^2 + \frac{\hbar^4}{4m_2 m_t} \left( \frac{\pi n_y}{d_y} \right)^4 \right] \]

\[ G_{12} = \left[ \frac{\hbar^2}{2m_2} \left( \frac{\pi n_x}{d_x} \right)^2 + \frac{\hbar^2}{2m_3} \left( \frac{\pi n_z}{d_z} \right)^2 + \frac{\hbar^4}{4m_2 m_t} \left( \frac{\pi n_y}{d_y} \right)^4 \right] \]

Following (391), the EMME in this case assumes the from

\[ m^*(E_{\text{FID}}, n_y) = m_t \left[ 1 - \frac{\alpha \hbar^2}{2m_2} \left( \frac{\pi n_y}{d_y} \right)^2 \right] \left[ 1 + 2\alpha E_{\text{FID}} - \frac{\hbar^2}{2m_2} \alpha E_{\text{FID}} \right] \]

Thus EMME in this case is a function of Fermi energy and the size quantum number \( n_y \) due to the presence of band-nonparabolicity.
Using (391), the 1D electron statistics can be expressed as

\[ n_{1D} = \frac{2g_e}{\pi} \sqrt{\frac{2m_1}{\hbar}} \sum_{n_y=1}^{\infty} \sum_{n_z=1}^{\infty} \left[ t_{27} \left( E_{F1D}, n_y, n_z \right) + t_{32} \left( E_{F1D}, n_y, n_z \right) \right] \]  

(393)

where

\[ t_{27} \left( E_{F1D}, n_y, n_z \right) = \left[ 1 - \frac{\alpha E_y}{2m_1 \hbar^2} \right]^2 \left[ E_{F1D} (1 + \alpha E_{1D}) - G_{14} - \frac{\hbar^2}{2M_2} \alpha E_{1D} \left( 1 - \left( \frac{m_1}{m_f} \right) \left( \frac{n_z}{d_y} \right)^2 \right) \right] \]

and \( t_{32} \left( E_{F1D}, n_y, n_z \right) = \sum_{r=1}^{N} Z_{1D} (r) \left[ t_{27} \left( E_{F1D}, n_y, n_z \right) \right] \)

(b) The Hybrid Model

The 1D dispersion relation in Bi in accordance with the Hybrid model [93] can be expressed as

\[ E \left( 1 + \alpha E \right) = \frac{\hbar^2 k_x^2}{2m_1} + G_{14} + \frac{\hbar^2}{2M_2} \left( \frac{\pi n_z}{d_y} \right)^2 \alpha E (1 - \gamma_0) \]  

(394)

where \( G_{14} = \left[ \frac{\hbar^2}{2m_3} \left( \frac{\pi n_z}{d_x} \right)^2 + \frac{\hbar^2}{2M_2} \left( \frac{\pi n_z}{d_y} \right)^2 \left( 1 + \beta_0 \right) + \frac{\alpha \gamma_0 \hbar^4}{4M_2} \left( \frac{\pi n_z}{d_y} \right)^4 \right] \).

Using this (394), the EMME can be expressed as

\[ m^*(E_{F1D}, n_y) = m_1 \left[ 1 + 2\alpha E_{1D} - \frac{\hbar^2}{2M_2} \alpha \left( 1 - \left( \frac{\gamma_0}{\gamma_0} \right) \left( \frac{\pi n_y}{d_y} \right)^2 \right) \right] \]  

(395)

Thus EMME in this case is a function of Fermi energy and the size quantum number \( n_y \) due to the presence of band-nonparabolicity.

The use of (394) leads to the expression for the electron concentration per unit length as

\[ n_{1D} = \frac{2g_e}{\pi} \sqrt{\frac{2m_1}{\hbar}} \sum_{n_y=1}^{\infty} \sum_{n_z=1}^{\infty} \left[ t_{31} \left( E_{F1D}, n_y, n_z \right) + t_{32} \left( E_{F1D}, n_y, n_z \right) \right] \]  

(396)

where \( t_{31} \left( E_{F1D}, n_y, n_z \right) = \left[ E_{F1D} \left( 1 + \alpha E_{1D} \right) - G_{14} - \frac{\hbar^2}{2M_2} \left( \frac{\pi n_z}{d_y} \right)^2 \alpha E_{1D} \left( 1 - \gamma_0 \right) \right]^{1/2} \),

and \( t_{32} \left( E_{F1D}, n_y, n_z \right) = \sum_{r=1}^{N} Z_{1D} (r) \left[ t_{31} \left( E_{F1D}, n_y, n_z \right) \right] \)
(c) The Cohen model

The 1D carrier dispersion law in this case can be written as

$$\alpha E^2 + E l_j - G_{15} = \frac{\hbar^2 k_x^2}{2m_1}$$  \hspace{1cm} (397)

where, $$L_j = \left[ 1 - \frac{\alpha h^2}{2m_z} \left( \frac{\pi n_x}{d_y} \right)^2 + \frac{\alpha h^2}{2m_1'} \left( \frac{\pi n_y}{d_y} \right)^2 \right]$$

and $$G_{15} = \left[ \frac{\hbar^2}{2m_z} \left( \frac{\pi n_z}{d_z} \right)^2 + \frac{\hbar^2}{2m_1} \left( \frac{\pi n_y}{d_y} \right)^2 + \frac{\alpha h^4}{4m_1 m_1'} \left( \frac{\pi n_y}{d_y} \right)^4 \right]$$

Using this (397), the EMME can be expressed as

$$m^* \left( E_{F1D}, n_y \right) = m_1 \left[ 1 + 2\alpha E_{F1D} + L_j \right]$$  \hspace{1cm} (398)

Thus EMME in this case is a function of Fermi energy and the size quantum number $$n_y$$ due to the presence of band-nonparabolicity.

The 1D electron concentration per unit length assumes the form

$$n_{1D} = \frac{2g_0}{\pi} \sqrt{2m_1} \sum_{n_x=-\infty}^{\infty} \sum_{n_y=-\infty}^{\infty} \left[ t_{35} \left( E_{F1D}, n_y, n_z \right) + t_{36} \left( E_{F1D}, n_y, n_z \right) \right]$$  \hspace{1cm} (399)

where, $$t_{35} \left( E_{F1D}, n_y, n_z \right) = \left[ \alpha E^2_{F1D} + E_{F1D} L_j - G_{15} \right]^{1/2}$$ and

$$t_{36} \left( E_{F1D}, n_y, n_z \right) = \sum_{r=1}^{k} Z_{1D} (r) \left[ t_{35} \left( E_{F1D}, n_y, n_z \right) \right]$$

(d) The Lax model

The 1D dispersion relation in this case can be expressed as

$$E \left( 1 + \alpha E \right) = \frac{\hbar^2 k_x^2}{2m_1} + G_{16}$$  \hspace{1cm} (400)

Using (400), the EMME can be expressed as

$$m^* \left( E_{F1D} \right) = m_1 \left[ 1 + 2\alpha E_{F1D} \right]$$  \hspace{1cm} (401)

Thus EMME in this case is a function of Fermi energy and is independent of the size quantum number $$n_y$$ due to the presence of band-nonparabolicity.
The 1D electron statistics is given by

\[ n_{1D} = \frac{2g_v}{\pi} \frac{\sqrt{2m_i}}{h} \sum_{n_1} \sum_{n_2} a_{nm} \left[ t_{37} \left( E_{F1D}, n_y, n_z \right) + t_{38} \left( E_{F1D}, n_y, n_z \right) \right] \]

where \( t_{37} \left( E_{F1D}, n_y, n_z \right) = \left[ E_{F1D} \left( 1 + \alpha E_{F1D} \right) - G_0 \right]^{1/2} \)

and \( t_{38} \left( E_{F1D}, n_y, n_z \right) = \sum_r Z_{1D}(r) \left[ t_{37} \left( E_{F1D}, n_y, n_z \right) \right] \)

It may be noted that under the conditions \( \alpha \to 0, M_2' \to \infty \) and isotropic effective electron mass at the edge of the conduction band, \( m \) all models of Bismuth convert into isotropic parabolic energy bands leading to the confirmatory test.

5.2.5 The EMME in nanowires of IV-VI materials

The 1D dispersion relation in this case in accordance with the Dimmock model can be expressed as

\[ k_x^2 = A_{23}(E, n_x, n_y) \]

where,

\[ A_{23}(E, n_x, n_y) = (2h \gamma)^2 \left[ h_0 \left( E, n_x, n_y \right) - \left[ h_0^2 \left( E, n_x, n_y \right) + 4h_1h_2 \left( E, n_x, n_y \right) \right]^{1/2} \right] = \frac{\alpha \gamma^2}{4x_3 x_6}, \]

\[ x_3 = \frac{3m_i^- m_i^+}{2m_i^- + m_i^+}, x_6 = \frac{3m_i^* m_i^+}{2m_i^* + m_i^+}, \]

\[ h_0 \left( E, n_x, n_y \right) = \frac{\alpha E \gamma^2}{2x_6} - \frac{\alpha \gamma^2}{2x_6} \left[ \frac{\pi n_x}{d_x} \right]^2 \left[ \frac{\pi n_y}{d_y} \right]^2 \frac{h^2}{2x_4} + \left( \frac{\pi n_y}{d_y} \right)^2 \frac{h^2}{2x_2} - \frac{h^2}{2m_i} \frac{1 + \alpha E \gamma^2}{2x_4}, \]

\[ \frac{\alpha \gamma^2}{2x_3} \left[ \frac{\pi n_x}{d_x} \right]^2 \left[ \frac{\pi n_y}{d_y} \right]^2 \frac{h^2}{2x_4} - \frac{h^2}{2m_i} \frac{1 + \alpha E \gamma^2}{2x_4}, \]

\[ x_1 = m_i^-, x_2 = \frac{m_i^- + 2m_i^+}{3}, x_4 = m_i^+, x_5 = \frac{m_i^+ + 2m_i^+}{3}, m_3 = \frac{3m_i^* m_i^+}{m_i^* + 2m_i^-}, \]

\[ S_{20}(E, n_i) = \left\{ \frac{1}{m_i} - \frac{\alpha E}{m_i^+} \left( \frac{m_i^+}{m_i} \right) + \frac{\alpha E}{2m_i^+ m_i^-} \left( n_i + \frac{1}{2} \right) T(E) + \frac{\alpha \gamma^2}{2m_i^+ m_i^-} \left( n_i + \frac{1}{2} \right) T(E) \right\} \]

157
The EMME can be written from (403) as

\[ m^* (E_{F1D}, n_x, n_y) = \frac{\hbar^2}{2} A_{23} (E_{F1D}, n_x, n_y) \]  

where \( A_{23} (E_{F1D}, n_x, n_y) = (2\hbar)^{-1} [h'_6 (E_{F1D}, n_x, n_y) h'_6 + 2 h'_6 h'_6 (E_{F1D}, n_x, n_y)] \)

\[ h'_6 (E_{F1D}, n_x, n_y) = [1 + 2\alpha E_{F1D} + \alpha (\frac{\hbar^2}{2x_5} (\frac{\pi n_x}{d_x})^2 + (\frac{\pi n_y}{d_y})^2 + (\frac{\pi n_y}{d_y})^2 h'_6 - \alpha (\frac{\hbar^2}{2x_5} (\frac{\pi n_x}{d_x})^2 + (\frac{\pi n_y}{d_y})^2 h'_6)] \]

The mass is a function of quantum numbers in addition to Fermi energy due to band non-parabolicity.

The electron concentration is given by

\[ n_e = \frac{2g_s}{\pi} \sum_{n_z} \sum_{n_r} [B_{23} (E_{F1D}, n_x, n_y) + B_{33} (E_{F1D}, n_x, n_y)] \]  

where \( B_{33} (E_{F1D}, n_x, n_y) = [A_{23} (E_{F1D}, n_x, n_y)]^{1/2} \)

\[ B_{23} (E_{F1D}, n_x, n_y) = \sum_{r=1}^{q} Z_{1D} (r) [B_{23} (E_{F1D}, n_x, n_y)] \]

5.2.6 The EMME in nanowires of stressed compounds

The 1D dispersion relation of the carriers in stressed materials in this case can be written as

\[ k^* = A_{24} (E, n_x, n_y) \]

where \( A_{24} (E, n_x, n_y) = [c^* (E)]^{-1} \bigg[ 1 - \left( \frac{\pi n_x}{d_x} \right)^2 \left[ a^* (E) \right]^2 - \left( \frac{\pi n_y}{d_y} \right)^2 \left[ b^* (E) \right]^2 \bigg] \)

The EMME can be written from (406) as
\[ m^*(E_{F1D}, n_x, n_y) = \frac{h^2}{2} A_{24}(E_{F1D}, n_x, n_y) \]  

where

\[
A_{24}(E_{F1D}, n_x, n_y) = \left[ \frac{K_0'(E_{F1D})}{L_0(E_{F1D})} - \frac{K_0(E_{F1D}) L_0'(E_{F1D})}{L_0^2(E_{F1D})} \right] \left( \frac{n_x \pi}{d_x} \right)^2 \left[ \frac{M_0'(E_{F1D}) + \frac{1}{2} N_0'(E_{F1D})}{L_0(E_{F1D})} \right] + \left( \frac{n_y \pi}{d_y} \right)^2 \left[ \frac{I_0'(E_{F1D})}{L_0^2(E_{F1D})} \right] \left[ \frac{M_0(E_{F1D}) - \frac{1}{2} N_0(E_{F1D})}{L_0(E_{F1D})} \right] + \frac{1}{L_0(E_{F1D})} [M_0'(E_{F1D}) - \frac{1}{2} N_0'(E_{F1D})] \]  

\[ K_0'(E_{F1D}) = \left[ 1 + \frac{2C_2^2 e_y^2}{3(E_{F1D})^2} \right] \frac{3E_{F1D}(E_{F1D})}{2B_2^2} + \left[ V_0 - C_1 e_y - \frac{2C_2^2 e_y^2}{3(E_{F1D})^2} \right] \left( \frac{3}{2B_2^2} \right) \]  

\[ M_0'(E_{F1D}) = \left[ \frac{(\bar{a} + C_1)e_y - \frac{3\bar{b} e_y}{2(E_{F1D})^2}}{(E_{F1D})^2} \right] + \frac{\bar{b} e_y}{2(E_{F1D})^2} \]  

and \[ N_0'(E_{F1D}) = -\left( \bar{d}_x \sqrt{3} \right) \frac{e_y}{(E_{F1D})^2} \]  

The mass is a function of quantum numbers in addition to Fermi energy due to stress.  

The sub-band energy \( E_{23} \) assumes the form

\[
\left( \frac{\pi n_x}{d_x} \right)^2 \left[ a^*(E_{23}) \right]^2 + \left( \frac{\pi n_y}{d_y} \right)^2 \left[ b^*(E_{23}) \right]^2 = 1 \]  

Using (406), the 1D electron statistics can be expressed as

\[
n_{1D} = \frac{2\bar{a} e_y}{\pi} \sum_{n_y} \sum_{n_x} B_{24} \left( E_{F1D}, n_y, n_x \right) + B_{23} \left( E_{F1D}, n_x, n_y \right) \]  

where \( B_{24} \left( E_{F1D}, n_y, n_x \right) = \sqrt{A_{24} \left( E_{F1D}, n_x, n_y \right)} \) and

\[
B_{23} \left( E_{F1D}, n_x, n_y \right) = \sum_{r=1}^{\bar{a}} Z_{1D}(r) \left[ B_{23} \left( E_{F1D}, n_x, n_y \right) \right] \]  

5.2.7 The EMME in nanowires of Tellurium

The 1-D dispersion relation can be written in accordance with the model of Bouat et.al [121] as
$$k_1^2 = A_{25,x}(E, n_x, n_y)$$  \hspace{1cm} (410)

where,

$$A_{25,x}(E, n_x, n_y) = [(\psi_3(E) - \psi_6 k_1^2 \pm \psi_7 [(\psi_6(E) - k_1^2)^{1/2}]], k_1^2 = \phi(n_x, n_y),$$

$$\psi_3(E) = \frac{E}{\psi_1} + \frac{E}{2\psi_1}, \psi_6 = \frac{\psi_6}{\psi_1}, \psi_7 = (2\psi_1^2)^{-1} [4\psi_3^2\psi_1\psi_2 - 4\psi_1^2\psi_4^2]^{1/2},$$

and

$$\psi_6(E) = \left[\frac{\psi_4^4 + 4E\psi_3^2\psi_1}{4\psi_1^2\psi_2 - 4\psi_1^2\psi_4^2}\right].$$

The EMME can be written from (406) as

$$m^*(E_{FID}, n_x, n_y) = \frac{h^2}{2} A_{25,k}(E_{FID}, n_x, n_y)$$  \hspace{1cm} (411)

where

$$A_{25,k}(E_{FID}, n_x, n_y) = \left[\frac{1}{\psi_1} \pm \frac{\psi_7}{2} (\psi_6(E_{FID}) - \phi(n_x, n_y))^{1/2}\psi_6(E_{FID})\right]$$

and

$$\psi_6(E_{FID}) = \left[\frac{4\psi_3^2\psi_1}{4\psi_1^2\psi_2 - 4\psi_1^2\psi_4^2}\right].$$

Thus the EMME is the function of the Fermi energy and the size quantum numbers which is the characteristic feature of such model.

The sub-band energies are given by

$$E_{26,x} = \psi_3 \phi(n_x, n_y) \pm \psi_4 (\phi(n_x, n_y))^{1/2}$$  \hspace{1cm} (412)

The electron concentration per unit length can be expressed as

$$n_0 = \frac{g_e}{\pi} \sum_{n_x=1}^{n_{max}} \sum_{n_y=1}^{n_{max}} \left[B_{26,x}(E_{FID}, n_x, n_y) + \theta_{26}\right]$$  \hspace{1cm} (413)

where

$$B_{26,x}(E_{FID}, n_x, n_y) = \sqrt{A_{25,x}(E_{FID}, n_x, n_y) + A_{25,x}(E_{FID}, n_x, n_y)}$$

and

$$\theta_{26} = \sum_{r=1}^{N} Z_{ID}(r) [B_{26,x}(E_{FID}, n_x, n_y) + B_{26,x}(E_{FID}, n_x, n_y)].$$

5.2.8 The EMME in nanowires of Gallium Phosphide

The 1D dispersion relation in this case it can be written in accordance with the model of Rees et. al [122] as

$$k_1^2 = A_{26}(E, n_x, n_y)$$  \hspace{1cm} (414)

where,

$$A_{26}(E, n_y, n_z) = \left[\frac{[2a^2]^{-1} \left\{2a(E - t_1) + D \right\}^2 - 4a^2 [(E - t_1)^2 - t_2]}{\left(\frac{n_y n_z}{\psi_1}\right)^2}\right].$$
The EMME can be written from (414) as

\[ m^* (E_{F1D}, n_y, n_z) = \frac{\hbar^2}{2} A'_{26} (E_{F1D}, n_y, n_z) \]  
(415)

where

\[ A'_{26} (E_{F1D}, n_y, n_z) = \frac{1}{a^2} \left[ a - \frac{2a (E_{F1D} - t_1) + D - 4a^2 (E_{F1D} - t_1)}{\sqrt{[2a (E_{F1D} - t_1) + D]^2 - 4a^2 [E_{F1D} - t_1]^2}} \right] \]

Thus the mass is a function of Fermi energy and the size quantum numbers which are the characteristic features of such model.

The sub-band energy \( E_{2l} \) can be written as

\[ A_{26} (E_{2l}, n_y, n_z) = 0 \]  
(416)

The electron concentration per unit length can be expressed as

\[ n_0 = \frac{2e}{\pi} \sum_{n_y=1}^{\infty} \sum_{n_z=1}^{\infty} B_{38} (E_{F1D}, n_y, n_z) + B_{39} (E_{F1D}, n_y, n_z) \]  
(417)

where, \( B_{38} (E_{F1D}, n_y, n_z) = \sqrt{A_{26} (E_{F1D}, n_y, n_z)} \) and

\[ B_{39} (E_{F1D}, n_y, n_z) = \sum_{n_y=1}^{\infty} Z_{1D} (r) \left[ B_{38} (E_{F1D}, n_y, n_z) \right] \]

5.2.9 The EMME in nanowires of Platinum Antimonide

The 1D dispersion relation in this case can be written as

\[ k^2 = \left[ 2A_y \right]^{-1} \left[ -A_{10} (E, n_z) + \left[ \overline{A_y}^2 (E, n_z) + 4 \left( \overline{A_y} A_{11} (E, n_z) \right) \right]^{1/2} - \left( \frac{n \pi}{d_y} \right)^2 \]  
(418)

where \( \overline{A_y} = (I_1 + \omega_1 \omega_3), I_1 = \frac{1}{16} \) \( \frac{a^4}{l} \),

\[ \overline{A_{10}} (E, n_z) = \left[ \omega_3 E + \omega_1 \left[ E + \delta_0 - \omega_4 \left( \frac{n \pi}{d_y} \right)^2 \right] + \omega_2 \omega_3 \left( \frac{n \pi}{d_y} \right)^2 \right] + 2l_1 \left( \frac{n \pi}{d_z} \right)^2 \],

\[ \omega_1 = \frac{a^2}{4} \left( \frac{\delta_0 - l}{\overline{\lambda_0}} \right), \omega_2 = \frac{a^2}{\overline{\lambda_0}} \right), \omega_3 = \frac{a^2}{4} (n + \nu), \omega_4 = \frac{a^2}{4} \nu \] and
The it can be expressed as

$$k_x^2 = A_{27}(E, n_y, n_z)$$

(419)

where

$$A_{27}(E, n_y, n_z) = \left[ 2\bar{A}_9 \right]^{-1} \left[ -\bar{A}_{10}(E, n_z) + \left( \bar{A}_{11}(E, n_y) + 4\left( \bar{A}_9 \bar{A}_{11}(E, n_z) \right) \right) \right]^{1/2} - \left( \frac{n_z \pi}{d_y} \right)^2$$

The EMME in this case can be written following (419) as

$$m^*(E_{FD}, n_y, n_z) = \frac{\hbar}{2} A_{27}(E_{FD}, n_y, n_z)$$

(420)

where

$$A_{27}'(E_{FD}, n_y, n_z) = \left[ 2\bar{A}_9 \right]^{-1} \left[ -\bar{A}_{10}(E_{FD}, n_z) + \left( \bar{A}_{11}(E_{FD}, n_y) + 4\left( \bar{A}_9 \bar{A}_{11}(E_{FD}, n_z) \right) \right) \right]^{1/2} - \left( \frac{n_z \pi}{d_y} \right)^2$$

(421)

Thus the mass is a function of Fermi energy and the size quantum numbers which are the characteristic features of such model.

The electron concentration per unit length can be written as

$$n_0 = \frac{2\gamma_s}{\pi} \sum_{n_x} \sum_{n_z} \left[ B_{40}(E_{FD}, n_x, n_z) + B_{41}(E_{FD}, n_x, n_z) \right]$$

(422)

where, $B_{40}(E_{FD}, n_x, n_z) = \sqrt{A_{27}(E_{FD}, n_x, n_z)}$ and

$$B_{41}(E_{FD}, n_x, n_z) = \sum_{r=1}^{q_4} Z_{rD}(r) \left[ B_{40}(E_{FD}, n_x, n_z) \right]$$

5.2.10 The EMME in nanowires of Bismuth Telluride

The 1D electron energy spectrum following (226) assumes the form

$$k_x^2 = A_{28}(E, n_z)$$

(423)
where \( A_{28}(E, n_x, n_y) = [E(1 + \alpha E) - \omega_2 \left( \frac{n_x \pi}{d_x} \right)^2 - \omega_3 \left( \frac{n_y \pi}{d_y} \right)^2 - \omega_4 \left( \frac{n_z \pi}{d_z} \right)^2]^{-1} \)

The sub-band energy \( E_{31} \) can be expressed as

\[
E_{31} = (2\alpha)^{-1} \left[ -\rho_{31}(n_y) + \sqrt{\rho_{31}(n_y)^2 - 4\alpha \rho_{32}(n_y)} \right]
\]

The EMME in this case can be written following (428) as

\[
m^*_{F1D}(n_x, n_y) = \frac{\hbar^2}{2\alpha} \left( 1 + 2\alpha E_{F1D} \right)
\]

The electron concentration per unit length is given by

\[
n_y = \frac{2e \alpha}{\pi} \sum_{n_x=1}^{\infty} \sum_{n_z=1}^{\infty} B_{42} \left( E_{F1D}, n_x, n_z \right) + B_{43} \left( E_{F1D}, n_x, n_z \right)
\]

The quantized energy levels \( E_{31} \) can be expressed through the equation

\[
E_{31} = (2\alpha)^{-1} \left[ -\rho_{31}(n_y) + \sqrt{\rho_{31}(n_y)^2 - 4\alpha \rho_{32}(n_y)} \right]
\]

The EMME in this case can be written following (427) as

\[
m^*_{F1D}(E_{F1D}, n_x, n_y) = \frac{\hbar^2}{2\alpha} \left( 1 + 2\alpha E_{F1D} \right)
\]

The sub-band energy \( E_{31} \) can be expressed as

\[
E_{31} = (2\alpha)^{-1} \left[ -\rho_{31}(n_y) + \sqrt{\rho_{31}(n_y)^2 - 4\alpha \rho_{32}(n_y)} \right]
\]

The EMME in this case can be written following (427) as
$m^*(E_{FD},n_x,n_y) = \frac{\hbar^2}{2} A'_{29}(E_{FD},n_x,n_y)$

where $A'_{29}(E_{FD},n_x,n_y) = \alpha^2\left[1 + 2\alpha E_{FD} - \left(\alpha - \frac{\hbar^2}{m^*_3} \frac{n_y \pi}{d_x} \right)^2 \left(2m^*_2 / \hbar^2\right) \right]$.

From (429) we observe that the EMME is a function of the Fermi energy and the size quantum number due to the presence of band-nonparabolicity only.

The electron concentration per unit length is given by

$$n_e = \frac{2g_s}{\pi} \sum_{n_x} \sum_{n_y} \left[ B_{44}(E_{FD},n_x,n_y) + B_{45}(E_{FD},n_x,n_y) \right]$$

where $B_{44}(E_{FD},n_x,n_y) = \sqrt{A_{29}(E_{FD},n_x,n_y)}$ and

$$B_{45}(E_{FD},n_x,n_y) = \sum_{n_z} Z_{ID}(r) \left[ B_{44}(E_{FD},n_x,n_y) \right]$$

b) The 1D electron energy spectrum for NWs of Ge in this case can be expressed as

$$k_x^2 = A_{30}(E,n_x,n_y)$$

where $A_{30}(E,n_x,n_y) = \left[ I_{29}(E,n_x) - \left(\frac{\hbar^2}{2m^*_1}\right)(\frac{n_y \pi}{d_x})^2 \left(2m^*_2 / \hbar^2\right) \right]$.

$$I_{29}(E,n_x) = \left[ 2C_i \right]^{-1} \left[ A_0(n_x) + \left[ \frac{A_0}{2}(n_x) - 4C_i E + 4 \left( C_i A_0(n_x) \right) \right] \right]$$

The quantized energy levels ($E_{32}$) can be expressed through the equation

$$E_{32} = \bar{A}_0(n_x) + \frac{1}{4C_i} \left[ \frac{C_i \hbar^2}{m^*_1} \left(\frac{n_y \pi}{d_x}\right)^2 - 2\bar{A}_0(n_x) \frac{C_i \hbar^2}{m^*_1} \left(\frac{n_y \pi}{d_x}\right)^2 \right]$$

The EMME in this case can be written following (431) as

$$m^*(E_{FD},n_x,n_y) = \frac{\hbar^2}{2} A'_{30}(E_{FD},n_x,n_y)$$

where $A'_{30}(E_{FD},n_x,n_y) = (2m^*_2 / \hbar^2)I'_{29}(E_{FD},n_x)$.
Thus the mass is a function of Fermi energy and the size quantum numbers due to the presence of band non-parabolicity only.

The electron concentration per unit length is given by

$$n_b = \frac{2g_s}{\pi} \sum_{n_x} \sum_{n_y} \left[ B_{46}(E_{FID}, n_x, n_y) + B_{47}(E_{FID}, n_x, n_y) \right]$$

where

$$B_{46}(E_{FID}, n_x, n_y) = \sqrt{A_{36}(E_{FID}, n_x, n_y)}$$

and

$$B_{47}(E_{FID}, n_x, n_y) = \sum_{r=1}^{n_{r}} Z_{ID}(r) \left[ B_{46}(E_{FID}, n_x, n_y) \right]$$

5.2.12 The EMME in nanowires of Gallium Antimonide

The 1D electron energy spectrum for NWs of GaSb can be expressed as

$$k_z^2 = A_{31}(E, n_x, n_y)$$

where

$$A_{31}(E, n_x, n_y) = ([I_{36}(E) - \phi(n_x, n_y)(2m_e / \hbar^2)]$$

The quantized energy levels ($E_{33}$) can be expressed through the equation

$$I_{36}(E_{33}) = \left(\frac{\hbar^2}{2m_e}\right) \phi(n_x, n_y)$$

The EMME in this case can be written following (435) as

$$m^*(E_{FID}) = \frac{\hbar^2}{2} A'_{31}(E_{FID})$$

where

$$A'_{31}(E_{FID}) = \left[1 - (m_e / m_0) (\bar{E}_{g0} / 2) \left( \frac{\bar{E}_{g0}}{2} \right)^2 + \left[ \left( \frac{\bar{E}_{g0}}{2} \right) \left( 1 - (m_e / m_0) \right) \right]^2 + E_{FID} \bar{E}_{g0} (1 - (m_e / m_0)) \right]^{1/2} (2m_e / \hbar^2)$$

The electron concentration per unit length is given by

$$n_b = \frac{2g_s}{\pi} \sum_{n_x} \sum_{n_y} \left[ B_{48}(E_{FID}, n_x, n_y) + B_{49}(E_{FID}, n_x, n_y) \right]$$

where

$$B_{48}(E_{FID}, n_x, n_y) = \sqrt{A_{31}(E_{FID}, n_x, n_y)}$$

and

$$B_{49}(E_{FID}, n_x, n_y) = \sum_{r=1}^{n_{r}} Z_{ID}(r) \left[ B_{48}(E_{FID}, n_x, n_y) \right]$$
5.2.13 The EMME in nanowires of II-V Materials

The 1D electron energy spectrum for NWs of II-V materials can be expressed as

\[ k_z^2 = A_{32±} (E, n_x, n_y) \]

(439)

where

\[ A_{32±} (E, n_x, n_y) = \alpha_{±} (n_x, n_y) + \beta_{±} (n_x, n_y) \]

(440)

\[ \alpha_{±} (n_x, n_y) = \left[ 2(\theta_3^2 - \theta_1^2) \right]^{-\frac{1}{2}} \left[ 2\theta_1 \alpha_3 (n_x, n_y) - 2\alpha_{±} (n_x, n_y) \theta_3 \right], \]

\[ \alpha_{±} (n_x, n_y) = \alpha_4 (\frac{\hbar^2}{2m}) + \theta_4 \left( \frac{\hbar^2}{2m} \right)^2 + \delta_4 \left( \frac{\hbar^2}{2m} \right)^2 \Delta_3, \]

and \[ \alpha_4 (n_x, n_y) = \left[ 2(\theta_3^2 - \theta_1^2) \right]^{-\frac{1}{2}} \left[ 2\theta_1 \alpha_3 (n_x, n_y) - 2\alpha_{±} (n_x, n_y) \theta_3 \right], \]

\[ \beta_4 = 2\theta_1 \left[ 2(\theta_3^2 - \theta_1^2) \right]^{-\frac{1}{2}}, \quad \beta_3 = 2\theta_1 \left[ 2(\theta_3^2 - \theta_1^2) \right]^{-\frac{1}{2}} \]

(441)

The EMME in this case can be written following (439) as

\[ m' (E_{FD}, n_x, n_y) = \frac{\hbar^2}{2} A_{32±} (E_{FD}, n_x, n_y) \]

(442)

where \[ A_{32±} (E_{FD}, n_x, n_y) = \left[ \beta_4 + \frac{1}{2} \left[ 2\beta_3 E_{FD} + \beta_6 (n_x, n_y) \right] \left[ 2\beta_3 E_{FD} + \beta_{±} (n_x, n_y) E_{FD} \right] \right] \]

Thus the mass is a function of Fermi energy and the size quantum numbers which are the characteristic features of such model.

The electron concentration per unit length is given by

\[ n_x = \frac{e_s}{\pi} \sum_{n_x} \sum_{n_y} \left[ B_{49} (E_{FD}, n_x, n_y) + B_{50} (E_{FD}, n_x, n_y) \right] \]

(443)

where \[ B_{49} (E_{FD}, n_x, n_y) = \left[ A_{32±} (E_{FD}, n_x, n_y) + A_{32±} (E_{FD}, n_x, n_y) \right] \]

(444)

and

\[ B_{50} (E_{FD}, n_x, n_y) = \sum_{n_z} Z_{1D} (r) \left[ B_{49} (E_{FD}, n_x, n_y) \right] \]

166
5.3 Results and Discussions:
The variation of the EMME in NWs of different materials along the transport direction has been exhibited in the figures below at various conditions. Throughout our formalism, we have assumed the NWs to be of rectangular cross-sectional dimensions so that the usual “particle-in-a-box” concept can be applied along the quantized directions. The NWs are assumed to be degenerately doped with a carrier density starting from $10^8$ m$^{-1}$. While deriving the closed form analytical solutions of EMME in all the materials, we have also assumed that the constants of the energy band structures of the materials are independent of thickness in the range beyond 5nm. Generally speaking, the “band-gap” is a strong function of cross-sectional dimension and its geometry, i.e., whether the cross-section is circular or triangular. For example in Si NW, the band gap is very high and even becomes direct rather than its usual indirect nature in the zone 1-4 nm cross-sectional dimensions and beyond this, the band gap is nearly equal to its bulk value.

Keeping this trend in view, we have assumed the invariant property of the material energy spectrum constants and evaluated the EMME in NWs of Cd$_3$As$_2$ and CdGeAs$_2$ as function of wire thickness along their respective transport directions in figs. 5.1 and 5.2.

![Fig. 5.1: Plot of the EMME as function of wire thickness for QWs of Cd$_3$As$_2$. The plots for three and two band models of Kane have also been exhibited with their corresponding anisotropic bulk values as presented in Fig. 5.1.](image-url)
Fig. 5.2: Plot of the EMME as a function of wire thickness for QWs of CdGeAs$_2$ for all the cases of Fig. 5.1

Fig. 5.3: Plot of the EMME as function of wire thickness for NWs of InAs considering the three and two band models of Kane with the corresponding isotropic bulk value.
Fig. 5.4: Plot of the EMME as function of wire thickness for NWs of InSb considering the three and two band models of Kane with the corresponding isotropic bulk value.

Fig. 5.5: Plot of the EMME as function of wire thickness for NWs of GaAs considering the three and two band models of Kane with the corresponding isotropic bulk value.
Fig. 5.6: Plot of the EMME as function of wire thickness for NWs of Hg$_{1-x}$Cd$_x$Te considering the three and two band models of Kane with the corresponding isotropic bulk value at $x = 0.3$.

Fig. 5.7: Plot of the EMME as function of wire thickness for NWs of In$_{1-x}$Ga$_x$As$_{1-y}$P$_y$ considering the three and two band models of Kane with the corresponding isotropic bulk value at $x = 0.3$. 
It appears from figs. 5.1 and 5.2 that the EMME at the lowest sub-band in both the cases are strong cross-sectional functions of the dimensions which converge to their corresponding bulk values at larger dimensions. The effect of crystal field splitting in case of dispersion relation of Cd$_3$As$_2$ lets the asymptotic fall to be closer to the bulk value. It should be noted that all the curves have been evaluated at $T = 4$ K where the average thermal energy i.e., $E_F + k_B T$ is very less than that of the difference of the adjacent sub-band energies. This leads the carrier to reside in the lowest sub-bands only. Figs. 5.4-5.5 exhibit the variation of the EMME with the wire thickness for III-V materials namely InAs, InSb and GaAs in accordance with the well-known standard non-parabolic dispersion relation of Kane. It appears from these figures that the difference in the energy band models in predicting the EMME is almost insignificant. Hence for all practical purposes for determination of EMME, the second order model of Kane can fit well. It should be noted that the EMME for these materials as presented here can be compared with that of the EMME in 2D systems as given in Section 1.

Fig. 5.8: Plot of the EMME as function of carrier degeneracy for n-Cd$_3$As$_2$ nanowire for all cases of Fig. 5.1.

A quick view can lead to interpret that the EMME for both the case are same. However, it should also be kept in mind about the difference in the wire thickness and carrier
concentration. All the curves in this chapter have been evaluated at those concentrations for which the EMME stand close to that of their corresponding 2D systems. Further in deriving the results, we have assumed that the conduction band valley does not split along the channel transport direction, which is a usual case with Silicon nanowire along [110] and [111] valleys.

Figs 5.6 and 5.7 exhibit the variation of the EMME for Hg$_{1-x}$Cd$_x$Te and In$_{1-x}$Ga$_x$As$_{1-y}$P$_y$ considering all the aforementioned cases at $x = 0.3$. In Figs 5.8-5.14, we have exhibited the variation of the EMME as function of carrier degeneracy. It appears from the said figures that the EMME increases with the increase in the degeneracy. The EMME rises sharply above $10^7$ m$^{-1}$ for all the materials in an exponential way due to the presence of the Fermi-Dirac probability factor in the respective carrier concentration equation. Fig. 5.15 exhibits the variation of the EMME as function of the alloy composition in ternary and quaternary systems. It appears that as in the previous cases of quantum confinements, the EMME in this case also exhibits an increasing variation with $x$.

Fig 5.9: Plot of the EMME as function of carrier degeneracy for NWs of CdGeAs$_2$ for all cases of Fig. 5.2
Fig 5.10: Plot of the EMME as function of carrier degeneracy for n-InAs nanowire for all cases of Fig. 5.3.

Fig 5.11: Plot of the EMME as function of carrier degeneracy for n-InSb nanowire for all cases of Fig 5.4.
Fig. 5.12: Plot of the EMME as function of carrier degeneracy for n-GaAs nanowire for all cases of Fig. 5.5.

Fig. 5.13: Plot of the EMME as function of carrier degeneracy for n-Hg$_{1-x}$Cd$_x$Te nanowire for all cases of Fig. 5.6
Fig. 5.14: Plot of the EMME as function of carrier degeneracy for n-In$_{1-x}$Ga$_x$As$_{1-y}$P$_y$ nanowire for all cases of Fig. 5.8.

Fig. 5.15: Plot of the EMME as function of alloy composition for n-Hg$_{1-x}$Cd$_x$Te and n-In$_{1-x}$Ga$_x$As$_{1-y}$P$_y$ nanowires.
We have also plotted the variation of the bulk effective mass as $x$ varies to present a comparative view. The influence of band non-parabolicity on the EMME in these two materials can easily be seen. In both the cases, we see that the EMME is a slow variation function of $x$ due to the change in band gap. The effect of dimensionality on nanowire of Bismuth has been exhibited in Figs. 5.16 and 5.17 for the energy band models of McClure and Choi, Hybrid, Cohen and Lax. It appears that as the dimension reduces in Fig. 5.16, the EMME increases which is generally accepted. However, using McClure and Choi model, the EMME tends to decrease in the sub-5 nm regime thereby unfolding the validity of the model in this zone. However at large thicknesses, the entire model tends to their corresponding bulk value $0.00194m_0$.

The variation of the EMME as function of carrier degeneracy has further been plotted using the aforementioned band structure models. It appears from the two figures that the effect of different band structure models has significantly less deviation from one another except for model of McClure and Choi. The EMME is found to increase almost linear with degeneracy $2 \times 10^8 \text{ m}^{-1}$ and beyond using the all the models. In case of PbTe, the EMME rises sharply with decreasing wire thickness below sub-15 nm from the bulk value $0.098m_0$ at carrier degeneracy of $10^9 \text{ m}^{-1}$, which can affect the carrier mobility strongly.
Fig. 5.17: Plot of the EMME as function of carrier degeneracy for Bismuth nanowire.

Fig. 5.18: Plot of the EMME as function of carrier degeneracy for PbTe nanowire
Fig. 5.19: Plot of the EMME as function of wire thickness for stressed InSb nanowire

Fig. 5.20: Plot of the EMME as function of wire thickness for Ge, GaSb and Bi$_2$Te$_3$ nano wires.
The effect of strain on stressed InSb nanowires has been exhibited in fig. 5.19 for two different momentum matrix elements to signify its importance as dimension reduces. A compressive strain of 3% has been applied along x and z directions to predict the variation of the EMME along the transport direction. At this point, we could not provide the influence of strain on the energy band structure of InSb nanowire due to the lack in both experimental and simulation investigations. It is also not very clear about the exact value of how the momentum matrix element $B_2$ will change in the definition of the strain. All the parameters together with the factor $a+C_1$, as given in the Section 1 is expected to modulate with the applied strain orientation. Fig. 5.20 exhibits the effect of cross-sectional dimension on the EMME in n-Ge, n-GaSb and topological insulators like $\text{n-Bi}_2\text{Te}_3$. We have used the Cardona model to evaluate the EMME in Ge nanowires. It appears that the EMME in n-Ge and $\text{Bi}_2\text{Te}_3$ evolve from its corresponding bulk value significantly from below 15 nm. However in case of GaSb, the EMME is almost invariant.
Study of the EMME in Quantum Confined Optoelectronic Materials in the Presence of Light Waves

6.1 Introduction
With the advent of nano-photonics, there has been a considerable interest in studying the optical processes in materials and their nanostructures [210]. It appears from the literature, that the investigations have been carried out on the assumption that the carrier energy spectra are invariant quantities in the presence of intense light waves, which is not fundamentally true. The physical properties of materials in the presence of light waves which change the basic dispersion relation have been relatively less investigated in the literature [211]. In this chapter we shall study the EMME in III-V, ternary and quaternary materials and their nano structures on the basis of newly formulated electron dispersion law in the presence of external photo excitation under different physical conditions.

