An important part of the present study is concerned with the fitting of Cobb-Douglas production function. As such it would not be irrelevant to refer to its advantages, applications and limitations. In other-words, the present Chapter aims at discussing some applications and limitations of the Cobb-Douglas function. An attempt is also made here to examine the relative strength and weaknesses of linear, log linear, and semilog forms of production functions.

A number of functional forms are used in the analysis of input-output data in agriculture. The functions that are generally used for agricultural data are: (i) Spillman function; (ii) quadratic function; (iii) square root function; (iv) linear function; (v) Cobb-Douglas function (log linear) and semilog form.

Of the different forms just mentioned above, the Cobb-Douglas form is widely used in the analysis of input-output data in the field of agriculture. This is because the function has some well-known properties that justify its wide application in agriculture. It is a homogeneous function that provides a scale factor enabling one to measure the returns to scale and interprets the elasticity-co-efficients with relative ease. Besides, this, the Cobb-Douglas
function is characterised by continuous production iso-quants so that for realising a given level of output there is a scope of varying prices. From the fitted functions inferences can be drawn about the possibility and the rate of substitution between different inputs.

The function was mathematically formulated first by Cobb on the basis of the empirical observations of Douglas. The function, in the form generally used is $Y = a X^b \ldots \ldots (i)$ where $X$ is the variable resource measured, $Y$ is output, $a$ is constant, and $b$ defines the transformation ratio when $X$ is at different magnitudes. The exponent or $b$ co-efficient is the elasticity of production and can be used directly. The equation is estimated in logarithmic form. This function allows either constant, increasing or decreasing marginal productivity. But it does not allow an input-output curve embracing all three. However, with all other inputs held in fixed magnitudes, the marginal product is expected to decline. The marginal product equation is

$$\frac{dY}{dX} = baX^{b-1} = \frac{bax}{X} \ldots \ldots (2)$$

indicating that if $b=1$, the marginal product (and also the average product) will be constant at the level $a$. Where $b>1$, the magnitude of marginal products will increase as $X$ increases, depending on the magnitude of $b$. Where $b<1$, the magnitude of marginal products will decline as $X$ increases since $X^b < X$. 
The function assumes a constant elasticity of production, \( E \), over the entire input-output curve, or that

\[
\frac{dy_1}{dx_1} \frac{x_1}{y_1} = \frac{dy_2}{dx_2} \frac{x_2}{y_2} \cdots \frac{dy_n}{dx_n} \frac{x_n}{y_n} \quad (3)
\]

where the subscripts refer to marginal products and total outputs corresponding to various magnitudes of \( X \). This condition of the equation, suggests that the successive equal increments of input add the same percentage to total output. This can be proved by multiplying the derivative or marginal product equation (2) by the inverse of the average product as shown below.

\[
E = (b_1 x_1^{b_1-1}) \frac{X}{Y} = \frac{b_1 x_1^b}{X} \cdot \frac{X}{Y} \quad (4)
\]

Now substituting the value of \( Y \) of equation (1) into equation (4) we obtain

\[
E = \frac{bY X}{X} \cdot \frac{X}{Y} \quad (5)
\]

and since the \( Y \)'s and \( X \)'s cancel, we have \( E = b \), or the elasticity is a constant equal to the exponent of \( X \) in equation (1).

In India, quite a few attempts had been made to use the Cobb-Douglas production function for analysing production conditions in agriculture. G.D. Agrawal and W.J. Foreman used this form.

They carried out three exercises in their research paper. They first related the total output with total inputs in a linear function and then they employed Cobb-Douglas production function and tried several alternative combinations of inputs as independent variables. They employed Cobb-Douglas production function for individual crops as well. In the aggregate function where total output was related with aggregate of inputs, they got a very low value of co-efficients. Cobb-Douglas type of production function gave a better fit on the whole but not for individual crops. The value of $R^2$ was found to be higher for polynomial function than that for Cobb-Douglas function.