In Section 6.2.1 of the theoretical background 6.2, we have formulated the dispersion relation of the conduction electrons of III-V, ternary and quaternary materials in the presence of light waves whose unperturbed electron energy spectrum is described by the three band model of Kane. In the same section, we have studied the dispersion relations for the said materials in presence of external photo-excitation when the unperturbed energy spectra are defined by the two band model of Kane and that of parabolic energy bands respectively for the purpose of relative comparison. In Section 6.2.2, we have derived the expressions for the electron statistics and the EMME for all the aforementioned cases. We have also investigated the EMME for the aforementioned band models in the absence of light waves consequently. In section 6.3, the EMME has been numerically investigated by taking n-InAs and n-InSb as examples of III-V materials, n-Hgl_xCd_xTe as an example of ternary compounds and n-In_{1-x}Ga_xAs_yP_{1-y} lattice matched to InP as an example of quaternary materials in accordance with the three and the two band models of Kane together with the models of parabolic energy bands respectively for the purpose of relative assessment.

6.2 Theoretical Background

6.2.1 The formulation of the electron dispersion relation in the presence of light waves in III-V, ternary and quaternary materials
The Hamiltonian \( \hat{H} \) of an electron in the presence of light wave characterized by the vector potential \( \vec{A} \) can be written following [212] as
\[
\hat{H} = \left[ \left( \hat{p} + |e| \vec{A} \right)^2 / 2m \right] + V(\vec{r}) \tag{443}
\]
in which, \( \hat{p} \) is the momentum operator, \( V(\vec{r}) \) is the crystal potential and \( m \) is the free electron mass.

(443) can be expressed as
\[
\hat{H} = \hat{H}_0 + \hat{H}'
\tag{444}
\]
where, \( \hat{H}_0 = \frac{\hat{p}^2}{2m} + V(\vec{r}) \) and
\[
\hat{H}' = \frac{|e|}{2m} \vec{A} \cdot \hat{p}
\tag{445}
\]
The perturbed Hamiltonian \( \hat{H}' \) can be written as
\[
\hat{H}' = \left( -\frac{i\hbar|e|}{2m} \right) \left( \vec{A} \cdot \nabla \right)
\tag{446}
\]
where, \( i = \sqrt{-1} \) and \( \hat{p} = -i\hbar \nabla \)

The vector potential \( \vec{A} \) of the monochromatic light of plane wave can be expressed as
\[
\vec{A} = A_0 \vec{e}_s \cos(\vec{e}_s \cdot \vec{r} - \omega t)
\tag{447}
\]
where \( A_0 \) is the amplitude of the light wave, \( \vec{e}_s \) is the polarization vector, \( \vec{e}_s \) is the momentum vector of the incident photon, \( \vec{r} \) is the position vector, \( \omega \) is the angular frequency of light wave and \( t \) is the time scale. The matrix element of \( \hat{H}' \) between initial state, \( \psi_i(\vec{q}, \vec{r}) \) and final state \( \psi_n(\vec{k}, \vec{r}) \) in different bands can be written as
\[
\hat{H}'_{nk} = \frac{|e|}{2m} \left\langle n\vec{k} | \vec{A} \cdot \nabla | l\vec{q} \right\rangle
\tag{448}
\]
Using (446) and (447), we can re-write (448) as
\[
\hat{H}'_{nk} = \left( -\frac{i\hbar|e|A_0}{4m} \right) \vec{e}_s \left[ \left\langle n\vec{k} | e^{i\vec{e}_s \cdot \vec{r}} | l\vec{q} \right\rangle e^{-i\omega t} \right] + \left\langle n\vec{k} | e^{-i\vec{e}_s \cdot \vec{r}} | l\vec{q} \right\rangle e^{i\omega t} \left\langle n\vec{k} | e^{i\vec{e}_s \cdot \vec{r}} | l\vec{q} \right\rangle e^{-i\omega t}
\tag{449}
\]
The first matrix element of (449) can be written as
\[
\left\langle n\vec{k} | e^{i\vec{e}_s \cdot \vec{r}} | l\vec{q} \right\rangle = \int e^{i[\vec{q} \cdot \vec{k} - \vec{q} \cdot \vec{r}]} u_n^*(\vec{k}, \vec{r}) u_l(\vec{q}, \vec{r}) d^3r + \int e^{i[\vec{q} \cdot \vec{k} - \vec{q} \cdot \vec{r}]} u_n^*(\vec{k}, \vec{r}) \nabla u_l(\vec{q}, \vec{r}) d^3r
\tag{450}
\]
181
The functions $u^*_n u_i$ and $u^*_n \nabla u_i$ are periodic. The integral over all space can be separated into a sum over unit cells times an integral over a single unit cell. It is assumed that the wave length of the electromagnetic wave is sufficiently large so that if $\vec{k}$ and $\vec{q}$ are within the Brillouin zone, $(\vec{q} + \vec{s}_0 - \vec{k})$ is not a reciprocal lattice vector.

Therefore, we can write (450) as

$$< \mathbf{n}\mathbf{k} | e^{(\mathbf{q} - \mathbf{k})} \nabla | \mathbf{n}\mathbf{k} > = \left[ \frac{(2\pi)^3}{\Omega} \right] \left\{ i\mathbf{q} \delta(\mathbf{q} + \mathbf{s}_0 - \mathbf{k}) \delta_{nl} + \delta(\mathbf{q} + \mathbf{s}_0 - \mathbf{k}) \int_{\text{cell}} u^*_n(\mathbf{k}, \mathbf{r}) \nabla u_i(\mathbf{q}, \mathbf{r}) d^3r \right\}$$

$$= \left[ \frac{(2\pi)^3}{\Omega} \right] \left\{ \delta(\mathbf{q} + \mathbf{s}_0 - \mathbf{k}) \int_{\text{cell}} u^*_n(\mathbf{k}, \mathbf{r}) \nabla u_i(\mathbf{q}, \mathbf{r}) d^3r \right\}$$

(451)

where, $\Omega$ is the volume of the unit cell and $\int_{\text{cell}} u^*_n(\mathbf{k}, \mathbf{r}) u_i(\mathbf{q}, \mathbf{r}) d^3r = \delta(\mathbf{q} - \mathbf{k}) \delta_{nl} = 0$, since $n \neq l$

The delta function expresses the conservation of wave vector in the absorption of light wave and $\mathbf{s}_0$ is small compared to the dimension of a typical Brillouin zone and we set $\mathbf{q} = \mathbf{k}$.

From (450) and (451), we can write,

$$\hat{H}'_{nl} = \frac{|e| A_{\mathbf{q}}}{2m} \hat{\mathbf{e}} \cdot \hat{\mathbf{p}}_n(\mathbf{k}) \delta(\mathbf{q} - \mathbf{k}) \cos(\alpha \mathbf{r})$$

(452)

where, $\hat{\mathbf{p}}_n(\mathbf{k}) = -i\hbar \int_{\text{cell}} u^*_n(\mathbf{k}, \mathbf{r}) \mathbf{u}_i(\mathbf{q}, \mathbf{r}) d^3r = \int_{\text{cell}} u^*_n(\mathbf{k}, \mathbf{r}) \mathbf{u}_i(\mathbf{q}, \mathbf{r}) d^3r$

Therefore, we can write

$$\hat{H}'_{nl} = \frac{|e| A_{\mathbf{q}}}{2m} \hat{\mathbf{e}} \cdot \hat{\mathbf{p}}_n(\mathbf{k})$$

(453)

where, $\hat{\mathbf{e}} = \mathbf{e}_1 \cos \alpha \mathbf{r}$.

When a photon interacts with a semiconductor, the carriers (i.e., electrons) are generated in the bands which are followed by the inter-band transitions. For example, when the carriers are generated in the valence band, the carriers then make inter-band transition to the conduction band. The transition of the electrons within the same band i.e., $\hat{H}'_{nn} = \langle \mathbf{n}\mathbf{k} | \hat{H}' | \mathbf{n}\mathbf{k} \rangle$ is neglected. Because, in such a case, i.e., when the carriers are generated within the same bands by photons, are lost by recombination within the aforementioned band resulting zero carriers.

Therefore, $< \mathbf{n}\mathbf{k} | \hat{H}' | \mathbf{n}\mathbf{k} > = 0$ (454)
With $n=c$ stands for conduction band and $l=v$ stand for valance band, the energy equation for the conduction electron can approximately be written as

$$I_{11}(E) = \left( \frac{\hbar^2 k^2}{2m_c} \right) + \frac{\left| e A_0 \right|^2}{2m} \left( \left| \mathcal{E} \mathcal{P}_{\sigma} (\vec{k}) \right|^2 \right)_{av} \tag{455}$$

where, $I_{11}(E) \equiv E(aE+1)(bE+1)/(cE+1)$, $a \equiv 1/E_{g0}$, $E_{g0}$ is the un-perturbed band-gap, $b \equiv 1/(E_{g0} + \Delta)$, $c \equiv 1/(E_{g0} + 2\Delta/3)$, and $\left( \left| \mathcal{E} \mathcal{P}_{\sigma} (\vec{k}) \right|^2 \right)_{av} \text{ represents the average of}$

the square of the optical matrix element (OME).

For the three-band model of Kane, we can write,

$$\xi_{ik} = E_c(\vec{k}) - E_v(\vec{k}) = (E_{g0}^2 + E_{g2})^{1/2} \tag{456}$$

where, $m_r$ is the reduced mass and is given by $m_r^{-1} = (m_c)^{-1} + m_h^{-1}$, and $m_v$ is the effective mass of the heavy hole at the top of the valance band in the absence of any field.

The doubly degenerate wave functions $u_1(\vec{k}, \vec{r})$ and $u_2(\vec{k}, \vec{r})$ can be expressed as [213]

$$u_1(\vec{k}, \vec{r}) = a_{k+} \left[ (is)^{\downarrow} \right] + b_{k+} \left[ X' - i Y' \right] + c_{k+} \left[ Z' \downarrow \right] \tag{457}$$

and

$$u_2(\vec{k}, \vec{r}) = a_{k-} \left[ (is)^{\uparrow} \right] - b_{k-} \left[ X' + i Y' \right] + c_{k-} \left[ Z' \uparrow \right] \tag{458}$$

$s$ is the s-type atomic orbital in both unprimed and primed coordinates, $\downarrow'$ indicates the spin down function in the primed coordinates,

$$a_{kz} = \beta \left[ E_{g0}^2 - (\gamma_{0xz}^2)^2 (E_{g0} - \delta') \right]^{-1/2} (E_{g0}^2 + \delta')^{-1/2}, \beta \equiv \left[ 6(E_{g0} + 2\Delta/3)(E_{g0} + \Delta) / \chi \right]^{1/2} \tag{459}$$

$$\chi = (6E_{g0} + 9E_{g2} \Delta + 4\Delta^2), \gamma_{0xz} = \left[ \frac{E_{g0}^2 + 2m_{k+}}{2(E_{g0} + \delta)} \right]^{1/2} \tag{460}$$

$$\xi_{0x} \equiv E_c(\vec{k}) - E_v(\vec{k}) = E_{g0} \left[ 1 + 2 \left( 1 + \frac{m^r}{m_v} \right) \frac{I_{11}(E)}{E_{g0}} \right]^{1/2}, \delta' \equiv \left( E_{g2} \Delta \right)^{-1}, X', Y', \text{ and } Z' \text{ are the }$$

p-type atomic orbitals in the primed coordinates, $\uparrow'$ indicates the spin-up function in the primed coordinates, $b_{k+} = \rho \gamma_{0xz}$, $\rho = \left( 4\Delta^2 / 3\chi \right)^{1/2}$, $c_{k+} = t \gamma_{0xz}$ and

$$t = \left[ 6(E_{g0} + 2\Delta/3)^2 / \chi \right]^{1/2} \tag{461}$$

We can, therefore, write the expression for the optical matrix element (OME) as
Since the photon vector has no interaction in the same band for the study of inter-band optical transition, we can therefore write
\[ \langle S | \hat{\rho} | S \rangle = \langle X | \hat{\rho} | X \rangle = \langle Y | \hat{\rho} | Y \rangle = \langle Z | \hat{\rho} | Z \rangle = 0 \]
and \[ \langle X | \hat{\rho} | Y \rangle = \langle Y | \hat{\rho} | Z \rangle = \langle Z | \hat{\rho} | X \rangle = 0 \]

There are finite interactions between the conduction band (CB) and the valence band (VB) and we can obtain
\[ \langle S | P | X \rangle = i \hat{\nu} \hat{P} = i \hat{\nu} \hat{\nu} \]
\[ \langle S | P | Y \rangle = j \hat{\nu} \hat{P} = j \hat{\nu} \hat{\nu} \]
\[ \langle S | P | Z \rangle = k \hat{\nu} \hat{P} = k \hat{\nu} \hat{\nu} \]

where, \( \hat{i}, \hat{j}, \) and \( \hat{k} \) are the unit vectors along \( x, y \) and \( z \) axes respectively.

It is well known that
\[ \begin{bmatrix} \uparrow \\ \downarrow \end{bmatrix} = \begin{bmatrix} e^{-i\theta/2}\cos(\theta/2) & e^{i\theta/2}\sin(\theta/2) \\ -e^{-i\theta/2}\sin(\theta/2) & e^{i\theta/2}\cos(\theta/2) \end{bmatrix} \begin{bmatrix} \uparrow \\ \downarrow \end{bmatrix} \]

\[ \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\phi \\ -\sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \]

Besides, the spin vector can be written as
\[ \vec{\sigma} = \frac{\hbar}{2} \vec{\sigma}, \text{ where, } \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \text{ and } \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \]

From above, we can write
\[ \hat{p}_{CV}(k) = \left\langle u_1(\vec{k}, \vec{\nu}) \right| \hat{P} \left| u_2(\vec{k}, \vec{\nu}) \right\rangle \]
\[ = \left\langle \left\{ a_k \left[ (iS) \downarrow \right] + b_k \left[ \frac{X'-iY'}{\sqrt{2}} \right] \uparrow + c_k \left[ Z' \downarrow \right] \right\} \right| \hat{P} \left| \left\{ a_k \left[ iS \right] \uparrow \right\} \right\rangle \]
\[ - b_k \left[ \frac{X'+iY'}{\sqrt{2}} \right] \downarrow + c_k \left[ Z' \uparrow \right] \right\rangle \]

Using above relations, we get
\[ \hat{p}_{CV}(k) = \left\langle u_1(\vec{k}, \vec{\nu}) \right| \hat{P} \left| u_2(\vec{k}, \vec{\nu}) \right\rangle \]

184
\[ \frac{b_k^* a_k}{\sqrt{2}} \{ \langle (X' - iY') | \hat{P} iS \rangle \langle \uparrow \uparrow \rangle \} + c_k a_k \{ \langle Z' | \hat{P} iS \rangle \langle \downarrow \uparrow \rangle \} - \frac{a_k b_k}{\sqrt{2}} \{ \langle iS | \hat{P} (X' + iY') \rangle \langle \downarrow \uparrow \rangle \} + a_k c_k \{ \langle iS | \hat{P} Z' \rangle \langle \downarrow \uparrow \rangle \} \]  

(460)

From (460), we can write
\[
\langle (X' - iY') | \hat{P} iS \rangle = \langle (X') | \hat{P} iS \rangle - \langle (iY') | \hat{P} iS \rangle = i \int u_x \hat{P} S - [ - iu_x \hat{P} iu_x ] = i \langle X' | \hat{P} S \rangle - \langle Y' | \hat{P} S \rangle
\]

From the above relations, for \( X', Y' \) and \( Z' \), we get

\[ |X'\rangle = \cos \theta \cos \phi |X\rangle + \cos \theta \sin \phi |Y\rangle - \sin \theta |Z\rangle \]

Thus,
\[ \langle X' | \hat{P} S \rangle = \cos \theta \cos \phi \langle X | \hat{P} S \rangle + \cos \theta \sin \phi \langle Y | \hat{P} S \rangle - \sin \theta \langle Z | \hat{P} S \rangle = \hat{P} \hat{r}_1 \]

where, \( \hat{r}_1 = i \cos \theta \cos \phi + j \cos \theta \sin \phi - \hat{k} \sin \theta \)

\[ |Y'\rangle = -\sin \phi |X\rangle + \cos \phi |Y\rangle + 0 |Z\rangle \]

Thus,
\[ \langle Y' | \hat{P} S \rangle = -\sin \phi \langle X | \hat{P} S \rangle + \cos \phi \langle Y | \hat{P} S \rangle + 0 \langle Z | \hat{P} S \rangle = \hat{P} \hat{r}_2 \]

where \( \hat{r}_2 = -\hat{i} \sin \phi + \hat{j} \cos \phi \)

so that \( \langle (X' - iY') | \hat{P} S \rangle = \hat{P} (\hat{r}_1 - \hat{r}_2) \)

Thus,
\[ \frac{a_k b_k}{\sqrt{2}} \langle (X' - iY') | \hat{P} S \rangle \langle \uparrow \uparrow \rangle = \frac{a_k b_k}{\sqrt{2}} \hat{P} (\hat{r}_1 - \hat{r}_2) \langle \uparrow \uparrow \rangle \]

(461)

Now since,
\[ \langle iS | \hat{P} (X' + iY') \rangle = i \langle S | \hat{P} X' \rangle - \langle S | \hat{P} Y' \rangle = \hat{P} (\hat{r}_1 - \hat{r}_2) \]

We can write,
\[ - \left[ \frac{a_k b_k}{\sqrt{2}} \{ \langle iS | \hat{P} (X' + iY') \rangle \langle \downarrow \uparrow \rangle \} \right] = - \left[ \frac{a_k b_k}{\sqrt{2}} \hat{P} (\hat{r}_1 - \hat{r}_2) \langle \downarrow \uparrow \rangle \right] \]

(462)

Similarly, we get
\[ |Z'\rangle = \sin \theta \cos \phi |X\rangle + \sin \theta \sin \phi |Y\rangle + \cos \theta |Z\rangle \]

So that,
\[ \langle Z' | \hat{P} iS \rangle = i \langle Z' | \hat{P} S \rangle = i \hat{P} \{ \sin \theta \cos \phi \hat{i} + \sin \theta \sin \phi \hat{j} + \cos \theta \hat{k} \} = i \hat{P} \hat{r}_3 \]

where, \( \hat{r}_3 = i \sin \theta \cos \phi + j \sin \theta \sin \phi + \hat{k} \cos \theta \)

Thus,
\[ c_k a_k \langle Z' | \hat{P} iS \rangle \langle \downarrow \uparrow \rangle = c_k a_k i \hat{P} \hat{r}_3 \langle \downarrow \uparrow \rangle \]

(463)

Similarly, we can write,
\[ c_k a_k \langle i S | \hat{P} | Z' \rangle \langle \downarrow | \uparrow \rangle = c_k a_k \hat{P} \rangle \langle \downarrow | \uparrow \rangle \]  

(464)

Therefore, we obtain

\[
\frac{a_k b_k}{\sqrt{2}} \{ \langle (X' - iY') | \hat{P} | S \rangle \langle \uparrow | \uparrow \rangle \} - \frac{a_k b_k}{\sqrt{2}} \{ \langle iS | \hat{P} | (X' + iY') \rangle \langle \downarrow | \downarrow \rangle \} \]

\[ = \frac{\hat{P}}{\sqrt{2}} (i \hat{r}_1 - \hat{r}_2) + a_k b_k (\langle \uparrow | \uparrow \rangle - \langle \downarrow | \downarrow \rangle) \]  

(465)

Also, we can write,

\[ c_k a_k \langle Z' | \hat{P} | iS \rangle \langle \downarrow | \uparrow \rangle + c_k a_k \langle iS | \hat{P} | Z' \rangle \langle \downarrow | \uparrow \rangle = i\hat{P} (c_k a_k + c_k a_k) \hat{r} \]  

(466)

Combining (465) and (466), we find

\[ \hat{P} \langle \downarrow | \downarrow \rangle \]

(467)

From the above relations, we obtain,

\[
\langle \downarrow | \uparrow \rangle = e^{-\frac{1}{2}} \cos (\theta / 2) \uparrow + e^{i \theta / 2} \sin (\theta / 2) \downarrow \\
\langle \downarrow | \downarrow \rangle = -e^{-\frac{1}{2}} \sin (\theta / 2) \uparrow + e^{i \theta / 2} \cos (\theta / 2) \downarrow 
\]

(468)

Therefore,

\[
\langle \downarrow | \uparrow \rangle = 0, \langle \downarrow | \uparrow \rangle = \frac{1}{2}, \langle \downarrow | \downarrow \rangle = \frac{1}{2} \text{ and } \langle \downarrow | \downarrow \rangle = 0
\]

But we know from above that

\[
\langle \downarrow | \uparrow \rangle = 0, \langle \downarrow | \uparrow \rangle = \frac{1}{2}, \langle \downarrow | \downarrow \rangle = 0
\]

Thus, from equation (469), we get

\[
\langle \downarrow | \uparrow \rangle = \frac{1}{2} [e^{-\frac{1}{2}} \cos (\theta / 2) - e^{i \theta / 2} \sin (\theta / 2)]
\]

(470)

Similarly, we obtain

\[
\langle \downarrow | \uparrow \rangle = \frac{1}{2} [i \cos \theta + \sin \theta \cos \theta] \text{ and } \langle \downarrow | \uparrow \rangle = \frac{1}{2} [-\sin \theta]
\]

Therefore,

\[
\langle \downarrow | \uparrow \rangle = i \langle \downarrow | \uparrow \rangle + j \langle \downarrow | \uparrow \rangle + \hat{k} \langle \downarrow | \uparrow \rangle
\]

(471)

Therefore,

\[
\langle \downarrow | \uparrow \rangle = \frac{1}{2} [\hat{i} (\cos \theta \cos \phi + \sin \theta \cos \phi) - i \sin \theta \phi]
\]

(472)

Similarly, we obtain

\[
\langle \downarrow | \uparrow \rangle = \frac{1}{2} [i (\cos \phi + \sin \phi \cos \theta) + \hat{j} - \sin \theta \hat{k}]
\]

Thus, we get

\[
\langle \downarrow | \uparrow \rangle = i (\langle \downarrow | \uparrow \rangle + \hat{k} \langle \downarrow | \uparrow \rangle)
\]

Therefore,

\[
= \frac{1}{2} [\hat{i} (\cos \theta \cos \phi) + (\sin \phi \cos \theta) \hat{j} - \sin \theta \hat{k}]
\]

(469)
Similarly, we can write
\[ \langle \uparrow \mid \uparrow \rangle = \frac{1}{2} [\sin \theta \cos \phi + \sin \theta \sin \phi + \hat{k} \cos \theta] = \frac{1}{2} \hat{r}_z \quad \text{and} \quad \langle \downarrow \mid \downarrow \rangle = -\frac{1}{2} \hat{r}_z \]

Using the above results we can write
\[
\hat{P}_{cv}(\vec{k}) = \frac{\hat{P}}{\sqrt{2}} (i \hat{r}_1 - \hat{r}_2) \left\{ (a_k, b_k) \langle \uparrow \mid \uparrow \rangle - (b_k, a_k) \langle \downarrow \mid \downarrow \rangle \right\} + i \hat{P}_r \left\{ (c_k, a_k - c_k, a_k) \langle \downarrow \mid \downarrow \rangle \right\}
\]

Thus, \( \hat{P}_{cv}(\vec{k}) = \frac{\hat{P}}{2} \hat{r}_1 (i \hat{r}_1 - \hat{r}_2) \left\{ (a_k, b_k) \left( \frac{b_k}{\sqrt{2}} + c_k \right) + a_k \left( \frac{b_k}{\sqrt{2}} + c_k \right) \right\} \) (471)

It can be written that,
\[ |\hat{\epsilon}| = |\hat{r}_1| = |\hat{r}_2| = 1 \], also, \( \hat{P}_r = \hat{P}_r \sin \theta \cos \phi \hat{I} + \hat{P}_s \sin \theta \sin \phi \hat{J} + \hat{P}_c \cos \theta \hat{K} \)

where, \( \hat{P} = \langle S \mid \hat{P} \mid X \rangle = \langle S \mid \hat{P} \mid Y \rangle = \langle S \mid \hat{P} \mid Z \rangle \),

\[ \langle S \mid \hat{P} \mid X \rangle = \int u_c^* (0, \vec{r}) \hat{P} u_v (0, \vec{r}) d^3r = \hat{P}_{cvx} (0) \quad \text{and} \quad \langle S \mid \hat{P} \mid Z \rangle = \hat{P}_{cvz} (0) \]

Thus, \( \hat{P} = \hat{P}_{cvx} (0) = \hat{P}_{cv} (0) = \hat{P}_{cvz} (0) = \hat{P}_{cv} (0) \)

where, \( \hat{P}_{cv} (0) = \int u_c^* (0, \vec{r}) \hat{P} u_v (0, \vec{r}) d^3r = \hat{P} \)

For a plane polarized light wave, we have the polarization vector \( \vec{\epsilon}, = \hat{k}, \) when the light wave vector is traveling along the z-axis. Therefore, for a plane polarized light-wave, we have considered \( \vec{\epsilon}, = \hat{k}. \)

Then, from (471) we get
\[
\langle \vec{\epsilon} \mid \hat{P}_{cv}(\vec{k}) \rangle = \frac{\hat{P}}{2} \hat{r}_1 (i \hat{r}_1 - \hat{r}_2) \left\{ A(\vec{k}) + B(\vec{k}) \right\} \cos \alpha \quad (472)
\]

and
\[
A(\vec{k}) = a_k \left( \frac{b_k}{\sqrt{2}} + c_k \right) \\
B(\vec{k}) = a_k \left( \frac{b_k}{\sqrt{2}} + c_k \right) \quad (473)
\]

Thus, \( |\vec{\epsilon} \rangle \hat{P}_{cv}(\vec{k}) |\rangle^2 = \frac{1}{4} |\hat{P}|^2 |i \hat{r}_1 - \hat{r}_2|^2 [A(\vec{k}) + B(\vec{k})]^2 \cos^2 \alpha = \frac{1}{4} |\hat{P}|^2 [A(\vec{k}) + B(\vec{k})]^2 \cos^2 \alpha \quad (474)\)

So, the average value of \( |\vec{\epsilon} \rangle \hat{P}_{cv}(\vec{k}) |\rangle^2 \) for a plane polarized light-wave is given by
\[
\mathcal{E}^{(r)}(\mathbf{k}) = -\frac{\alpha^2}{2} \left[ A(\mathbf{k}) + B(\mathbf{k}) \right] \left( \frac{2}{3} \int_0^{\pi} \sin^2 \theta \sin \theta d\theta \right) \left( \frac{1}{2} \right) = \frac{2\pi}{3} \left| \mathbf{k} \right|^2 \left[ A(\mathbf{k}) + B(\mathbf{k}) \right]^2
\]  
(475)

where, \[ |\mathbf{k}|^2 = \left( \frac{1}{2} \right) \left[ \mathbf{k} \cdot \mathbf{\hat{p}}_c(0) \right]^2 \] and

\[
\left| \mathbf{k} \cdot \mathbf{\hat{p}}_c(0) \right|^2 = \frac{m^2}{4m_{\nu}} \frac{E_{\nu_0} (E_{\nu_0} + \Delta)}{E_{\nu_0}^2 + \frac{2}{3} \Delta}
\]  
(476)

We shall express \[ A(\mathbf{k}) \] and \[ B(\mathbf{k}) \] in terms of constants of the energy spectra in the following way:

Substituting \[ a_k^\nu, b_k^\nu, c_k^\nu, \] and \[ \nu_{k\nu} \] in \[ A(\mathbf{k}) \] and \[ B(\mathbf{k}) \] in (473) we get

\[
A(\mathbf{k}) = \beta \left( t + \frac{\rho}{\sqrt{2}} \right) \left[ \left( \frac{E_{\nu_0}}{E_{\nu_0} + \Delta} \right) \left( \frac{E_{\nu_0} - \Delta}{E_{\nu_0} + \Delta} \right)^{1/2} \right]
\]  
(477)

\[
B(\mathbf{k}) = \beta \left( t + \frac{\rho}{\sqrt{2}} \right) \left[ \left( \frac{E_{\nu_0}}{E_{\nu_0} + \Delta} \right) \left( \frac{E_{\nu_0} - \Delta}{E_{\nu_0} + \Delta} \right)^{1/2} \right]
\]  
(478)

in which, \[ \nu_{k\nu} = \frac{E_{\nu_0} - \Delta}{2(\xi_{k\nu} + \Delta)} = \frac{1}{2} \left[ 1 - \left( \frac{E_{\nu_0} + \Delta}{\xi_{k\nu} + \Delta} \right) \right] \] and

\[
\nu_{k\nu} = \frac{\xi_{k\nu} + E_{\nu_0}}{2(\xi_{k\nu} + \Delta)} = \frac{1}{2} \left[ 1 + \left( \frac{E_{\nu_0} - \Delta}{\xi_{k\nu} + \Delta} \right) \right]
\]

Substituting \[ x = \xi_{k\nu} + \Delta \] in \[ \nu_{k\nu} \], we can write,

\[
A(\mathbf{k}) = \beta \left( t + \frac{\rho}{\sqrt{2}} \right) \left[ \left( \frac{E_{\nu_0}}{E_{\nu_0} + \Delta} \right) \left( \frac{E_{\nu_0} - \Delta}{x} \right) \frac{1}{2} \left( 1 - \frac{E_{\nu_0} + \Delta}{x} \right) \left( 1 + \frac{E_{\nu_0} - \Delta}{x} \right) \right]
\]  
(479)

Thus, \[ A(\mathbf{k}) = \frac{\beta}{2} \left( t + \frac{\rho}{\sqrt{2}} \right) \left[ 1 - \frac{a_0}{x} + \frac{a_1}{x^2} \right]^{1/2} \]

where, \[ a_0 \equiv \left( E_{\nu_0}^2 + \Delta^2 \right) \left( E_{\nu_0} + \Delta \right)^{-1} \] and \[ a_1 \equiv \left( E_{\nu_0} - \Delta \right)^2 \].

After tedious algebra, one can show that

\[
A(\mathbf{k}) = \frac{\beta}{2} \left( t + \frac{\rho}{\sqrt{2}} \right) \left( E_{\nu_0} - \Delta \right) \left[ \frac{1}{\xi_{k\nu} + \Delta} \left( E_{\nu_0} + \Delta \right) \right]^{1/2} \left[ \frac{1}{\xi_{k\nu} + \Delta} \left( E_{\nu_0} - \Delta \right)^2 \right]^{1/2}
\]  
(479)

Similarly, from (479), we can write,
Using (475), (476), (479) and (480), we can write
\[
\left\langle \psi_\nu(E) \right| \left\langle \psi_\nu(E) \right\rangle_{av} = \frac{\left| e_\nu A_0 \right|^2}{2m} \left\langle E_\nu(k) - E_\nu(k') \right\rangle_{av} = \frac{2\pi}{3} \frac{\beta^2}{t + \frac{\rho}{\sqrt{2}}} \int_{-\infty}^{\infty} \left\{ \left( 1 + \frac{E_{s0} - \delta'}{E_{s1} + \delta'} \right) + \frac{1}{\phi_0(E) + \delta'} \frac{1}{\phi_0(E) + \delta'} \right\}^{1/2} \right\{ \left( 1 + \frac{E_{s0} - \delta'}{E_{s1} + \delta'} \right) + \frac{1}{\phi_0(E) + \delta'} \frac{1}{\phi_0(E) + \delta'} \right\}^{1/2} \right\}^{2}
\]

Following Nag [214], it can be shown that
\[
A_0^2 = \frac{I \lambda^2}{2n^2 c^3 \sqrt{\varepsilon_0}}
\]

where, \( I \) is the light intensity of wavelength \( \lambda \), \( \varepsilon_0 \) is the permittivity of free space and \( c \) is the velocity of light. Thus, the simplified electron energy spectrum in III-V, ternary and quaternary materials in the presence of light waves can approximately be written as
\[
\frac{\hbar^2 k^2}{2m^*} = \beta_0(E, \lambda)
\]

where, \( \beta_0(E, \lambda) \equiv \left[ I_{11}(E) - \theta_0(E, \lambda) \right] \).

Thus, under the limiting condition \( k \to 0 \), from (483), we observe that \( E \neq 0 \) and is positive. Therefore, in the presence of external light waves, the energy of the electron...
does not tend to zero when $\bar{k} \to 0$, whereas for the un-perturbed three band model of Kane, $I_{11}(E) = \left[\frac{\hbar^2 k^2}{2m_c}\right]$ in which $E \to 0$ for $\bar{k} \to 0$. As the conduction band is taken as the reference level of energy, therefore the lowest positive value of $E$ for $\bar{k} \to 0$ provides the increased band gap ($\Delta E_g$) of the semiconductor due to photon excitation. The values of the increased band gap can be obtained by computer iteration processes for various values of $I$ and $\lambda$ respectively.

**Special Cases:**

1) For the two-band model of Kane, we have $\Delta \to 0$. Under this condition, 
\[ I_{11}(E) \to E(1+aE)=\frac{\hbar^2 k^2}{2m_c}. \]
Since, $\beta \to 1, \ t \to 1, \ \rho \to 0, \ \delta' \to 0$ for $\Delta \to 0$, from equation (483), we can write the energy spectrum of III-V, ternary and quaternary materials in the presence of external photo-excitation whose unperturbed conduction electrons obey the two band model of Kane as
\[ \frac{\hbar^2 k^2}{2m_c} = \tau_0(E, \lambda) \tag{484} \]
where, $\tau_0(E, \lambda) = E(1+aE) - B_0(E, \lambda)$,
\[ B_0(E, \lambda) = \frac{e^2}{384\pi^3 c^3 m_e} \left[ 1 + \frac{E}{E_0} \right] \left[ \left( 1 + \frac{E}{E_0} \right) + E_0 \left( \frac{1}{\phi(E)} - \frac{1}{E_0} \right) \right]^2, \]
\[ \phi(E) = E_0 \left[ 1 + \frac{2m_e}{m_e} \right]^{3/2}. \]
2) For relatively wide band gap materials, one can write, $a \to 0, b \to 0, c \to 0$ and $I_{11}(E) \to E$.

Thus, from (484), we get,
\[ \frac{\hbar^2 k^2}{2m_c} = \rho_0(E, \lambda) \tag{485} \]
\[ \rho_0(E, \lambda) = E - \frac{e^2}{96\pi^3 c^3 m_e \sqrt{E_e E_0}} \left[ 1 + \left( \frac{2m_e}{m_e} \right) \right]^{-3/2} \tag{486} \]

6.2.2 The formulation of the EMME in the presence of light waves in III-V, ternary and quaternary materials

The EMME can, in general, be written as
where, $E_F$ is the Fermi energy in the present case. Using (483) and (487) we get,

$$m_s(E_F) = m_s \left[ I_{11} (E_F) - \beta'_0 (E_F, \lambda) \right]$$  \hspace{1cm} (488)$$

where, the primes indicate the differentiation of the differentiable functions with respect to $E_F$. It appears then that the formulation of the EMME requires an expression of electron statistics, which, in turn, is determined by the density-of-states function. Using (483), the density-of-states function for III-V, ternary and quaternary materials in the presence of light waves whose unperturbed conduction electrons obey the three band model of Kane can be expressed as

$$D_0(E) = 4\pi \left( \frac{2m^*_s}{\hbar^2} \right)^{\frac{3}{2}} g_s \sqrt{\beta_0(E, \lambda) \beta'_0 (E, \lambda)}$$  \hspace{1cm} (489)$$

where \( \beta'_0 (E, \lambda) = \frac{\partial}{\partial E} [\beta_0 (E, \lambda)] \).

Combining (489) with the Fermi-Dirac occupation probability factor and using the generalized Sommerfeld's lemma [116], the electron concentration can be written as

$$n_0 = \left( \frac{3\pi^2}{2m^*_s} \right)^{\frac{3}{2}} \left[ M_1 (E_F, \lambda) + N_1 (E_F, \lambda) \right]$$  \hspace{1cm} (490)$$

where, $M_1(E_F, \lambda) \equiv [\beta_1 (E_F, \lambda)]^{\frac{3}{2}}$, $N_1(E_F, \lambda) = \sum_{r=1}^{L(r)} M_1 (E_F, \lambda)$ and

$$L(r) = \left[ 2(k_B T)^2 (1-2^{1-2r}) \xi (2r) \right] \left( \frac{\partial^{2r}}{\partial E^{2r}} \right)_{E=E_F}$$

The expressions of EMME and $n_0$, for III-V, ternary and quaternary materials in the presence of light waves whose unperturbed conduction electrons obey the two band model of Kane can be expressed as

$$m^*(E_F) = m_s \left[ (1+2\alpha E_F) - \beta'_0 (E_F, \lambda) \right]$$  \hspace{1cm} (491)$$

$$n_0 = \left( \frac{3\pi^2}{2m^*_s} \right)^{\frac{3}{2}} g_s \left[ M_2 (E_F, \lambda) + N_2 (E_F, \lambda) \right]$$  \hspace{1cm} (492)$$

where $M_2(E_F, \lambda) \equiv [\alpha q (E_F, \lambda)]^{\frac{3}{2}}$ and $N_2(E_F, \lambda) = \sum_{r=4}^{L(r)} M_2 (E_F, \lambda)$. 

191
The expression of EMME for III-V, ternary and quaternary materials in the presence of light waves whose unperturbed conduction electrons obey the parabolic energy bands can be expressed as

\[ m'(E_F) = m_c \left[ \rho'_0(E_F, \lambda) \right] \quad (493) \]

\[ n_0 = (3\pi^2)^{-1} \left( \frac{2m_c}{h^2} \right)^{3/2} g_v \left[ M_4(E_F, \lambda) + N_4(E_F, \lambda) \right] \quad (494) \]

where \( M_4(E_F, \lambda) = [\rho(E_F, \lambda)]^{3/2} \) and \( N_4(E_F, \lambda) = \sum_{r=1}^{i} L(r)M_5(E_F, \lambda) \).

In the absence of external photo-excitations, the expressions of the EMME and \( n_0 \) in accordance with the three band model of Kane assume the forms

\[ m'(E_{F_0}) = m_c \left[ I'_{11}(E_{F_0}) \right] \quad (495) \]

\[ n_0 = g_v \left( 3\pi^2 \right)^{-1} \left( \frac{2m_c}{h^2} \right)^{3/2} \left[ M_4(E_{F_0}) + N_4(E_{F_0}) \right] \quad (496) \]

where \( E_{F_0} \) is the Fermi energy in the absence of photo-excitation, \( M_4(E_{F_0}) = \left[ I'_{11}(E_{F_0}) \right]^{3/2} \) and \( N_4(E_{F_0}) = \sum_{r=1}^{i} L(r)M_5(E_{F_0}) \).

In accordance with the two band model of Kane, the corresponding expressions of the EMME and \( n_0 \) are given by

\[ m'(E_{F_0}) = m_c \left[ 1 + 2\alpha E_{F_0} \right] \quad (497) \]

\[ n_0 = (3\pi^2)^{-1} \left( \frac{2m_c}{h^2} \right)^{3/2} g_v \left[ M_5(E_{F_0}) + N_5(E_{F_0}) \right] \quad (498) \]

where \( M_5(E_{F_0}) = \left[ E_{F_0}(1+\alpha E_{F_0}) \right]^{3/2} \) and \( N_5(E_{F_0}) = \sum_{r=1}^{i} L(r)M_6(E_{F_0}) \).

Under the constraints \( \Delta >> E_{s_0} \) or \( \Delta << E_{s_0} \) together with the condition \( \alpha E_{F_0} << 1 \), the (498) assumes the form

\[ n_0 = N_c \left[ F_{\gamma_2}(\eta) + \left( \frac{15\alpha k_BT}{4} \right) F_{\gamma_2}(\eta) \right] \quad (499) \]

where \( \eta = \frac{E_{F_0}}{k_BT} \). For relatively wide gap materials \( E_{s_0} \to \infty \) and the (497) and (499) get simplified as to the well known results [214] as
\[ m^\ast (E_F) = m^\ast \] 
and
\[ n_0 = N_c F_{\gamma^2} (\eta) \]

6.3 Results and Discussions:

Using the appropriate equations and the values of the energy band constants from table 1, we have plotted in figs 6.1a to 6.1d, the EMME as functions of electron concentration at \( T = 4.2 \, K \) by taking n-InAs, n-InSb, n-Hg_{1-x}Cd_xTe and n-In_{1-x}Ga_{x}As_{y}P_{1-y} lattice matched to InP as examples of III-V, ternary and quaternary materials which are used for the purpose of numerical computations in accordance with the perturbed three and two band models of Kane and that of perturbed parabolic energy bands respectively. In figs.6.2a to 6.2d, we have plotted the EMME as a function of intensity. In Figs 6.3a to 6.3d we have plotted the EMME as a function of wavelength. In figs 6.4 and 6.5, we have plotted the EMME as functions of the alloy composition for ternary and quaternary materials respectively.

From fig 6.1a, it appears that the EMME increases with the increasing electron concentration for n-InAs and the numerical values of the EMME in the presence of light waves in accordance with all the band models are relatively larger than that of the same in the absence of the external photo-excitation excluding curve (e) where for parabolic energy band the EMME is numerically concentration invariant due to large band gap. The reason behind such behavior is the fact that the Fermi energy is the monotonic increasing function of electron concentration and the EMME increases monotonically with increasing Fermi energy both in the presence and absence of light waves. By comparing the plot (a) with plot (c) in fig 6.1a, we observe that the presence of spin-orbit splitting in curve (a) decreases the value of the EMME as compared with curve (c) in the whole range of carrier degeneracy as considered here. For relatively low values of \( n_0 \), the curves (a),(b),(c) and (d) exhibit converging tendency whereas they differ with each other for relatively higher values of carrier degeneracy.

The curve (e) of fig 6.1a represents the EMME both in the presence and absence of external light waves for the relatively wide band-gap model which is independent of doping.

Plots (a) and (c) of fig 6.1b diverge for relatively low values of \( n_0 \), intersect each other for a particular zone of concentration and then exhibit small difference although both of
Fig. 6.1a: Plot of the normalized EMME as a function of electron concentration for n-InAs in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.

Fig. 6.1b: Plot of the normalized EMME as a function of electron concentration for n-InSb in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.
Fig. 6.1c: Plot of the normalized EMME as a function of electron concentration for n-Hg$_{1-x}$Cd$_x$Te in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.

Fig. 6.1d: Plot of the normalized EMME as a function of electron concentration for In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.
them increase with increasing degeneracy. The numerical values of the EMME for the curves (a), (b), (c) and (d) of n-InSb as given in fig 1.1b are greater as compared with the same for n-InAs as given in Fig. 6.1a, although the nature of the curve (e) is same for both figs 6.1a and 6.1b respectively. The curves (a) and (c) of fig 6.1c explore the fact that the influence of the spin-orbit splitting on the EMME for n-Hg\textsubscript{1-x}Cd\textsubscript{x}Te in the presence of light waves decreases significantly the same mass as compared with the perturbed two band model of Kane and the two curves exhibit wide difference with each other for relatively low values of doping. In the absence of light wave, the effect of $\Delta$ on the EMME is much less. It appears by comparing the figs 6.1a, 6.1b and 6.1c that the EMME for n-Hg\textsubscript{1-x}Cd\textsubscript{x}Te in the presence of external photo-excitation is much more as that of n-InSb and n-InAs respectively.

![Image](image.png)

Fig. 6.2a: Plot of the normalized EMME as a function of light intensity for n-InAs in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.
From plots (a) and (c) of fig 6.1d, it appears for In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP that both the curves maintain constant wide difference under photo-excitation with respect to electron concentration in the whole range of electron degeneracy as considered here. The influence of $\Delta$ on the EMME in the absence of external light waves for three and two band models of Kane is very small as evident from the curves (b) and (d), although the EMME increases with $n_0$ as usual.

The influence of the energy band constants on the EMME is apparent from all the plots of figs 6.1a to 6.1d and the numerical values of the EMME is greatest for ternary alloys and the least for quaternary systems under light waves.

Fig. 6.2b: Plot of the normalized EMME as a function of light intensity for n-InSb in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.

The curves (a) and (c) of fig 6.2a explore that the EMME increases with increasing light intensity for n-InAs in the presence of light waves for both perturbed three and two band models of Kane whereas for perturbed parabolic energy bands the EMME is intensity invariant. The reason behind such behavior is that the Fermi energy increases with
increasing light intensity and the EMME is the function of Fermi energy. For perturbed parabolic energy bands, due to very large band gap the EMME is numerically independent of light intensity. In the absence of light waves the EMME is naturally independent of $I$ which is apparent from the curves (b) and (d) of fig. 6.2a.

The curves (a) and (c) of fig 6.2b for n-InSb reflects the fact that the influence of spin-orbit splitting increases rapidly with increasing intensity and for low values of light intensity the EMME decreases for both perturbed three and two band models of Kane, whereas for higher values of $I$ the EMME increases significantly.

![Normalized Effective Mass](image)

Fig. 6.2c: Plot of the normalized EMME as a function of light intensity for n-Hg$_{1-x}$Cd$_x$Te in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.
Fig. 6.2d: Plot of the normalized EMME as a function of light intensity for In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.

From the plot (a) of fig 6.2c, one can infer that the EMME for n-Hg$_{1-x}$Cd$_x$Te in the presence of light waves increases with increasing $I$, in a more or less linear fashion in accordance with the perturbed three band model of Kane whereas from plot(c), one observes that the EMME on the basis of the perturbed two band model of Kane is greater as compared with plot (a) in the whole range of $I$. For low values of $I$, the curves (a) and (c) exhibits converging tendency. It is important to note that with respect to light intensity, the numerical values of the EMME in the absence of light waves, in the case of n-Hg$_{1-x}$Cd$_x$Te are greater for both types of band models (curves (b) and (d)) as compared with that of (a) and (c) when $I\neq0$.

It appears from the plots (a) and (c) of fig 6.2d that the EMME increases with $I$ for both types of perturbed band models with different small curvatures and the difference in numerical values of EMME increases with increasing in $I$, although they converge to a particular value for $I=10^{-6}$ nWm$^{-2}$. 

199
Fig. 6.3a: Plot of the normalized EMME as a function of wavelength for n-InAs in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.

Fig. 6.3b: Plot of the normalized EMME as a function of wavelength for n-InSb in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.
The curves (a) and (c) of fig. 6.3a exhibit the fact that the EMME for n-InAs increases with increasing wave length in the presence of light waves for both perturbed three and two band models of Kane since the Fermi energy increases with increasing wave length and the EMME is the function of Fermi energy. For perturbed parabolic energy bands, the EMME is numerically independent of wave length due to large band gap. The curves (a) and (c) maintain wide difference with increasing wave length and the spin-orbit splitting decreases the EMME in the whole range of $\lambda$. The curves (b) and (d) exhibit the same variation in the absence of photo excitation and are independent of wave length of the incident light. From the plots (a) and (c) of fig 6.3b for n-InAs, one can infer that the EMME increases with increasing wave length in the presence of light waves for both the cases and the exhibit the diverging tendency for relatively low values of $\lambda$ where as for higher values of the wave length exhibit the converging nature. From plots (b) and (d), we observe that the numerical values of the EMME for both three and two band models of Kane are much larger as compared with the same under photo-excitation for relatively low values of $\lambda$. The influence of $\Delta$ on the EMME for perturbed three band model as observed in fig 6.3b is less as compared with the same as given in plot (a) of fig 6.3a.

Fig. 6.3c: Plot of the normalized EMME as a function of wavelength for n- Hg$_{1-x}$Cd$_x$Te in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.
Fig 6.3d: Plot of the normalized EMME as a function of wavelength for In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.

The curves (b) and (c) for n-Hg$_{1-x}$Cd$_x$Te of fig 6.3c increase with increasing $\lambda$ and after intersection they exhibit wide difference with each other. The spin-orbit splitting decreases the EMME remarkably for relatively higher values of $\lambda$. From fig 6.3d, we can write that the EMME, in this case, is much less when compared with the same as given in, the Figs.6.3a, 6.3b and 6.3c. under external photo-excitation. From plots (a) and (c) of fig 6.3d, it appears for In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP that both the curves maintain wide difference under photo-excitation with respect to $\lambda$ in the whole range of wavelength as considered here.

As the wave length increases, from plots (a) and (c) of fig 6.3d, we infer that the difference also increases. The influence of $\Lambda$ on the EMME in the absence of external light waves for three and two band models of Kane is very small as evident from the curves (b) and (d).
It appears that the EMME increases as the wavelength shifts from violet to red. The influence of light is immediately apparent from the plots in the figs 6.2a to 6.3d since the EMME depends strongly on $I$ and $\lambda$ for the three and the two band models of Kane which is in direct contrast with that for the bulk specimens of the said compounds in the absence of external photo-excitation. The variations of the EMME in the figs 6.2a to 6.3d reflect the direct signature of the light wave on the band structure dependent physical properties of materials in general in the presence of external photo-excitation and the photon assisted transport for the corresponding photonic devices. The numerical values of the EMME in the presence of the light waves are larger than that of the same in the absence of light wave for both the three and the two band models of Kane. Although, the EMME tends to increase with the intensity and the wavelength but the rate of increase is totally and structure dependent. It appears that the numerical values of the EMME are greatest for ternary materials and least for quaternary compounds.

![Graph](image)

Fig 6.4: Plot of the normalized EMME as a function of alloy composition for Hg$_{1-x}$Cd$_x$Te in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.
In figs. 6.4 and 6.5, the EMME has been plotted as a function of alloy composition for $n$-Hg$_{1-x}$Cd$_x$Te and $n$-In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP respectively in which all the cases of fig. 6.1 have further been plotted for the purpose of relative comparison. The Fermi energy decreases with increasing alloy composition and the EMME is a function of Fermi energy.

From fig 6.4, we can write that the EMME in ternary compounds decreases with increasing alloy composition. The numerical values of EMME in the presence of light waves are greater for both the models as appears from the plots (a), (b), (c) and (d). As alloy composition increases, the EMME for all the cases exhibit the converging tendency. The plots of the Fig. 6.4 are valid for $x>0.17$, since for $x<0.17$, the band gap becomes negative in $n$-Hg$_{1-x}$Cd$_x$Te leading to semi-metallic state.

![Normalized Effective Mass (m(Ep) / m_0)](image)

Fig. 6.5: Plot of the normalized EMME as a function of alloy composition for In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP in the presence of light waves in which the curves (a) and (c) represent the three and two band models of Kane respectively. The curves (b) and (d) exhibit the same variation in the absence of external photo-excitation. The curve (e) represents the parabolic energy band model both in the presence and in the absence of the external photo-excitation.

The plots of the fig 6.5 exhibit the variation of the EMME with $y$ for $n$-In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP. As the Fermi energy increases with the $y$, from the curves (a),
(b),(c) and (d) of Fig 6.5 we observe that the EMME increases with increasing y. These four plots also exhibit the fact that the influence of the spin-orbit splitting constant in the presence of light waves is much greater as compared with the same in the absence of photo excitation.

The theoretical results as presented here will be useful in determining the mobility even for relatively wide gap compounds whose energy band structures can be approximated by the parabolic energy bands both in the presence and absence of light waves. It is worth remarking that our basic equation covers various materials having different energy band structures. In this section, the concentration, alloy composition, light intensity and the wavelength dependences of EMME in bulk specimens of n-InAs, n-InSb, n-Hg_{1-x}Cd_xTe and n-In_{1-x}Ga_xAs_yP_{1-y} lattice matched to InP have been studied. Thus, we have covered a wide class of optoelectronic and allied compounds whose energy band structures are defined by the three and two band models of Kane in the absence of photon field. Under certain limiting conditions, all the results of the EMME for different materials having various band structures lead to the well-known expression of the EMME for degenerate compounds having parabolic energy band. This indirect test not only exhibits the mathematical compatibility of our formulation but also shows the fact that our simple analysis is a more generalized one, since one can obtain the corresponding results for the relatively wide gap materials having parabolic energy bands under certain limiting conditions from our present derivation.

6.4 The formulation of the EMME in the presence of quantizing magnetic field under external photo excitation in III-V, ternary and quaternary materials

6.4.1 Introduction

It is well known that the band structure of electronic materials can be dramatically changed by applying the external fields [216]. In the section 6.4.2 of theoretical background we study the effective electron mass in III-V, ternary and quaternary materials under magnetic quantization in the presence of light waves. The section 6.5 explores results and discussions in this context.

6.4.2 Theoretical Background
The magneto-dispersion law, in the absence of spin, for III-V, ternary and quaternary materials, in the presence of photo-excitation, whose unperturbed conduction electrons obey the three band model of Kane, is given by

$$\beta_b(E, \lambda) = \left(n + \frac{1}{2}\right) \hbar \omega_b + \frac{\hbar^2 k^2}{2m_e}$$  \hspace{1cm} (502)

where, $n$ is the Landau quantum number and $\omega_b = eB/m_i$.

Using (502), the density-of-states function in the present case can be expressed as

$$D_b(E, \lambda) = \frac{g_e |\lambda| \sqrt{2m_e}}{2\pi \hbar^2} \sum_{n=0}^{\infty} \left[ \beta_b(E, \lambda) \right]^n \frac{H \left(E - E_{n+i}\right)}{(n + 1/2) \hbar \omega_b}$$  \hspace{1cm} (503)

in which $E_{n+i}$ is the positive lowest root of the equation.

$$\beta_b(E_{n+i}, \lambda) = \left(n + \frac{1}{2}\right) \hbar \omega_b$$  \hspace{1cm} (504)

The EMME in this case assumes the form

$$m^* \left(E_{F_{n+i}}, \lambda\right) = \frac{\hbar^2 k^2}{2E} \left.E \frac{\partial k^2}{\partial E}\right|_{E=E_{n+i}} = m_e \left[ \beta_b(E_{F_{n+i}}, \lambda) \right]^n$$  \hspace{1cm} (505)

where, $E_{F_{n+i}}$ is the Fermi energy under quantizing magnetic field in the presence of light waves as measured from the edge of the conduction band in the vertically upward direction in the absence of any quantization. Combining equation (504) with the Fermi-Dirac occupation probability factor and using the generalized Sommerfeld's lemma, the electron concentration can be written as

$$n = \frac{g_e |\lambda| eB}{\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ M_{13} \left(E_{F_{n+i}}, B, \lambda\right) + N_{13} \left(E_{F_{n+i}}, B, \lambda\right) \right]$$  \hspace{1cm} (506)

where, $M_{13}(E, B, \lambda) = \left[ \beta_b(E, \lambda) - \left(n + \frac{1}{2}\right) \hbar \omega_b \right]^{1/2}$ and $N_{13}(E_{F_{n+i}}, B, \lambda) = \sum_{r=1}^{\infty} L(r) M_{13}(E_{F_{n+i}}, B, \lambda)$.

The magneto-dispersion law in the absence of spin, for III-V, ternary and quaternary materials, in the presence of photo-excitation, whose unperturbed conduction electrons obey the two band model of Kane, is given by

$$\tau_0(E, \lambda) = \left(n + \frac{1}{2}\right) \hbar \omega_b + \frac{\hbar^2 k^2}{2m_e}$$  \hspace{1cm} (507)

Using (507), the density-of-states function in this case can be written as

$$D_b(E, \lambda) = \frac{g_e |\lambda| \sqrt{2m_e}}{2\pi \hbar^2} \sum_{n=0}^{\infty} \left[ \tau_0(E, \lambda) \right]^n \frac{H \left(E - E_{n+i}\right)}{(n + 1/2) \hbar \omega_b}$$  \hspace{1cm} (508)
where, \( E_{n_1} \) is the Landau sub-band energies and can be expressed as
\[
\tau_0(E_{n_1}, \lambda) = \left(n + \frac{1}{2}\right) \hbar \alpha_b \tag{509}
\]

The EMME assumes the form
\[
m^*\left(E_{F_n}, \lambda\right) = m \left\{ \tau_0\left(E_{F_n}, \lambda\right) \right\}' \tag{510}
\]

Thus, the electron concentration can be written as
\[
n_0 = \frac{g_s |e| B \sqrt{2m_r}}{\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ M_{14}\left( E_{F_n}, B, \lambda \right) + N_{14}\left( E_{F_n}, B, \lambda \right) \right] \tag{511}
\]

where, \( M_{14}\left( E_{F_n}, B, \lambda \right) \equiv \left[ \tau_0\left(E_{F_n}, \lambda\right) - \left(n + \frac{1}{2}\right) \hbar \alpha_b \right]^{1/2} \) and
\[
N_{14}\left( E_{F_n}, B, \lambda \right) \equiv \sum_{r=1}^{r} L(r) M_{14}\left( E_{F_n}, B, \lambda \right) .
\]

The magneto-dispersion law in the absence of spin, for III-V, ternary and quaternary materials, in the presence of photo-excitation, whose unperturbed conduction electrons obey the parabolic energy bands, is given by
\[
\rho_0\left(E, \lambda\right) = \left(n + \frac{1}{2}\right) \hbar \alpha_b + \frac{\hbar^2 k^2}{2m_r} \tag{512}
\]

Using (512), the density-of-states function in this case can be written as
\[
D_{gs}\left(E, \lambda\right) = \frac{g_s |e| B \sqrt{2m_r}}{2 \pi^2 \hbar^2} \sum_{n=0}^{\infty} \left\{ \rho_0\left(E, \lambda\right) \right\}' \left\{ \rho_0\left(E, \lambda\right) - \left(n + \frac{1}{2}\right) \hbar \alpha_b \right\}^{-1/2} H\left(E - E_{n_1}\right) \tag{513}
\]

where, \( E_{n_1} \) is the Landau sub-band energies and is given by
\[
\rho_0\left(E_{n_1}, \lambda\right) = \left(n + \frac{1}{2}\right) \hbar \alpha_b \tag{514}
\]

The EMME assumes the form
\[
m^*\left(E_{F_n}, \lambda\right) = m \left\{ \rho_0\left(E_{F_n}, \lambda\right) \right\}' \tag{515}
\]

Thus, the electron concentration in this case can be written as
\[
n_0 = \frac{g_s |e| B \sqrt{2m_r}}{\pi^2 \hbar^2} \sum_{n=0}^{\infty} \left[ M_{15}\left( E_{F_n}, B, \lambda \right) + N_{15}\left( E_{F_n}, B, \lambda \right) \right] \tag{516}
\]

where,
\[
M_{15}\left( E_{F_n}, B, \lambda \right) \equiv \left[ \rho_0\left(E_{F_n}, \lambda\right) - \left(n + \frac{1}{2}\right) \hbar \alpha_b \right]^{1/2} \text{ and } N_{15}\left( E_{F_n}, B, \lambda \right) \equiv \sum_{r=1}^{r} L(r) M_{15}\left( E_{F_n}, B, \lambda \right) .
\]

207
In the absence of light waves all the results of this section agrees with the corresponding results of section 1.

6.5 Results and Discussions:

Using the values of the energy band constants from table 1, we have plotted the EMME along the direction of z as functions of 1/B, electron concentration, intensity and wavelength (as shown in fig 6.6, 6.7, 6.8, and 6.9) at $T = 4.2 \, K$ by taking n-InSb and n-InAs which are used for the purpose of numerical computations in accordance with the perturbed three and two band models of Kane and that of perturbed parabolic energy bands respectively. It appears from fig. 6.6 that the EMME is an oscillatory function of inverse quantizing magnetic field. The oscillatory dependence is due to the crossing over of the Fermi level by the Landau sub-bands in steps resulting in successive reduction the number of occupied Landau levels as the magnetic field is increased.

![Fig 6.6: Plot of the normalized EMME as a function of inverse magnetic field for n-InSb and n-InAs in the presence of light waves in accordance with the three, the two band models of Kane and the parabolic energy band model in the presence of external photo-excitation.](image-url)
Fig 6.7: Plot of the normalized EMME as a function of carrier concentration for n-InSb and n-InAs in the presence of light waves in accordance with the three, the two band models of Kane and the parabolic energy band model in the presence of external photo-excitation.

Fig 6.8: Plot of the normalized EMME as a function of light intensity for n-InSb and n-InAs in the presence of light waves in accordance with the three, the two band models of Kane and the parabolic energy band model in the presence of external photo-excitation.
Fig 6.9: Plot of the normalized EMME as a function of light wavelength for n-InSb and n-InAs in the presence of light waves in accordance with the three, the two band models of Kane and the parabolic energy band model in the presence of external photo-excitation.

For each coincidence of a Landau level, with the Fermi level, there would be a discontinuity in the density-of-states function resulting in a peak of oscillation. Thus the peaks should occur whenever the Fermi energy is a multiple of energy separation between the two consecutive Landau levels and it may be noted that the origin of oscillations in the EMME is the same as that of the Shubnikov-de Hass oscillations. With increase in magnetic field, the amplitude of the oscillation will increase and, ultimately, at very large values of the magnetic field, the conditions for the quantum limit will be reached (neglecting magnetic freeze out) when the EMME will be found to decrease monotonically with increase in magnetic field. In fig 6.7, the concentration dependence of the magneto-EMME has been plotted for all the cases of fig 6.6 for both n-InSb and n-InAs. The EMME again shows oscillatory dependence with different numerical values exhibiting the signature of the SdH effect. Although the rate of variations are different, the influence of the energy band constants in accordance with all the type of the band models is apparent from the figs. One can observe from fig 6.8 that the EMME has a steady increase with the increase of the light intensity although the same EMME increases sharply with the increase in wavelength in different ways, as appears from figs
6.8 and 6.9 respectively. The nature of variations in all the cases depends strongly on the energy spectrum constants of the respective materials and the external physical conditions. It should be noted that the numerical value of the EMME in the presence of light waves is relatively much higher even at smaller value of the magnetic field, than that in the absence of the magnetic field. Such a high value in the EMME can cause a drastic effect by reducing the electron mobility under the application of a quantized magnetic field and the contribution of the oscillatory mass or the oscillatory mobility would be is more important.

6.6 The Formulation of the EMME for the ultrathin films of III-V, Ternary and Quaternary Materials under external photo-excitation

6.6.1 Introduction

It is well known that the concept of reduction of symmetry of the wave-vector space and its consequence can unlock the physics of low dimensional structures. In section 6.6.2 of theoretical background we shall study the EMME in ultrathin films of III-V, ternary and quaternary materials in the presence of external light waves. The section 6.7 contains result and discussions in this context.

6.6.2 Theoretical Background

The 2D electron energy spectrum in ultra-thin films of III-V, ternary and quaternary materials, whose unperturbed band structure is defined by the three band model of Kane, in the presence of light waves, can be expressed as

\[
\frac{\hbar^2 k_x^2}{2 m_e} + \frac{\hbar^2}{2 m_e} \left( \frac{n \pi}{d_z} \right)^2 = \beta_0 (E, \lambda)
\]  

(517)

The sub band energies \((E_{n\tau})\) can be written as

\[
\beta_0(E_{n\tau}, \lambda) = \frac{\hbar^2}{2 m_e} (n \pi / d_z)^2
\]

(518)

The expression of the EMME in this case is given by

\[
m^* (E_{F2D\lambda}, n\tau, \lambda) = \hbar^2 k_z \left. \frac{\partial k}{\partial E} \right|_{E=E_{F2D\lambda}} = m_e \left\{ \beta_0 (E_{F2D\lambda}, \lambda) \right\}'
\]

(519)
where, \( E_{F2DL} \) is the Fermi energy in the present case as measured from the edge of the conduction band in the vertically upward direction in absence of any quantization.

The density-of-states function can be written as

\[
N_{2D}(E, \lambda) = \left( \frac{m^* g_v}{\pi \hbar^2} \right) \sum_{n_z=1}^{\infty} \left[ \beta_0(E, \lambda) \right]' \cdot H \left( E - E_{n_z} \right)
\]  

(520)

Combining (520), with the Fermi Dirac occupation probability factor, the two dimensional electron concentration can be expressed as

\[
n_{2D} = \frac{m^* g_v}{\pi \hbar^2} \sum_{n_z=1}^{\infty} \left[ M_{18}(n_z, E_{F2DL}, \lambda) + N_{18}(n_z, E_{F2DL}, \lambda) \right]
\]  

(521)

where,

\[
M_{18}(n_z, E_{F2DL}, \lambda) = \left[ \beta_0(E_{F2DL}, \lambda) - \frac{\hbar^2}{2m_e} \left( \frac{n_z \pi}{d_z} \right)^2 \right],
\]

and

\[
N_{18}(n_z, E_{F2DL}, \lambda) = \sum_{r=1}^{r} L(r) M_{18}(n_z, E_{F2DL}, \lambda) \]

The expressions for the 2D dispersion relation, the sub-band energies, the EMME, the density of states function and the electron concentration for ultra-thin films of III-V, ternary and quaternary materials whose unperturbed band structure is defined by the two band model of Kane, can respectively be written in the presence of photo-excitation as

\[
\frac{\hbar^2 k_z^2}{2m_e} + \frac{\hbar^2}{2m_e} \left( \frac{n_z \pi}{d_z} \right)^2 = \tau_0(E, \lambda)
\]  

(522)

\[
\tau_0(E, \lambda) = \frac{\hbar^2}{2m_e} (n_z \pi / d_z)^2
\]  

(523)

\[
m^*(E_{F2DL}, n_z, \lambda) = m_e \left\{ \tau_0(E_{F2DL}, \lambda) \right\}'
\]  

(524)

\[
N_{2D}(E, \lambda) = \left( \frac{m^* g_v}{\pi \hbar^2} \right) \sum_{n_z=1}^{\infty} \left[ \tau_0(E, \lambda) \right]' \cdot H \left( E - E_{n_z} \right)
\]  

(525)

\[
n_{2D} = \frac{m^* g_v}{\pi \hbar^2} \sum_{n_z=1}^{\infty} \left[ M_{19}(n_z, E_{F2DL}, \lambda) + N_{19}(n_z, E_{F2DL}, \lambda) \right]
\]  

(526)

where

\[
M_{19}(n_z, E_{F2DL}, \lambda) = \left[ \tau_0(E_{F2DL}, \lambda) - \frac{\hbar^2}{2m^*} \left( \frac{n_z \pi}{d_z} \right)^2 \right],
\]

and

\[
N_{0}(n_z, E_{F2DL}, \lambda) = \sum_{r=1}^{r} L(r) M_{18}(n_z, E_{F2DL}, \lambda)
\]
The expressions for the 2D dispersion relation, the sub-band energies, the EMME, the
density of states function and the electron concentration for ultra-thin films of III-V,
ternary and quaternary materials, whose unperturbed band structure is defined by the
parabolic energy bands, can respectively be written in the presence of photo-excitation as

\[ \frac{\hbar^2 k^2}{2m_c} + \frac{\hbar^2}{2m_c} \left( \frac{n \pi}{d_z} \right)^2 = \rho_0(E, \lambda) \]  \hspace{1cm} (527)

\[ \rho_0(E_{\text{ns}}, \lambda) = \frac{\hbar^2}{2m_c} \left( \frac{n \pi}{d_z} \right)^2 \]  \hspace{1cm} (528)

\[ m^* (E_{F2DL}, n_z, \lambda) = m_c \left\{ \rho_0(E_{F2DL}, \lambda) \right\} \]  \hspace{1cm} (529)

\[ N_{2D}(E, \lambda) = \left( \frac{m^*}{\pi \hbar^2} \right) \sum_{n=1}^{N_{\text{em}}} \left[ \rho_0(E, \lambda) \right] H(E - E_{\text{ns}}) \]  \hspace{1cm} (530)

\[ n_{2D} = \frac{m^* g_s}{\pi \hbar^2} \sum_{n=1}^{N_{\text{em}}} \left[ M_{20}(n_z, E_{F2DL}, \lambda) + N_{20}(n_z, E_{F2DL}, \lambda) \right] \]  \hspace{1cm} (531)

where, \( M_{20}(n_z, E_{F2DL}, \lambda) \) and \( N_{20}(n_z, E_{F2DL}, \lambda) \) are defined as

\[ N_{20}(n_z, E_{F2DL}, \lambda) = \sum_{r=1}^{N_{\text{em}}} L(r) M_{20}(n_z, E_{F2DL}, \lambda). \]

6.7 Results and Discussions:

Using the values of the energy band constants from table 1, we have plotted the EMME
in the \( k_s \) plane as functions of film thickness, surface electron concentration, intensity
and wavelength at \( T = 4.2 \) K by taking ultra thin films of ternary materials which are used
for the purpose of numerical computations in accordance with the perturbed three two
and band models of Kane and that of perturbed parabolic energy bands as shown in figs
6.10, 6.11, 6.12 and 6.13 respectively. The influence of carrier confinement in 2D under
the presence of an external photo-excitation on the behavior of EMME can be understood from the figs 6.10 to 6.13. The effect of quantum confinement is immediately
apparent from all the curves of fig 6.10, since, the 2D EMME depend strongly on the
nano-thickness, which is in direct contrast with the corresponding bulk specimens which
is also the direct signature of quantum confinement. It appears from the said figures that
the EMME in this case decreases with the increasing film thickness in a step like manner
as considered here although the numerical values vary widely and determined by the
constants of the energy spectra. The oscillatory dependence is due to the crossing over of
Fig 6.10: Plot of the normalized EMME as a function of film thickness for n-InSb and n-InAs in the presence of light waves in accordance with the three and the two band models of Kane together with the parabolic energy band model in the presence of external photo-excitation.

Fig 6.11: Plot of the normalized EMME as a function of surface electron concentration for n-InSb and n-InAs in the presence of light waves in accordance with the three and the two band models of Kane together with the parabolic energy band model in the presence of external photo-excitation.
Fig 6.12: Plot of the normalized EMME as a function of light intensity for n-InSb and n-InAs in the presence of light waves in accordance with the three and the two band models of Kane together with the parabolic energy band model in the presence of external photo-excitation.

Fig 6.13: Plot of the normalized EMME as a function of wavelength for n-InSb and n-InAs in the presence of light waves in accordance with the three and the two band models of Kane together with the parabolic energy band model in the presence of external photo-excitation.
the Fermi level by the size quantized levels. For each coincidence of a size quantized level with the Fermi level, there would be a discontinuity in the density-of-states function resulting in a peak of oscillations. With large values of film thickness, the height of the steps decreases and the EMME decreases with increasing film thickness in non-oscillatory manner and exhibit monotonic decreasing dependence. The height of step size and the rate of decrement are totally dependent on the band structure. The influence of energy band non-parabolicity is immediately apparent by comparing the curves of the said figures. The energy band non-parabolicity and the spin orbit splitting constant significantly enhances the numerical values of the EMME both the cases of the materials. The numerical values of the EMME in accordance with the three band model of Kane are different as compared with the corresponding two band model, which reflects that fact that the presence of the spin orbit splitting constant changes the magnitude of the EMME. It may be noted that the presence of the band non-parabolicity in accordance with the two-band model of Kane further changes the peaks of the oscillatory EMME for all cases of quantum confinements.

In fig 6.11, we have plotted the EMME as a function of surface electron concentration per unit area for all cases of fig 6.10. It appears that the EMME increases with increasing carrier degeneracy and also reflects the signature of the 1D confinement through the non-linear dependence with the 2D electron statistics. Since, most of the electrons at low temperatures occupy the lowest sub-band level; we have plotted the EMME by considering the lowest sub-band energy in fig 6.11 to 6.13. If more sub-bands were considered, the oscillatory dependence will be less and less prominent with increasing carrier concentration and ultimately, for bulk specimens of the same material, the EMME will be found to increase continuously with increasing electron concentration in a non-oscillatory manner. The effects of the light intensity and wavelength on the EMME has been exhibited in figs 6.12 and 6.13 respectively in the regime of very low temperatures. The EMME increases with both the variables with different slopes. It appears that the EMME in the case of parabolic energy band varies extremely slowly both with the light intensity and wavelength, although, the sharp and significant variations are exhibited for both three and the two band energy models.

6.8 Investigation of the EMME in n-channel inversion layars of III-V, Ternary and Quaternary materials under external photo-excitation
6.8.1 Introduction

In section 6.8.2 of theoretical background, we shall study the EMME for both weak and strong electric field limits in n-channel inversion layers of III-V, ternary and quaternary materials both in the presence and absence of external light waves. The section 6.9 contains result and discussions in this context.

6.8.2 Theoretical Background

In the presence of a surface electric field $F_s$ along $z$ direction and perpendicular to the surface, in this case we can write

$$\frac{\hbar^2 k^2}{2m_c} = \beta_0 \left( E - |e| F_s z, \lambda \right)$$

(532)

where, for this section, $E$ represents the electron energy as measured from the edge of the conduction band at the surface in the vertically upward direction. The 2D electron dispersion relation in n-channel inversion layers of III-V, ternary and quaternary materials (whose unperturbed electrons obey the three band model of Kane) in the presence of light waves under the condition of weak electric field limit, assumes the form

$$\beta_0 (E, \lambda) = \frac{\hbar^2 k^2}{2m_c} + S \left[ \frac{\hbar |e| F_s \left[ \beta_0 (E, \lambda) \right]^*}{2m_c} \right]^{2/3}$$

(533)

The EMME can be expressed as

$$m^* (E_{F,\text{weak}}, i) = \hbar^2 k^* \left[ \frac{\partial \beta_0}{\partial E}_{E=E_{F,\text{weak}}} \right] = m_c \left[ P_{3L} (E_{F,\text{weak}}, i) \right]$$

(534)

where $E_{F,\text{weak}}$ is the Fermi energy under the weak electric field limit as measured from the edge of the conduction band at the surface in the vertically upward direction in the presence of light waves and

$$P_{3L} (E, i) = \left\{ \beta_0 (E, \lambda) \right\}^* - \frac{2}{3} S \left[ \frac{\hbar |e| F_s}{2m_c} \right]^{2/3} \left\{ \left[ \beta_0 (E, \lambda) \right]^* \right\}^{-1/3} \left[ \beta_0 (E, \lambda) \right]^*.$$

Thus, one can observe that the EMME is a function of the sub-band index, surface electric field, the Fermi energy and the other spectrum constants due to the combined influence of $E_s$, $A$ and $\lambda$. 

217
The sub-band energy \( (E_{n\omega L}) \) in this case can be obtained from (533) as

\[
\beta_0 (E_{n\omega L}, \lambda) = S_i \left[ \frac{\hbar |e| F_x \left[ \beta_0 (E_{n\omega L}, \lambda) \right]}{\sqrt{2m_c}} \right]^{2/3}
\]

(535)

The 2D total density-of-states function in weak electric field limit can be expressed as

\[
N_{2D} (E) = \frac{m_c^2 e^2}{\pi \hbar^2} \sum_{i=0}^{N_L} P_{3L} (E, i) H \left( E - E_{n\omega L} \right)
\]

(536)

Using (536) and the occupation probability, the \( n_{2Dw} \) in the present case can be written as

\[
n_{2Dw} = \frac{g_c m_c^2}{\pi \hbar^2} \sum_{i=0}^{N_L} \left[ P_{3L} (E_{FwL}, i) + Q_{4L} (E_{FwL}, i) \right]
\]

(537)

where, \( P_{4L} (E_{FwL}, i) \equiv \left[ \beta_0 (E_{FwL}, \lambda) - S_i \left[ \frac{\hbar e F_x \left[ \beta_0 (E_{FwL}, \lambda) \right]}{\sqrt{2m_c}} \right]^{2/3} \right] \)

and

\[
Q_{4L} (E_{FwL}, i) = \sum_{r=1}^{L} \left[ L (r) \left[ P_{4L} (E_{FwL, i}) \right] \right].
\]

The 2D electron dispersion relation in n-channel inversion layers of III-V, ternary and quaternary materials in the presence of light waves under the condition of strong electric field limit, assumes the form

\[
\beta_0 (E, \lambda) - \left[ \frac{\hbar e F_x \left[ \beta_0 (E, \lambda) \right]}{\sqrt{2m_c}} \right] = \frac{\hbar^2 k^2}{2m_c}
\]

(538)

The EMME can be expressed as

\[
m^* (E_{Fad}, i) = \hbar^2 k^* \frac{\partial^2 E}{\partial F_x} \bigg|_{F_x=E_{Fad}} = m_c \left[ P_{3L} (E, i) \right]_{E=E_{Fad}}
\]

(539)

where, \( E_{Fad} \) is the Fermi energy under the strong electric field limit as measured from the edge of the conduction band at the surface in the presence of light waves and

\[
P_{3L} (E, i) \equiv \left[ \beta_0 (E, \lambda) \right]^{2} \left[ \frac{\hbar e F_x \left[ \beta_0 (E, \lambda) \right]}{\sqrt{2m_c}} \right]^{2/3} \left[ \beta_0 (E, \lambda) \right]^{2/3} \left[ \beta_0 (E, \lambda) \right]^{2/3}.
\]

Thus, one can observe that the EMME is a function of the sub-band index, surface electric field, the Fermi energy and the other spectrum constants due to the combined influence of \( E_{Fad}, \lambda \) and \( \lambda \)
The sub-band energy $E_{n_{2D}}$ in this case can be obtained from (538) as

$$\beta_0(E_{n_{2D}}, \lambda) - \left[ \frac{\mathcal{F}_s \hbar \left( \frac{2 \sqrt{2} (S_s) \sqrt{3}}{3} \right)}{\sqrt{2m_e}} \left[ \beta_0(E_{n_{2D}}, \lambda) \right] \right] = 0 \quad (540)$$

The total 2D density-of-states function under the strong electric field limit can be expressed as

$$N_{2D}(E) = \frac{m_e g_s m^*}{\pi \hbar^2} \sum_{i=0}^{i=m} \left[ P_{sl}(E_{n_{2D}}, i) \right] (541)$$

Using (541) and the Fermi-Dirac occupation probability factor, the $n_{2D}$ in the present case under the strong electric field can be written as

$$n_{2D} = \frac{g_s m^*}{\pi \hbar^2} \sum_{i=0}^{i=m} \left[ P_{sl}(E_{n_{2D}}, i) + Q_{sl}(E_{n_{2D}}, i) \right] \quad (542)$$

where,

$$P_{sl}(E_{n_{2D}}, i) = \left[ \beta_0(E_{n_{2D}}, \lambda) - \left( \frac{2 \sqrt{2} (S_s) \sqrt{3}}{3} \right) \frac{\mathcal{F}_s \hbar}{\sqrt{2m_e}} \left[ \beta_0(E_{n_{2D}}, \lambda) \right] \right]$$

For n-channel inversion layers, the opto dispersion relation for the perturbed two band model of Kane can be written as

$$\frac{\hbar^2 k^2}{2m_e} = \tau_0(E - |e| F_z, \lambda) \quad (543)$$

The 2D electron dispersion relation in n-channel inversion layers of III-V, ternary and quaternary materials (whose unperturbed electrons obey the two band model of Kane) in the presence of light waves under the condition of weak electric field limit, assumes the form

$$\tau_0(E, \lambda) = \frac{\hbar^2 k^2}{2m_e} + S_1 \left[ \frac{\mathcal{F}_s \hbar \left[ \tau_0(E, \lambda) \right]}{\sqrt{2m_e}} \right]^{2/3} \quad (544)$$

The EMME can be expressed as

$$m^* (E_{Fnl2}, i) = m_e \left[ P_{L2}(E_{Fnl2}, i) \right] \quad (545)$$

$E_{Fnl2}$ is the Fermi energy under the weak electric field limit as measured from the edge of the conduction band at the surface in the vertically upward direction in the presence of light waves in this case and
Thus, one can observe that the EMME is a function of the sub-band index, surface electric field, the Fermi energy and the other spectrum constants due to the combined influence of $E_g$ and $\lambda$.

The sub-band energy $\tau_0 (E_{nwl2})$ in this case can be obtained from (544) as

\[
\tau_0 (E_{nwl2}, \lambda) = S_i \left( \frac{\hbar |e| F_i \tau_0 (E_{nwl2}, \lambda)}{\sqrt{2m_e}} \right)^{2/3}
\]  
(546)

The 2D total density-of-states function in weak electric field limit can be expressed as

\[
N_{2D} (E) = \frac{m_e g_e \pi \hbar^2}{2} \sum_{i=0}^{\infty} \left[ P_{4iL2} (E_{Fnl2}, i) H \left( E - E_{nwl2} \right) \right]
\]  
(547)

Using (547) and the occupation probability, the $n_{2Dw}$ in the present case can be written as

\[
n_{2Dw} = \frac{g_e m_e}{\pi \hbar^2} \sum_{i=0}^{\infty} \left[ P_{4iL2} (E_{Fnl2}, i) + Q_{4iL2} (E_{Fnl2}, i) \right]
\]  
(548)

where,

\[
P_{4iL2} (E_{Fnl2}, i) = \left\{ \tau_0 (E_{Fnl2}, \lambda) - S_i \left[ \frac{\hbar e F_i \tau_0 (E_{Fnl2}, \lambda)}{\sqrt{2m_e}} \right] \right\}^{2/3}
\]

and

\[
Q_{4iL2} (E_{Fnl2}, i) = \sum_{r=0}^{L} \left\{ \left[ P_{4iL2} (E_{Fnl2}, i) \right] \right\}.
\]

Using the appropriate equations, the 2D electron dispersion relation in n-channel inversion layers of III-V, ternary and quaternary materials (whose unperturbed electrons obey the two band model of Kane) in the presence of light waves under the condition of strong electric field limit, assumes the form

\[
\tau_0 (E, \lambda) = \left[ \frac{|e| F i h}{\sqrt{2m_e}} \frac{2 \sqrt{2} (S_i)^{3/2}}{3} \right] \tau_0 (E, \lambda) = \frac{h^2 k_i^2}{2m_e}
\]  
(549)

The EMME can be expressed as

\[
m^* (E_{Fnl2}, i) = m_e \left[ P_{4iL2} (E, i) \right]_{E=E_{Fnl2}}
\]  
(550)
where, \( E_{\text{Ful2}} \) is the Fermi energy under the strong electric field limit as measured from the edge of the conduction band at the surface in the presence of light waves in the present case and

\[
P_{5L2}(E, i) = \left\{ \tau_0(E, \lambda) \right\}^* - \left[ \frac{|e| F \hbar}{\sqrt{2m_c}} \left( \frac{\sqrt{2} (S_i)^{3/2}}{3} \right) \left[ \tau_0(E, \lambda) \right]^* \right]^{1/2} \left[ \tau_0(E, \lambda) \right]^* \right\}.
\]

Thus, one can observe that the EMME is a function of the sub-band index, surface electric field, the Fermi energy and the other spectrum constants due to the combined influence of \( E_g \) and \( \lambda \).