Ram Saran\textsuperscript{2} carried out an intensive study of output-input relationship based on farm management data. He employed three different types of functions: (i) Spillman function; (ii) Quadratic function and (iii) Cobb-Douglas function. He employed these functions for individual crops as well as for combined output of important crops taken together.

\textsuperscript{2} Ram Saran "Production Function Approach to Measurement in Agriculture - Agricultural Situation in India, August 1964 pp. 413-418.
He first used the Spillman and Quadratic type of functions to study the relationship between output of individual crops such as rice and wheat and inputs of nitrogenous and other fertilizers. He also employed the Cobb-Douglas function. Based on this function, he computed the values of marginal products of different inputs and for different states and compared them with the factor costs. He also measured the returns to scale. He found returns to scale to be constant in all the cases but the values of the marginal products were found to be different from the factor costs. The results related to the years 1953-54 and 1954-55.

In his article entitled "Some Production Functions for Punjab", Raj Krishna used Cobb-Douglas production functions. Based on the production functions, he worked out the values of the marginal product of different inputs and compared them with the factor costs. In the same article, Raj Krishna examined the relationship between the total costs and total output as well as the average product of land and labour with that of the scale of operation measured in terms of area of land holdings, as well as total output.

He examined these relationships both linearly and non-linearly. In these exercises he obtained better fit in terms of value of $R^2$ for Cobb-Douglas production function than that for other relationships. He found returns to scale to be close to one.

C.H. Hanumantha Rao employed Cobb-Douglas production function to analyse agricultural data. His contribution was in the adoption of disaggregated approach. He ran regression for three natural regions of the erstwhile Hyderabad State. Land and labour were included as independent variables. He had used land as measured in units of area standardized by using land assessment per acre. He found the production elasticity of labour to be higher than that of land in two relatively less fertile regions and a reverse situation in the fertile tract of Marathwada.

W. David Hopper investigated into the problems of allocation efficiency with the help of Cobb-Douglas type of production function.


He examined the relationship between the value of marginal product of inputs and the prices of the different inputs. He found that the value of marginal product and the prices of the services of different inputs were very closely related. He also found that the relative prices and market prices of factors and commodities were very close to each other.

A more detailed and somewhat comprehensive analysis had been undertaken by Gian Sahota. He employed the Cobb-Douglas production function. His study covered 859 observations and 13 crops for two years. He used Farm Management data. He examined isolation of inter-seasonal, inter year and inter regional differences in addition to the measurement of relationship between the value of the marginal product and the prices of inputs. The value of marginal product of fertilizers and irrigation water charges was above their respective costs. His study showed that there were no significant inter-seasonal differences but regional differences were quite significant. Again, though the impact of the size of the farm was observed in the case of some crops, the patterns differed for different crops. Thus, no conclusive conclusion emerged from his study relating to the impact of the size of the farm on general production efficiency.

Saini's study employed the Cobb-Douglas production function. Saini examined the problem of multicollinearity among the independent variables. He did not find the problem serious enough to vitiate the results. He found constant returns to scale for all the years included in the study. His results corroborated to a large extent the results of Raj Krishna. He found the ratio of the marginal returns to factor costs to be close to one.

Meghnad Desai and Dipak Majumdar employed linear as well as loglinear (i.e. Cobb-Douglas type) production functions to give a new thrust to the analysis of allocative efficiency. In particular, they examined the relationship between the value of marginal product of labour and wage rate of labour employed in agriculture. They divided the sample farms into two sub-groups consisting of (I) farms who employed hired labour and (II) farms who did not employ hired labour. Owned and hired labour bullocks, implements and land were included as independent variables. The dependent variables were in turn output and yield per acre. The


findings of the authors were illuminating. They found the co-efficients of labour to be positive and statistically significant in 9 cases out of 12 cases in the case of farmers hiring labour on wages. But in the remaining three cases though positive, it was found to be not significant. In contrast, for the groups of farmers not hiring labour on wages, out of 12 equations in 8, the co-efficient of labour was not significantly different from zero and in the remaining 4 cases, though statistically significant, the co-efficient was negative. In their exercise, the value of $R^2$ had been as low as 0.3 to 0.2 in the case of the farmers employing hired labour and 0.5 to 0.6 and even 0.9 to 0.8 (in three cases) for other group.

Ashok Parikh\(^9\) combines cross-section and time series data for ten districts of Madras for Paddy, groundnut, cholam cambu and ragi. He employed the Cobb-Douglas function to the data for different years. In his exercise the value of $R^2$ varied very widely. It was low for groundnut and paddy and high for generally dry crops. He found that the area under crop explained to a great extent the variation in production. In the exercise relating to the variations in the first order differences, he found that only

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9. Parikh Ashok "Crop wise Districtwise Production Functions"
the area explained the variation in the production changes from year to year.

Pan A. Yotopoulos, Lawrence J. Lau and Kutlu Somel first a regression of semilog equation with output per acre and inputs other than land per acre as dependent and independent variables respectively. On the basis of results of this study they got the value of

\[
\sigma = \frac{\frac{\partial Q}{\partial L} \cdot \frac{\partial Q}{\partial C}}{\frac{\partial L}{\partial \text{other than land}}} \quad \text{and} \quad Q = \text{output,} \quad L = \text{labour,} \quad C = \text{cost (other than land)}
\]

They also ran CES function and found the value of \( \sigma = 0.295 \) (for small farms) and still a lower elasticity of substitution between land and other inputs \( (R^2 = 0.981) \). It varied from 0.265 for large farms \( (R^2 = 0.974) \) to 0.942 \( (R^2 = 0.959) \) for small farms. They had used Farm Management data. The authors had not accepted the result as they considered this value to be low. They had then proceeded with Cobb-Douglas type of production function.

Inspite of wide spread use of this technique, one cannot but note a number of difficulties connected with it. The most serious drawback of the Cobb-Douglas function lies in the fact that the function cannot allow of any of its variables to take zero value. If any of its input variables $X_i$ takes the zero value, output has automatically to become zero. This is far from realistic in agriculture. Indeed, production can take place without some inputs which may be used by some of the farms. For example, many farms use fertilizers and irrigation water and many farms do not. If one chooses the Cobb-Douglas form to represent a common production function for all such farms and if one identifies separately such inputs as irrigation and fertilizers, one would be assuming that the farmers using no irrigation water and no fertilizers would be having zero output. This is quite absurd. There are many farms producing crops without using any fertilizers or any irrigation water. This logical absurdity makes statistical estimation by the usual least squares method impossible. Thus, at the very outset one is obstructed by the impossibility of obtaining the logarithm of zero.

Labour and Capital are assumed to be homogeneous in Cobb-Douglas function. Atleast labour cannot be treated as homogeneous in the Indian situation. In the agriculture of an under-developed country like India there are several factors such as
class and easte and traditions which make different types of labour associated with different categories of labourers and there is no one labour market for all kinds of labour. However, overlooking this important phenomenon and working with production function or supply function involving a single homogeneous category of labour may lead to serious mistake in analysis and understanding.

The conceptual difficulties arise from the aggregation and measurement of the heterogeneous input items. If any of the variables are measured in value terms in Cobb-Douglas function the fitted functions suffer from a definite departure from a theoretical concept of a production function which physically is a technological concept - a relation between inputs and outputs in physical terms. The conceptual difficulties mainly arise in the field of income distribution in factor shares if the input items are measured in current prices.

The Cobb-Douglas function implies certain restraints on the nature of the variation of the marginal productivity of any inputs. Thus, if the elasticity of any input lies between zero and one, it assumes that the marginal productivity of that input is always positive and always decreasing. If the elasticity is positive but greater than one, it implies that the marginal productivity is positive and always increasing. With the Cobb-Douglas type of production function, it is, therefore, considered that the
elasticity of any input should lie between zero and one. In economies, it is usually assumed that the marginal productivity of any input should at first be increasing and then decreasing. But the Cobb-Douglas function does not allow for such a possibility.