The sub-band energy \( E_{s+11} \) in this case can be obtained from (549) as

\[
\tau_0 \left( E_{s+11}, \lambda \right) - \left[ \frac{|e| F \hbar}{\sqrt{2m_c}} \left( \frac{\sqrt{2} (S_i)^{3/2}}{3} \right) \left[ \tau_0 \left( E_{s+11}, \lambda \right) \right]^* \right]^{1/2} \left[ \tau_0 \left( E_{s+11}, \lambda \right) \right]^* = 0 \tag{551}
\]

The total 2D density-of-states function under the strong electric field limit can be expressed as

\[
N_{2D}(E) = \frac{g_s m_r}{\pi \hbar^2} \sum_{i=0}^{\infty} P_{5L2}(E, i) H \left( E - E_{s+11} \right) \tag{552}
\]

Using (552) and the Fermi-Dirac occupation probability factor, the \( n_{2D_0} \) in the present case under the strong electric field can be written as

\[
n_{2D_0} = \frac{g_s m_r}{\pi \hbar^2} \sum_{i=0}^{\infty} \left[ P_{6L2}(E_{\text{Ful2}}, i) + Q_{6L2}(E_{\text{Ful2}}, i) \right] \tag{553}
\]

where, \( P_{6L2}(E_{\text{Ful2}}, i) \equiv \tau_0 \left( E_{\text{Ful2}}, \lambda \right) - \left[ \frac{2 \sqrt{2} (S_i)^{3/2}}{3} \frac{|e| F \hbar}{\sqrt{2m_c}} \left[ \tau_0 \left( E_{\text{Ful2}}, \lambda \right) \right]^* \right]^{1/2} \left[ \tau_0 \left( E_{\text{Ful2}}, \lambda \right) \right]^* \right\} \) and

\[
Q_{6L2}(E_{\text{Ful2}}, i) \equiv \sum_{r=1}^{3} \left\{ L(r) \left[ P_{5L2}(E_{\text{Ful2}}, i) \right] \right\}.
\]

In this case we can write

\[
\frac{\hbar^2 k^2}{2m_c} = \rho_0 \left( E - |e| F \hbar z, \lambda \right) \tag{554}
\]

The 2D electron dispersion relation in n-channel inversion layers of III-V, ternary and quaternary materials (whose unperturbed electrons obey the parabolic energy bands) in the presence of light waves under the condition of weak electric field limit, assumes the form
\[ \rho_0(E, \lambda) = \frac{\hbar^2 k^2}{2m_c} + S_i \left[ \frac{\hbar |e| F_i \left[ \rho_0(E, \lambda) \right]^r}{\sqrt{2m_c}} \right]^{2/3} \]  

(555)

The EMME can be expressed as

\[ m' (E_{\text{Foil}}, i) = m_c \left[ P_{3Li} (E_{\text{Foil}}, i) \right] \]  

(556)

\[ E_{\text{Foil}} \] is the Fermi energy under the weak electric field limit as measured from the edge of the conduction band at the surface in the vertically upward direction in the presence of light waves in this case and

\[ P_{3Li} (E, i) = \left[ \rho_0(E, \lambda) \right]^r - \left[ \frac{2}{3} S_i \left[ \frac{\hbar |e| F_i}{\sqrt{2m_c}} \right]^{2/3} \left[ \rho_0(E, \lambda) \right]^r \right] \left[ \rho_0(E, \lambda) \right]^r \].

Thus, one can observe that the EMME is a function of the sub-band index, surface electric field, the Fermi energy and the other spectrum constants due to the combined influence of \( E_g \) and \( \lambda \). The sub-band energy \( E_{\text{Foil}} \) in this case can be written as

\[ \rho_0(E_{\text{Foil}}, \lambda) = S_i \left[ \frac{\hbar |e| F_i \left[ \rho_0(E_{\text{Foil}}, \lambda) \right]^r}{\sqrt{2m_c}} \right]^{2/3} \]  

(557)

The 2D total density-of-states function in weak electric field limit can be expressed as

\[ N_{2D_i}(E) = \frac{m_e g_F}{\pi \hbar^2} \sum_{i=0}^{2m} P_{3Li} (E, i) H \left( E - E_{\text{Foil}} \right) \]  

(558)

Using (558) and the occupation probability, the \( n_{2Dw} \) in the present case can be written as

\[ n_{2Dw} = \frac{g_F m_e}{\pi \hbar^2} \sum_{i=0}^{2m} P_{4Li} (E_{\text{Foil}}, i) + Q_{4Li} (E_{\text{Foil}}, i) \]  

(559)

where, \( P_{4Li} (E_{\text{Foil}}, i) \equiv \left\{ \rho_0(E_{\text{Foil}}, \lambda) - S_i \left[ \frac{\hbar e F_i \left[ \rho_0(E_{\text{Foil}}, \lambda) \right]^r}{\sqrt{2m_c}} \right] \right\} \) and

\[ Q_{4Li} (E_{\text{Foil}}, i) \equiv \sum_{r=1}^{4} L(r) \left[ P_{4Li} (E_{\text{Foil}}, i) \right] \].

6.9 Results and Discussions:

Using the values of the energy band constants from table 1, we have plotted the EMME for the first two sub bands as functions of surface electric field, wave length and intensity.
at $T=4.2\, K$ by taking n-channel inversion layers of n-InSb and n-InAs materials which are used for the purpose of numerical computations in accordance with the perturbed three and two band models of Kane and that of perturbed parabolic energy bands for weak electric field limit respectively. In fig 6.14, we have presented the variation of the EMME in the n-channel inversion layers of n-InSb and n-InAs as function of surface electric field in accordance with the three band model of Kane, the two band model of Kane and the parabolic energy bands respectively under weak electric field by considering the effect of electric sub-bands.

It appears from fig 6.14 that the EMME in the n-channel inversion layers increases with increase in surface electric field for weak electric field in a step-like manner with different numerical values and the influence of the energy band constants can also be assessed from the said figures. The EMME depends on the electric sub-band index, surface electric field, the Fermi energy and the other spectrum constants due to the combined influence of $E_g, \Delta$ and $\lambda$ which is the characteristic feature of such 2D systems under radiation. In figs 6.15 and 6.16, the effect of surface concentration and wavelength on the EMME for all the cases of fig 6.14 has been considered under the quantum limit approximation.

Fig 6.14: Plot of the normalized EMME as a function of surface electric field for n-channel inversion layers of n-InSb and n-InAs in the presence of light waves in accordance with the three, the two band models of Kane and the parabolic energy band model in the presence of external photo-excitation.
Fig 6.15: Plot of the normalized EMME as a function of surface electron concentration for n-channel inversion layers of n-InSb and n-InAs in the presence of light waves in accordance with the three, the two band models of Kane and the parabolic energy band model in the presence of external photo-excitation.

Fig 6.16: Plot of the normalized EMME as a function of electron wavelength for n-channel inversion layers of n-InSb and n-InAs in the presence of light waves in accordance with the three, the two band models of Kane and the parabolic energy band model in the presence of external photo-excitation.
It appears from the fig 6.15 and 6.16 that the EMME increases with both the variables in the straight lines segmentation fashion with different increasing slopes at particular values of surface electron concentration and light wave length. The values of EMME for InSb are greater than of InAs. The slopes are entirely determined by the spectra constants of n channel InSb and InAs respectively.

It may be noted that if the direction of application of the surface electric field applied perpendicular to the surface be taken in an arbitrary direction and not as $k_z$ as assumed in the present work, the EMME would be different analytically for both the limits. Nevertheless, the arbitrary choice of the direction normal to the surface would not result in a change of the basic qualitative feature of the EMME in n-channel inversion layers of materials in the presence of photo excitation. The approximation of the potential well at the surface by a triangular well introduces some errors, as for instance the omission of the free charge contribution to the potential. Our simplified approach will be useful for the purpose of comparison, when, the methods of tackling of the aforementioned formidable problems for the present generalized system appear and we admit the fact that the inclusion of the said effects would certainly increase the accuracy of our results.

6.10 Investigation of the EMME in nipi structures of III-V, Ternary and Quaternary materials under external photo-excitation

6.10.1 Introduction
In section 6.10.2 of theoretical background we shall study the EMME in nipi structures of III-V, ternary and quaternary materials both in the presence and absence of external light waves. The section 6.11 contains result and discussions in this context.

6.10.2 Theoretical Background
The 2D electron dispersion relation in nipi structures of III-V, ternary and quaternary materials (whose unperturbed electrons obey the three band model of Kane) in the presence of light waves can be written as

$$\beta_0(E, \lambda) = \left( n_i + \frac{1}{2} \right) \hbar \omega_{fe}(E, \lambda) + \frac{\hbar^2 k_z^2}{2m_e} \quad (560)$$

where $n_i (=0,1,2,...)$ is the mini-band index for nipi structures, $\epsilon_{sc}$ is the semiconductor permittivity and
\[ a_{bl}(E, \lambda) = \left( \frac{n_0 |e|^2}{\varepsilon \pi \beta_0'(E, \lambda) m_e} \right)^{1/2}. \]

The EMME in this case can be written as

\[ m'(E_{Fad}, n_i) = m_e R_{\text{EMME}}(E, n_i) \bigg|_{E=E_{Fad}} \]

in which, \( E_{Fad} \) is the Fermi energy in the present case as measured from the edge of the conduction band in vertically upward direction in the absence of any quantization and

\[ R_{\text{EMME}}(E, n_i) = \left[ \beta_0(E, \lambda) \right]' - \left( n_i + \frac{1}{2} \right) \hbar \sigma_{bl}(E, \lambda)' \].

From (561), we observe that the EMME in this case is a function of the Fermi energy, wavelength, nipi sub-band index and the other material constants which is the characteristic feature of nipi structures of III-V, ternary and quaternary compounds in the presence of light waves whose bulk dispersion relation in the absence of any field is defined by the three band models of Kane.

The sub-band energies \( E_{2nl} \) can be written as

\[ \beta_0(E_{2nl}, \lambda) = \left( n_i + \frac{1}{2} \right) \hbar \sigma_{bl}(E_{2nl}, \lambda) \] (562)

The density-of-states function in this case can be expressed as

\[ N_{\text{spad}}(E) = \frac{m_e g}{\pi \hbar^2 d_0} \sum_{n=0}^{n_{\text{max}}} R_{\text{EMME}}(E, n_i) H(E - E_{2nl}) \] (563)

in which \( d_0 \) is the super-lattice period.

The use of (563) leads to the expression of the electron concentration as

\[ n_0 = \frac{m_e g}{\pi \hbar^2 d_0} \sum_{n=0}^{n_{\text{max}}} \left[ T_{\text{KL}}(E_{Fad}, n_i) + T_{\text{BL}}(E_{Fad}, n_i) \right] \] (564)

where,

\[ T_{\text{KL}}(E_{Fad}, n_i) = \left[ \beta_0(E_{Fad}, \lambda) - \left( n_i + \frac{1}{2} \right) \hbar \sigma_{bl}(E_{Fad}, \lambda) \right] \]

and

\[ T_{\text{BL}}(E_{Fad}, n_i) = \sum_{r=1}^{L} L(r) T_{\text{KL}}(E_{Fad}, n_i). \]

Again, the 2D electron dispersion relation in nipi structures of III-V, ternary and quaternary materials (whose unperturbed electrons obey the two band model of Kane) in the presence of light waves can be written as,
\[ \tau_0(E, \lambda) = \left( n_i + \frac{1}{2} \right) \hbar \omega_{y_{1/2}}(E, \lambda) + \frac{\hbar^2 k_i^2}{2m_c} \]  

(565)

where \( \omega_{y_{1/2}}(E, \lambda) \equiv \frac{n_i |e|^2}{E_i \tau_0'(E, \lambda) m_e} \). 

The EMME in this case can be written as

\[ m_i(E_{pd,2}, n_i) = m_i R_{2L2}(E, n_i) \bigg|_{E=E_{pd,2}} \]  

(566)

in which, \( R_{2L2}(E, n_i) = \left\{ \tau_0'(E, \lambda) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{y_{1/2}}(E, \lambda) \right\} \).

The sub-band energies \( (E_{2mL2}) \) can be written as

\[ \tau_0(E_{2mL2}, \lambda) = \left( n_i + \frac{1}{2} \right) \hbar \omega_{y_{1/2}}(E_{2mL2}, \lambda) \]  

(567)

The density-of-states function in this case can be expressed as

\[ N_{spqL2}(E) = \frac{m_i g_r}{\pi \hbar^2 d_0} \sum_{n=0}^{\infty} R_{qL2}(E, n_i) H(E - E_{2mL2}) \]  

(568)

The use of (568) leads to the expression of the electron concentration as

\[ n_o = \frac{m_i g_r}{\pi \hbar^2 d_0} \sum_{n=0}^{\infty} T_{3L2}(E_{pd,2}, n_i) + T_{8L2}(E_{pd,2}, n_i) \]  

(569)

where, \( T_{3L2}(E_{pd,2}, n_i) = \left\{ \tau_0(E_{pd,2}, \lambda) - \left( n_i + \frac{1}{2} \right) \hbar \omega_{y_{1/2}}(E_{pd,2}, \lambda) \right\} \).

\( E_{pd,2} \) is the Fermi energy in the present case as measured from the edge of the conduction band in vertically upward direction in the absence of any quantization.

and \( T_{8L2}(E_{pd,2}, n_i) = \sum_{r=1}^{L} L(r) T_{3L2}(E_{pd,2}, n_i) \).

Besides, the 2D electron dispersion relation in nipi structures of III-V, ternary and quaternary materials (whose unperturbed electrons obey the parabolic energy bands) in the presence of light waves can be written as

\[ \rho_0(E, \lambda) = \left( n_i + \frac{1}{2} \right) \hbar \omega_{y_{1/2}}(E, \lambda) + \frac{\hbar^2 k_i^2}{2m_c} \]  

(570)

where, \( \omega_{y_{1/2}}(E, \lambda) \equiv \frac{n_i |e|^2}{E_i \rho_0'(E, \lambda) m_e} \). 

The EMME in this case can be written as
in which $E_{F \alpha 1}$ is the Fermi energy in the present case as measured from the edge of the conduction band in vertically upward direction in the absence of any quantization and

$$R_{\alpha 2 \alpha 1}(E, n_e) = \left[ \rho_0(E, \lambda) \right]' - \left( n_e + \frac{1}{2} \right) \hbar^2 \alpha_{h \alpha 1}(E, \lambda)' \right].$$

The sub-band energies $(E_{2n+1})$ can be written as

$$\rho_0(E_{2n+1}, \lambda) = \left( n_e + \frac{1}{2} \right) \hbar^2 \alpha_{h \alpha 1}(E_{2n+1}, \lambda) \quad (572)$$

The density-of-states function in this case can be expressed as

$$N_{sdpd \alpha}(E) = \frac{m_e g_{s \alpha}}{\pi \hbar^2 \alpha_{00}} \sum_{\alpha=0}^{\infty} R_{\alpha 2 \alpha 1}(E, n_e) H(E - E_{2n+1}) \quad (573)$$

The use of (573) leads to the expression of the electron concentration as

$$n_e = \frac{m_e g_{s \alpha}}{\pi \hbar^2 \alpha_{00}} \sum_{\alpha=0}^{\infty} \left[ T_{\alpha 2 \alpha 1}(E_{F \alpha 1}, n_e) + T_{\alpha 4 \alpha 1}(E_{F \alpha 1}, n_e) \right] \quad (574)$$

where,

$$T_{\alpha 2 \alpha 1}(E_{F \alpha 1}, n_e) = \left[ \rho_0(E_{F \alpha 1}, \lambda) - \left( n_e + \frac{1}{2} \right) \hbar^2 \alpha_{h \alpha 1}(E_{F \alpha 1}, \lambda) \right]$$

and

$$T_{\alpha 4 \alpha 1}(E_{F \alpha 1}, n_e) = \sum_{r=3}^{\infty} L(r) T_{\alpha 2 \alpha 1}(E_{F \alpha 1}, n_e)$$

### 6.11 Results and Discussions:

Using the values of the energy band constants from table 1, we have plotted the EMME for the first two sub bands as functions of wave length, intensity, thickness and electron concentration at $T= 4.2 \, K$ by taking nip1 structures of ternary materials which are used for the purpose of numerical computations in accordance with the perturbed three, two band models of Kane and that of perturbed parabolic energy bands respectively.

Using the multiple sub-bands, one can numerically evaluate the EMME as a function of electron concentration and wavelength in nip1 structures of III-V compounds by using the nip1 structures of InSb and InAs as shown in figs 6.17 and 6.18 respectively, in accordance with three and two band models of Kane. The occurrence of the humps in fig 6.17 has been explained earlier in the context of ultrathin films. The effect of nip1 structure tailoring increases the EMME to an extremely high value which severely affects the electron mobility in such structures.
Fig 6.17: Plot of the normalized EMME as a function of electron concentration for nipi structures of n-InSb and n-InAs in the presence of light waves in accordance with the three and the two band models of Kane in the presence of external photo-excitation.

Fig 6.18: Plot of the normalized EMME as a function of wavelength for nipi structures of n-InSb and n-InAs in the presence of light waves in accordance with the three and the two band models of Kane in the presence of external photo-excitation.
The effect of increasing wavelength aids to increase the EMME in a linear way, however in case of increasing the light intensity, the tendency of increase in EMME is extremely slow. We have not considered the effect of the light intensity and wavelength on the EMME governed by the parabolic energy band due to its slow variation from the value 1.

6.12 Investigation of the Emme in Nano Wires of III-V, Ternary and Quaternary materials under external photo-excitation

6.12.1 Introduction
In section 6.12.2 of theoretical background, we shall study the EMME in nano wires of III-V, ternary and quaternary materials both in the presence and absence of external light waves. The section 6.13 contains result and discussions in this context.

6.12.2 Theoretical Background
The 1D electron energy spectrum in nano wires of III-V, ternary and quaternary materials, whose unperturbed band structure is defined by the three band model of Kane, in the presence of light waves, can be expressed as

\[ \frac{\hbar^2 k^2}{2m_c} = \beta_0(E, \lambda) - \left[ \frac{\hbar^2}{2m_c} \left( \frac{n_y \pi}{d_y} \right)^2 + \frac{\hbar^2}{2m_c} \left( \frac{n_z \pi}{d_z} \right)^2 \right] \]  (575)

The sub band energies \( E_{n_{\text{ho}}} \) can be expressed as

\[ \beta_0(E_{n_{\text{ho}}}, \lambda) = \left[ \frac{\hbar^2}{2m_c} \left( n_y \pi/d_y \right)^2 + \frac{\hbar^2}{2m_c} \left( n_z \pi/d_z \right)^2 \right] \]  (576)

The EMME in the free direction \( k' \) in this case can be written from (575) as

\[ m' \left( E_{\text{FIDL}}, n_y, n_z, \lambda \right) = m_c \left\{ \beta_0(E_{\text{FIDL}}, \lambda) \right\}' \]  (577)

where, \( E_{\text{FIDL}} \) is the Fermi energy in the present case as measured from the edge of the conduction band in the vertically upward direction in absence of any quantization.

The one-dimensional density-of-states function \( N_{1D}(E, \lambda) \) is given by

\[ N_{1D}(E, \lambda) = \left( \frac{8 \sqrt{2m_c}}{\pi \hbar} \right) \sum_{n_y=-\infty}^{n_y=\infty} \sum_{n_z=-\infty}^{n_z=\infty} \left[ \beta_0(E, \lambda) - \phi(n_y, n_z) \right]^{1/2} H(E - E_{n_{\text{ho}}}) \]  (578)
where, $\phi(n_x, n_z) = \frac{\hbar^2 \pi^2}{2m_c} \left[ \left( \frac{n_x}{d_x} \right)^2 + \left( \frac{n_z}{d_z} \right)^2 \right]$.

Combining (578), with the Fermi Dirac occupation probability factor, the one dimensional electron concentration ($n_{1D}$) can thus be written as

$$n_{1D} = \frac{2g_x}{\pi} \sqrt{\frac{2m_c}{\hbar^2}} \sum_{n_x=1}^{\infty} \sum_{n_z=1}^{\infty} \left[ M_{21}(n_x, n_z, E_{F1DL}, \lambda) + N_{21}(n_x, n_z, E_{F1DL}, \lambda) \right]$$ (579)

where, $M_{21}(n_x, n_z, E_{F1DL}, \lambda) = \left[ \beta_{E_{F1DL}} - \phi(n_x, n_z) \right]^{1/2}$ and

$N_{21}(n_x, n_z, E_{F1DL}, \lambda) = \sum_{r=1}^{\infty} L(r) M_{21}(n_x, n_z, E_{F1DL}, \lambda)$

The expressions for the 1D dispersion relation, the sub-band energies, the EMME, the density-of-states function and the electron concentration for nano wires of III-V, ternary and quaternary materials, whose unperturbed band structure is defined by the two band model of Kane, can, respectively, be written in the presence of photo-excitation as

$$\frac{\hbar^2 k^2}{2m_c} = \left\{ \tau_0(E, \lambda) - \phi(n_x, n_z) \right\}$$ (580)

$$\tau_0(E_{ni}, \lambda) = \phi(n_x, n_z)$$ (581)

$$m^*(E_{F1DL}, n_x, n_z, \lambda) = m_c \left\{ \tau_0(E_{F1DL}, \lambda) \right\}'$$ (582)

$$N_{1D}(E, \lambda) = \left( \frac{g_x \sqrt{2m_c}}{\pi \hbar} \right) \sum_{n_x=1}^{\infty} \sum_{n_z=1}^{\infty} \left[ \tau_0(E, \lambda) \right]' \left[ \tau_0(E, \lambda) - \phi(n_x, n_z) \right]^{1/2} H(E - E_{ni})$$ (583)

$$n_{1D} = \frac{2g_x}{\pi} \sqrt{\frac{2m_c}{\hbar^2}} \sum_{n_x=1}^{\infty} \sum_{n_z=1}^{\infty} \left[ M_{22}(n_x, n_z, E_{F1DL}, \lambda) + N_{22}(n_x, n_z, E_{F1DL}, \lambda) \right]$$ (584)

where, $M_{22}(n_x, n_z, E_{F1DL}, \lambda) = \left[ \beta_{E_{F1DL}} - \phi(n_x, n_z) \right]^{1/2}$ and

$N_{22}(n_x, n_z, E_{F1DL}, \lambda) = \sum_{r=1}^{\infty} L(r) M_{22}(n_x, n_z, E_{F1DL}, \lambda)$.

The expressions for the 1D dispersion relation, the sub-band energies, the EMME, the density of states function and the electron concentration for nano wires of III-V, ternary and quaternary materials, whose unperturbed band structure is defined by the parabolic energy bands, can respectively be written in the presence of photo-excitation as
\[
\frac{\hbar^2 k^2}{2m_e} = \left[ \rho_0(E, \lambda) - \phi(n_x, n_z) \right]
\]

(585)

\[
\rho_0(E_{ni}, \lambda) = \phi(n_y, n_z)
\]

(586)

\[
m'(E_{FIDL}, n_y, n_z, \lambda) = m_e \{ \rho_0(E_{FIDL}, \lambda) \}'
\]

(587)

\[
N_{ID}(E, \lambda) = \left( \frac{g_s \sqrt{2m_e}}{\pi \hbar} \right) \sum_{n_y=1}^{\infty} \sum_{n_z=1}^{\infty} \left[ \rho_0(E, \lambda) \right]' \left[ \rho_0(E, \lambda) - \phi(n_y, n_z) \right]^{-1/2} H(E - E_{nn})
\]

(588)

\[
n_{ID} = \frac{2g_s \sqrt{2m_e}}{\pi \hbar} \sum_{n_y=1}^{\infty} \sum_{n_z=1}^{\infty} \left[ M_{23} \left( n_y, n_z, E_{FIDL}, \lambda \right) + N_{23} \left( n_y, n_z, E_{FIDL}, \lambda \right) \right]
\]

(589)

where,

\[
M_{23} \left( n_y, n_z, E_{FIDL}, \lambda \right) = \left[ \rho_0(E_{FIDL}, \lambda) - \phi(n_y, n_z) \right]^{-1/2}, N_{23} \left( n_y, n_z, E_{FIDL}, \lambda \right) = \sum_{r=3}^{l} L(r) M_{23} \left( n_y, n_z, E_{FIDL}, \lambda \right)
\]

6.13 Results and Discussions:

Using the values of the energy band constants from table 1, we have plotted the EMME as functions of thickness and electron concentration per unit length at \(T= 4.2 \text{ K}\) by taking nano wires of ternary materials which are used for the purpose of numerical computations in accordance with the perturbed three and two band models of Kane respectively. In figs 6.19 and 6.20, we have plotted the EMME in nano wires of III-V materials as function of lateral dimension and electron concentration respectively. The effect of external photo-excitation increases the EMME significantly.

![Fig 6.19: Plot of the normalized EMME as a function of lateral film thickness field for nano wires of n-InSb and n-InAs in the presence of light waves in accordance with the three and the two band models of Kane in the presence of external photo-excitation.](image_url)
The influence of quantum confinement is immediately apparent from all the curves of figs 6.19 since, the 1D EMME depends strongly on the nano-thickness, which is in direct contrast with the corresponding bulk specimens. It appears from the said figures that the 1D EMME decreases with the increasing film thickness in a step like manner as considered here although the numerical values vary widely and determined by the constants of the energy spectra. The oscillatory dependence is due to the crossing over of the Fermi level by the quantized levels. For each coincidence of a quantized level with the Fermi level, there would be a discontinuity in the density-of-states function resulting in a peak of oscillations. With large values of film thickness, the height of the steps decreases and the EMME decreases with increasing film thickness in non-oscillatory manner and exhibit monotonic decreasing dependence. The height of step size and the rate of decrement are totally dependent on the band structure. In fig 6.20, we have exhibited the effect of the lowest sub-band on the EMME when the electron concentration is varied. A direct assessment of the effect of light intensity and electron wavelength can be procured form the said figures.
6.14 The EMME in Effective Mass Superlattices of Optoelectronic materials under magnetic quantization under external photo-excitation

6.14.1 Introduction

In recent years, modern fabrication techniques have generated altogether a new dimension in the arena of quantum effect devices through the experimental realization of an important artificial structure known as semiconductor super-lattice (SL) by growing two similar but different semiconducting materials in alternate layers with finite thicknesses. The materials forming the alternate layers have the same kind of band structure but different energy gaps. The concept of SL was developed for the first time by Keldysh [217] and was successfully fabricated by Esaki and Tsu [218]. The SLs are being extensively used in thermal sensors [219], quantum cascade lasers [220], photo-detectors [221], light emitting diodes [222], multiplication [223], frequency multiplication [224], photo-cathodes [225], thin film transistor [226], solar cells [227], infrared imaging [228], and thermal imaging [229], infrared sensing [230] and also in other microelectronic devices.

The most extensively studied III-V SL is the one consisting of alternate layers of GaAs and Ga_{1-x}Al_xAs owing to the relative easiness of fabrication. The GaAs and Ga_{1-x}Al_xAs layers form the quantum wells and the potential barriers respectively. The III-V SL's are attractive for the realization of high speed electronic and optoelectronic devices [231]. In addition to SLs with usual structure, other types of SLs such as II-VI [232], IV-VI [233] and HgTe/CdTe [234] SL's have also been investigated in the literature. The IV-VI SLs exhibit quite different properties as compared to the III-V SL due to the specific band structure of the constituent materials [235]. The epitaxial growth of II-VI SL is a relatively recent development and the primary motivation for studying the mentioned SLs made of materials with the large band gap is in their potential for optoelectronic operation in the blue [236]. HgTe/CdTe SL's have raised a great deal of attention since 1979, when as a promising new materials for long wavelength infrared detectors and other electro-optical applications [237]. Interest in Hg-based SL's has been further increased as new properties with potential device applications were revealed. These features arise from the unique zero band gap material HgTe and the direct band gap semiconductor CdTe which can be described by the three band mode of Kane. The combination of the aforementioned materials with specified dispersion relation makes
HgTe/CdTe SL very attractive, especially because of the tailoring of the material properties for various applications by varying the energy band constants of the SLs. In addition to it, for effective mass SLs, the electronic sub-bands appear continually in real space [238].

In section 6.14 of theoretical background we shall study the EMME in effective mass super-lattices of optoelectronic materials under magnetic quantization in the presence of light waves .The section 6.15 explores result and discussion for this section.

6.14.2 Theoretical Background

The dispersion relation of the conduction electrons in effective mass super-lattices of III-V materials (whose constituent materials obey the three band models of Kane) in the presence of external light waves can be expressed following Sasaki [238] as

\[ a_i \cos[c_{i5}(E, \lambda, E_{g0}, \Delta_i) a_0 + c_{25}(E, \lambda, E_{g1}, \Delta_2) b_0] - a_i \cos[c_{i5}(E, \lambda, E_{g0}, \Delta_i) a_0 - c_{25}(E, \lambda, E_{g1}, \Delta_2) b_0] = \cos(I_0 k) \]

(590)

where \( L_0 = a_0 + b_0 \) is the period length, \( a_0 \) and \( b_0 \) are the widths of the barrier and the well respectively,

\[
a_i = \left[ 1 + \frac{m_{z_i}}{\sqrt{m_{e_i}}} \right]^{-1} \left[ 4 \left( \frac{m_{z_i}}{m_{e_i}} \right)^2 \right]^{-1}, \quad a_2 = \left[ 1 + \frac{m_{z_2}}{\sqrt{m_{e_2}}} \right]^{-1} \left[ 4 \left( \frac{m_{z_2}}{m_{e_2}} \right)^2 \right]^{-1},
\]

\[
c_{i5}^2 (E, \lambda, E_{g0}, \Delta_i) = \frac{2m_{z_i}}{\hbar^2} \left[ \beta_{i50} (E, \lambda, E_{g0}, \Delta_i) - k_i^2 \right], i = 1, 2, k_i^2 = k_y^2 + k_z^2
\]

\[
\beta_{i50} (E, \lambda, E_{g0}, \Delta_i) = \left[ I_{i50} (E, E_{g0}, \Delta_i) \right] - \theta_{i50} (E, \lambda, E_{g0}, \Delta_i),
\]

\[
I_{i50} (E, E_{g0}, \Delta_i) = \frac{E (E + E_{g0}) (E + E_{g0} + \Delta_i) (E_{g0} + \frac{2}{3} \Delta_i)}{E_{g0} (E_{g0} + \Delta_i) (E + E_{g0} + \frac{2}{3} \Delta_i)},
\]

\[
\theta_{i50} (E, \lambda, E_{g0}, \Delta_i) = \frac{C_{i50} (\lambda, E_{g0}, \Delta_i)}{\phi_{i50} (E)} \psi_{i50}^2 (E),
\]

\[
C_{i50} (\lambda, E_{g0}, \Delta_i) = \frac{e^2}{96n \pi c^3} \frac{L_0^2}{\sqrt{E_{sc} \epsilon_0}} \frac{E_{g0} (E_{g0} + \Delta_i)}{E_{g0} + \frac{2}{3} \Delta_i} \frac{2}{4} \left( t_{i50} + \frac{\beta_{i50}}{2} \right)^2, \quad m_n \text{ is the reduced mass and is given by } m_n^{-1} = \left( m_0 a \right)^{-1} + m_w^{-1}, \quad m_w \text{ is the effective mass of the heavy hole at the top of the valance band in the absence of any field},
\]

235
\[ \bar{\beta}_{s0} = \left[ (6E_{x_0} + 2\Delta / 3)(E_{x_0} + \Delta_j) / \chi_{s0} \right]^{1/2}, \chi_{s0} = \left( 6E_{x_0}^2 + 9E_{x_0}\Delta_j + 4\Delta_j^2 \right), \]
\[ t_{s0} = \left[ (6E_{x_0}^2 + 2\Delta / 3)^2 / \chi_{s0} \right]^{1/2}, \bar{\rho}_{s0} = \left( 4\Delta^2 / 3 \chi_{s0} \right)^{1/2}, \]
\[ \phi_{s0}(E) = E_{x_0} \left( 1 + 2 \left( 1 + \frac{m_s}{m_e} \right) \frac{t_{s0}(E)}{E_{x_0}} \right)^{1/2}, \]
\[ \psi_{s0}(E) = \left[ \left( 1 + \frac{E_{x_0} - \delta_{s0}}{\phi_{s0}(E) + \delta_{s0}} \right) + (E_{x_0} - \delta_{s0}) \right] \left[ \frac{1}{\phi_{s0}(E) + \delta_{s0}} \frac{1}{E_{x_0} + \delta_{s0}} \right]^{1/2} \frac{1}{\phi_{s0}(E) + \delta_{s0}} \left( \frac{E_{x_0} + \delta_{s0}}{(E_{x_0} - \delta_{s0})^2} \right)^{1/2} \]
\[ \delta_{s0} = (E_{x_0}\Delta_j)(\chi_{s0})^{1/2}. \]

In the presence of a quantizing magnetic field \( B \), along x-direction the magneto-energy spectrum assumes the form
\[ k_s^2 = \bar{\alpha}_{s0}(E, \lambda, n) \quad (591) \]

where \( \bar{\alpha}_{s0}(E, \lambda, n) = \frac{1}{L_o^2} \left[ \cos^{-1}[\bar{f}_{s0}(E, \lambda, n)] \right]^2 \frac{2eB}{\hbar} \left( n + \frac{1}{2} \right) L_o^2 \]
\[ \bar{f}_{s0}(E, \lambda, n) = \left[ a_1 \cos[\bar{c}_{s0}(E, \lambda, E_{g_{10}}, \Delta_j, n)]a_o + b_o \bar{c}_{s0}(E, \lambda, E_{g_{02}}, \Delta_j, n) - a_2 \cos[\bar{c}_{s0}(E, \lambda, E_{g_{10}}, \Delta_j, n)]a_o - b_o \bar{c}_{s0}(E, \lambda, E_{g_{02}}, \Delta_j, n) \right], \]
\[ \bar{c}_{s0}(E, \lambda, E_{g_{10}}, \Delta_j, n) = \left( \frac{2m_{s1}}{\hbar^2} \right) \left( \beta_{s0}(E, \lambda, E_{g_{10}}, \Delta_j) \right) \frac{2eB}{\hbar} \left( n + \frac{1}{2} \right) L_o^2 \]
\[ \bar{c}_{s0}(E, \lambda, E_{g_{02}}, \Delta_j, n) = \left( \frac{2m_{s2}}{\hbar^2} \right) \left( \beta_{s0}(E, \lambda, E_{g_{02}}, \Delta_j) \right) \frac{2eB}{\hbar} \left( n + \frac{1}{2} \right) L_o^2 \]

The EMME along x-direction in this case can be expressed as
\[ m^*(V_0, \lambda, n) = (\hbar^2 / 2) \left[ \bar{\alpha}_{s0}(V_0, \lambda, n) \right]^{1/2} \quad (592) \]

where \( V_0 \) is the Fermi energy in this case and
\[ \{\bar{\alpha}_{s0}(V_0, \lambda, n) \}' = 2[\bar{f}_{s0}(V_0, \lambda, n)] L_o^{-2} \cos^{-1}[\bar{f}_{s0}(V_0, \lambda, n)] [1 - \bar{f}_{s0}(V_0, \lambda, n)]^{-1/2} \]
\[ \bar{f}_{s0}(V_0, \lambda, n) = \left[ a_1 \sin[\bar{c}_{s0}(V_0, \lambda, E_{g_{10}}, \Delta_j, n)]a_o + b_o \bar{c}_{s0}(V_0, \lambda, E_{g_{02}}, \Delta_j, n) \right] \left[ \bar{c}_{s0}(V_0, \lambda, E_{g_{10}}, \Delta_j, n) \right] a_o + b_o \bar{c}_{s0}(V_0, \lambda, E_{g_{02}}, \Delta_j, n) \]
\[ \{\bar{c}_{s0}(V_0, \lambda, E_{g_{10}}, \Delta_j, n) \}' = \frac{m_{s1}}{\hbar^2} \left( \beta_{s0}(V_0, \lambda, E_{g_{10}}, \Delta_j) \right) \}
\[ \{\bar{c}_{s0}(V_0, \lambda, E_{g_{02}}, \Delta_j, n) \}' = \frac{m_{s2}}{\hbar^2} \left( \beta_{s0}(V_0, \lambda, E_{g_{02}}, \Delta_j) \right) \}
The electron concentration in this case assumes the form

\[
n_0 = \frac{g_e e B}{\pi^2 h L_0} \sum_{n=0}^{\infty} [S_{54}(V_0, \lambda, n) + T_{54}(V_0, \lambda, n)]
\]

where \(S_{54}(V_0, \lambda, n)\) and \(T_{54}(V_0, \lambda, n)\) are given by equations (593) and (594), respectively.

The electron concentration in this particular case when the dispersion relations of the constituent materials are defined by the perturbed two band model of Kane, can be expressed as

\[
n_0 = \frac{g_e e B}{\pi^2 h L_0} \sum_{n=0}^{\infty} \left[ \cos^{-1} \bar{f}_{151}(V_0, \lambda, n) \right]^2 - \frac{2eB}{\hbar} (n + \frac{1}{2}) I_0^2 \]

where \(S_{55}(V_0, \lambda, n)\) and \(T_{55}(V_0, \lambda, n)\) are given by equations (593) and (594), respectively.
The electron concentration when the dispersion relations of the constituent materials are defined by the perturbed parabolic energy bands can be expressed as

\[ n_0 = \frac{g_e B}{\pi^2 h L_0} \sum_{n=0}^{\infty} \left[ S_{50}(V_0, \lambda, n) + T_{50}(V_0, \lambda, n) \right] \]

where \( S_{50}(V_0, \lambda, n) = \left[ \cos^{-1} (\bar{f}_{50}(V_0, \lambda, n)) \right]^2 - \frac{2eB}{h} (n + \frac{1}{2})^2 \), and \( T_{50}(V_0, \lambda, n) = \sum_{n=1}^{\infty} Z_0(r) \left[ S_{50}(E_{FB}, \lambda, n) \right] \).
\[ c_{152}(E, \lambda, E_{g1}, n) = \left\{ \frac{2m_{c1}}{\hbar} \right\} \left[ \rho_{150}(E, \lambda, E_{g1}) - \frac{2eB}{\hbar} (n + \frac{1}{2}) \right]^2, \]

\[ c_{252}(E, \lambda, E_{g2}, n) = \left\{ \frac{2m_{c2}}{\hbar} \right\} \left[ \rho_{250}(E, \lambda, E_{g2}) - \frac{2eB}{\hbar} (n + \frac{1}{2}) \right]^2, \]

\[ \rho_{30}(E,E_{g1},\lambda) = E - C_{35}(\lambda) \left[ 1 + \left( \frac{2m_{c1}}{m_{h}} \right) \frac{E}{E_{g1}} \right] \left( \frac{E}{E_{g1}} \right)^{\frac{3}{2}}, \]

\[ C_{152}(\lambda) = \frac{\epsilon^2 I_0 \lambda^2}{96\pi c^2 m_{h} \sqrt{E_{g1} E_{0}}}, \alpha = \frac{1}{E_{g1}}, \]

\[ T_{56}(E_{FB}, \lambda, n) = \sum_{n=1}^{N} \epsilon_{B}(r) \left[ S_{56}(E_{FB}, \lambda, n) \right] \]

The EMME in this case can be written as

\[ m^*(V_0, \lambda, n) = \left( \frac{\hbar^2}{2} \right) \left[ \alpha_{17}(V_0, \lambda, n) \right] \]

where,

\[ \{ \alpha_{17}(V_0, \lambda, n) \} = 2 \left[ f_{152}(V_0, \lambda, n) \right] \left( \frac{e^{-2}}{E_{B}} \right) \cos^{-1} \left[ f_{152}(V_0, \lambda, n) \right] \left( 1 - f_{152}(V_0, \lambda, n) \right)^{-1/2}, \]

\[ f_{152}(V_0, \lambda, n) = -a \sin \left[ c_{152}(V_0, \lambda, E_{g1}, n) \right] - b \cos \left[ c_{252}(V_0, \lambda, E_{g2}, n) \right] \]

\[ \frac{f_{152}(V_0, \lambda, E_{g1}, n)}{f_{252}(V_0, \lambda, E_{g2}, n)} \]

\[ \{ c_{152}(V_0, \lambda, E_{g1}, n) \} = \frac{m_{c1}}{\hbar^2} \left[ \rho_{150}(V_0, \lambda) \right] \left[ c_{152}(V_0, \lambda, E_{g1}, n) \right] \]

\[ \{ c_{252}(V_0, \lambda, E_{g2}, n) \} = \frac{m_{c2}}{\hbar^2} \left[ \rho_{250}(V_0, \lambda) \right] \left[ c_{252}(V_0, \lambda, E_{g2}, n) \right] \]

\[ \rho_{30}(E,E_{g1},\lambda) = 1 + C_{35}(\lambda) \left( \frac{2m_{c1}}{m_{h}} \right) \left[ 1 + \left( \frac{2m_{c1}}{m_{h}} \right) \frac{E}{E_{g1}} \right] \left( \frac{E}{E_{g1}} \right)^{\frac{3}{2}}, \]

6.15 Results and Discussions:

Using the values of the energy band constants from table 1, we have plotted the EMME as functions of 1/B and electron concentration at \( T = 4.2 \) K by taking effective mass super-lattices of optoelectronic materials under magnetic quantization in accordance with the perturbed three, two band models of Kane and that of perturbed parabolic energy bands respectively. In figs. 6.21 and 6.22, we have plotted the effect of magnetic field and carrier concentration on the EMME of effective mass super-lattices in GaAs/AlGaAs structures. The effect of SdH oscillations has been exhibited in this case for multi sub-band generation. In this case, the EMME is a sub-band index dependent and we have plotted the EMME by considering the lowest sub-band index. In both the figures we see that the effect of the external photo-excitation on the EMME dominated by the parabolic...
Fig 6.21: Plot of the normalized EMME for the lowest sub-band index as a function of inverse magnetic field of effective mass super-lattices of GaAs/AlGaAs in the presence of light waves in accordance with the three, the two band models of Kane and parabolic energy band model in the presence of external photo-excitation.

Fig 6.22: Plot of the normalized EMME for the lowest sub-band index as a function of carrier concentration of effective mass super-lattices of GaAs/AlGaAs in the presence of light waves in accordance with the three, the two band models of Kane and parabolic energy band model in the presence of external photo-excitation.
energy law does not tend to modulate with either of the variable compared to the bulk value of the EMME for InAs and InSb, we find that the EMME in GaAs/AlGaAs structures are extremely low and therefore the mobility in superlattices are very large as compared with the value of the mobility of the constituent materials which is very important from the application point of view for modern devices made of superlattices.

6.16 The EMME in Nanowire Effective Mass Superlattices of Optoelectronic materials in the presence of external photo-excitation

6.16.1 Introduction

We shall study the EMME in nano wire effective mass super-lattices of optoelectronic materials in the presence of photo excitation in section 6.16.2 of theoretical background and the section 6.17 explores the result and discussions pertaining to section 6.16.2.

6.16.2 Theoretical Background

The dispersion relation of the conduction electrons for nano wire effective mass super­lattices in accordance with the perturbed three-band model of Kane in the presence of light waves is given by

\begin{equation}
\alpha_{3}(E, \lambda, n_{y}, n_{z}) = \frac{1}{L_{0}} \left[ \cos^{-1}\left( f_{3}(E, \lambda, n_{y}, n_{z}) \right) \right] ^{2} - H(n_{y}, n_{z})
\end{equation}

where, \( \alpha_{3}(E, \lambda, n_{y}, n_{z}) = [a_{y} \cos[\epsilon_{y}(E, \lambda, E_{g}, \Delta_{y}, n_{y}, n_{z})]b_{y} \epsilon_{y}(E, \lambda, E_{g}, \Delta_{y}, n_{y}, n_{z})] \)

\( f_{3}(E, \lambda, n_{y}, n_{z}) = [a_{y} \cos[\epsilon_{y}(E, \lambda, E_{g}, \Delta_{y}, n_{y}, n_{z})]b_{y} \epsilon_{y}(E, \lambda, E_{g}, \Delta_{y}, n_{y}, n_{z})] \),

\( \epsilon_{y}^{2}(E, \lambda, E_{g}, \Delta_{y}, n_{y}, n_{z}) = \left[ \frac{2m_{y} \pi}{h^{2}} \right] \beta_{y} \left[ \epsilon_{y}(E, \lambda, E_{g}, \Delta_{y}, n_{y}, n_{z}) \right] - H(n_{y}, n_{z}) \) and

\[ H(n_{y}, n_{z}) = \left[ \left( \frac{n_{y} \pi}{d_{y}} \right)^{2} + \left( \frac{n_{z} \pi}{d_{z}} \right)^{2} \right] \]

The expression of the electron concentration in this case can be written as

\[ n_{0} = \frac{2g_{s}}{\pi} \sum_{n_{y}=1}^{n_{ymax}} \sum_{n_{z}=1}^{n_{zmax}} [\bar{Q}_{23}(V_{1}, \lambda, n_{y}, n_{z}) + \bar{Q}_{24}(V_{1}, \lambda, n_{y}, n_{z})] \]

(599)

where \( V_{1} \) is the Fermi energy in this case, \( \bar{Q}_{23}(V_{1}, \lambda, n_{y}, n_{z}) = \sqrt{\alpha_{23}(V_{1}, \lambda, n_{y}, n_{z})} \),

241
\overline{Q}_{24} (V_1, \lambda, n_y, n_z) = \sum_{R=0}^{R=R_F} L(R) \overline{Q}_{23} (V_1, \lambda, n_y, n_z), \text{and}

L(R) = 2 (k_0^2 R)^{2R} (1 - 2^{1-2R}) \frac{\partial^{2R}}{\partial V_1^{2R}}.

The EMME along the x-direction in this case can be expressed as

\[ m^* (V_1, \lambda, n_y, n_z) = \frac{h^2}{2} \left[ \overline{a}_b (V_1, \lambda, n_y, n_z) \right] \]

where,

\[ [\overline{a}_b (V_1, \lambda, n_y, n_z)]^T = [1 - f_5^0 (V_1, \lambda, n_y, n_z)] [2 f_5^0 (V_1, \lambda, n_y, n_z)] \cos \left[ f_5^0 (V_1, \lambda, n_y, n_z) \right] \]

\[ [f_5^0 (V_1, \lambda, n_y, n_z)]^T = -a_1 \sin \left[ a_0 \overline{a}_1 (V_1, \lambda, E_{g_1}, \Delta_1, n_y, n_z) + b_0 \overline{a}_2 (V_1, \lambda, E_{g_2}, \Delta_2, n_y, n_z) \right] \]

\[ \left[ a_0 \overline{a}_1 (V_1, \lambda, E_{g_1}, \Delta_1, n_y, n_z) \right] + \left[ b_0 \overline{a}_2 (V_1, \lambda, E_{g_2}, \Delta_2, n_y, n_z) \right] \]

\[ + a_2 \sin \left[ a_0 \overline{a}_1 (V_1, \lambda, E_{g_1}, \Delta_1, n_y, n_z) - b_0 \overline{a}_2 (V_1, \lambda, E_{g_2}, \Delta_2, n_y, n_z) \right] \]

\[ \left[ a_0 \overline{a}_1 (V_1, \lambda, E_{g_1}, \Delta_1, n_y, n_z) \right] - \left[ b_0 \overline{a}_2 (V_1, \lambda, E_{g_2}, \Delta_2, n_y, n_z) \right] \]

and \[ [\overline{a}_1 (V_1, \lambda, E_{g_{01}}, \Delta_1, n_y, n_z)]^T = \frac{\frac{\partial \beta_{01}}{\partial V_1}}{h_2} \left[ \overline{a}_1 (V_1, \lambda, E_{g_{01}}, \Delta_1, n_y, n_z) \right] \]

In accordance with the perturbed two-band model of Kane the electron concentration per unit length given by,

\[ n_b = \frac{2 g_x}{\pi} \sum_{n_y=1}^{n_{y_{max}}} \sum_{n_z=1}^{n_{z_{max}}} \left[ \overline{Q}_{25} (V_1, \lambda, n_y, n_z) + \overline{Q}_{26} (V_1, \lambda, n_y, n_z) \right] \]

where, \[ \overline{Q}_{25} (V_1, \lambda, n_y, n_z) = \left[ \sqrt{\overline{a}_2 (V_1, \lambda, n_y, n_z)} \right], \]

\[ \overline{Q}_{26} (V_1, \lambda, n_y, n_z) = \sum_{R=0}^{R=R_F} L(R) \overline{Q}_{25} (V_1, \lambda, n_y, n_z), \overline{a}_b (V_1, \lambda, n_y, n_z) = \left[ \frac{1}{f_0^0} \cos \left[ f_6^0 (V_1, \lambda, n_y, n_z) \right] - H(n_y, n_z) \right], \]

\[ f_5 (V_1, \lambda, n_y, n_z) = [a_1 \cos \left[ a_0 \overline{a}_1 (V_1, \lambda, E_{g_1}, n_y, n_z) - b_0 \overline{a}_2 (V_1, \lambda, E_{g_2}, n_y, n_z) \right] \]

\[ - a_2 \cos \left[ a_0 \overline{a}_1 (V_1, \lambda, E_{g_1}, n_y, n_z) - b_0 \overline{a}_2 (V_1, \lambda, E_{g_2}, n_y, n_z) \right] \]

and \[ \overline{a}_1 (V_1, \lambda, E_{g_{01}}, n_y, n_z) = \left[ \frac{2 m_{01}^*}{h_2^2} \omega_{01} (V_1, \lambda, E_{g_{01}}) - H(n_y, n_z) \right] \]

The EMME in this case can be expressed as

\[ m^* (V_1, \lambda, n_y, n_z) = \frac{h^2}{2} \left[ \overline{a}_2 (V_1, \lambda, n_y, n_z) \right] \]
In accordance with the perturbed parabolic energy bands, the electron concentration per unit length is given by,

\[
n_0 = \frac{2g_e}{\pi} \sum_{n_{\text{val}}} \sum_{n_{\text{con}}} \left[ \tilde{Q}_{251} (V_1, \lambda, n_x, n_z) + \tilde{Q}_{261} (V_1, \lambda, n_x, n_z) \right]
\]

(604)

where, \( \tilde{Q}_{251} (V_1, \lambda, n_x, n_z) = \sqrt{\tilde{\omega}_{201} (V_1, \lambda, n_x, n_z)} \).

\[
\tilde{Q}_{261} (V_1, \lambda, n_x, n_z) = \sum_{n_{\text{con}}} L(R) \left[ \tilde{Q}_{251} (V_1, \lambda, n_x, n_z) \right]
\]

\[
\tilde{\omega}_{201} (V_1, \lambda, n_x, n_z) = \left[ \frac{1}{E_0} \left[ \cos^{-1} f_{41} (V_1, \lambda, n_x, n_z) \right] \right] - H(n_y, n_z)
\]

(605)

The EMME in this case can be expressed as

\[
m^*(V_1, \lambda, n_x, n_z) = \frac{\hbar^2}{2} \left[ \tilde{\omega}_{201} (V_1, \lambda, n_x, n_z) \right]
\]

where,

\[
\left[ \tilde{\omega}_{201} (V_1, \lambda, n_x, n_z) \right] = \frac{2f_{41} (V_1, \lambda, n_x, n_z)}{\sqrt{1 - f_{41}^2 (V_1, \lambda, n_x, n_z)}} \cos^{-1} f_{41} (V_1, \lambda, n_x, n_z)
\]

(606)
Results and Discussions:

Using the values of the energy band constants from table 1, we have plotted the EMME for the $n_y = 1$ and $n_z = 1$ as a function of the film thickness at $T = 4.2 K$ by taking nano wire effective mass super lattices of optoelectronic materials in accordance with the perturbed three, two band models of Kane and that of perturbed parabolic energy bands respectively in fig 6.23. Fig 6.23 exhibits the variation of EMME in the nano wire effective mass super-lattices of GaAs/AlGaAs by considering the quantum limit approximation.

Fig 6.23: Plot of the normalized EMME ($n_y = 1$ and $n_z = 1$) as a function of lateral film thickness of nano wire effective mass super-lattices of GaAs/AlGaAs in the presence of light waves in accordance with the three, the two band models of Kane and parabolic energy band model in the presence of external photo-excitation.
The EMME is greatest for the lowest sub-bands and for higher sub-bands the numerical values of the EMMEs will be less. It appears that the EMME in such structure decreases with the increase in the film thickness in a non-linear way for the three and the two band energy models. Although, it appears that the EMME in case of parabolic energy band is linear, however, depends on the photo-excitation factor, which makes the slow variation of EMME with both intensity and wavelength.

6.18 The EMME in Superlattices of Optoelectronic materials with graded interfaces under magnetic quantization in the presence of external photo-excitation

6.18.1 Introduction

We note that all the aforementioned SLs have been proposed with the assumption that the interfaces between the layers are sharply defined, of zero thickness, i.e., devoid of any interface effects. The SL potential distribution may be then considered as a one dimensional array of rectangular potential wells. The aforementioned advanced experimental techniques may produce SLs with physical interfaces between the two materials crystallographically abrupt; adjoining their interface will change at least on an atomic scale. As the potential form changes from a well (barrier) to a barrier (well), an intermediate potential region exists for the electrons. The influence of finite thickness of the interfaces on the electron dispersion law is very important, since; the electron energy spectrum governs the electron transport in SLs [237]

We shall study the EMME in super-lattices of optoelectronic materials with graded interfaces under magnetic quantization in the presence of photo excitation in section 6.18.2 of theoretical background and the section 6.19 explores the result and discussions pertaining to section 6.18.2.

6.18.2 Theoretical Background

The energy spectrum in super-lattices of III-V compounds with graded interfaces in the presence of light waves whose constituent materials are defined by perturbed three-band model of Kane can be written following [237] as

\[ \cos(I_\alpha k) = \frac{1}{2} \Phi_{115}(E,k_x) \]  

(606)

where,

\[ \Phi_{115}(E,k_x) = 2 \cosh \{X_{215}(E,k_x)\} \cos \{Y_{215}(E,k_x)\} + e_{215}(E,k_x) \sinh \{X_{215}(E,k_x)\} \sin \{Y_{215}(E,k_x)\} \]
In the presence of a quantizing magnetic field $\mathbf{B}$ along z-direction, the simplified magneto dispersion relation can be written as

\[ k_z = \omega_{215}(E, \lambda, n) \]

where $\omega_{215}(E, \lambda, n) = \left[ \frac{1}{L_0} \cos^{-1} \left[ \frac{1}{2} f_{215}(E, \lambda, n) \right] \right]^2 - \frac{2|\mathbf{B}|}{\hbar} \left( n + \frac{1}{2} \right),

\[ f_{215}(E, \lambda, n) = \left[ 2 \cosh \left( M_{215}(n, E) \right) \cos \left( N_{215}(n, E) \right) + Z_{215}(n, E) \sinh \left( M_{215}(n, E) \right) \right] \sin \left( N_{215}(n, E) \right) + \Delta_{21} \left[ \left( \frac{I_{215}(n, E)}{I_{225}(n, E)} \right)^2 - 3I_{225}(n, E) \right] \cosh \left( M_{215}(n, E) \right) \]

\[ \sin \left( N_{215}(n, E) \right) + \Delta_{21} \left[ 3I_{215}(n, E) - \frac{I_{225}(n, E)}{I_{215}(n, E)} \right] \sinh \left( M_{215}(n, E) \right) \cos \left( N_{215}(n, E) \right) \]

\[ + \Delta_{21} \left[ 2 \left( \frac{I_{215}(n, E)}{I_{225}(n, E)} \right)^2 - \frac{I_{225}(n, E)}{I_{215}(n, E)} \right] \cosh \left( M_{215}(n, E) \right) \cos \left( N_{215}(n, E) \right) \]

\[ + \frac{1}{12} \left[ \frac{5I_{225}(n, E)}{I_{215}(n, E)} \right]^3 - \frac{5I_{215}(n, E)}{I_{225}(n, E)} \left[ \frac{34I_{225}(n, E)I_{215}(n, E)}{I_{215}(n, E)} \right] \sinh \left( M_{215}(n, E) \right) \sin \left( N_{215}(n, E) \right) \]
\[ Z_{215}(n,E) = \left[ \frac{I_{215}(n,E)}{I_{215}(n,E)} \right], \]

\[ I_{215}(n,E) = \frac{2m_e^2}{h^2} \beta_{215}(E-V_2, \lambda, E_{\text{tot}}, \Delta_i) + \frac{2|e|B}{h} \left( n + \frac{1}{2} \right) \]

\[ N_{215}(n,E) = I_{225}(n,E)[h - \Delta_2] \quad \text{and} \quad I_{225}(n,E) = \left[ \frac{2m_e^2}{h^2} \beta_{215}(E, \lambda, E_{\text{tot}}, \Delta_i) - \frac{2|e|B}{h} \left( n + \frac{1}{2} \right) \right]^{1/2} \]

The electron concentration is given by

\[ n_0 = \frac{g_e B}{\pi^2 \hbar} \left[ \sum_{n=0}^{2m_e} \mathcal{Q}_{27}(V_2, \lambda, n) + \mathcal{Q}_{28}(V_2, \lambda, n) \right] \]

where \( \mathcal{Q}_{27}(V_2, \lambda, n) = [\omega_{215}(V_2, \lambda, n)]^{1/2} \)

\( V_2 \) is the Fermi energy in the present case,

\[ \mathcal{Q}_{28}(V_2, \lambda, n) = \sum_{k=1}^{k_f} L(R) \left[ \mathcal{Q}_{27}(V_2, \lambda, n) \right] \]

\[ L(R) = 2(k_c)\xi(2R) \frac{\partial^{2R}}{\partial V_2^{2R}}. \]

The EMME along the z-direction in this case can be expressed as

\[ m^*(V_2, \lambda, n) = (\hbar^2 / 2) [\omega_{215}(V_2, \lambda, n)] \]

where

\[ \omega_{215}(V_2, \lambda, n) = \left[ 2M'_{215}(n,V_2) \sinh \left\{ M_{215}(n,V_2) \right\} \cos \left\{ N_{215}(n,V_2) \right\} \right] + \]

\[ Z_{215}(n,V_2) M'_{215}(n,V_2) \cosh \left\{ M_{215}(n,V_2) \right\} \sin \left\{ N_{215}(n,V_2) \right\} - 2N_{215}(n,V_2) \sinh \left\{ N_{215}(n,V_2) \right\} \cosh \left\{ M_{215}(n,V_2) \right\} + \]

\[ Z_{225}(n,V_2) \sinh \left\{ M_{215}(n,V_2) \right\} \sin \left\{ N_{215}(n,V_2) \right\} \]

\[ + Z_{215}(n,V_2) N'_{215}(n,V_2) \cos \left\{ N_{215}(n,V_2) \right\} \sinh \left\{ M_{215}(n,V_2) \right\} \]

\[ + A_{215} \left[ \left\{ \frac{2I_{215}(n,V_2) I_{215}'(n,V_2)}{I_{225}(n,V_2)} \right\} - \left\{ \frac{I_{215}(n,V_2) I_{225}'(n,V_2)}{I_{225}(n,V_2)} \right\} - 3I_{225}'(n,V_2) \right] \times \]

\[ \cosh \left\{ M_{215}(n,V_2) \right\} \sin \left\{ N_{215}(n,V_2) \right\} \]

\[ + \left\{ -3I_{225}(n,V_2) + \left\{ \frac{I_{215}(n,V_2)}{I_{225}(n,V_2)} \right\}^2 \right\} \left\{ M_{215}'(n,V_2) \sin \left\{ M_{215}(n,V_2) \right\} \sin \left\{ N_{215}(n,V_2) \right\} \right\] \]

\[ + \left\{ N_{215}'(n,V_2) \cos \left\{ M_{215}(n,V_2) \right\} \cos \left\{ N_{215}(n,V_2) \right\} \right\} \]

247
For perturbed two band model of Kane the forms of the electron concentration and the EMME remain same where
\[ I_{215}(V_2, n) = \left[ \frac{2eB}{\hbar} \left( n + \frac{1}{2} \right) - \frac{2m_{c2}}{\hbar^2} \omega_{250}(V_2, \lambda, E_{g21}) \right]^2, \]
\[ I_{225}(V_2, n) = \left[ \frac{2m_{c1} \omega_{50}(V_2, \lambda, E_{g01})}{\hbar^2} - \frac{2eB}{\hbar} \left( n + \frac{1}{2} \right) \right]^2, \]
\[ I'_{215}(V_2, n) = \frac{m_{c2} \omega_{250}(V_2, \lambda, E_{g21})}{\hbar^2 I_{215}(V_2, n)}, \]
and
\[ I'_{225}(V_2, n) = \frac{m_{c1} \omega_{50}(V_2, \lambda, E_{g01})}{\hbar^2 I_{225}(V_2, n)}. \]

For perturbed parabolic energy bands, the forms of the electron concentration and the EMME remain same where
\[ I_{215}(V_2, n) = \left[ \frac{2eB}{\hbar} \left( n + \frac{1}{2} \right) - \frac{2m_{c2}}{\hbar^2} \rho_{250}(V_2, \lambda, E_{g21}) \right]^2, \]
\[ I_{225}(V_2, n) = \left[ \frac{2m_{c1} \rho_{50}(V_2, \lambda, E_{g01})}{\hbar^2} - \frac{2eB}{\hbar} \left( n + \frac{1}{2} \right) \right]^2, \]
\[ I'_{215}(V_2, n) = \frac{m_{c2} \rho_{250}(V_2, \lambda, E_{g21})}{\hbar^2 I_{215}(V_2, n)}, \]
and
\[ I'_{225}(V_2, n) = \frac{m_{c1} \rho_{50}(V_2, \lambda, E_{g01})}{\hbar^2 I_{225}(V_2, n)}. \]

## 6.19 Results and Discussions:
Using the values of the energy band constants from table 1, we have plotted the EMME for the first sub band as functions of light intensity as shown in fig 6.24 at \( T = 4.2 \) \( K \) by taking super lattices of optoelectronic materials with graded interfaces under magnetic quantization in accordance with the perturbed three band model of Kane. It appears that with the increase in the light intensity, the EMME increases sharply as the wavelength varies for the present case.

With the incorporation of different sub-bands, discontinuous behavior in the EMME would be expected due to the generation of the Landau sub-bands. Incidentally, in this case, we have limited ourselves with the lowest energy sub-band at low temperatures where the electrons will be mostly occupied for prominent quantum effects.
6.20 The EMME in Quantum Wire Superlattices of Optoelectronic materials with graded interfaces in the presence of external photo-excitation

6.20.1 Introduction

We shall study the EMME in quantum wire super-lattices of optoelectronic materials with graded interfaces in the presence of photo excitation in section 6.20.2 of theoretical background and the section 6.21 explores the result and discussions pertaining to section 6.20.2

6.20.2 Theoretical Background

The dispersion relation in accordance with the perturbed three band model of Kane, in this case is given by

\[ k_i^2 = \omega_{223}(E, \lambda, n_y, n_z) \]  \hspace{1cm} (610)

where \( \omega_{223}(E, \lambda, n_y, n_z) = \left[ \frac{1}{\epsilon_0} \left[ \cos^{-1} \left( \frac{1}{2} f_{i33}(E, \lambda, n_y, n_z) \right) \right]^2 - H(n_y, n_z) \right] \),
\[ f_{355}(E, \lambda, n_y, n_z) = \left[ 2 \cosh \left( M_{315} (n_y, n_z, E) \right) \cos \left( N_{315} (n_y, n_z, E) \right) + \sinh \left( M_{315} (n_y, n_z, E) \right) \right] + \Delta_{21} \left[ \left( \frac{\left\{ I_{315} (n_y, n_z, E) \right\}^2}{I_{355} (n_y, n_z, E)} - 3 I_{355} (n_y, n_z, E) \right) \cosh \left( M_{315} (n_y, n_z, E) \right) \right] + \sin \left( N_{315} (n_y, n_z, E) \right) \right] \]

\[ + \Delta_{21} \left[ 2 \left\{ I_{315} (n_y, n_z, E) \right\}^2 - \left\{ I_{355} (n_y, n_z, E) \right\} \right] \cosh \left( M_{315} (n_y, n_z, E) \right) \cos \left( N_{315} (n_y, n_z, E) \right) \]

\[ + \frac{1}{12} \left[ \frac{S I_{355} (n_y, n_z, E)}{I_{355} (n_y, n_z, E)} \right] + \frac{S I_{355} (n_y, n_z, E)}{I_{355} (n_y, n_z, E)} \right] \right] \sin \left( M_{355} (n_y, n_z, E) \right) \cos \left( N_{355} (n_y, n_z, E) \right) \]

\[ Z_{355}(n_y, n_z, E) = \left[ \frac{I_{315} (n_y, n_z, E)}{I_{355} (n_y, n_z, E)} \right] M_{315} (n_y, n_z, E) = I_{315} (n_y, n_z, E) \left[ \alpha_0 - \Delta_{21} \right] \]

The electron concentration per unit length is given by

\[ n_0 = \frac{2 g_e}{\hbar} \sum_{n_y=1}^{\infty} \sum_{n_z=1}^{\infty} \left[ \bar{Q}_{355} (V, \lambda, n_y, n_z) + \bar{Q}_{30} (V, \lambda, n_y, n_z) \right] \]

where, \[ \bar{Q}_{355} (V, \lambda, n_y, n_z) = \left[ \omega_{255} (V, \lambda, n_y, n_z) \right] \]

\[ \bar{Q}_{30} (V, \lambda, n_y, n_z) = \sum_{R=1}^{R_{max}} L(R) \left[ \bar{Q}_{30} (V, \lambda, n_y, n_z) \right] \]

\[ L(R) = 2 \left( k_F \right)^2 \left( 1 - 2^{1-2R} \right) \xi(2R) \frac{\partial^{2R}}{\partial V_3^{2R}} \]

and \[ V_3 \] is the Fermi energy in the present case.

The EMME along x-direction in this case can be expressed as

\[ m^* (V_3, \lambda, n_y, n_z) = \left( \hbar^2 / 2 \right) \left[ \omega_{255} (V_3, \lambda, n_y, n_z) \right] \]

The electron concentration per unit length is given by

\[ n_0 = \frac{2 g_e}{\hbar} \sum_{n_y=1}^{\infty} \sum_{n_z=1}^{\infty} \left[ \bar{Q}_{355} (V, \lambda, n_y, n_z) + \bar{Q}_{30} (V, \lambda, n_y, n_z) \right] \]

where, \[ \bar{Q}_{355} (V, \lambda, n_y, n_z) = \left[ \omega_{255} (V, \lambda, n_y, n_z) \right] \]

\[ \bar{Q}_{30} (V, \lambda, n_y, n_z) = \sum_{R=1}^{R_{max}} L(R) \left[ \bar{Q}_{30} (V, \lambda, n_y, n_z) \right] \]

\[ L(R) = 2 \left( k_F \right)^2 \left( 1 - 2^{1-2R} \right) \xi(2R) \frac{\partial^{2R}}{\partial V_3^{2R}} \]

and \[ V_3 \] is the Fermi energy in the present case.

The EMME along x-direction in this case can be expressed as

\[ m^* (V_3, \lambda, n_y, n_z) = \left( \hbar^2 / 2 \right) \left[ \omega_{255} (V_3, \lambda, n_y, n_z) \right] \]

The electron concentration per unit length is given by

\[ n_0 = \frac{2 g_e}{\hbar} \sum_{n_y=1}^{\infty} \sum_{n_z=1}^{\infty} \left[ \bar{Q}_{355} (V, \lambda, n_y, n_z) + \bar{Q}_{30} (V, \lambda, n_y, n_z) \right] \]

where, \[ \bar{Q}_{355} (V, \lambda, n_y, n_z) = \left[ \omega_{255} (V, \lambda, n_y, n_z) \right] \]

\[ \bar{Q}_{30} (V, \lambda, n_y, n_z) = \sum_{R=1}^{R_{max}} L(R) \left[ \bar{Q}_{30} (V, \lambda, n_y, n_z) \right] \]

\[ L(R) = 2 \left( k_F \right)^2 \left( 1 - 2^{1-2R} \right) \xi(2R) \frac{\partial^{2R}}{\partial V_3^{2R}} \]

and \[ V_3 \] is the Fermi energy in the present case.

The EMME along x-direction in this case can be expressed as

\[ m^* (V_3, \lambda, n_y, n_z) = \left( \hbar^2 / 2 \right) \left[ \omega_{255} (V_3, \lambda, n_y, n_z) \right] \]
where, 

\[
\alpha_{225} (V_3, \lambda, n_x, n_z) = \frac{2 f'_{135} (V_3, \lambda, n_x, n_z) \left[ \cos^{-1} \left( \frac{1}{2} f_{135} (V_3, \lambda, n_x, n_z) \right) \right]}{\sqrt{4 - f_{135}^2 (V_3, \lambda, n_x, n_z)}}.
\]

\[
f_{135} (V_3, \lambda, n_x, n_z) = 2 M'_{135} (n_x, n_z, V_3) \sinh \left[ M_{135} (n_x, n_z, V_3) \right] \cos \left[ N_{135} (n_x, n_z, V_3) \right] + Z_{135} (n_x, n_z, V_3) M'_{135} (n_x, n_z, V_3).
\]

\[
\cosh \left[ M_{135} (n_x, n_z, V_3) \right]
\]

\[
\sin \left[ N_{135} (n_x, n_z, V_3) \right] - 2 N'_{135} (n_x, n_z, V_3) \sin \left[ N_{135} (n_x, n_z, V_3) \right] \cosh \left[ M_{135} (n_x, n_z, V_3) \right]
\]

\[
+ Z_{135} (n_x, n_z, V_3) \sin \left[ N_{135} (n_x, n_z, V_3) \right]
\]

\[
+ Z_{135} (n_x, n_z, V_3) N'_{135} (n_x, n_z, V_3) \cos \left[ N_{135} (n_x, n_z, V_3) \right] \sin \left[ M_{135} (n_x, n_z, V_3) \right]
\]

\[
+ \Delta_{21} \left[ \left( \frac{2 I_{135} (n_x, n_z, V_3) I'_{135} (n_x, n_z, V_3)}{I_{135} (n_x, n_z, V_3)} \right) \left( \frac{I_{135}^2 (n_x, n_z, V_3) I'_{135} (n_x, n_z, V_3)}{I_{135}^2 (n_x, n_z, V_3)} \right) \right] - 3 I_{135} (n_x, n_z, V_3)
\]

\[
\cosh \left[ M_{135} (n_x, n_z, V_3) \right] \sin \left[ N_{135} (n_x, n_z, V_3) \right]
\]

\[
+ \left[ -3 I_{135} (n_x, n_z, V_3) \left( \frac{I_{135} (n_x, n_z, V_3)}{I_{135} (n_x, n_z, V_3)} \right) \right] \left( M'_{135} (n_x, n_z, V_3) \sin \left[ M_{135} (n_x, n_z, V_3) \right] \sin \left[ N_{135} (n_x, n_z, V_3) \right] \right)
\]

\[
+ \left[ -2 I_{135} (n_x, n_z, V_3) I'_{135} (n_x, n_z, V_3) \right] \left( \frac{I_{135}^2 (n_x, n_z, V_3) I'_{135} (n_x, n_z, V_3)}{I_{135}^2 (n_x, n_z, V_3)} \right) + 3 I_{135} (n_x, n_z, V_3) \sin \left[ M_{135} (n_x, n_z, V_3) \right] \sin \left[ N_{135} (n_x, n_z, V_3) \right]
\]

\[
\cos \left[ N_{135} (n_x, n_z, V_3) \right]
\]

\[
+ \left[ 3 I_{135} (n_x, n_z, V_3) \right] \left( \frac{I_{135} (n_x, n_z, V_3)}{I_{135} (n_x, n_z, V_3)} \right) \left[ M'_{135} (n_x, n_z, V_3) \cos \left[ M_{135} (n_x, n_z, V_3) \right] \cos \left[ N_{135} (n_x, n_z, V_3) \right] \right]
\]

\[
-N'_{135} (n_x, n_z, V_3) \sin \left[ N_{135} (n_x, n_z, V_3) \right] \sin \left[ M_{135} (n_x, n_z, V_3) \right]
\]

\[
\Delta_{21} \left[ \left( 4 \left( I_{135} (n_x, n_z, V_3) I'_{135} (n_x, n_z, V_3) \right) \right) \left( I_{235} (n_x, n_z, V_3) I'_{235} (n_x, n_z, V_3) \right) \right] \cos \left[ M_{135} (n_x, n_z, V_3) \right] \cos \left[ N_{135} (n_x, n_z, V_3) \right]
\]

\[
- \left( I_{235} (n_x, n_z, V_3) I'_{235} (n_x, n_z, V_3) \right) \cos \left[ M_{135} (n_x, n_z, V_3) \right] \cos \left[ N_{135} (n_x, n_z, V_3) \right]
\]

\[
+ 2 \left( \left( I_{135} (n_x, n_z, V_3) \right) ^2 \right) \left( I_{235} (n_x, n_z, V_3) \right) \sin \left[ M_{135} (n_x, n_z, V_3) \right] \cos \left[ N_{135} (n_x, n_z, V_3) \right]
\]

\[
- N'_{135} (n_x, n_z, V_3) \cos \left[ M_{135} (n_x, n_z, V_3) \right] \sin \left[ N_{135} (n_x, n_z, V_3) \right]
\]

\[
\frac{1}{12} \left( 15 \left( I_{235} (n_x, n_z, V_3) \right) ^2 \right) + \frac{5}{12} \left( I_{235} (n_x, n_z, V_3) \right) \left( I_{135} (n_x, n_z, V_3) \right) \left( I_{235} (n_x, n_z, V_3) \right)
\]

\[
+ \frac{1}{12} \left( I_{235} (n_x, n_z, V_3) \right) \left( I_{135} (n_x, n_z, V_3) \right) \left( I_{235} (n_x, n_z, V_3) \right)
\]

\[
\frac{1}{2} \left( I_{135} (n_x, n_z, V_3) \right) \left( I_{235} (n_x, n_z, V_3) \right) \left( I_{235} (n_x, n_z, V_3) \right)
\]

\[
+ \frac{1}{12} \left( I_{135} (n_x, n_z, V_3) \right) \left( I_{135} (n_x, n_z, V_3) \right) \left( I_{235} (n_x, n_z, V_3) \right)
\]

\[
252
\]
\[
\frac{5[I_{315}(n_y,n_z,V_3)]^3}{I_{325}^2(n_y,n_z,V_3)} - \left\{34I_{315}(n_y,n_z,V_3)I_{315}(n_y,n_z,V_3)\right\} \\
-34I_{325}(n_y,n_z,V_3)I_{315}(n_y,n_z,V_3) \sinh \left\{M_{315}(n_y,n_z,V_3)\right\} \sin \left\{N_{315}(n_y,n_z,V_3)\right\} \\
+ \left\{5[I_{325}(n_y,n_z,V_3)]^3 + 5[I_{315}(n_y,n_z,V_3)]^3 \right\} - \left\{34I_{325}(n_y,n_z,V_3)I_{325}(n_y,n_z,V_3)\right\} \\
\{M_{315}(n_y,n_z,V_3)\cosh \left\{M_{315}(n_y,n_z,V_3)\right\} \sin \left\{N_{315}(n_y,n_z,V_3)\right\} + \\
N_{315}(n_y,n_z,V_3)\sin \left\{M_{315}(n_y,n_z,V_3)\right\} \cos \left\{N_{315}(n_y,n_z,V_3)\right\}\right]\]

\[M'_{315}(n_y,n_z,V_3) = I'_{315}(n_y,n_z,V_3) \left[ \alpha_0 - \Delta_3 \right], \quad I'_{325}(V_3,n_y,n_z) = \frac{m_c \beta_{350}(V_3,\lambda, E_{g02}, \Delta_2)}{\hbar^2 I_{325}(V_3,n_y,n_z)} \]

\[N_{315}(n_y,n_z,V_3) = I_{325}(n_y,n_z,V_3) \left[ b_0 - \Delta_3 \right], \quad I'_{315}(V_3,n_y,n_z) = \frac{m_c \beta_{350}(V_3,\lambda, E_{g02}, \Delta_2, n_y, n_z)}{\hbar^2 I_{325}(V_3,n_y,n_z)} \]

\[I_{315}(n_y,n_z,V_3) = \left[ -\frac{2m_c^2}{\hbar^2} \beta_{350}(V_3,\lambda, E_{g02}, \Delta_2) + H(n_y,n_z) \right]^{1/2} \]

\[N_{315}(n_y,n_z,V_3) = I_{325}(n_y,n_z,V_3) \left[ b_0 - \Delta_3 \right], \quad I'_{315}(V_3,n_y,n_z) = \frac{m_c \beta_{350}(V_3,\lambda, E_{g02}, \Delta_2, n_y, n_z)}{\hbar^2 I_{325}(V_3,n_y,n_z)} \]

\[I'_{325}(V_3,n_y,n_z) = \frac{2m_c^2}{\hbar^2} \beta_{350}(V_3,\lambda, E_{g01}, \Delta_3) - H(n_y,n_z) \right]^{1/2} \]

\[Z_{315}(V_3,n_y,n_z) = \frac{Z_{315}(V_3,n_y,n_z)}{I_{315}(V_3,n_y,n_z)} \frac{Z_{325}(V_3,n_y,n_z)}{I_{325}(V_3,n_y,n_z)} + \left( I_{315}(V_3,n_y,n_z)I_{325}(V_3,n_y,n_z) \right)^{1/2} \left[ 2I'_{315}(V_3,n_y,n_z)I_{315}(V_3,n_y,n_z) - 2I'_{325}(V_3,n_y,n_z)I_{325}(V_3,n_y,n_z) \right] \]

For perturbed two band model of Kane, the form of electron concentration per unit length and the field emitted current remain same where

\[I_{315}(n_y,n_z,V_3) = \left[ H(n_y,n_z) - \frac{2m_c^2}{\hbar^2} \alpha_{350}(V_3,\lambda, E_{g02}) \right]^{1/2}, \quad I'_{315}(V_3,n_y,n_z) = \frac{m_c \alpha_{350}(V_3,\lambda, E_{g02})}{\hbar^2 I_{315}(V_3,n_y,n_z)} \]

\[I_{325}(n_y,n_z,V_3) = \left[ -H(n_y,n_z) + \frac{2m_c^2}{\hbar^2} \alpha_{350}(V_3,\lambda, E_{g01}) \right]^{1/2}, \quad I'_{325}(V_3,n_y,n_z) = \frac{m_c \alpha_{350}(V_3,\lambda, E_{g01})}{\hbar^2 I_{325}(V_3,n_y,n_z)} \]

For perturbed parabolic energy bands, the form of electron concentration per unit length and the field emitted current remain same where
\[ I_{315}(n_x, n_z, V_3) = \left[ \frac{H(n_x, n_z) - \frac{2m_z}{\hbar^2} \rho_{250}(V_3, \lambda, E_{go2})}{\hbar^2} \right]^{1/2}, \quad I'_{315}(V_3, n_x, n_z) = \frac{m_z \rho_{150}(V_3, \lambda, E_{go2})}{\hbar^2 I_{315}(V_3, n_x, n_z)}, \]

\[ I_{325}(n_x, n_z, V_3) = \left[ -H(n_x, n_z) + \frac{2m_z}{\hbar^2} \rho_{150}(V_3, \lambda, E_{go1}) \right]^{1/2}, \quad I'_{325}(V_3, n_x, n_z) = \frac{m_z \rho_{150}(V_3, \lambda, E_{go1})}{\hbar^2 I_{325}(V_3, n_x, n_z)}. \]

6.21 Results and Discussions:

Using the values of the energy band constants from table 1, we have plotted the EMME for \( n_x = 1 \) and \( n_z = 1 \) as a function of electron concentration at \( T = 4.2 \) K by taking quantum wire super lattices of optoelectronic materials with graded interfaces in accordance with the perturbed two band model of Kane. In fig. 6.25, we have plotted the EMME as function of electron concentration per unit length in quantum wires of GaAs/AlGaAs super-lattices with graded interfaces. It appears from fig 6.25 that the effect of a single sub-band linearly increases the EMME for low value of wavelength.

![Graph showing EMME as function of electron concentration](image)

Fig 6.25: Plot of the normalized EMME as a function of linear electron concentration of quantum wire super-lattices of GaAs/AlGaAs with graded interfaces in the presence of light waves in accordance with the two band models of Kane in the presence of external photo-excitation. It may be noted that with the increase in the wavelength, the EMME tends to coincide with that of the lower wavelength values at higher carrier concentration.
SECTION 7

Applications in the Realm of Quantum Science and Technology

The investigations as presented in this section find ten different applications in the realm of modern quantum effect devices:

7.1 Thermoelectric Power: In recent years, with the advent of Quantum Hall Effect (QHE) [239], there has been considerable interest in studying the thermoelectric power under strong magnetic field (TPSM) in various types of nanostructured materials having quantum confinement of their charge carriers in one, two and three dimensions of the respective wave vector space leading to different carrier energy spectra [240-247]. The classical TPSM equation as mentioned in the preface is valid only under the condition of carrier non-degeneracy, being independent of carrier concentration and reflects the fact that the signature of the band structure of any material is totally absent in the same.

Zawadzki [242] demonstrated that the TPSM for electronic materials having degenerate electron concentration is essentially determined by their respective energy band structures. It has, therefore, different values in different materials and changes with the doping, magnitude of the reciprocal quantizing magnetic field under magnetic quantization, quantizing electric field as in inversion layers, nanothickness as in quantum wells, wires and dots, with superlattice period as in quantum confined semiconductor superlattices with graded interfaces having various carrier energy spectra and also in other types of field assisted nanostructured materials.

The magnitude of the thermoelectric power $G$ can be written as [243]

$$ G = \frac{1}{|e| T n_0} \int (E - E_F) R(E) \left[ -\frac{\partial f_0}{\partial E} \right] dE $$  \hspace{1cm} (613)

where $R(E)$ is the total number of states. The (613) can be written under the condition of carrier degeneracy as

$$ G = \left( \frac{\pi^2 k_B^2 T}{3 |e| n_0} \right) \left( \begin{array}{c} \frac{\partial n_0}{\partial E_F} \end{array} \right) $$  \hspace{1cm} (614)

For inversion layers, heavily doped materials and their nanostructures and the nipi superlattices, under the condition of electric quantum limit, (613) assumes the form
Thus, we can use the carrier statistics for different low dimensional materials to investigate the TPSM in such compounds and for the purpose of completeness we present few results of TPSM for bulk specimens as written below:

(i) **Nonlinear optical materials and Cd$_3$As$_2**

The electron concentration of bulk specimens in this case can be expressed following (2) of Section 1 as

\[ n_0 = g_v \left( \frac{3\pi^2}{2} \right)^{-1} \left[ M_{la} \left( E_{F_s} \right) + N_{la} \left( E_{F_s} \right) \right] \]  

(616 a)

where \( M_{la} \left( E_{F_s} \right) = \left[ \frac{\gamma \left( E_{F_s} \right)}{f_1 \left( E_{F_s} \right)} \right]^3 \) \( f_2 \left( E_{F_s} \right) \), \( E_{F_s} \) is the Fermi energy as measured from the edge of the conduction band in the vertically upward direction in the absence of any quantization, \( N_{la} \left( E_{F_s} \right) = \sum_{r=1}^{\infty} Z_{la}(r) M_{la} \left( E_{F_s} \right) \) and

\[ Z_{la}(r) = \left[ 2 \left( k_B T \right)^{2r} \left( 1 - 2^{1-2r} \right) \xi(2r) \right] \left[ \frac{\partial^{2r} \xi}{\partial E_{F_s}^{2r}} \right]. \]

Using (614) and (616), the TPSM in this case is given by

\[ G_0 = \frac{\pi^2 k_B^2 T}{3e} \left[ M_{la} \left( E_{F_s} \right) + N_{la} \left( E_{F_s} \right) \right]\left[ M_{la} \left( E_{F_s} \right) + N_{la} \left( E_{F_s} \right) \right]^{-1} \]  

(616 b)

(ii) **III-V materials**

(a) **Three band model of Kane**

In accordance with this model the electron concentration can be expressed as

\[ n_0 = \frac{g_v}{3\pi^2} \left( \frac{2m^*}{h^2} \right)^{\frac{3}{2}} \left[ \tilde{M}_A \left( E_{F_s} \right) + \tilde{N}_A \left( E_{F_s} \right) \right] \]  

(617)

where, \( \tilde{M}_A \left( E_{F_s} \right) = \left[ \frac{E_{F_s} \left( E_{F_s} + E_\Delta \right) \left( E_{F_s} + E_\Delta + \frac{2}{3} \Delta \right)}{E_\Delta \left( E_\Delta + \Delta \right) \left( E_{F_s} + E_\Delta + \frac{2}{3} \Delta \right)} \right]^{\frac{3}{2}} \) and
\[
\bar{N}_A(E_f) = \sum_{r=1}^{i} 2(k_B T)^{3r} (1 - 2^{1-2r}) \zeta(2r) \frac{\partial^{2r}}{\partial E_r^{2r}} [\bar{M}_A(E_f)]
\]

Using (614) and (617), the TPSM in this case can be written as

\[
G_0 = \left( \frac{\pi k_B^2 T}{3e} \right) \left[ \frac{\bar{M}_A(E_f)'}{\bar{M}_A(E_f) + \bar{N}_A(E_f)} \right] (618)
\]

(b) The model of Stillman et. al.