The cross-section production function has often been put in with regard to the concept of 'returns of scale'. The procedure followed is to test for the hypothesis of the sum of the estimated constant elasticities not being different from unity. A serious difficulty here is that the result of such a test would depend on the input variables included and the way they are aggregated. There is no satisfactory means of judging that the explanatory variables used in a fit are the only variables that need to be taken into account and that they are aggregated and measured in the best possible manner. As such, the result of a test with the help of a fitted function involving a certain number of arbitrarily chosen variables defined in certain arbitrary ways cannot be regarded as definitive. However, a problem of inference in this particular problem would be in the choice of the null hypothesis. The same fitted function might with \( \sum \beta_j < 1 \) indicate constant or decreasing returns to scale depending on whether one chooses the null hypothesis to be \( \sum \beta_j = \sum \hat{\beta}_j \) where \( \lambda_j \) are the elasticities of output with respect to the input items and \( \hat{\beta}_j \) the corresponding estimates. This particular problem can be solved if we make prior theoretical or empirical
reasons for constant returns to scale. It has no justification if there are such prior reasons.

A standard practice is to accept a production function fit as sufficiently good as long as it gives a significant $R^2$ value. But it is conceded by econometricians that the statistical significance of $R^2$ is not sufficient to treat fit as 'good'. Because, a significant $R^2$ only means that it is significantly different from zero. In fact, a statistically significant $R^2$ only rejects the null hypothesis of the total absence of any correlation between output and all the inputs considered. Thus, it has been correctly observed, "a model that makes entrepreneurial decisions unpredictable in any systematic fashion is an unlikely candidate for acceptance over almost any other alternative". Even a large $R^2$ is not the sufficient basis for considering a production function fit to be 'good'. It is quite possible for two very different functional forms involving different sets of input variables each to give fits with high $R^2$, but they may carry totally different economic interpretations. Indeed, when we choose the functional form and explanatory variables, there is not much in statistical theory that can provide a very good guidance.

To choose a fit that makes economic sense depends on one's judgement about the economic problem at hand. Thus, one way of judging the goodness of the production function fit would be to examine the numerical magnitudes of the estimated coefficients from the point of view of their stability and plausibility.

The negative co-efficients in a Cobb-Douglas function are also a problem. It is remarkable fact that many a production function fit to cross-section farm data is presented with negative co-efficients - sometimes significantly negative co-efficients. Nothing can be economically more absurd than a cross-section production function with a negative co-efficients for an input variable.

In spite of the problems just mentioned above, we have employed Cobb-Douglas function in the present study to examine the different aspects mentioned in the introduction. Indeed, the Cobb-Douglas function to cross-section data can be advantageously used to examine output changes in the accompaniment to input combinations. The Cobb-Douglas fit may be used to calculate the ratios of marginal value products and market prices.
RELATIVE STRENGTH AND WEAKNESSES OF LINEAR, LOG-LINEAR AND SEMI-LOG FORMS OF PRODUCTION FUNCTIONS; A PRELIMINARY INVESTIGATION

We started with three forms of production functions (i.e., linear, loglinear and semi-log forms) in order to assess the suitability of each of the three different forms of production functions. The following diagram (based on a similar one by Bronfenbrenner)* assuming one output and one input might clear why a preliminary discussion of linear, log linear and semilog forms was considered necessary.

In the above diagram, the intra-firm production $Y = f(X)$, has a non-linear shape in the case of both farms A and B. But in choosing the points on the production function, a and b they are guided by the price line OC, the slope of the line being the

price of Y divided by the Price of X. Naturally a and b are the points where the price line is tangential to the production curve. Since the points lie on a straight line, here a linear regression of Y on X will give a good fit, though the production function is non-linear. All these show that when we try to apply an estimate by single equation method in a situation characterised by more than an equations, we are really getting a kind of mongrel relation between the explained variable and the set of explanatory variables. On an a - priori grounds, there is, therefore, no reason for excluding linear and other forms of inter-farm production function. In the usual studies, log-linear form is fitted and if it gives a good fit, then this is considered a very desirable state of affairs. Thus, to get a proper perspective it might be better to fit not only the log-linear form but also other forms such as linear and semilog etc. Thus, we started with linear, loglinear and semilog forms of production functions in our preliminary investigation. These three forms were examined in the different stages of our enquiry to see which form gave the best fit to our data.