The expression of electron concentration in this case can be written as

\[
n_0 = \frac{g_v}{3\pi^2 \hbar^2} \left[ M_{\Lambda_0}(E_f) + N_{\Lambda_0}(E_f) \right]
\]

where,

\[
M_{\Lambda_0}(E_f) = [I_{11}(E_f)]^{\frac{1}{2}} \text{ and }
\]

\[
N_{\Lambda_0}(E_f) = \sum_{r=1}^{i} 2(k_B T)^{3r} (1 - 2^{1-2r}) \zeta(2r) \frac{\partial^{2r}}{\partial E_r^{2r}} [M_{\Lambda_0}(E_f)]
\]

Using (614) and (619), the TPSM can be expressed as

\[
G_0 = \left( \frac{\pi k_B^2 T}{3e} \right) \left[ \frac{M_{\Lambda_0}'(E_f) + N_{\Lambda_0}'(E_f)}{M_{\Lambda_0}(E_f) + N_{\Lambda_0}(E_f)} \right] (620)
\]

(e) The model of Palik et. al.

In accordance with this model the electron concentration can be expressed as

\[
n_0 = \frac{g_v}{3\pi^2 \hbar^2} \left[ \bar{M}_{12\Lambda}(E_f) + \bar{N}_{12\Lambda}(E_f) \right]
\]

where, \(\bar{M}_{12\Lambda}(E_f) = [I_{12}(E_f)]^{\frac{1}{2}}\) and

\[
\bar{N}_{12\Lambda}(E_f) = \sum_{r=1}^{i} 2(k_B T)^{3r} (1 - 2^{1-2r}) \zeta(2r) \frac{\partial^{2r}}{\partial E_r^{2r}} [\bar{M}_{12\Lambda}(E_f)]
\]

Using (614) and (621), the TPSM in this case can be written as
\[ G_0 = \left( \frac{\pi^2 k_B^2 T}{3e} \right) \left[ \frac{\left( \bar{M}_{\alpha h} \left( E_{F_s} \right) \right)'}{\bar{M}_{\alpha h} \left( E_{F_s} \right)} + \frac{\left( \bar{N}_{\alpha h} \left( E_{F_s} \right) \right)'}{\bar{N}_{\alpha h} \left( E_{F_s} \right)} \right] \]  

(622)

(f) Model of Johnson and Dicley

The expressions of the electron concentration and the TPSM for this model are given by

\[ n_0 = \frac{g_v}{3\pi^2} \left[ M_{13h} \left( E_{F_s} \right) + N_{13h} \left( E_{F_s} \right) \right] \]  

(623)

and

\[ G_0 = \left( \frac{\pi^2 k_B^2 T}{3e} \right) \left[ \frac{M'_{\nu h} \left( E_{F_s} \right) + N'_{\nu h} \left( E_{F_s} \right)}{M_{\nu h} \left( E_{F_s} \right) + N_{\nu h} \left( E_{F_s} \right)} \right] \]  

(624)

where

\[ M_{\nu h} \left( E_{F_s} \right) = \left[ \bar{e}_h \left( E_{F_s} \right) \right]^\frac{1}{2}, \]

\[ N_{\nu h} \left( E_{F_s} \right) = \sum_{r=1}^{N_{\nu h}} 2(k_B T)^{2r} \left( 1 - 2^{-2r} \right) \zeta \left( 2r \right) \frac{\partial^{2r}}{\partial E_{F_s}^{2r}} \left[ M_{\nu h} \left( E_{F_s} \right) \right], \]

\[ \bar{e}_h \left( E_{F_s} \right) = \left[ E_{h0} + 2E_{F_s} \right] \frac{e^2}{4} e_\gamma \left( E_{F_s} \right), \]

\[ e_\gamma \left( E_{F_s} \right) = \frac{\hbar^2}{2} \left[ \frac{1}{m^*} - \frac{1}{m_0} \right], \]

\[ e_\gamma \left( E_{F_s} \right) = 2\hbar^2 \phi_\gamma \left( E_{F_s} \right), \]

\[ \phi_\gamma \left( E_{F_s} \right) = \frac{\left( E_{F_s} + \Delta \right) \left( E_{F_s} + E_{F_s} + \frac{2}{3} \Delta \right)}{\left( E_{F_s} + \frac{2}{3} \Delta \right) \left( E_{F_s} + E_{F_s} + \Delta \right)} \]

(iii) n-type Gallium Phosphide

In this case, the electron concentration and the TPSM can, respectively, be written as

\[ n_0 = \frac{2g_v}{3\pi^2} \left[ M_A \left( E_{F_s} \right) + N_A \left( E_{F_s} \right) \right] \]  

(625)

\[ G_0 = \left( \frac{\pi^2 k_B^2 T}{3e} \right) \left[ \frac{M'_A \left( E_{F_s} \right) + N'_A \left( E_{F_s} \right)}{M_A \left( E_{F_s} \right) + N_A \left( E_{F_s} \right)} \right] \]  

(626)

where
The expressions of electron concentration and the TPSM for II-VI materials assume the forms

\[
M(E_b) = \left[ t_1(E_b) \theta(E_b) + t_2(E_b) \theta(E_b)^2 + t_3(E_b) \theta(E_b)^3 \right] /
\left( \theta(E_b) \theta(E_b)^2 + t_2(E_b) \theta(E_b)^3 \right)
\]

\[
t_a = \frac{1}{a}, \quad a = \left( \frac{h^2}{2m_\varepsilon} + \frac{A}{2m_\varepsilon} \right), \quad b = \frac{\hbar^2}{2m_\varepsilon}, \quad c = \frac{\hbar^2 k^2}{2m_\varepsilon}, \quad D = |V_a|^2, \quad t_a = \left[ \frac{g_1}{2a^2} \right], \quad t_s = \left( \frac{b}{a} \right)
\]

\[
t_a = \left( \frac{g_3}{2a^2} \right), \quad g_1 = (2aD - c), \quad g_2 = 4ab^2 + c^2 - 4acD, \quad g_3 = 4abc + 4a^2 c
\]

\[
t_a \left( E_b \right) = \left[ \frac{g_2 - (4ac) \theta(E_b)}{g_3} \right]
\]

\[
t_a = \left( t_a^2 + 2t_a t_s \right), \quad t_s = \left( 2t_a t_s \right), \quad t_s = \left[ t_a^2 + 4t_a^2 t_s + \left( 4t_a^2 t_s g_2 / g_3 \right) \right],
\]

\[
t_s = \left[ 4t_a t_s t_a^2 + 8t_a^2 t_s t_a^2 - \left( 16t_a^2 t_s^2 a c / g_3 \right) \right]
\]

and

\[
\theta \left( E_b \right) = \left( t_a \sqrt{2} \right)^{-1} \left( t_a + \left( E_b \right) \left( t_a \right) - \left( t_a + \left( t_s \right) \left( E_b \right) \right)^{1/2} \right)
\]

(iv) II-VI materials

The expressions of electron concentration and the TPSM for II-VI materials assume the forms

\[
n_0 = \frac{1}{2} \left( \frac{k_B T}{\pi B_0} \right)^{1/2} \left( \frac{B_0}{A_0} \right) \left[ \frac{F_1 (\eta)}{2} + \frac{C_0^2}{2A_0 k_B T} F_3 (\eta) \right]
\]

\[
G_0 = \left( \frac{\pi^2 k_B}{3e} \right) \left[ \frac{F_1 (\eta)}{2} + \left( \frac{C_0^2}{2A_0 k_B T} \right) F_3 (\eta) \right]
\]

(v) Stressed Materials

In this case electron concentration and TPSM assume the forms

\[
n_0 = \frac{1}{2} \left( \frac{k_B T}{\pi B_0} \right)^{1/2} \left( \frac{B_0}{A_0} \right) \left[ \frac{F_1 (\eta)}{2} + \frac{C_0^2}{2A_0 k_B T} F_3 (\eta) \right]
\]

\[
G_0 = \left( \frac{\pi^2 k_B}{3e} \right) \left[ \frac{F_1 (\eta)}{2} + \left( \frac{C_0^2}{2A_0 k_B T} \right) F_3 (\eta) \right]
\]
\[ n_0 = g_v (3\pi^2)^{-1} \left[ M_{\alpha} (E_{F_s}) + N_{\alpha} (E_{F_s}) \right] \]  
(629)

\[ G_0 = \left( \frac{\pi^2 k_B T}{3e} \right) \left[ \frac{M'_{\alpha} (E_{F_s}) + N'_{\alpha} (E_{F_s})}{M_{\alpha} (E_{F_s}) + N_{\alpha} (E_{F_s})} \right] \]  
(630)

where

\[ M_{\alpha} (E_{F_s}) = \left[ a^* (E_{F_s}) b^* (E_{F_s}) c^* (E_{F_s}) \right] \]

and

\[ N_{\alpha} (E_{F_s}) = \sum_{r=1}^{s} 2 (k_B T)^{2r} (1 - 2^{1-2r}) \xi (2r) \frac{\partial^{2r}}{\partial E_{F_s}^{2r}} \left[ M_{\alpha} (E_{F_s}) \right] \]

(vi) IV-VI Semiconductors

(a) Bangert and Kästner model

In this case electron concentration and the TPSM can, respectively, be expressed as

\[ n_0 = \left( \frac{2 \pi}{3 \pi^2} \right) \left[ M_{\alpha} (E_{F_s}) + N_{\alpha} (E_{F_s}) \right] \]  
(631)

and

\[ G_0 = \left( \frac{\pi^2 k_B T}{3e} \right) \left[ \frac{M'_{\alpha} (E_{F_s}) + N'_{\alpha} (E_{F_s})}{M_{\alpha} (E_{F_s}) + N_{\alpha} (E_{F_s})} \right] \]  
(632)

where

\[ M_{\alpha} (E_{F_s}) = \tau_{\alpha} (E_{F_s}) \frac{1}{\sqrt{F_1 (E_{F_s}) \sqrt{F_2 (E_{F_s})}}} \]  
(633)

and

\[ N_{\alpha} (E_{F_s}) = \sum_{r=1}^{s} 2 (k_B T)^{2r} (1 - 2^{1-2r}) \xi (2r) \frac{\partial^{2r}}{\partial E_{F_s}^{2r}} \left[ M_{\alpha} (E_{F_s}) \right] \]

(b) Cohen Model

In this case electron concentration and the TPSM can, respectively, be written as

\[ n_0 = \left( \frac{g_v \sqrt{m_i m_e}}{\pi^2 \hbar} \right) \left[ M_{\alpha} (E_{F_s}) + N_{\alpha} (E_{F_s}) \right] \]  
(633)
\[ G_0 = \left( \frac{\pi^2 k_B^2 T}{3e} \right) \left[ \frac{M'_A(E_F) + N_A'(E_F)}{M_A(E_F) + N_A(E_F)} \right] \]  

(634)

where

\[ M_A(E_F) = \tau_A(E_F) \left[ E_F \left(1 + \alpha E_F\right) - \frac{\tau_A^2(E_F) + \frac{\alpha E_F}{2m_2^*}}{6m_2^*} \right] \]

\[ \tau_A(E_F) = \left[ \frac{\alpha}{2m_2^*} \right]^{1/2} \left[ \left( \frac{1 + \alpha E_F}{2m_2^*} \right)^{1/2} + \left( \frac{1 + \alpha E_F}{2m_2^*} \right)^{3/2} \right]^{1/2} \]

and

\[ N_A(E_F) = \sum_{r=1}^{\infty} 2(k_B T)^2 \left[ \frac{1}{2} - 2^{1-2r} \right] \xi(2r) \frac{\partial^2 \xi}{\partial E_F^{2r}} \left[ M_A(E_F) \right] \]

(c) Dimmock Model

In this case electron concentration and the TPSM assume the forms

\[ n_0 = \left( \frac{g_s}{2\pi^2} \right) \left[ M_A(E_F) + N_A(E_F) \right] \]  

(635)

\[ G_0 = \left( \frac{\pi^2 k_B^2 T}{3e} \right) \left[ \frac{M'_A(E_F) + N_A'(E_F)}{M_A(E_F) + N_A(E_F)} \right] \]  

(636)

where,

\[ M_A(E_F) = \left[ \alpha_s J_A(E_F) - \alpha_s(E_F) \bar{\tau}_A(E_F) - \frac{\alpha_s}{3} \left[ \bar{\tau}_A(E_F) \right]^3 \right], \alpha_s = \left[ \frac{2m^*_1 m^*_2}{\alpha h^2} \omega_A \right], \]

\[ \omega_A = \left[ \frac{\alpha^2}{16} \left( \frac{1}{m^*_1 m^*_2} + \frac{1}{m^*_1 m^*_2} \right) \right]^{1/2} \left[ \frac{\alpha^2}{4m^*_1 m^*_2 m^*_1 m^*_1} \right] \]

\[ J_A(E_F) = \frac{A_s(E_F)}{3} \left[ \left( A_s^2(E_F) + B_s^2(E_F) \right) E(\lambda, q) + 2B_s^2(E_F) F(\lambda, q) \right] + \frac{\bar{\tau}_A(E_F)}{3} \]

\[ \left[ \left( \bar{\tau}_A(E_F) \right)^2 + A_s^2(E_F) + 2B_s^2(E_F) \right] \left[ A_s^2(E_F) + \bar{\tau}_A^2(E_F) \right]^{1/2} \left[ B_s^2(E_F) + \bar{\tau}_A^2(E_F) \right]^{1/2} \]

\[ \lambda = \tan^{-1} \frac{\bar{\tau}_A(E_F)}{B_s(E_F)}, q = \frac{\sqrt{A_s^2(E_F) - B_s^2(E_F)}}{A_s(E_F)} \]
\[ A_{\lambda}(E_{x}) = \left[ \frac{1}{2} \left( \frac{E_{x}}{m_{1}^{*}} + 1 \right) + \left( \frac{1}{2m_{1}^{*}} + \frac{1}{2m_{2}^{*}} \right) \frac{E_{x}}{m_{1}^{*} m_{2}^{*}} + \left( \frac{1}{2m_{1}^{*}} + \frac{1}{2m_{2}^{*}} \right) \frac{E_{x}}{m_{1}^{*} m_{2}^{*}} \right]^{\frac{1}{2}}, \]

\[ B_{\lambda}(E_{x}) = \left[ \frac{1}{2} \left( \frac{E_{x}}{m_{1}^{*}} + 1 \right) + \left( \frac{1}{2m_{1}^{*}} + \frac{1}{2m_{2}^{*}} \right) \frac{E_{x}}{m_{1}^{*} m_{2}^{*}} + \left( \frac{1}{2m_{1}^{*}} + \frac{1}{2m_{2}^{*}} \right) \frac{E_{x}}{m_{1}^{*} m_{2}^{*}} \right]^{\frac{1}{2}}, \]

\[ \omega_{\lambda}(E_{x}) = \frac{\alpha}{2m_{2}^{*} + \frac{1}{2m_{1}^{*} m_{2}^{*}} + \frac{1}{2m_{1}^{*} m_{2}^{*}}} \left[ \frac{1}{m_{2}^{*} m_{2}^{*}} + \frac{1}{2m_{1}^{*} m_{2}^{*}} \right] + \frac{\alpha}{2m_{1}^{*} + \frac{1}{2m_{1}^{*} m_{2}^{*}} + \frac{1}{2m_{1}^{*} m_{2}^{*}}} \left[ \frac{1}{m_{2}^{*} m_{2}^{*}} + \frac{1}{2m_{1}^{*} m_{2}^{*}} \right] \]

\[ \alpha_{3} = \frac{\alpha \hbar^{2}}{4} \left[ \frac{1}{m_{2}^{*} m_{2}^{*}} + \frac{1}{m_{1}^{*} m_{2}^{*}} \right], \]

\[ \tau_{\lambda}(E_{x}) = \left[ \frac{2m_{1}^{*} m_{2}^{*}}{\alpha^{2}} \right]^{\frac{1}{2}} \left[ \left( \frac{1}{2m_{1}^{*} m_{2}^{*}} + \frac{1}{2m_{1}^{*} m_{2}^{*}} \right) \frac{E_{x}}{m_{1}^{*} m_{2}^{*}} + \left( \frac{1}{2m_{1}^{*} m_{2}^{*}} + \frac{1}{2m_{1}^{*} m_{2}^{*}} \right) \frac{E_{x}}{m_{1}^{*} m_{2}^{*}} \right]^{\frac{1}{2}}, \]

\[ E(\lambda, q) = \int_{0}^{1} \frac{1 - q^{2} \sin^{2} \alpha}{1 - q^{2} \sin^{2} \alpha} d\alpha \]

is the complete Elliptic integral of second kind,

\[ F(\lambda, q) = \int_{0}^{1} \frac{d\alpha}{\sqrt{1 - q^{2} \sin^{2} \alpha}} \]

is the complete Elliptic integral of first kind

and

\[ N_{\lambda}(E_{x}) = \sum_{r=1}^{\infty} 2(k_{B}T)^{2r}(1 - 2^{1-2r}) \zeta(2r) \frac{\partial^{2r}}{\partial E_{x}^{2r}} \left[ M_{\lambda}(E_{x}) \right] \]

\[ (d) \quad \text{Foley and Langenberg Model} \]

In this case electron concentration and the TPSM can, respectively, be expressed as

\[ n_{0} = \left( \frac{2g_{s}}{4\pi^{2}} \right) \left[ h_{\lambda}(E_{x}) + h_{\mu}(E_{x}) \right] \]  \hspace{1cm} (637)

\[ G_{0} = \left( \frac{\pi^{2}k_{B}^{2}T}{3e} \right) \left[ h_{\lambda}(E_{x}) + h_{\mu}(E_{x}) \right] \]  \hspace{1cm} (638)

where,

\[ h_{\lambda}(E_{x}) = \left[ \frac{1}{3} \delta_{\lambda} h_{\lambda}(E_{x}) - \delta_{\mu} h_{\lambda}(E_{x}) h_{\mu}(E_{x}) + \delta_{\lambda} J_{\lambda}(E_{x}) \right], \]
The expressions for the electron concentration and the TPSM can be written as

\[ n_0 = N_{c0} \left[ F_1(\eta) + \bar{\alpha}_c F_1(\eta) - \bar{\alpha}_c F_1(\eta) \right] \]

(vii) \( n \)-Ge

(a) Model of Cardona et. al

The expressions for the electron concentration and the TPSM can be written as
\[ G_0 = \left( \frac{\pi^2k_b}{3e} \right) \left[ \frac{F_{12}(\eta) + \bar{\alpha}_2F_{12}(\eta) - \bar{\alpha}_3F_{12}(\eta)}{F_{12}(\eta) + \bar{\alpha}_2F_{12}(\eta) - \bar{\alpha}_3F_{12}(\eta)} \right] \] (640)

where, \( N_{c0} = 2g_s \left( \frac{2\pi m^*_D k_B T}{\hbar^2} \right)^{3/2}, m^*_D = \left( \frac{m^*_e}{m^*_b} \right)^{3/2}, \bar{\alpha}_2 = \frac{4\alpha k_B T}{24} \) and

\[ \bar{\alpha}_s = \frac{189}{8} \alpha(k_B T)^2 \left( \frac{k_B T (m^*_b)^2}{\hbar^4} \right) \]

(b) Model of Wang and Ressler

The expressions for the electron concentration and the TPSM assume the forms

\[ n_0 = \left( \frac{m^*_e}{\pi^2 \hbar^2} \right) \left[ M_0(E_n) + N_{c0}(E_n) \right] \]
(641)

\[ G_0 = \left( \frac{\pi^2k_B T}{3e e} \right) \left[ \frac{M_0(E_n) + N_{c0}(E_n)}{M_0(E_n) + N_{c0}(E_n)} \right] \]
(642)

where \( M_0(E_n) = \left[ \bar{\alpha}_s \rho_0(E_n) - \frac{\bar{\alpha}_s}{3} \rho_0^3(E_n) - \bar{\alpha}_0 J_{\lambda_0}(E_n) \right], \bar{\alpha}_4 = \beta_4 \left( \frac{2m^*_b}{h^2} \right)^2, \beta_4 = 1.4 \beta_3, \rho_0 = \frac{1}{\bar{\alpha}_4} \left( \frac{2m^*_b}{h^2} \right)^2 \left( \frac{1-\bar{\alpha}_4}{\bar{\alpha}_4} \right)^2, \alpha_5 = \bar{\alpha}_5 \left( \frac{4m^*_b m^*_b}{h^2} \right) \), \( \alpha_7 = 0.8 \beta_5 \),

\[ \alpha_5 = 0.005 \beta_5 \left( \frac{2m^*_b}{h^2} \right)^2, \alpha_10 = \left( \frac{1}{2\bar{\alpha}_4} \right) \left( \frac{2m^*_b}{h^2} \right)^2 \left[ \bar{\alpha}_5^2 - 4\bar{\alpha}_5 \bar{\alpha}_6 \right] \]

\[ \bar{\alpha}_11 = \left( \frac{2m^*_b}{h^2} \right)^2 \left[ 4\bar{\alpha}_4 - 2\bar{\alpha}_5 \right], \bar{\alpha}_12(E_n) = \left( \frac{2m^*_b}{h^2} \right)^2 \left[ \frac{1-4\bar{\alpha}_4 E_n}{\bar{\alpha}_5^2 - 4\bar{\alpha}_5 \bar{\alpha}_6} \right]. \]

\[ \rho_\lambda(E_n) = \frac{1}{h} \left( \frac{m^*_b}{\bar{\alpha}_0} \right)^{3/2} \left[ 1 - \sqrt{1 - 4\bar{\alpha}_5 E_n} \right]^{3/2}, \bar{A}_\lambda(E_n) = \frac{1}{2} \left[ \bar{\alpha}_11 - \left[ \bar{\alpha}_11^2 - 4\bar{\alpha}_12(E_n) \right]^{1/2} \right]. \]

\[ \bar{B}_\lambda(E_n) = \frac{1}{2} \left[ \bar{\alpha}_11 - \left[ \bar{\alpha}_11^2 - 4\bar{\alpha}_12(E_n) \right]^{1/2} \right]. \]

\[ \bar{J}_\lambda(E_n) = \frac{\bar{A}_\lambda(E_n)}{3} \left[ -E(\lambda, q_3) \left[ \bar{A}_\lambda^2(E_n) + \bar{B}_\lambda^2(E_n) \right] + 2\bar{B}_\lambda^2(E_n) F(\lambda, q_3) \right] + \frac{\bar{A}_\lambda(E_n)}{3} \left[ \bar{\rho}_\lambda^2(E_n) + \bar{A}_\lambda^2(E_n) + 2\bar{B}_\lambda^2(E_n) \right] \left[ \frac{\bar{A}_\lambda^2(E_n) + \rho_\lambda^2(E_n)}{\bar{B}_\lambda^2(E_n) + \rho_\lambda^2(E_n)} \right]^{1/2} \]

264
\[ \lambda = \tan^{-1} \frac{\rho_{\lambda}(E_{F})}{B_{\lambda}(E_{F})}, \quad q_{2} = \left[ \frac{A_{\lambda}^{2}(E_{F}) - B_{\lambda}^{2}(E_{F})}{A_{\lambda}(E_{F})} \right] \text{ and} \]

\[ N_{\lambda}(E_{F}) = \sum_{r=1}^{2} 2(k_{B}T)^{2r} (1 - 2^{1-2r}) \zeta(2r) \frac{\partial^{2r}}{\partial E_{F}^{2r}} \left[ M_{\lambda}(E_{F}) \right] \]

(viii) Platinum Antimonide

The expressions for the electron concentration and the TPSM can be written as

\[ n_{0} = \left( \frac{g_{e}}{2\pi^{2}} \right) \left[ M_{\lambda}(E_{F}) + N_{\lambda}(E_{F}) \right] \]

\[ G_{0} = \left( \frac{\pi^{2}k_{B}^{2}T}{3e} \right) \left[ \frac{M_{\lambda}'(E_{F}) + N_{\lambda}'(E_{F})}{M_{\lambda}(E_{F}) + N_{\lambda}(E_{F})} \right] \]

where,

\[ M_{\lambda}(E_{F}) = \left[ T_{\lambda}(E_{F}) \rho_{\lambda}(E_{F}) - T_{\lambda}(E_{F}) \frac{\rho_{\lambda}^{3}(E_{F})}{3} - T_{\lambda} J_{\lambda}(E_{F}) \right], \]

\[ T_{\lambda} = [1 + \omega_{1}(\lambda)], T_{\lambda}(E_{F}) = \left[ -E_{F} \omega_{1} + \omega_{1} (E_{F} + \delta_{0}) \right], \quad T_{\lambda} = [2I_{1} + \omega_{1}(\lambda)], \]

\[ T_{\lambda} = [1 + \omega_{1}(\lambda)], T_{\lambda}(E_{F}) = \omega_{1} (E_{F} + \delta_{0}), \quad T_{\lambda}(E_{F}) = \left[ E_{F} (E_{F} + \delta_{0}) - E_{F} \omega_{1} \right], \]

\[ \bar{T}_{\lambda} = [T_{\lambda}^{2} - 4T_{\lambda} T_{\lambda}], \quad T_{\lambda}(E_{F}) = \left[ 2T_{\lambda} T_{\lambda}(E_{F}) - 4T_{\lambda} T_{\lambda}(E_{F}) \right], \]

\[ T_{\lambda}(E_{F}) = [T_{\lambda}^{2}(E_{F}) + 4T_{\lambda} T_{\lambda}(E_{F})], \quad T_{\lambda}(E_{F}) = \frac{T_{\lambda}(E_{F})}{2T_{\lambda}}, \quad T_{\lambda} = \left[ T_{\lambda}/2T_{\lambda} \right], \]

\[ T_{\lambda} = \frac{\sqrt{T_{\lambda}}}{2T_{\lambda}}, \quad T_{\lambda}(E_{F}) = \left[ T_{\lambda}(E_{F})/T_{\lambda} \right], \quad T_{\lambda}(E_{F}) = T_{\lambda}(E_{F})/T_{\lambda}, \]

\[ \rho_{\lambda}(E_{F}) = \left[ T_{\lambda}(E_{F}) - \sqrt{T_{\lambda}^{2}(E_{F}) + 4T_{\lambda} T_{\lambda}(E_{F})} \right]^{\frac{1}{2}}, \]

\[ \beta_{\lambda}'(E_{F}) = \frac{1}{2} T_{\lambda}(E_{F}) + \sqrt{T_{\lambda}^{2}(E_{F}) + 4T_{\lambda} T_{\lambda}(E_{F})}, \quad B_{\lambda}'(E_{F}) = \frac{1}{2} \left[ T_{\lambda}(E_{F}) - \sqrt{T_{\lambda}^{2}(E_{F}) + 4T_{\lambda} T_{\lambda}(E_{F})} \right], \]

\[ J_{\lambda}(E_{F}) = \frac{\rho_{\lambda}(E_{F})}{3} \left[ \beta_{\lambda}'(E_{F}) - B_{\lambda}'(E_{F}) \right]^{2}, \quad \beta_{\lambda}'(E_{F}) = \frac{1}{2} \left[ T_{\lambda}(E_{F}) - \sqrt{T_{\lambda}^{2}(E_{F}) + 4T_{\lambda} T_{\lambda}(E_{F})} \right], \]

\[ \eta_{1} = \tan^{-1} \left[ \frac{\rho_{\lambda}(E_{F})}{B_{\lambda}(E_{F})} \right], \quad \eta_{1} = \left[ B_{\lambda}(E_{F}) / \beta_{\lambda}(E_{F}) \right], \]

and

\[ N_{\lambda}(E_{F}) = \sum_{r=1}^{2} 2(k_{B}T)^{2r} (1 - 2^{1-2r}) \zeta(2r) \frac{\partial^{2r}}{\partial E_{F}^{2r}} \left[ M_{\lambda}(E_{F}) \right] \]

(ix) n-GaSb
In accordance of model of Mathur and Jain, the electron concentration and the TPSM can be expressed as

\[ n_0 = \frac{e_T}{3\pi\gamma^2}\left[\delta_h(E_{F_n}) + \delta_h(E_{F_n})\right] \]  \quad (645)

\[ G_0 = \left(\frac{\pi^2 k_B T}{3e}\right)\left[\delta_h(E_{F_n}) + \delta_h(E_{F_n})\right] \]  \quad (646)

where \( \delta_h(E_{F_n}) = \left[\delta_h(E_{F_n})\right]^\gamma \)

\[ \delta_h(E_{F_n}) = \left[ E_{F_n} + E_{F_n} - \frac{m^* E_{F_n}}{m_0} \right] \left[ \left( \frac{E_{F_n}}{2} \right)^2 + \left( \frac{E_{F_n}}{2} \left( \frac{1 - m^*}{m_0} \right) \right)^2 + \left( \frac{E_{F_n}}{2} \left( \frac{1 - m^*}{m_0} \right) \right)^2 \right] \]

and \( \delta_h(E_{F_n}) = \frac{\sum_{r=1}^{r_3} 2(k_BT)^{2r} (1-2^{1-2r}) \xi(2r) \frac{\partial^{2r}}{\partial E_{F_n}^{2r}} \left[ \delta_h(E_{F_n}) \right]}{2E_{F_n}} \)

\( (x) n\text{-Te} \)

The electron concentration and TPSM in n-Te in accordance with the model of Bouat et al can be written as

\[ n_0 = \frac{e_T}{3\pi\gamma^2}\left[ M_h(E_{F_n}) + N_h(E_{F_n}) \right] \]  \quad (647)

\[ G_0 = \left(\frac{\pi^2 k_B T}{3e}\right)\left[ M_h(E_{F_n}) + N_h(E_{F_n}) \right] \]  \quad (648)

\[ M_h(E_{F_n}) = \left[ 3\psi_3(E_{F_n}) \Gamma_3(E_{F_n}) - \psi_6 \Gamma_3(E_{F_n}) \right], \psi_3(E_{F_n}) = \frac{E_{F_n} + \psi_4^2}{2\psi_2^2} \]

\[ \Gamma_3(E_{F_n}) = \left[ 2\psi_1 \right]^3 \left[ \sqrt{\psi_3^2 + 4\psi_1 E_{F_n}} - \psi_3 \right], \psi_6 = (\psi_4/\psi_2) \]

\[ N_h(E_{F_n}) = \frac{\sum_{r=1}^{r_3} 2(k_BT)^{2r} (1-2^{1-2r}) \xi(2r) \frac{\partial^{2r}}{\partial E_{F_n}^{2r}} \left[ M_h(E_{F_n}) \right]}{2E_{F_n}}, \psi_1 = A_h, \psi_2 = A_r, \psi_3 = A_h \]

and \( \psi_4^2 = A_9 \)

\( (xi) Bismuth \)

\( (a) McClure and Choi model \)
The electron concentration and TPSM in Bi in accordance with this model can be written as

\[ n_0 = \left( \frac{g_e}{4\pi^2} \right) h_\kappa \left[ h_{\kappa_0} \left( E_{R_0} \right) + h_{\kappa_1} \left( E_{R_1} \right) \right] \]

(649)

\[ G_0 = \left( \frac{\pi^2 k_B^2 T}{3e} \right) \left[ \frac{h'_{\kappa_0} \left( E_{R_0} \right) + h'_{\kappa_1} \left( E_{R_1} \right)}{h_{\kappa_0} \left( E_{R_0} \right) + h_{\kappa_1} \left( E_{R_1} \right)} \right] \]

(650)

where \( h_\kappa = \frac{4\pi^2 \sqrt{m_1 m_2}}{h^2 \theta_\kappa} \), \( \theta_\kappa = \frac{\alpha h^2}{2m_2} \), \( \theta_\kappa = \frac{\alpha h^4}{4m_2 m'_2} \), \( \theta_{\kappa} \left( E_{R_0} \right) = \frac{\alpha E_{R_0} h^4}{2m_2} \left[ 1 - \frac{m_2}{m'_2} \right] \),

\[ \theta_{\kappa} = \frac{1}{\theta_\kappa} \cdot \]

\[ h_{\kappa_0} \left( E_{R_0} \right) = \left[ \frac{h_{\kappa_0} \left( E_{R_0} \right) - \theta_\kappa \left( E_{R_0} \right) \theta_{\kappa}^2 - \theta_\kappa \theta_{\kappa}^4}{2\theta_\kappa} \right] \]

\[ h_{\kappa_1} \left( E_{R_1} \right) = \sum_{r=1}^{2} 2 \left( k_2 T \right)^{2r} \left( 1 - 2^{1-2r} \right) \zeta \left( 2r \right) \frac{\partial^{2r}}{\partial E_{R_1}^{2r}} \left[ h_{\kappa_0} \left( E_{R_0} \right) \right] \]

(b) Hybrid model

In accordance with Hybrid model, the expressions for \( n_0 \) and \( G_0 \) are given by

\[ n_0 = \left( \frac{8\pi}{2\pi^2} \right) h_{\kappa_1} \left( E_{R_1} \right) + h_{\kappa_0} \left( E_{R_0} \right) \]

(651)

\[ G_0 = \left( \frac{\pi^2 k_B^2 T}{3e} \right) \left[ \frac{h'_{\kappa_1} \left( E_{R_1} \right) + h'_{\kappa_0} \left( E_{R_0} \right)}{h_{\kappa_1} \left( E_{R_1} \right) + h_{\kappa_0} \left( E_{R_0} \right)} \right] \]

(652)

where
\[ h_\lambda(E_k) = \left[ E_k \left(1 + \alpha E_k^2 \right) \frac{L_\lambda(E_k) \rho_\lambda^2(E_k)}{GM_2} \right] \]  

\[ L_\lambda(E_k) = \left[ 1 + \alpha E_k \left(1 - L_\lambda \right) \right], \quad L_\lambda = M_2/m_2, \quad L_\nu = M_2/M_2' \]  

\[ I_\lambda(E_k) = \left[ \frac{L_\lambda}{2E_\lambda M_2^2} \right] \left[ \frac{-L_\lambda(E_k)}{2M_2} + \frac{L_\lambda^2(E_k)}{4M_2^2} + \frac{L_\lambda E_k \left(1 + \alpha E_k \right)}{4E_\lambda M_2^2} \right]^{\frac{1}{2}} \]  

and \[ h_\lambda(E_k) = \sum_{r=1}^{d} 2(k_b T)^{2r} \left(1 - 2^{1-2r}\right) \zeta(2r) \frac{2r}{\partial E_\lambda^{2r}} \left[ h_\lambda(E_k) \right] \]

Thus by using the expressions of carrier statistics as given in section 1 to section 6 we can study the Thermoelectric power under strong magnetic field for various quantized structures under different physical conditions.

7.2 Debye screening length: The Debye screening length (DSL) of the carriers in the materials is a fundamental quantity, characterizing the screening of the Coulomb field of the ionized impurity centers by the free carriers. It affects many special features of the modern semiconductor devices, the carrier mobility under different mechanisms of scattering, and the carrier plasmas in materials [248, 249].

The DSL \( L_D \) can, in general, be written as [248]

\[ L_D = \left( \frac{\left| e \right|^2}{\epsilon_e} \frac{\partial n_0}{\partial E_F} \right)^{-\frac{1}{2}} \]  \hspace{1cm} (653)

where \( n_0 \) and \( E_F \) are applicable for bulk samples.

Using (614) and (653), we obtain

\[ L_D = \left(3|e|^3 n_0 G / \epsilon_e \sigma^2 \kappa \right)^{-\frac{1}{2}} \]  \hspace{1cm} (654)

Therefore, we can experimentally determine \( L_D \) by knowing the experimental curve of \( G \) versus carrier concentration at a fixed temperature. It is evident that the DSL for a system can be investigated if the functional dependence between the electron concentration and the Fermi energy of that particular material is known. Thus we can
determine the DSL for all the appropriate cases from the expressions of the electron 
statistics as formulated in various sections of this chapter.

7.3 Carrier contribution to the elastic constants: The knowledge of the carrier 
contribution to the elastic constants is important in studying the mechanical properties of 
the materials and has been investigated in the literature [250-251]. The electronic 
contribution to the second- and third- order elastic constants can be written as [250-251]

\[
\Delta C_{44} = -\frac{(G_0)^2}{9} \frac{\partial n_0}{\partial E_F},
\]

and

\[
\Delta C_{456} = \frac{(G_0)^3}{27} \frac{\partial^2 n_0}{\partial E_F^2},
\]

where \( G_0 \) is the deformation potential constant. Thus we can write

\[
\Delta C_{44} = \left[ -n_0 (G_0)^2 \epsilon G_0 / \left( 3\pi^2 k_B T \right) \right]
\]

and

\[
\Delta C_{456} = \left( n_0 \epsilon (G_0)^3 G_0 / \left( 3\pi^4 k_B^3 T \right) \right) \left( 1 + \frac{n_0 \partial G_0}{G_0 \partial n_0} \right)
\]

Thus, again the experimental graph of \( G_0 \) versus \( n_0 \) allows us to determine the electronic 
contribution to the elastic constants for materials having arbitrary spectra. We present a 
few results in this context:

The expressions for \( \Delta C_{44} \) and \( \Delta C_{456} \) in quantum wires of non-linear optical materials, 
III-V, II-VI, Bismuth, IV-VI, stressed semiconductors, Te, n-GaP, PtSb₂Bi₂Te₃, n-Ge, 
and II-V can, respectively, be expressed as

a) Non-linear optical materials:

\[
\Delta C_{44} = -\frac{(2(G_0)^2 \xi)}{9\pi} \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} [B_{11}'(E_{F1D}, n_x, n_y) + B_{12}'(E_{F1D}, n_x, n_y)]
\]
\[ \Delta C_{456} = \left( \frac{2(G_0^2 g_x)}{27 \pi} \right) \sum_{n_{x_1}=1}^{n_{\text{max}}} \sum_{n_{x_2}=1}^{n_{\text{max}}} \left[ B_{11}^{'''}(E_{FID}, n_x, n_y) + B_{12}^{'''}(E_{FID}, n_x, n_y) \right] \]  \hspace{1cm} (660)

b) III-V materials:

1. Three band model of Kane:

\[ \Delta C_{44} = \left( \frac{2(G_0^2 g_x)}{9 \pi} \right) \sum_{n_{x_1}=1}^{n_{\text{max}}} \sum_{n_{x_2}=1}^{n_{\text{max}}} \left[ T_{65}^{'}(E_{FID}, n_x, n_y) + T_{66}^{'}(E_{FID}, n_x, n_y) \right] \]  \hspace{1cm} (661)

\[ \Delta C_{456} = \left( \frac{2(G_0^2 g_x)}{27 \pi} \right) \sum_{n_{x_1}=1}^{n_{\text{max}}} \sum_{n_{x_2}=1}^{n_{\text{max}}} \left[ T_{65}^{'''}(E_{FID}, n_x, n_y) + T_{66}^{'''}(E_{FID}, n_x, n_y) \right] \]  \hspace{1cm} (662)

2. Two band model of Kane:

\[ \Delta C_{44} = \left( \frac{2(G_0^2 g_x)}{9 \pi} \right) \sum_{n_{x_1}=1}^{n_{\text{max}}} \sum_{n_{x_2}=1}^{n_{\text{max}}} \left[ T_{65}^{'}(E_{FID}, n_x, n_y) + T_{66}^{'}(E_{FID}, n_x, n_y) \right] \]  \hspace{1cm} (663)

\[ \Delta C_{456} = \left( \frac{2(G_0^2 g_x)}{27 \pi} \right) \sum_{n_{x_1}=1}^{n_{\text{max}}} \sum_{n_{x_2}=1}^{n_{\text{max}}} \left[ T_{65}^{'''}(E_{FID}, n_x, n_y) + T_{66}^{'''}(E_{FID}, n_x, n_y) \right] \]  \hspace{1cm} (664)

3. The model of Stillman et al.:

\[ \Delta C_{44} = \left( \frac{2(G_0^2 g_x)}{9 \pi} \right) \sum_{n_{x_1}=1}^{n_{\text{max}}} \sum_{n_{x_2}=1}^{n_{\text{max}}} \left[ B_{17}^{'}(E_{FID}, n_x, n_y) + B_{18}^{'}(E_{FID}, n_x, n_y) \right] \]  \hspace{1cm} (665)

\[ \Delta C_{456} = \left( \frac{2(G_0^2 g_x)}{27 \pi} \right) \sum_{n_{x_1}=1}^{n_{\text{max}}} \sum_{n_{x_2}=1}^{n_{\text{max}}} \left[ B_{17}^{'''}(E_{FID}, n_x, n_y) + B_{18}^{'''}(E_{FID}, n_x, n_y) \right] \]  \hspace{1cm} (666)

4. The model of Newson and Kurobe:

\[ \Delta C_{44} = \left( \frac{2(G_0^2 g_x)}{9 \pi} \right) \sum_{n_{x_1}=1}^{n_{\text{max}}} \sum_{n_{x_2}=1}^{n_{\text{max}}} \left[ B_{19}^{'}(E_{FID}, n_x, n_y) + B_{20}^{'}(E_{FID}, n_x, n_y) \right] \]  \hspace{1cm} (667)

\[ \Delta C_{456} = \left( \frac{2(G_0^2 g_x)}{27 \pi} \right) \sum_{n_{x_1}=1}^{n_{\text{max}}} \sum_{n_{x_2}=1}^{n_{\text{max}}} \left[ B_{19}^{'''}(E_{FID}, n_x, n_y) + B_{20}^{'''}(E_{FID}, n_x, n_y) \right] \]  \hspace{1cm} (668)

5. The model of Palik et al.:

\[ \Delta C_{44} = \left( \frac{2(G_0^2 g_x)}{9 \pi} \right) \sum_{n_{x_1}=1}^{n_{\text{max}}} \sum_{n_{x_2}=1}^{n_{\text{max}}} \left[ B_{21}^{'}(E_{FID}, n_x, n_y) + B_{22}^{'}(E_{FID}, n_x, n_y) \right] \]  \hspace{1cm} (669)
\[ \Delta C_{456} = \frac{(G_0)^3}{27 \pi} \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \left[ B_{21}''(E_FID, n_x, n_y) + B_{22}''(E_FID, n_x, n_y) \right] \]  

(670)

b) II-VI materials:

\[ \Delta C_{44} = -\frac{(G_0)^3}{27 \pi \sqrt{B_0}} \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \left[ t'_7(E_FID, n_x, n_y) + t''_8(E_FID, n_x, n_y) \right] \]  

(671)

\[ \Delta C_{456} = \frac{(G_0)^3}{27 \pi \sqrt{B_0}} \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \left[ t''_7(E_FID, n_x, n_y) + t''_8(E_FID, n_x, n_y) \right] \]  

(672)

c) Bismuth:

1. The model of McClure and Choi:

\[ \Delta C_{44} = -\frac{2(G_0)^3}{9 \pi} \frac{\sqrt{2m}}{\hbar} \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \left[ t''_{21}(E_FID, n_x, n_y) + t'_{23}(E_FID, n_x, n_y) \right] \]  

(673)

\[ \Delta C_{456} = \frac{2(G_0)^3}{27 \pi} \frac{\sqrt{2m}}{\hbar} \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \left[ t''_{21}(E_FID, n_x, n_y) + t''_{23}(E_FID, n_x, n_y) \right] \]  

(674)

2. Hybrid model:

\[ \Delta C_{44} = -\frac{2(G_0)^3}{9 \pi} \frac{\sqrt{2m}}{\hbar} \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \left[ t'_{31}(E_FID, n_x, n_y) + t''_{32}(E_FID, n_x, n_y) \right] \]  

(675)

\[ \Delta C_{456} = \frac{2(G_0)^3}{27 \pi} \frac{\sqrt{2m}}{\hbar} \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \left[ t'_{31}(E_FID, n_x, n_y) + t''_{32}(E_FID, n_x, n_y) \right] \]  

(676)

3. Cohen model:

\[ \Delta C_{44} = -\frac{2(G_0)^3}{9 \pi} \frac{\sqrt{2m}}{\hbar} \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \left[ t''_{35}(E_FID, n_x, n_y) + t'_{36}(E_FID, n_x, n_y) \right] \]  

(677)

\[ \Delta C_{456} = \frac{2(G_0)^3}{27 \pi} \frac{\sqrt{2m}}{\hbar} \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \left[ t''_{35}(E_FID, n_x, n_y) + t''_{36}(E_FID, n_x, n_y) \right] \]  

(678)

4. Lax model:
\[ \Delta C_{44} = -\frac{(G_0)^3 g_z}{9\pi} \frac{\sqrt{2m_i}}{\hbar} \sum_{n_{a,m}} \sum_{n_{a,m}} \left[ t_{37}''(E_{FD,D},n_x,n_y) + t_{38}''(E_{FD,D},n_x,n_y) \right] \] (679)

\[ \Delta C_{456} = \frac{2(G_0)^3 g_z}{27\pi} \frac{\sqrt{2m_i}}{\hbar} \sum_{n_{a,m}} \sum_{n_{a,m}} \left[ t_{37}''(E_{FD,D},n_x,n_y) + t_{38}''(E_{FD,D},n_x,n_y) \right] \] (680)

d) IV-VI materials:

**Dimmock model:**

\[ \Delta C_{44} = -\frac{(G_0)^3 g_z}{9\pi} \frac{\sqrt{2m_i}}{\hbar} \sum_{n_{a,m}} \sum_{n_{a,m}} \left[ B_{32}'(E_{FD,D},n_x,n_y) + B_{33}'(E_{FD,D},n_x,n_y) \right] \] (681)

\[ \Delta C_{456} = \frac{2(G_0)^3 g_z}{27\pi} \frac{\sqrt{2m_i}}{\hbar} \sum_{n_{a,m}} \sum_{n_{a,m}} \left[ B_{32}'(E_{FD,D},n_x,n_y) + B_{33}'(E_{FD,D},n_x,n_y) \right] \] (682)

e) Stressed materials:

\[ \Delta C_{44} = -\frac{(G_0)^3 g_z}{9\pi} \frac{\sqrt{2m_i}}{\hbar} \sum_{n_{a,m}} \sum_{n_{a,m}} \left[ B_{34}'(E_{FD,D},n_x,n_y) + B_{35}'(E_{FD,D},n_x,n_y) \right] \] (683)

\[ \Delta C_{456} = \frac{2(G_0)^3 g_z}{27\pi} \frac{\sqrt{2m_i}}{\hbar} \sum_{n_{a,m}} \sum_{n_{a,m}} \left[ B_{34}'(E_{FD,D},n_x,n_y) + B_{35}'(E_{FD,D},n_x,n_y) \right] \] (684)

f) Tellurium:

\[ \Delta C_{44} = \frac{(G_0)^3 g_z}{9\pi} \frac{\sqrt{2m_i}}{\hbar} \sum_{n_{a,m}} \sum_{n_{a,m}} \left[ B_{36,\pm}'(E_{FD,D},n_x,n_y) + \theta_{5,\pm}' \right] \] (685)

\[ \Delta C_{456} = \frac{(G_0)^3 g_z}{27\pi} \frac{\sqrt{2m_i}}{\hbar} \sum_{n_{a,m}} \sum_{n_{a,m}} \left[ B_{36,\pm}'(E_{FD,D},n_x,n_y) + \theta_{5,\pm}' \right] \] (686)

e) Gallium phosphide:

\[ \Delta C_{44} = -\frac{2(G_0)^3 g_z}{9\pi} \frac{\sqrt{2m_i}}{\hbar} \sum_{n_{a,m}} \sum_{n_{a,m}} \left[ B_{38}'(E_{FD,D},n_x,n_y) + B_{39}'(E_{FD,D},n_x,n_y) \right] \] (687)

\[ \Delta C_{456} = \frac{2(G_0)^3 g_z}{27\pi} \frac{\sqrt{2m_i}}{\hbar} \sum_{n_{a,m}} \sum_{n_{a,m}} \left[ B_{38}'(E_{FD,D},n_x,n_y) + B_{39}'(E_{FD,D},n_x,n_y) \right] \] (688)

f) Platinum Antimonide:
\[ \Delta C_{44} = -\frac{2\left(\bar{G}_0\right)^2 g_x}{9\pi} \sum_{n_{\alpha}} \sum_{n_{\beta}} \left[ B_4^{44} (E_{FID}, n_x, n_z) + B_4^{44} (E_{FID}, n_x, n_z) \right] \]  

\[ \Delta C_{456} = \frac{2\left(\bar{G}_0\right)^3 g_x}{27\pi} \sum_{n_{\alpha}} \sum_{n_{\beta}} \left[ B_6^{44} (E_{FID}, n_x, n_z) + B_6^{44} (E_{FID}, n_x, n_z) \right] \]  

**g) Bismuth Telluride:**

\[ \Delta C_{44} = -\frac{2\left(\bar{G}_0\right)^2 g_x}{9\pi} \sum_{n_{\alpha}} \sum_{n_{\beta}} \left[ B_4^{44} (E_{FID}, n_x, n_z) + B_4^{44} (E_{FID}, n_x, n_z) \right] \]  

\[ \Delta C_{456} = \frac{2\left(\bar{G}_0\right)^3 g_x}{27\pi} \sum_{n_{\alpha}} \sum_{n_{\beta}} \left[ B_6^{44} (E_{FID}, n_x, n_z) + B_6^{44} (E_{FID}, n_x, n_z) \right] \]  

**h) Germanium:**

1. The model of Cardona et al:

\[ \Delta C_{44} = -\frac{2\left(\bar{G}_0\right)^2 g_x}{9\pi} \sum_{n_{\alpha}} \sum_{n_{\beta}} \left[ B_4^{44} (E_{FID}, n_x, n_z) + B_4^{44} (E_{FID}, n_x, n_z) \right] \]  

\[ \Delta C_{456} = \frac{2\left(\bar{G}_0\right)^3 g_x}{27\pi} \sum_{n_{\alpha}} \sum_{n_{\beta}} \left[ B_6^{44} (E_{FID}, n_x, n_z) + B_6^{44} (E_{FID}, n_x, n_z) \right] \]  

2. The model of Wang and Ressler:

\[ \Delta C_{44} = -\frac{2\left(\bar{G}_0\right)^2 g_x}{9\pi} \sum_{n_{\alpha}} \sum_{n_{\beta}} \left[ B_4^{44} (E_{FID}, n_x, n_z) + B_4^{44} (E_{FID}, n_x, n_z) \right] \]  

\[ \Delta C_{456} = \frac{2\left(\bar{G}_0\right)^3 g_x}{27\pi} \sum_{n_{\alpha}} \sum_{n_{\beta}} \left[ B_6^{44} (E_{FID}, n_x, n_z) + B_6^{44} (E_{FID}, n_x, n_z) \right] \]  

**i) Gallium Antimonide:**

\[ \Delta C_{44} = -\frac{2\left(\bar{G}_0\right)^2 g_x}{9\pi} \sum_{n_{\alpha}} \sum_{n_{\beta}} \left[ B_4^{44} (E_{FID}, n_x, n_y) + B_4^{44} (E_{FID}, n_x, n_y) \right] \]  

\[ \Delta C_{456} = \frac{2\left(\bar{G}_0\right)^3 g_x}{27\pi} \sum_{n_{\alpha}} \sum_{n_{\beta}} \left[ B_6^{44} (E_{FID}, n_x, n_y) + B_6^{44} (E_{FID}, n_x, n_y) \right] \]  

**j) II-V materials:**
\[ \Delta C_{44} = -\frac{(\tilde{G}_6)^2}{9\pi} \sum_{n_x=1}^{n_{\text{max}}} \sum_{n_y=1}^{n_{\text{max}}} \left[ B'_4 \left( E_{F1D}, n_x, n_y \right) + B'_5 \left( E_{F1D}, n_x, n_y \right) \right] \]  

\[ \Delta C_{45} = \frac{(\tilde{G}_6)^3}{27\pi} \sum_{n_x=1}^{n_{\text{max}}} \sum_{n_y=1}^{n_{\text{max}}} \left[ B''_4 \left( E_{F1D}, n_x, n_y \right) + B''_5 \left( E_{F1D}, n_x, n_y \right) \right] \]  

### 7.4 Diffusivity-mobility ratio

The diffusivity \( D \) to mobility \( \mu \) ratio (DMR) of the carriers in semiconductor devices is known to be very useful [252] since the diffusion constant (a quantity often used in device analysis but whose exact experimental determination is rather difficult) can be obtained from this ratio by knowing the experimental values of the mobility. In addition, it is more accurate than any of the individual relation for the diffusivity or the mobility, which is the two widely used quantities of carrier transport of modern nanostructured materials and devices. The classical DMR equation is valid for both types of carriers. In its conventional form, it appears that, the DMR increases linearly with the temperature \( T \) being independent of the carrier concentration. This relation holds only under the condition of carrier non-degeneracy although its validity has been suggested erroneously for degenerate materials [253]. The performance of the electron devices at the device terminals and the speed of operation of modern switching transistors are significantly influenced by the degree of carrier degeneracy present in these devices [254]. The simplest way of analyzing them under degenerate condition is to use the appropriate DMR to express the performance of the devices at the device terminals and the switching speed in terms of the carrier concentration [254].

It is well known from the fundamental work of Landsberg [255-257] that the DMR for electronic materials having degenerate electron concentration is essentially determined by their respective energy band structures. This relation is useful for semiconductor homostructures [258, 259], semiconductor-semiconductor heterostructures [260, 261], metals-semiconductor heterostructures [262-265] and insulator-semiconductor heterostructures [266-269]. It has different values in different materials and varies with the doping, with the magnitude of the reciprocal quantizing magnetic field under magnetic quantization, with the quantizing electric field as in inversion layers, with the nanothickness as in quantum wells and quantum well wires and with superlattice period as in the quantum confined superlattices of small gap materials with graded interfaces.
having various carrier energy spectra [270-281]. It can, in general, be proved that for bulk specimens the DMR is given by [270]

\[
\frac{D}{\mu} = \left( \frac{n_0}{|e|} \right) \left( \frac{\partial n_0}{\partial E_F} \right) \tag{701}
\]

The electric quantum limit as in inversion layers and nipi structures refers to the lowest electric sub-band as

\[
\frac{D}{\mu} = \left( \frac{\bar{n}_0}{|e|} \right) \left( \frac{\partial \bar{n}_0}{\partial (E_{F0} - \bar{E}_0)} \right) \tag{702}
\]

where \( \bar{n}_0 \), \( E_{F0} \) and \( \bar{E}_0 \) are the electron concentration, the energy of the electric sub-band and the Fermi energy in the electric quantum limit.

Using the appropriate equations one obtains

\[
\frac{D}{\mu} = \left( \frac{\pi^2 k_B^2 T}{3|e|^2 G} \right) \tag{703}
\]

Thus, the DMR for degenerate materials can be determined by knowing the experimental values of \( G \).

The suggestion for the experimental determination of DMR for degenerate materials having arbitrary dispersion laws as given in (703) does not contain any energy band constants. For a fixed temperature, the DMR varies inversely as \( G \). Only the experimental values of \( G \) for any material as a function of electron concentration will generate the experimental values of the DMR for that range of \( n_0 \) for that system. Since \( G \) decreases with increasing \( n_0 \), from (703) one can infer that the DMR will increase with increase in \( n_0 \). This statement is the compatibility test so far as the suggestion for the experimental determination of DMR for degenerate materials is concerned. Thus by using the various expressions for carrier concentration we can study the DMR in different cases.

7.5 Measurement of Bandgap in the presence of Light Waves: Using the appropriate equations, the normalized incremental band gap (\( \Delta E_g \)) has been plotted as a function of normalized \( I_0 \) (for a given wavelength and considering red light for which \( \lambda = 660nm \)) at \( T = 4.2 K \) in figs. 7.1 and 7.2 for n-Hg\(_{1-x}\)Cd\(_x\)Te and n-In\(_{1-x}\)Ga\(_x\)As\(_y\)P\(_{1-y}\) lattice matched to
InP in accordance with the perturbed three and two band models of Kane and that of perturbed parabolic energy bands respectively. In figs. 7.3 and 7.4, the normalized incremental band gap has been plotted for the aforementioned optoelectronic compounds as a function of $\lambda$. It is worth remarking that the influence of an external photo-excitation is to change radically the original band structure of the material. Because of this change, the photon field causes to increase the band gap of semiconductors. We propose the following two experiments for the measurement of band gap of semiconductors under photo-excitation.

A) A white light with color filter is allowed to fall on a semiconductor and the optical absorption coefficient $\alpha_0$ is being measured experimentally. For different colors of light, $\alpha_0$ is measured and $\alpha_0$ versus $h\omega$ (the incident photon energy) is plotted and we extrapolate the curve such that $\alpha_0 \rightarrow 0$ at a particular value $h\omega_i$. This $h\omega_i$ is the unperturbed band gap of the semiconductor. During this process, we vary the wavelength with fixed $I_0$. From our present study, we have observed that the band gap of the semiconductor increases for various values of $\lambda$ when $I_0$ is fixed (from Figs. 7.3 and 7.4). This implies that the band gap of the semiconductor measured (i.e. $h\omega_i = E_g$) is not the unperturbed band gap $E_{gs}$ but the perturbed band gap $E_g$; where $E_g = E_{gs} + \Delta E_g$, $\Delta E_g$ is the increased band gap at $h\omega_i$. Conventionally, we consider this $E_g$ as the unperturbed band gap of the semiconductor and this particular concept needs modification. Furthermore, if we vary $I_0$ for a monochromatic light (when $\lambda$ is fixed) the band gap of the semiconductor will also change consequently (Fig. 7.1 and 7.2). Consequently, the absorption coefficient will change with the intensity of light. For the overall understanding, the detailed theoretical and experimental investigations are needed in this context for various materials having different band structures.

B) The conventional idea for the measurement of the band gap of the semiconductors is the fact that the minimum photon energy $h\nu$ (where $\nu$ is the frequency of the monochromatic light) should be equal to the band gap $E_{gs}$ (unperturbed) of the semiconductor, i.e.,
\[ h\nu = E_g \]  
(704)

In this case, \( \lambda \) is fixed for a given monochromatic light and the semiconductor is exposed to a light of wavelength \( \lambda \). Also the intensity of the light is fixed. From Figs. 7.3 and 7.4, we observe that the band gap of the semiconductor is not \( E_{g0} \) (for a minimum value of \( h\nu \)) but \( E_g \), the perturbed band gap. Thus, we can rewrite the above equality as

\[ h\nu = E_g \]  
(705)

Furthermore, if we vary the intensity of light (Figs. 7.1 and 7.2) for the study of photoemission, the minimum photon energy should be

\[ h\nu_1 = E_{g1} \]  
(706)

where \( E_{g1} \) is the perturbed band gap of the semiconductor due to various intensity of light when \( \nu \) and \( \nu_1 \) are different.

Fig. 7.1: Plots of the normalized incremental band gap \( (\Delta E_g) \) for n-Hg\(_{1-x}\)Cd\(_{x}\)Te as a function of normalized light intensity in which the curves (a) and (b) represent the perturbed three and two band models of Kane respectively. The curve (c) represents the same variation in n-Hg\(_{1-x}\)Cd\(_{x}\)Te in accordance with the perturbed parabolic energy bands.
Fig. 7.2: Plots of the normalized incremental band gap $\left(\Delta E_g\right)$ for In$_{1-x}$Ga$_x$As$_y$P$_{1-y}$ lattice matched to InP as a function of normalized light intensity for all cases of Fig. 7.1.

Fig. 7.3: Plots of the normalized incremental band gap $\left(\Delta E_g\right)$ for Hg$_{1-x}$Cd$_x$Te as a function of wavelength for all cases of Fig. 7.1.
Fig. 7.4: Plots of the normalized incremental band gap \( \Delta E_{\text{g}} \) for \( \text{In}_{1-x}\text{Ga}_{x}\text{As}_{y}\text{P}_{1-y} \) lattice matched to \( \text{InP} \) as a function of wavelength for all cases of Fig. 7.2.

Thus, we arrive at the following conclusions:

a) Under different intensity of light, keeping \( \lambda \) fixed, the condition of band gap measurement is given by

\[ h\nu = E_g = E_{g_0} + \Delta E', \]

(707)

b) Under different color of light, keeping the intensity fixed, the condition of band gap measurement assumes the form

\[ h\nu = E_g = E_{g_0} + \Delta E, \]

(708)

and not the conventional result as given by (704).

7.6 Diffusion coefficient of the minority carriers: This particular coefficient in quantum confined lasers can be expressed as

\[ \frac{D_i}{D_0} = \frac{dE_{F_i}}{dE_F} \]

(709)

where \( D_i \) and \( D_0 \) are the diffusion coefficients of the minority carriers both in the presence and absence of quantum confinements and \( E_{F_i} \) and \( E_F \) are the Fermi energies in the respective cases. It appears then that, the formulation of the above ratio requires a
relation between $E_F$ and $E_{F_i}$, which, in turn, is determined by the appropriate carrier statistics. Thus, our present study plays an important role in determining the diffusion coefficients of the minority carriers of quantum-confined lasers with materials having arbitrary band structures. Therefore in the investigation of the optical excitation of the optoelectronic materials which lead to the study of the ambipolar diffusion coefficients the present results contribute significantly.

7.7 Nonlinear optical response: The nonlinear response from the optical excitation of the free carriers is given by [282]

$$Z_0 = -\frac{e^2}{\omega^2 \hbar^2} \int \left( k_z \frac{\partial k_z}{\partial E} \right)^{-1} f(E) N(E) dE \quad (710)$$

where $\omega$ is the optical angular frequency, $N(E)$ is the density-of-states function. From the various E-k relations of different materials under different physical conditions, we can formulate the expression of $N(E)$ and from band structure we can derive the term $\left( k_z \frac{\partial k_z}{\partial E} \right)$ and thus by using the density-of-states function as formulated, we can study the $Z_0$ for all types of materials as considered in this thesis.

7.8 Third order nonlinear optical susceptibility: This particular susceptibility can be written as [283]

$$\chi^3_{np}(\omega_1, \omega_2, \omega_3) = \frac{n_0 e^4 \langle \epsilon^4 \rangle}{24 \omega_1 \omega_2 \omega_3 (\omega_1 + \omega_2 + \omega_3) \hbar^4} \quad (711)$$

where $n_0 \langle \epsilon^4 \rangle = \int \frac{\partial^4 E}{\partial k_z^4} N(E) f(E) dE$ and the other notations are defined in [283]. The term $\left( \frac{\partial^4 E}{\partial k_z^4} \right)$ can be formulated by using the dispersion relations of different materials as given in appropriate sections of this thesis. Thus one can investigate the $\chi^3_{np}(\omega_1, \omega_2, \omega_3)$ for all materials as considered in this chapter.

7.9 Generalized Raman gain: The generalized Raman gain in optoelectronic materials can be expressed as [284]
\[
R_c = \overline{T} \left( \frac{16\pi^2 c^2}{\hbar \omega_{p} g \omega_n n_p^2} \left( \frac{\Gamma_{p}}{\Gamma} \right) \left( \frac{e^2}{mc^2} \right)^2 m^2 R^2 \right)
\]

(712)

where, \( \overline{T} = \sum_{n,k_z} \left[ f_{0}(n,k_z \uparrow) - f_{0}(n,k_z \downarrow) \right] \), \( f_{0}(n,k_z \uparrow) \) is the Fermi factor for spin-up Landau levels, \( f_{0}(n,k_z \downarrow) \) is the Fermi factor for spin down Landau levels, \( n \) is the Landau quantum number and the other notations are defined in [284]. It appears then the formulation of \( R_c \) is determined by the appropriate derivation requires the magneto-dispersion relations. By using the different appropriate formulas as formulated in various sections of this chapter \( R_c \) can, in general, be investigated.

7.10 The EEM in Heavily Doped Compound Semiconductors: It is well known that the band tails are being formed in the forbidden zone of heavily doped semiconductors and can be explained by the overlapping of the impurity band with the conduction and valence bands [285]. Kane [286] and Bonch Bruevich [287] have independently derived the theory of band tailing for semiconductors having unperturbed parabolic energy bands. Kane’s model [286] was used to explain the experimental results on tunneling [288] and the optical absorption edges [289, 290] in this context. Halperin and Lax [291] developed a model for band tailing applicable only to the deep tailing states. Although Kane’s concept is often used in the literature for the investigation of band tailing [292, 293], it may be noted that this model [286, 294] suffers from serious assumptions in the sense that the local impurity potential is assumed to be small and slowly varying in space coordinates [293]. In this respect, the local impurity potential may be assumed to be a constant. In order to avoid these approximations, we have developed in this chapter the electron energy spectra for heavily doped semiconductors for studying the EEM based on the concept of the variation of the kinetic energy [285, 293] of the electron with the local point in space coordinates. This kinetic energy is then averaged over the entire region of variation using a Gaussian type potential energy. On the basis of the \( E-k \) dispersion relation, we have obtained the electron statistics for different heavily doped materials for the purpose of numerical computation of the respective EEMs. It may be noted that, a more general treatment of many-body theory for the density-of-states of heavily doped semiconductors merges with one-electron theory under macroscopic conditions [285]. Also, the experimental results for the Fermi energy and others are the average effect of this macroscopic case. So, the present treatment of the one-electron
system is more applicable to the experimental point of view and it is also easy to understand the overall effect in such a case [295]. In a heavily-doped semiconductors, each impurity atom is surrounded by the electrons, assuming a regular distribution of atoms, and it is screened independently [292, 294, 296]. The interaction energy between electrons and impurities is known as the impurity screening potential. This energy is determined by the inter-impurity distance and the screening radius, which is known as the screening length. The screening radius grows with the electron concentration and the effective mass. Furthermore, these entities are important for heavily doped materials in characterizing the semiconductor properties [297, 298] and the devices [292, 299]. The works on Fermi energy and the screening length in an n-type GaAs have already been initiated in the literature [300, 301, 302], based on Kane’s model. Incidentally, the limitations of Kane’s model [293], as mentioned above, are also present in the studies.

At this point, it may be noted that many band tail models are proposed using the Gaussian distribution of the impurity potential variation [286, 293]. In this chapter, we have used the Gaussian band tails to obtain the exact $E-k$ dispersion relations for heavily doped tetragonal, III-V, II-VI, IV-VI and stressed Kane type compounds. Our method is not at all related with the density-of-states (DOS) technique as used in the aforementioned works. From the electron energy spectrum, one can obtain the DOS but the DOS technique, as used in the literature cannot provide the $E-k$ dispersion relation. Therefore, our study is more fundamental than those in the existing literature, because the Boltzmann transport equation, which controls the study of the charge transport properties of the semiconductor devices, can be solved if and only if the $E-k$ dispersion relation is known. We wish to note that the Gaussian function for the impurity potential distribution has been used by many authors. It has been widely used since 1963 when Kane first proposed it. We will also use the Gaussian distribution for the present study.

In the following we shall study the EEM in heavily doped tetragonal materials. The results for heavily doped III-V, ternary and quaternary compounds whose undoped conduction electrons obeys the three and the two band models of Kane together with parabolic energy bands and they form the special cases of the generalized analysis. We have also studied the EEM for heavily doped II-VI, IV-VI and stressed Kane type semiconductors respectively.

a) Study of the EEM in heavily doped tetragonal materials forming Gaussian band-tails
The generalized unperturbed electron energy spectrum for the bulk specimens of the
tetragonal materials in the absence of any doping can be expressed following (2) of
Section 1 as

\[
\frac{h^2 k^2}{2m_{\text{eff}}} + \left( \frac{b_{\|}}{b_1} \frac{c_{\|}}{c_1} \right) \frac{h^2 k^2}{2m_{\perp}} = \left\{ \frac{E(\alpha E + 1)(b_{\|}E + 1)}{c_{\|}E + 1} + \frac{\alpha b_{\|}}{c_1} \right\}
\]

\[
\left[ \delta E + \frac{2}{9} \left( \Delta_1^2 - \Delta_1^2 \right) \right] \left\{ \frac{2}{9} \left( \frac{\alpha b_{\|}}{c_{\|}E + 1} \right) \right\} + \left( \frac{\Delta_1^2 - \Delta_1^2}{6\Delta_1} \right) \left\{ \frac{\alpha b_{\|}}{c_{\|}E + 1} \right\}
\]

\[
\frac{\delta}{2} \left( \frac{\Delta_1^2 - \Delta_1^2}{6\Delta_1} \right) \left\{ \frac{c_{\|}}{c_{\|}E + 1} \right\} \right]
\]

(713)

where \( b_{\|} \equiv \frac{1}{E + \Delta_2} \), \( c_{\|} \equiv \frac{1}{E + \Delta_3} \), \( b_{\perp} \equiv \frac{1}{E + \Delta_1} \), \( c_{\perp} \equiv \frac{1}{E + \Delta_4} \) and

\( \alpha \equiv \frac{1}{E} \).

The Gaussian distribution \( F(V) \) of the impurity potential is given by

\[
F(V) = \left( \pi \eta^2 \right)^{1/2} \exp \left( -V^2 / \eta^2 \right)
\]

(714)

where, \( \eta \) is the impurity scattering potential. It appears from (714) that the variance
parameter \( \eta \) is not equal to zero, but the mean value is zero. Further, the impurities are
assumed to be uncorrelated and the band mixing effect has been neglected in this
simplified theoretical formalism.

We have to average the Kinetic energy in the order to obtain the E-k dispersion relation
in tetragonal materials including the band tailing effect. Using the (713) and (714), we
get

\[
\left[ \frac{h^2 k^2}{2m_{\perp}} \int F(V) dV \right] + \left[ \frac{b_{\|}}{b_1} \frac{c_{\|}}{c_1} \right] \frac{h^2 k^2}{2m_{\perp}} \int F(V) dV
\]

\[
\frac{\delta}{2} \left( E - V \right) \left[ \frac{\alpha(E - V) + 1}{c_{\|}(E - V) + 1} \right] F(V) dV
\]

\[
+ \frac{\alpha b_{\|}}{c_1} \left( E - V \right) F(V) dV + \frac{2}{9} \left( \Delta_1^2 - \Delta_1^2 \right) \int F(V) dV
\]

\[
- \left( \frac{2}{9} \right) \frac{\alpha b_{\|}}{c_1} \left( \Delta_1^2 - \Delta_1^2 \right) \left[ \frac{F(V) dV}{c_1(E - V) + 1} \right]
\]

283
\[
\frac{\hbar^2 k^2}{2m_1} \left[ \left( \frac{b_1 c_1}{b_1 c_1} \right) \left( \frac{\delta + \Delta_1^2 - \Delta_2^2}{6 \Delta_1^2} \right) \alpha \int \frac{F(V)dV}{[\alpha(E-V)+1]} \right] + \left( \frac{\delta - \Delta_1^2 - \Delta_2^2}{6 \Delta_1^2} \right) \int \frac{F(V)dV}{\left[ q_1 (E-V) + 1 \right]} \right] \]  

(715)

can be rewritten as

\[
\frac{\hbar^2 k^2}{2m_1} I(1) + \left( \frac{b_1 c_1}{b_1 c_1} \right) \frac{\hbar^2 k^2}{2m_1} I(1) = \left\{ I_3 \left( q_1 \right) + \frac{\alpha h}{c_1} \left[ \delta I(4) \right] \right\} + \left( \frac{2}{9} \right) \frac{\alpha h}{c_1} \left( \frac{\Delta_1^2 - \Delta_2^2}{I(1)} \right) I_6 \left( q_1 \right) 
\]

\[
\left( \frac{\hbar^2 k^2}{2m_1} \right) \left[ \left( \frac{b_1 c_1}{b_1 c_1} \right) \left( \frac{\delta + \Delta_1^2 - \Delta_2^2}{6 \Delta_1^2} \right) \alpha I(\alpha) + \left( \frac{\delta - \Delta_1^2 - \Delta_2^2}{6 \Delta_1^2} \right) q_1 I(q_1) \right] \]  

(716)

where,

\[
I(1) \equiv \int F(V)dV 
\]

(717)

\[
I_3 \left( q_1 \right) \equiv \int \frac{E-V}{[\alpha(E-V)+1]} \left[ \frac{b_1(E-V)+1}{c_1 (E-V)+1} \right] F(V)dV 
\]

(718)

\[
I(4) \equiv \int (E-V) F(V)dV 
\]

(719)

\[
I(\alpha) \equiv \int \frac{F(V)dV}{[\alpha(E-V)+1]} 
\]

(720)

Let us substitute \( E - V = x \) and \( x/\eta_g = t_0 \), we get from (717)

\[
I(1) = \left( \exp(-E^2/\eta_g^2)/\sqrt{\pi} \right) \int_0^\infty \exp \left[ -t_0^2 + \left( 2Et_0/\eta_g \right) \right] dt_0 
\]

Thus,

\[
I(1) = \left[ \frac{1 + Erf \left( E/\eta_g \right)}{2} \right] 
\]

(721)

From (719), one can write
\[ I(4) = \frac{1}{\eta_s \sqrt{\pi}} \int_{-\infty}^{\infty} (E-V) \exp(-V^2/\eta_s^2) dV \]

\[ = \frac{E}{2} \left[ 1 + \text{Erf}(E/\eta_s) \right] - \left\{ \frac{1}{\sqrt{\pi \eta_s^2}} \int_{-\infty}^{\infty} V \exp(-E^2/\eta_s^2) dV \right\} \]

After computing this simple integration, one obtains

Thus, \[ I(4) = \eta_s \exp(-E^2/\eta_s^2) \left( 2 \sqrt{\pi} \right)^{-1} + \frac{E}{2} \left( 1 + \text{Erf}(E/\eta_s) \right) = \gamma_0(E, \eta_s) \] (722)

From (720), we can write

\[ I(\alpha) = \frac{1}{\sqrt{\pi \eta_s^2}} \int_{-\infty}^{\infty} \frac{\exp(-V^2/\eta_s^2) dV}{[\alpha(E-V)+1]} \] (723)

when, \( V \to \pm \infty, \frac{1}{[\alpha(E-V)+1]} \to 0 \) and \( \exp(-V^2/\eta_s^2) \to 0 \); therefore, using (723) one can write

\[ I(\alpha) = \frac{1}{\sqrt{\pi \eta_s^2}} \int_{-\infty}^{\infty} \frac{\exp(-V^2/\eta_s^2) dV}{[\alpha E + 1 - \alpha V]} \] (724)

The (724) can be expressed as

\[ I(\alpha) = \frac{1}{\alpha \eta_s \sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-t^2)(u-t)^{-\alpha} dt \] (725)

where, \( \frac{V}{\eta_s} \equiv t \) and \( u \equiv \left( \frac{1 + \alpha E}{\alpha \eta_s} \right) \).

We know that

\[ W(Z) = \left( \frac{i}{\pi} \right) \int_{-\infty}^{\infty} (Z-t)^{-\alpha} \exp(-t^2) dt \] (726)

in which \( i = \sqrt{-1} \) and \( Z \) is, in general, a complex number.

We also know,
\[ W(Z) = \exp(-Z^2) \text{Erfc}(-iZ) \tag{727} \]

where, \( \text{Erfc}(Z) = 1 - \text{Erf}(Z) \).

Thus, \( \text{Erfc}(-iu) = 1 - \text{Erf}(-iu) \)

Since, \( \text{Erf}(-iu) = -\text{Erf}(iu) \)

Therefore, \( \text{Erfc}(-iu) = 1 + \text{Erf}(iu) \).

Thus,

\[ I(\alpha) = \left[ -i \sqrt{\pi/\alpha \eta_k} \right] \exp(-u^2) [1 + \text{Erf}(iu)] \tag{728} \]

We also know that

\[ \text{Erf}(x+iy) = \text{Erf}(x) + \left( \frac{e^{-x^2}}{2\pi x} \right) \left[ (1 - \cos(2xy)) + i \sin(2xy) + \frac{2}{\pi} e^{-x^2} \sum_{p=1}^{\infty} \frac{\exp(-p^2/4)}{p} \frac{1}{p^2 + 4x^2} \right] \]

\[ f_p(x, y) + ig_p(x, y) + \mathcal{E}(x, y) \tag{729} \]

where, \( f_p(x, y) \equiv \left[ 2x - 2x \cosh(py) \cos(2xy) + p \sinh(py) \sin(2xy) \right] \),

\( g_p(x, y) \equiv \left[ 2x \cosh(py) \sin(2xy) + p \sinh(py) \cos(2xy) \right] \),

\[ |\mathcal{E}(x, y)| \approx 10^{-16} |\text{Erf}(x+iy)| \]

Substituting \( x = 0 \) and \( y = u \) in (729), one obtains,

\[ \text{Erf}(iu) = \left( \frac{2i}{\pi} \right) \sum_{p=1}^{\infty} \left\{ \frac{\exp(-p^2/4)}{p} \right\} \sinh(pu) \tag{730} \]

Therefore, one can write

\[ I(\alpha) = C_{21}(\alpha, E, \eta_k) - iD_{21}(\alpha, E, \eta_k) \tag{731} \]

where,
\[ C_{21}(\alpha, E, \eta_s) = \left[ \frac{2}{\alpha \eta_s \sqrt{\pi}} \right] \exp(-u^2) \left\{ \sum_{n=1}^{\infty} \left( \frac{\exp(-p^2/4)}{p} \right) \sinh(npu) \right\} \text{ and} \]
\[ D_{21}(\alpha, E, \eta_s) = \left[ \frac{\sqrt{\pi}}{\alpha \eta_s} \exp(-u^2) \right] \]

The (731) has both real and imaginary parts and therefore, \( I(\alpha) \) is complex, which can also be prove by using the method of analytic continuation.

The integral \( I_3(q_g) \) in (718) can be written as
\[ I_3(q_g) = \left( \frac{\alpha_1}{q_1} \right) I_5(5) + \left( \frac{\alpha_1 + \eta E - \alpha_1}{q_1} \right) I(q) + \frac{1}{q_1} \left( \frac{1 - \alpha^2}{q_1} \right) I(1) - \frac{1}{q_1} \left( \frac{1 - \alpha}{q_1} \right) I(\bar{q}) \]  \( (732) \)

where
\[ I_5(\varepsilon) = \int (E - V)^2 F(V) dV \]  \( (733) \)

From (733) one can write
\[ I_5(\varepsilon) = \int E^2 \exp \left( -\frac{V^2}{\eta_s^2} \right) dV - 2E \int \exp \left( -\frac{V^2}{\eta_s^2} \right) dV + \int \exp \left( -\frac{V^2}{\eta_s^2} \right) dV \]

The evaluations of the component integrals lead us to write
\[ I_5(\varepsilon) = \eta_s \frac{E}{2\sqrt{\pi}} \exp \left( -\frac{E^2}{\eta_s^2} \right) + \frac{1}{4} \left( \eta_s^2 + 2E^2 \right) \left[ 1 + \text{Erf} \left( \frac{E}{\eta_s} \right) \right] = \theta_0(E, \eta_s) \]  \( (734) \)

Thus combining the aforementioned equations, \( I_3(q_g) \) can be expressed as
\[ I_3(q_g) = A_{21}(E, \eta_s) + iB_{21}(E, \eta_s) \]  \( (735) \)

where, \( A_{21}(E, \eta) \) is defined as:
\[ A_{21}(E, \eta) = \left[ \frac{\alpha b_{||}}{c_{||}} \left[ \frac{\eta_s E}{2\sqrt{\pi}} \exp \left( -\frac{E^2}{\eta_s^2} \right) + \frac{1}{4} \left( \eta_s^2 + 2E^2 \right) \left[ 1 + \text{Erf} \left( \frac{E}{\eta_s} \right) \right] \right] \right] + \left[ \frac{\alpha c_{g} + b_{||} c_{||} - \alpha b_{||}}{c_{||}^2} \right] \left\{ \frac{E}{2} \left[ 1 + \text{Erf} \left( E/\eta \right) \right] + \frac{\eta_s \exp \left( -\frac{E^2}{\eta_s^2} \right)}{2\sqrt{\pi}} \right\} \]

287
Therefore, the combination of all the appropriate equations together with the algebraic manipulations lead to the dispersion relation of the conduction electrons of heavily doped tetragonal materials forming Gaussian band tails as

\[
\frac{\hbar^2 k_\perp^2}{2m_2 T_{21}(E, \eta_g)} + \frac{\hbar^2 k_\parallel^2}{2m_2 T_{22}(E, \eta_g)} = 1
\]  

(736)

where, \( T_{21}(E, \eta_g) \) and \( T_{22}(E, \eta_g) \) have both real and complex parts and they are given by

\[
T_{21}(E, \eta_g) \equiv [T_{27}(E, \eta_g) + iT_{28}(E, \eta_g)], \quad T_{27}(E, \eta_g) \equiv \left[ \frac{T_{23}(E, \eta_g)}{T_5(E, \eta_g)} \right],
\]

\[
T_{23}(E, \eta_g) \equiv \left[ A_{21}(E, \eta_g) + \frac{\alpha b_\parallel}{c_\parallel} \left[ \delta g(E, \eta_g) + \frac{1}{9} (\Delta_\perp^2 - \Delta_\parallel^2) \left[ 1 + Erf \left( E/\eta_g \right) \right] \right] \right.
\]

\[
- \left\{ \frac{2}{9} \left( \frac{\alpha b_\parallel}{c_\parallel} \right) (\Delta_\parallel^2 - \Delta_\perp^2) G_{21}(c_\parallel, E, \eta_g) \right\} \right],
\]

\[
G_{21}(E, \eta_g) \equiv \frac{2}{c_\parallel \eta_g \sqrt{\pi}} \exp(-u_\perp^2) \sum_{p=1}^{\infty} \left\{ \frac{\exp(-p^2/4)}{p} \sinh(p u_\parallel) \right\},
\]

\[
T_5(E, \eta_g) = \frac{1}{2} \left[ 1 + Erf \left( E/\eta_g \right) \right],
\]

\[
T_{28}(E, \eta_g) = \left[ \frac{T_{24}(E, \eta_g)}{T_5(E, \eta_g)} \right],
\]

\[
T_{24}(E, \eta_g) \equiv \left[ B_{21}(E, \eta_g) + \frac{2}{9} \frac{\alpha b_\parallel}{c_\parallel} (\Delta_\parallel^2 - \Delta_\perp^2) H_{21}(c_\parallel, E, \eta_g) \right],
\]
From (736), it appears that the energy spectrum in heavily doped tetragonal semiconductors is complex. The complex nature of the electron dispersion law in heavily doped semiconductors occurs from the existence of the essential poles in the corresponding undoped electron energy spectrum. It may be noted that the complex band structures have already been studied for bulk semiconductors and superlattices without heavy doping and bears no relationship with the complex electron dispersion law as indicated by (736). The physical picture behind the formulation of the complex energy spectrum in heavily doped tetragonal semiconductors is the interaction of the impurity atoms in the tails with the splitting constants of the valance bands. More is the interaction causes more prominence of the complex part than the other case. When there is no heavy doping, \( \eta_g \to 0 \), and there is no interaction of the impurity atoms in the tails with the spin orbit constants. As a result, there exist no complex energy spectrum and (736) gets converted into (724) when \( \eta_g \to 0 \). Besides, the complex spectra are not related to same evanescent modes in the band tails and the conduction bands. The transverse and the longitudinal EEMs at the Fermi energy \( (E_{F_1}) \) of heavily doped tetragonal materials can be expressed respectively as

\[
H_{21}(c_{11}, E, \eta_g) = \left[ \sqrt{\frac{\pi}{\eta_g c_{11}}} \exp\left(-u^2\right) \right],
\]

\[
T_{22}(E, \eta_g) = \frac{T_{23}(E, \eta_g)T_{25}(E, \eta_g) - T_{24}(E, \eta_g)T_{26}(E, \eta_g)}{\left[ T_{25}(E, \eta_g)^2 + T_{26}(E, \eta_g)^2 \right]},
\]

\[
T_{25}(E, \eta_g) = \left[ \frac{\delta}{2} \frac{\Delta_0^2 - \Delta_\perp^2}{6\Delta_\parallel} \right] \alpha C_{21}(\alpha, E, \eta_g) + \left( \frac{\delta}{2} \frac{\Delta_0^2 - \Delta_\perp^2}{6\Delta_\parallel} \right) C_{21}(\alpha, E, \eta_g)
\]

\[
T_{26}(E, \eta_g) = \frac{T_{23}(E, \eta_g)T_{25}(E, \eta_g) + T_{24}(E, \eta_g)T_{26}(E, \eta_g)}{\left[ T_{25}(E, \eta_g)^2 + T_{26}(E, \eta_g)^2 \right]}
\]

and

\[
T_{30}(E, \eta_g) = \frac{T_{34}(E, \eta_g)T_{25}(E, \eta_g) + T_{32}(E, \eta_g)T_{26}(E, \eta_g)}{\left[ T_{25}(E, \eta_g)^2 + T_{26}(E, \eta_g)^2 \right]}
\]
\[ m^*_x(E_{\text{F}}, \eta_g) = m^*_x \{ T_{29}(E, \eta_g) \} \bigg|_{E=E_{\text{F}}} \]

and

\[ m^*_y(E_{\text{F}}, \eta_g) = m^*_y \{ T_{27}(E, \eta_g) \} \bigg|_{E=E_{\text{F}}} \]

In the absence of band tailing effects \( \eta_g \rightarrow 0 \) and we get

\[ m^*_x(E_{\text{F}}, 0) = \frac{\hbar^2}{2} \left[ \frac{\psi_2(E)\{\psi_1(E)\}' - \psi_1(E)\{\psi_2(E)\}'}{\psi_2(E)^2} \right] \bigg|_{E=E_{\text{F}}} \]

and

\[ m^*_y(E_{\text{F}}, 0) = \frac{\hbar^2}{2} \left[ \frac{\psi_3(E)\{\psi_1(E)\}' - \psi_1(E)\{\psi_3(E)\}'}{\psi_3(E)^2} \right] \bigg|_{E=E_{\text{F}}} \]

Comparing the aforementioned equations, one can infer that the effective masses exist in the forbidden zone, which is impossible without the effect of band tailing. For undoped semiconductors, the effective mass in the band gap is infinity.

The density-of-states function is given by

\[ N_{\text{HD}}(E, \eta_g) = \frac{2g_m m^*_x \sqrt{2m^*_x}}{3\pi^2 \hbar^3} R_1(E, \eta_g) \cos \left[ \psi_1(E, \eta_g) \right] \]

where,

\[ R_1(E, \eta_g) = \left[ \left| T_{29}(E, \eta_g) \right| \sqrt{x(E, \eta_g)} + \left| T_{29}(E, \eta_g) \right| \sqrt{y(E, \eta_g)} \right]^2 \]

\[ + \left[ \left| T_{29}(E, \eta_g) \right| \sqrt{x(E, \eta_g)} + \left| T_{29}(E, \eta_g) \right| \sqrt{y(E, \eta_g)} \right]^2 \]
The oscillatory nature of the DOS for heavily doped tetragonal materials is apparent from (741). For, $\psi_{11}(E, \eta_e) \geq \pi$, the cosine function becomes negative leading to the negative values of the DOS. The electrons cannot exist for the negative values of the DOS and therefore, this reason is forbidden for electrons, which indicates that in the band tail, there appears a new forbidden zone in addition to the normal band gap of the semiconductor.

The use of (741) the electron concentration at low temperatures can be expressed as

$$n_0 = \frac{2g_s m^*}{3\pi^2 h^3} \left[ I_{11} \left( E_{K_1}, \eta_e \right) \right]$$

(742)

where, $I_{11} \left( E_{K_1}, \eta_e \right) \equiv \left[ T_{29} \left( E_{K_1}, \eta_e \right) \left[ x \left( E_{K_1}, \eta_e \right) - T_{30} \left( E_{K_1}, \eta_e \right) \right] \right].$

For heavily doped tetragonal semiconductors, $E_{md}$ is the smallest negative root of the equation:

$$\left[ T_{27} \left( E_{md}, \eta_e \right) T_{29} \left( E_{md}, \eta_e \right) - T_{28} \left( E_{md}, \eta_e \right) T_{30} \left( E_{md}, \eta_e \right) \right] = 0$$

(743)

b) Study of the EEM in heavily doped III-V, ternary and quaternary materials forming Gaussian band tails
(a) Under the conditions, $\delta = 0$, $m^* = m^* = m^*$ and $\Delta_0 = \Delta_0 = \Delta_0$, the electron dispersion law in this case assumes the form

$$\frac{\hbar^2 k^2}{2m^*} = T_{31} (E, \eta_\delta) + i T_{32} (E, \eta_\delta)$$

(744)

where,

$$T_{31} (E, \eta_\delta) = \left( \frac{2}{1 + Ef (E/\eta_\delta)} \right)^{\alpha \beta} \frac{\alpha \beta (E, \eta_\delta) + \alpha \beta \eta_\delta (E, \eta_\delta) + \left[ \frac{\alpha \beta - \alpha \beta}{c^2} \right] \eta_\delta (E, \eta_\delta) + \frac{1}{c} \left[ \frac{1 - \frac{\beta}{c}}{c} \right] \frac{1}{2} \left[ 1 + Ef \left( \frac{E}{\eta_\delta} \right) \right]$$

$$- \frac{1}{c} \left[ 1 - \frac{\beta}{c} \right] \left[ 1 - \frac{\beta}{c} \right] \frac{2}{c \eta_\delta} \exp \left[ - \frac{\sqrt{\pi}}{c \eta_\delta} \right] \left[ \sum_{p=1}^{\infty} \frac{\exp \left( \frac{p^2}{4} \right)}{p} \sinh \left( p u_2 \right) \right], \quad \epsilon = \left( \frac{1}{E^* + \frac{2}{3} \Delta} \right),$$

$$u_2 = \frac{1 + \epsilon E}{c \eta_\delta} \quad \text{and} \quad T_{32} (E, \eta_\delta) = \left( \frac{2}{1 + Ef (E/\eta_\delta)} \right)^{\alpha \beta} \frac{1}{c} \left[ 1 - \frac{\beta}{c} \right] \frac{1}{c \eta_\delta} \exp \left( - u_2^2 \right).$$

Thus, the complex energy spectrum occurs due to the term $T_{32} (E, \eta_\delta)$ and this imaginary band is quite different from the forbidden energy band.

The EEM at the Fermi level is given by

$$m^* \left( E_F, \eta_\delta \right) = m^* \left[ T_{31} (E, \eta_\delta) \right] \bigg|_{E = E_F}$$

(745)

Thus, the EEM in heavily doped III-V, ternary and quaternary materials exists in the band gap, which is the new attribute of the theory of band tailing. In the absence of band tailing, $\eta_\delta \to 0$ and the EEM assumes the form

$$m^* (E_F) = m^* \left[ I \left( E \right) \right] \bigg|_{E = E_F}$$

(746)

The density-of-states function in this case assumes the form

$$N_{nd} (E, \eta_\delta) = \frac{g_\nu}{3 \pi^2} \left( \frac{2 m^*}{\hbar^2} \right)^{3/2} R_{21} (E, \eta_\delta) \cos \left[ \varphi_{21} (E, \eta_\delta) \right]$$

(747)
where, $R_{21}(E, \eta_e) = \left[ \begin{array}{c} \{\alpha_1(E, \eta_e)\} \\ \{\beta_1(E, \eta_e)\} \end{array} \right] + \left[ \begin{array}{c} \{\alpha_1(E, \eta_e)\} \\ \{\beta_1(E, \eta_e)\} \end{array} \right]^{\prime}$.

$\alpha_1(E, \eta_e) = \frac{1}{2} \left[ T_{33}(E, \eta_e) + \sqrt{T_{33}(E, \eta_e)^2 + T_{34}(E, \eta_e)^2} \right]$, 

$T_{33}(E, \eta_e) = \left[ T_{33}(E, \eta_e) \right]^3 - 3T_{33}(E, \eta_e)T_{34}(E, \eta_e)^2$, 

$T_{34}(E, \eta_e) = \left[ 3T_{33}(E, \eta_e)T_{33}(E, \eta_e)^2 - T_{33}(E, \eta_e)^3 \right]$, 

$\beta_1(E, \eta_e) = \frac{1}{2} \left[ \sqrt{T_{33}(E, \eta_e)^2 + T_{34}(E, \eta_e)^2} - T_{33}(E, \eta_e) \right]$ and 

$\vartheta_{21}(E, \eta_e) = \tan^{-1} \left[ \frac{\{\alpha_1(E, \eta_e)\}'}{\{\alpha_1(E, \eta_e)\}'} \sqrt{\frac{\{\beta_1(E, \eta_e)\}'}{\{\beta_1(E, \eta_e)\}'}} \right]$. 

Thus, the oscillatory density-of-states function becomes negative for $\vartheta_{21}(E, \eta_e) \geq \pi$ and a new forbidden zone will appear in addition to the normal band gap.

The electron concentration in the zone of low temperatures can be written as

$$n_0 = \frac{g_e}{3\pi^2} \left( \frac{2m^*}{\hbar^2} \right)^{3/2} \frac{1}{\sqrt{2}} \left[ T_{33}(E, \eta_e) + \sqrt{T_{33}(E, \eta_e)^2 + T_{34}(E, \eta_e)^2} \right]$$

(748)

In this case, $E_{sd}$ is given by

$$T_{31}(E_{sd}, \eta_e) = 0$$

(749)

(b) The dispersion relation in heavily doped III-V, ternary and quaternary materials whose undoped energy spectrum obeys the two band model of Kane is given by

$$\frac{\hbar^2 k^2}{2m^*} = \gamma_2(E, \eta_e)$$

(750)
where, 
\[ \gamma_2(E, \eta_s) = \left[ \frac{2}{1 + \text{Erf}(E/\eta_s)} \right] \left[ \gamma_0(E, \eta_s) + \alpha \theta_0(E, \eta_s) \right]. \]

Since, the original two band Kane model is an all zero and no pole function, therefore, the heavily doped counterpart will be totally real and the complex band vanishes.

The EEM in this case can be written as

\[ m^* (E_{ks}, \eta_s) = \left. m^* \left\{ \gamma_2(E, \eta_s) \right\} \right|_{E=E_{ks}} \] \hspace{1cm} (751)

Thus, one again observes that the EEM in this case exists in the band gap.

In the absence of band tailing, \( \eta_s \rightarrow 0 \) and the EEM assumes the form

\[ m^* (E_F) = \left. m^* \left\{ 1 + 2\alpha E \right\} \right|_{E=E_F} \] \hspace{1cm} (752)

The density-of-states function in this case can be written as

\[ N_{HD}(E, \eta_s) = \frac{g_v}{2\pi^2 \hbar^2} \left\{ \frac{2m^*}{\hbar^2} \right\}^{3/2} \sqrt{\gamma_2(E, \eta_s)} \left\{ \gamma_2(E, \eta_s) \right\} \] \hspace{1cm} (753)

Since, the original two band Kane model is an all zero and no pole function, therefore, the heavily doped counterpart will be totally real and the complex band vanishes.

The electron concentration at low temperatures is given by

\[ n_0 = \frac{g_v}{3\pi^2 \hbar^2} \left\{ \frac{2m^*}{\hbar^2} \right\}^{3/2} \left\{ \gamma_2(E_{ks}, \eta_s) \right\}^{3/2} \] \hspace{1cm} (754)

In this case, \( \bar{E}_{sd} \) is given by

\[ \gamma_2(\bar{E}_{sd}, \eta_s) = 0 \] \hspace{1cm} (755)

(c) The dispersion relation in heavily doped semiconductors whose unperturbed conduction electrons obeys parabolic energy bands is given by

\[ \frac{\hbar^2 k^2}{2m^*} = \gamma_3(E, \eta_s) \] \hspace{1cm} (756)
where, $\gamma_3(E, \eta_g) \equiv \left[ \frac{2}{1 + \text{erf} \left( \frac{E}{\eta_g} \right)} \right] \gamma_6(E, \eta_g).

Since, the original parabolic energy band is no pole function, therefore, the heavily doped counterpart will be totally real, which is also apparent form the expression (756).

The EEM in this case can be written as

$$m^*(E_{k_x}, \eta_g) = m^* \left\{ \gamma_3(E, \eta_g) \right\}_{E=E_{k_x}}$$ (757)

In the absence of band tailing, $\eta_g \to 0$ and the EEM assumes the form

$$m^*(E_g) = m^*$$ (758)

It is well-known that the EEM in undoped parabolic energy bands is a constant quantity in general excluding cross-field configuration. But, the same mass in the corresponding heavily doped bulk counterpart is complicated functions of Fermi energy and the impurity potential together with the fact that the EEM also exists in the band gap.

The density-of-states function in this case can be written as

$$N_{HO} (E, \eta_g) = \frac{g_v}{2\pi^2} \left( \frac{2m^*}{h^2} \right)^{3/2} \sqrt{\gamma_3 (E, \eta_g)} \left\{ \gamma_3(E, \eta_g) \right\}$$ (759)

Since, the original parabolic energy band model is a no pole function, therefore, the heavily doped counterpart will be totally real and the complex band vanishes.

The electron concentration at low temperatures is given by

$$n_0 = \frac{g_v}{3\pi^2} \left( \frac{2m^*}{h^2} \right)^{3/2} \left\{ \gamma_3(E_{k_x}, \eta_g) \right\}^{3/2}$$ (760)

In this case, $\bar{E}_{md}$ is given by

$$\gamma_3(\bar{E}_{md}, \eta_g) = 0$$ (761)

c) Study of the EEM in heavily doped II-VI materials forming Gaussian band tails
Using (40) and (714), the dispersion relation of the carriers in heavily doped II-VI materials in the presence of Gaussian band tails can be expressed as

$$\gamma_3(E, \eta_g) = a'_0 k_x^2 + b'_0 k_z^2 + \lambda k_z$$  

(762)

Thus, the energy spectrum in this case is real since the corresponding undoped case as given by (40) is a no pole function.

The transverse and the longitudinal EEMs masses are respectively given by

$$m^*_x(E_F, \eta_g) = m^*_l \left\{ \gamma_3(E, \eta_g) \right\} \left[ 1 - \left( \frac{\lambda_0}{\sqrt{(\lambda_0)^2 + 4a'_0 \gamma_3(E, \eta_g)}} \right) \right]_{E=E_F}$$  

(763)

and

$$m^*_y(E_F, \eta_g) = m^*_l \left\{ \gamma_3(E, \eta_g) \right\} \left[ \right]_{E=E_F}$$  

(764)

In the absence of band tailing effects $\eta_g \rightarrow 0$, we get

$$m^*_x(E_F) = m^*_l \left\{ \right\} \left[ \right]_{E=E_F}$$  

(765)

and

$$m^*_y(E_F) = m^*_l$$  

(766)

Thus, the in heavily doped II-VI materials, both the transverse and the longitudinal EEM exist in the band gap.

The volume in k-space can be enclosed by the (762) can be expressed as

$$V(E, \eta_g) = \frac{4 \pi}{3 \lambda_0 \sqrt{a'_0}} \left\{ \chi(E, \eta_g) \right\}^{\frac{3}{2}} + \frac{3 \lambda_0}{8} \left( \frac{\chi(E, \eta_g)}{a'} \right) \left( \frac{3 \lambda_0}{4a'_0} \chi(E, \eta_g) \right) \left( \frac{a'_0}{4} \right) \sin^{-1} \left( \frac{\chi(E, \eta_g)}{a'_0} \right) + \left( \frac{a'_0}{4} \right) \right]$$  

(767)

Using (767), the density-of-states function in this case can be written as
Therefore, the electron concentration in the zone of low temperatures can be expressed as

\[
\rho_b = \frac{\varepsilon}{3\pi^2 k_B^2} \left[ \chi(E_b, \eta_b) \right]^{3/2} \left[ \frac{3}{8} \chi(E_b, \eta_b) \right]^{3/2} \left[ \frac{1}{2} \sqrt{\frac{E_b}{\eta_b}} \right] \left[ \frac{1}{4\epsilon_0} \right] \sin^4 \left[ \frac{\chi(E_b, \eta_b)}{4\epsilon_0} \right] \]  \tag{769}
\]

In this case, \( E_b \) is given by

\[
\{ \chi(E_b, \eta_b) \} = 0 \]  \tag{770}
\]

d) Study of the EEM in heavily doped IV-VI materials forming Gaussian band tails

From (63), we can write

\[
\frac{\alpha h^4 k_4^4}{4 m^4 m_i^4} + \hbar^2 k_4^2 \left[ \frac{1}{2 m_i^*} - \frac{1}{2 m_i} \right] + \varepsilon E \left[ \frac{1}{2 m_i^*} - \frac{1}{2 m_i} \right] + \frac{\alpha h^4 k_4^4}{4 m_i^4 m_i^4} \\
+ \left[ \frac{\hbar^2 k_4^2}{2 m_i^*} \right] + \alpha E \frac{h^2 k_4^2}{2} \left[ \frac{1}{m_i^*} - \frac{1}{m_i} \right] + \frac{\alpha h^4 k_4^4}{4 m_i^4 m_i^4} \right] = 0  \tag{771}
\]

Using (714) and (771), the dispersion relation of the conduction electrons in heavily doped IV-VI materials can be expressed as

\[
\frac{\alpha h^4 k_4^4}{4 m_i^4 m_i^4} Z_0(E, \eta_b) + \hbar^2 k_4^2 \left[ \lambda_4(E, \eta_b) k_4^2 + \lambda_2(E, \eta_b) \right] + \lambda_4(E, \eta_b) k_4^4 - 4 \lambda_4(E, \eta_b) = 0  \tag{772}
\]
where, \( Z_0(E, \eta_g) = \frac{1}{2} \left[ 1 + \text{Erf} \left( \frac{E}{\eta_g} \right) \right] \), \( \lambda_{\eta_0}(E, \eta_g) = \frac{\alpha}{4m_i^* m_i^*} Z_0(E, \eta_g) \),

\[ \lambda_{\eta_1}(E, \eta_g) = \left[ \frac{\alpha \hbar^2}{4m_i^* m_i^*} Z_0(E, \eta_g) + \frac{\alpha \hbar^2}{4m_i^* m_i^*} Z_0(E, \eta_g) \right], \]

\[ \lambda_{\eta_2}(E, \eta_g) = \left[ \left( \frac{1}{2m_i^*} - \frac{1}{2m_i^*} \right) Z_0(E, \eta_g) + \alpha \left( \frac{1}{2m_i^*} - \frac{1}{2m_i^*} \right) \gamma_0(E, \eta_g) \right]. \]

\[ \lambda_{\eta_3}(E, \eta_g) = \left[ \left( \frac{\hbar^2}{2m_i^*} + \frac{\hbar^2}{2m_i^*} \right) Z_0(E, \eta_g) + \frac{\alpha \hbar^2}{2} \left( \frac{1}{m_i^*} - \frac{1}{2m_i^*} \right) \gamma_0(E, \eta_g) \right]. \]

\[ \lambda_{\eta_4}(E, \eta_g) = \frac{\alpha \hbar^4 Z_0(E, \eta_g)}{4m_i^* m_i^*} \text{ and } \lambda_{\eta_5}(E, \eta_g) = \left[ \gamma_0(E, \eta_g) + \alpha \theta_0(E, \eta_g) \right]. \]

Thus, the energy spectrum in this case is real since the corresponding undoped material as given by (772) is a pole-less function. The respective transverse and the longitudinal EEM in this case can be written as

\[ n^i_1(E_g, \eta_g) \left[ Z_0(E, \eta_g) \right] \left[ \lambda_{\eta_2}(E, \eta_g) \right] = \left[ \frac{\lambda_{\eta_2}(E, \eta_g)}{2\lambda_{\eta_0}(E, \eta_g)} \right] \left[ Z_0(E, \eta_g) \right] \left[ \lambda_{\eta_3}(E, \eta_g) \right] \left[ \lambda_{\eta_4}(E, \eta_g) \right] \left[ \lambda_{\eta_5}(E, \eta_g) \right] \]

(773)

where, \( \lambda_{\eta_0}(E, \eta_g) = \left[ 4 \lambda_{\eta_0}(E, \eta_g) \lambda_{\eta_5}(E, \eta_g) \right] \)

and \( m_i^* (E_g, \eta_g) = \frac{\hbar^2}{4} \left[ \lambda_{\eta_0}(E, \eta_g) \right] \left[ \lambda_{\eta_4}(E, \eta_g) \right] \left[ \lambda_{\eta_5}(E, \eta_g) \right] \left[ \lambda_{\eta_5}(E, \eta_g) \right] \left[ \lambda_{\eta_5}(E, \eta_g) \right] \]

(774)

in which, \( \lambda_{\eta_4}(E, \eta_g) = \frac{\lambda_{\eta_4}(E, \eta_g)}{\lambda_{\eta_4}(E, \eta_g)} \) and \( \lambda_{\eta_5}(E, \eta_g) = \frac{\lambda_{\eta_5}(E, \eta_g)}{\lambda_{\eta_4}(E, \eta_g)} \).

Thus, we can see that the both the EEMs in this case exist in the band gap.

In the absence of band tailing effects \( \eta_g \to 0 \), we get
\[ m^*_1(E_F) = \frac{\hbar^2}{2} \left[-\left\{ \alpha_{11}(E) \right\}' + \frac{\alpha_{211} \{ T_{311}(E) \}'}{2\sqrt{T_{311}(E)}} \right]_{E=E_F} \]  

where, \( \alpha_{11}(E) \equiv \frac{2m^*_m^*}{\alpha E^2} \), \( \alpha_{211}(E) \equiv \left[ \frac{1}{2m^*_m^*} \frac{2m^*_m^*}{2m^*_m^*} + \frac{1}{2m^*_m^*} \right] \), \( \alpha_{311} \equiv \frac{2m^*_m^*}{\alpha E^2} \). 

\[
\begin{align*}
(\omega_{11}) & = \left[ \frac{\alpha^2}{16} \left( \frac{1}{m^*_m^*} + \frac{1}{m^*_m^*} \right)^2 - \frac{\alpha^2}{4m^*_m^* m^*_m^*} \right]^{1/2}, \\
T_{311}(E) & \equiv \frac{\omega_{311}(E)}{(\omega_{11})^2} \\
\omega_{311}(E) & \equiv \frac{\alpha E (1 + \alpha E)}{m^*_m^*} + \left( \frac{\alpha E}{2m^*_m^*} \right)^2 
\end{align*}
\]

and

\[ m^*_1(E_F) = \left( \frac{m^*_m^*}{\alpha} \right) \left( \frac{\alpha}{2m^*_m^*} - \frac{\alpha}{2m^*_m^*} \right)^{1/2} \left[ \frac{1}{2m^*_m^*} + \frac{1}{m^*_m^*} \right] \left( \frac{1}{m^*_m^*} \right)^{1/2} \left[ \frac{1}{2m^*_m^*} + \frac{1}{m^*_m^*} \right]^2 \left( \frac{1}{m^*_m^*} \right) \right]_{E=E_F} \]  

The volume in \( k \)-space can be enclosed by the \( 772 \) can be written through the integral as

\[ V(E, \eta) = 2\pi \int_{E_F}^{E\eta} \left[ -\lambda_0(E, \eta) \right] d\xi + \lambda_0(E, \eta) \left[ \sqrt{\lambda_0(E, \eta)} \right]^2 \]  

where, \( \lambda_0(E, \eta) \equiv \frac{\lambda_{11}(E, \eta)}{2n^2 Z_0(E, \eta)} \).

\[ \lambda_{79}(E, \eta) = \frac{\lambda_{71}(E, \eta)}{2n^2 Z_0(E, \eta)}, \quad \lambda_{90}(E, \eta) = \frac{\lambda_{91}(E, \eta)}{2n^2 Z_0(E, \eta)}, \quad \lambda_{80}(E, \eta) = \frac{\lambda_{81}(E, \eta)}{4n^2 Z_0(E, \eta)}, \quad \lambda_{86}(E, \eta) = \frac{\lambda_{86}(E, \eta)}{4n^2 Z_0(E, \eta)} \]

\[ \lambda_{82}(E, \eta) = \frac{\lambda_{97}(E, \eta)}{9n^4 Z_0(E, \eta)} \]
\( \lambda_{71} (E, \eta_g) = \left[ 2 \lambda_{71} (E, \eta_g) \lambda_{72} (E, \eta_g) - 4 \lambda_{70} (E, \eta_g) \lambda_{73} (E, \eta_g) - 4 \lambda_{70} (E, \eta_g) \lambda_{76} (E, \eta_g) \right] . \)

\( \lambda_{83} (E, \eta_g) = \frac{\lambda_{78} (E, \eta_g)}{9 \hbar^4 \left[ Z_0 (E, \eta_g) \right]^2} \) and
\( \lambda_{78} (E, \eta_g) = \left[ 4 \lambda_{70} (E, \eta_g) \lambda_{75} (E, \eta_g) \right] . \)

Thus,
\[ V (E, \eta_g) = \left[ \lambda_{71} (E, \eta_g) \right] \int_0^{E_{\eta_g}} \left[ \sqrt{k_g^2 + \lambda_{78} (E, \eta_g) k_g^2 + \lambda_{70} (E, \eta_g) - \lambda_{76} (E, \eta_g)} \right] dk_g . \tag{778} \]

where, \( \lambda_{72} (E, \eta_g) = 2 \pi \sqrt{\lambda_{71} (E, \eta_g)} \), \( \lambda_{83} (E, \eta_g) = \frac{\lambda_{72} (E, \eta_g)}{\lambda_{71} (E, \eta_g)} \)

\( \lambda_{89} (E, \eta_g) = \frac{\lambda_{83} (E, \eta_g)}{\lambda_{81} (E, \eta_g)} \)

and \( \lambda_{90} (E, \eta_g) = 2 \pi \left[ \frac{\lambda_{79} (E, \eta_g) \lambda_{86} (E, \eta_g)}{3} + \lambda_{80} (E, \eta_g) \lambda_{89} (E, \eta_g) \right] . \)

The (778) can be written as
\[ V (E, \eta_g) = \left[ \lambda_{71} (E, \eta_g) \lambda_{86} (E, \eta_g) - \lambda_{80} (E, \eta_g) \right] \tag{779} \]

in which,
\( \lambda_{93} (E, \eta_g) = \left[ \frac{\lambda_{91} (E, \eta_g)}{3} \right] \left[ E_i \left[ \lambda_{93} (E, \eta_g), \lambda_{94} (E, \eta_g) \right] \right] \)

\[ \left[ \lambda_{91} (E, \eta_g) \right]^3 + \left[ \lambda_{92} (E, \eta_g) \right]^3 + 2 \left[ \lambda_{93} (E, \eta_g) \right]^2 \left[ \lambda_{93} (E, \eta_g), \lambda_{94} (E, \eta_g) \right] \]

\[ + \left[ \frac{\lambda_{96} (E, \eta_g)}{3} \right] \left[ \left[ \lambda_{96} (E, \eta_g) \right]^2 + \left[ \lambda_{91} (E, \eta_g) \right]^2 + 2 \left[ \lambda_{92} (E, \eta_g) \right]^2 \right] \]

\[ \left[ \left[ \lambda_{91} (E, \eta_g) \right]^2 + \left[ \lambda_{96} (E, \eta_g) \right]^2 \right]^{1/2} \rightarrow \left[ \lambda_{92} (E, \eta_g) \right]^2 + \left[ \lambda_{96} (E, \eta_g) \right]^2 \right]^{1/2} \],

\( \left\{ \lambda_{91} (E, \eta_g) \right\}^2 = \frac{1}{2} \left[ \sqrt{\left[ \lambda_{93} (E, \eta_g) \right]^2 - 4 \lambda_{99} (E, \eta_g) + \lambda_{98} (E, \eta_g)} \right] . \)
\[ E, [\lambda_9 (E, \eta_t), \lambda_{10} (E, \eta_t)] \] is given by,

\[ E, [\lambda_9 (E, \eta_t), \lambda_{10} (E, \eta_t)] = \int_0^{\theta_0 (E, \eta_t)} \left[ 1 - \{\lambda_{10} (E, \eta_t)^2 \sin^2 \xi \}^{1/2} \right] d\xi, \xi \text{ is the variable of integration in this case}, \lambda_{10} (E, \eta_t) = \tan^{-1} \left( \frac{\lambda_{10} (E, \eta_t)}{\lambda_{10} (E, \eta_t)} \right) \]

\[ \left[ \lambda_{10} (E, \eta_t) \right]^2 = \frac{1}{2} \lambda_{10} (E, \eta_t) - \sqrt{\left( \lambda_{10} (E, \eta_t) \right)^2 - 4 \lambda_{10} (E, \eta_t)} \]

\[ \lambda_{10} (E, \eta_t) = \frac{\sqrt{\left[ \lambda_{10} (E, \eta_t) \right]^2 - \left[ \lambda_{10} (E, \eta_t) \right]^2}} {\lambda_{10} (E, \eta_t)} \]

\[ F_i, [\lambda_9 (E, \eta_t), \lambda_{10} (E, \eta_t)] \]

Using (779), the density-of-states function is given by

\[ N_{40} (E, \eta_t) = \frac{g_s}{4 \pi^2} \left[ \left\{ \lambda_{10} (E, \eta_t) \right\} \lambda_{10} (E, \eta_t) + \left\{ \lambda_{10} (E, \eta_t) \right\} \lambda_{10} (E, \eta_t) - \{\lambda_{10} (E, \eta_t) \} \right] \] (780)

Therefore the electron concentration at low temperature can be expressed as

\[ n_0 = \frac{g_s}{4 \pi^2} \left[ \{\lambda_{10} (E, \eta_t) \} \lambda_{10} (E, \eta_t) - \{\lambda_{10} (E, \eta_t) \} \right] \] (781)

In this case, \( \bar{E}_n \) is given by \( \lambda_{10} (\bar{E}_n, \eta_t) = 0 \) (782)

e) Study of the EEM in heavily doped stressed materials forming Gaussian band tails

The use of (77) leads us to write

\[ (E - \alpha_1)k_x^2 + (E - \alpha_2)k_y^2 + (E - \alpha_3)k_z^2 = t_1, t_2, t_3, t_4 \] (783)

where,

\[ \alpha_1 = \left[ E_z - C_e (\bar{E}_l - C_e) + \frac{3}{2} B_s e^2 \frac{B_n}{2} e + \left( \frac{\sqrt{3}}{2} \right) e + \bar{E}_n \right], \]

\[ \alpha_2 = \left[ E_z - C_e (\bar{E}_l - C_e) + \frac{3}{2} B_s e^2 \frac{B_n}{2} e - \left( \frac{\sqrt{3}}{2} \right) e + \bar{E}_n \right], \]

\[ \alpha_3 = \left[ E_z - C_e (\bar{E}_l - C_e) + \frac{3}{2} B_s e^2 \frac{B_n}{2} e - \left( \frac{\sqrt{3}}{2} \right) e + \bar{E}_n \right], \]

\[ \alpha_4 = \left[ E_z - C_e (\bar{E}_l - C_e) + \frac{3}{2} B_s e^2 \frac{B_n}{2} e - \left( \frac{\sqrt{3}}{2} \right) e + \bar{E}_n \right], \]

\[ t_1 = \left( \frac{3}{2} B_s \right), t_2 = \left( \frac{1}{2} B_s \right) \left[ 6 (E_z - C_e) + 3 C_e \right], \]
Using (714) and (783), we can write,

\[ I(4)k^2 - T_{17}I(1)k_x^2 - T_{27}I(1)k_y^2 - T_{37}k_z^2 I(1) = \left[ q_{g1}I(6) - R_{g1}I(5) + V_{g1}I(4) + \rho_{g1}I(1) \right] \]

(784)

where \( T_{17} = \alpha_1, T_{27} = \alpha_2, T_{37} = \alpha_3, t_1 = q_{g7}, t_2 = R_{g7}, t_3 = V_{g7}, t_4 = \rho_{g7} \) and

\[ I(6) = \int_{-\infty}^{\infty} (E - V)^3 F(V) dV \]

(785)

(785) can be written as

\[ I(6) = E^3 I(1) - 3E^2 I(7) + 3EI(8) - I(9) \]

(786)

In which,

\[ I(7) = \int_{-\infty}^{\infty} VF(V) dV \]

(787)

\[ I(8) = \int_{-\infty}^{\infty} V^2 F(V) dV \]

(788)

\[ I(9) = \int_{-\infty}^{\infty} V^3 F(V) dV \]

(789)

Using (787) together with simple algebraic manipulations, one obtains

\[ I(7) = \frac{-\eta_s^2}{2\sqrt{\pi}} \exp \left( \frac{-E^2}{\eta_s^2} \right) \]

(790)

\[ I(8) = \frac{\eta_s^2}{4} \left[ 1 + \text{Erf} \left( \frac{E}{\eta_s} \right) \right] \]

(791)

and

\[ I(9) = \frac{-\eta_s^2}{2\sqrt{\pi}} \exp \left( \frac{-E^2}{\eta_s^2} \right) \left[ 1 + \frac{E^2}{\eta_s^2} \right] \]

(792)

Thus, (786) can be written as

\[ I(6) = \left[ \frac{E}{2} \left[ 1 + \text{Erf} \left( \frac{E}{\eta_s} \right) \right] \left[ E^2 + \frac{3}{2} \eta_s^2 \right] + \frac{\eta_s}{2\sqrt{\pi}} \exp \left( \frac{-E^2}{\eta_s^2} \right) \left[ 4E^2 + \eta_s^2 \right] \right] \]

(793)

Thus, combining the appropriate equations, the dispersion relations of the conduction electrons in heavily doped stressed materials can be expressed as

\[ P_{11} \left( E, \eta_s \right) k_x^2 + Q_{11} \left( E, \eta_s \right) k_y^2 + S_{11} \left( E, \eta_s \right) k_z^2 = 1 \]

(794)
where, \( P_{1i}(E, \eta_g) = \left[ \frac{\gamma_0(E, \eta_g) - (T_{\nu}/2)[1 + \text{Erf}(E/\eta_g)]}{\Delta_4(E, \eta_g)} \right] \),

\[
\Delta_4(E, \eta_g) = \left[ q_{67} \left( \frac{E}{2} \left[ 1 + \text{Erf} \left( \frac{E}{\eta_g} \right) \right] \left[ E^2 + \frac{3}{2} \eta_g^2 \right] + \frac{E}{2 \sqrt{\pi}} \exp \left( -\frac{E^2}{2 \eta_g^2} \right) \right],
\]

\[
-\left[ R_{67} \gamma_0(E, \eta_g) + V_{67} \gamma_0(E, \eta_g) + \frac{P_{67}}{2} \left[ 1 + \text{Erf}(E/\eta_g) \right] \right],
\]

\[
Q_{1i}(E, \eta_g) = \left[ \frac{\gamma_0(E, \eta_g) - (T_{\nu}/2)[1 + \text{Erf}(E/\eta_g)]}{\Delta_4(E, \eta_g)} \right] \text{ and }
\]

\[
S_{1i}(E, \eta_g) = \left[ \frac{\gamma_0(E, \eta_g) - (T_{\nu}/2)[1 + \text{Erf}(E/\eta_g)]}{\Delta_4(E, \eta_g)} \right].
\]

Thus, the energy spectrum in this case is real since the corresponding undoped material as given by (794) is a pole-less function.

The EEMs along x, y and z directions in this case can be written as

\[
m_{x}(E_x, \eta_g) = \frac{\hbar^2}{2} \left[ \frac{\gamma_0(E_x, \eta_g) - (T_{\nu}/2)[1 + \text{Erf}(E_x/\eta_g)]}{\Delta_4(E_x, \eta_g)} \right]^2 \left[ (\Delta_4(E_x, \eta_g))' \left[ \gamma_0(E_x, \eta_g) - (T_{\nu}/2)[1 + \text{Erf}(E_x/\eta_g)] \right] \right]

- \Delta_4(E_x, \eta_g) \left[ \frac{1}{2} \left[ 1 + \text{Erf} \left( \frac{E_x}{\eta_g} \right) \right] - \frac{T_{\nu}}{\eta_g \sqrt{\pi}} \exp \left( -\frac{E_x^2}{2 \eta_g^2} \right) \right].
\]

(795)

\[
m_{y}(E_y, \eta_g) = \frac{\hbar^2}{2} \left[ \frac{\gamma_0(E_y, \eta_g) - (T_{\nu}/2)[1 + \text{Erf}(E_y/\eta_g)]}{\Delta_4(E_y, \eta_g)} \right]^2 \left[ (\Delta_4(E_y, \eta_g))' \left[ \gamma_0(E_y, \eta_g) - (T_{\nu}/2)[1 + \text{Erf}(E_y/\eta_g)] \right] \right]

- \Delta_4(E_y, \eta_g) \left[ \frac{1}{2} \left[ 1 + \text{Erf} \left( \frac{E_y}{\eta_g} \right) \right] - \frac{T_{\nu}}{\eta_g \sqrt{\pi}} \exp \left( -\frac{E_y^2}{2 \eta_g^2} \right) \right].
\]

(796)

and

\[
m_{z}(E_z, \eta_g) = \frac{\hbar^2}{2} \left[ \frac{\gamma_0(E_z, \eta_g) - (T_{\nu}/2)[1 + \text{Erf}(E_z/\eta_g)]}{\Delta_4(E_z, \eta_g)} \right]^2 \left[ (\Delta_4(E_z, \eta_g))' \left[ \gamma_0(E_z, \eta_g) - (T_{\nu}/2)[1 + \text{Erf}(E_z/\eta_g)] \right] \right]

- \Delta_4(E_z, \eta_g) \left[ \frac{1}{2} \left[ 1 + \text{Erf} \left( \frac{E_z}{\eta_g} \right) \right] - \frac{T_{\nu}}{\eta_g \sqrt{\pi}} \exp \left( -\frac{E_z^2}{2 \eta_g^2} \right) \right].
\]

(797)

Thus, we can see that the EEMs in this case exist within the band gap.
In the absence of band tailing effects \( \eta_s \to 0 \), we get

\[
m_\alpha^*(E_F) = \hbar^2 \bar{a}_0 (E_F) \left\{ \bar{a}_0 (E_F) \right\}'
\]

(798)

\[
m_\alpha^*(E_F) = \hbar^2 \bar{b}_0 (E_F) \left\{ \bar{b}_0 (E_F) \right\}'
\]

(799)

and

\[
m_\alpha^*(E_F) = \hbar^2 \bar{c}_0 (E_F) \left\{ \bar{c}_0 (E_F) \right\}'
\]

(800)

The density-of-states function in this case can be written as

\[
N_{\nu}(E, \eta_s) = \frac{g_e}{3\pi^2} \left[ \frac{3}{2} \left\{ \Delta_0 (E, \eta_s) \right\} \sqrt{\Delta_0 (E, \eta_s) \left\{ \Delta_0 (E, \eta_s) \right\}' \left\{ \Delta_0 (E, \eta_s) \right\}''} \right]^{3/2} \Delta_0 (E, \eta_s) \left\{ \Delta_0 (E, \eta_s) \right\}'
\]

(801)

where,

\[
\Delta_15 (E, \eta_s) = \left[ \gamma_0 (E, \eta_s) - \left( T_1 / 2 \right) \left[ 1 + \text{Erf} \left( E / \eta_s \right) \right] \right] \left[ \gamma_0 (E, \eta_s) - \left( T_1 / 2 \right) \left[ 1 + \text{Erf} \left( E / \eta_s \right) \right] \right]^{1/2}.
\]

Using (801), the electron concentration at low temperatures can be written as

\[
\kappa = \frac{g_e}{3\pi^2} \left[ \frac{\Delta_{14} (E_{R}, \eta_s)}{\Delta_{15} (E_{R}, \eta_s)} \right]^{3/2}
\]

(802)

In this case, \( \bar{E}_{ad} \) is given by

\[
\left\{ \Delta_{14} (\bar{E}_{ad}, \eta_s) \right\} = 0
\]

(803)

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