* The word 'stage' is used here in its literal sense and should not be confused with the meaning attached to it in the multi-stage estimation procedure of the simultaneous equation system.
In the first stage of our enquiry, land and labour were the explanatory variables and the explained variable was gross output. In the second stage of enquiry, seed was included along with land and labour variables. In the third stage, another additional variable i.e. manures and fertilizers was included along with land, labour and seed. Lastly, in the fourth stage, irrigation water as an input was included along with land, labour, seeds and manures and fertilizers.

When we were judging the goodness of fit among these forms we followed the following criteria. Generally, $R^2$ and $F$ values are the only criteria to judge the goodness of fit. This procedure would be appropriate if in all the cases the estimated co-efficients would come out with the correct signs. However, signs may be wrong in the sense that they are not what we should expect from the economic analysis. Such a situation might be due to either multi-collinearity or the inappropriateness of single equation method of estimation. If fact, if we judge what is more appropriate form of regression equation by $R^2$ and $F$ values alone, we might get one answer and if we take in consideration the correctness of regression co-efficients and their level of significance we might get a quite different answer. Thus, our criteria of choosing the best form includes $R^2$ + other considerations such as level of significance and the correctness of the signs associated with the variables.)
As an illustration, we present the estimates of the three forms (i.e. linear, log linear and semilog forms). The estimates were made using the data for 120 traditional 'treated' farms relating to the year 1963-64 in Period I.

**STAGE I**

(1) $Y = -782.34 + 220.23 X_1 + 3.4896 X_2$  \[ R^2 = .7666 \]

(101.37) \[ (.4558) \]

F (2,117) = 85.50

(2) $\log Y = 1.3181 + .3949 \log X_1 + .6614 \log X_2$

\[ R^2 = .8156 \]

(170.6) \[ (.0921) \]

F (2,117) = 118.99

(3) $Y = -29295.0 + 2754.4 \log X_1 + 10535.0 \log X_2$

\[ R^2 = .6939 \]

(1840.5) \[ (.0670) \]

F (2,117) = 55.90

**STAGE II**

(1) $Y = -645.32 + 402.00 X_1 + .2008 X_2 + 12.268 X_3$

\[ R^2 = .8925 \]

(71.95) \[ (.4282) \]

F (3,116) = 156.09

(2) $\log Y = 1.7493 + .4772 \log X_1 + .1655 \log X_2 + .4481 \log X_3$

\[ R^2 = .8958 \]

(1845.2) \[ (.1342) \]

F (3,116) = 161.88

(3) $Y = -22229.0 + 4103.8 \log X_4 + .2407.6 \log X_2 + 7310.3 \log X_3$

\[ R^2 = .8006 \]

(1895.1) \[ (1004.1) \]

F (3,116) = 71.497
STAGE - III

(1) \[ Y = -188.08 + 216.75 X_1 + 1.0204 X_2 + 1.0387 X_3 + 5.4520 X_4 \]
\[ (49.30) \quad (1.2876) \quad (1.1429) \quad (1.4357) \]
\[ R^2 = .8948 \]
\[ F(4,115) = 32.23 \]

(2) \[ \log Y = 1.5441 + .4048 \log X_1 + .2283 \log X_2 + .2544 \log X_3 + .1989 \log X_4 \]
\[ (.0701) \quad (.0914) \quad (.0667) \]
\[ R^2 = .9061 \]
\[ F(4,115) = 137.38 \]

(3) \[ Y = -25676.0 + 2887.0 \log X_1 + 3463.3 \log X_2 + 4091.0 \log X_3 + 3340.8 \log X_4 \]
\[ (1410.2) \quad (1838.1) \quad (1340.8) \]
\[ R^2 = .8195 \]
\[ F(4,115) = 61.59 \]

STAGE - IV

(1) \[ Y = -145.19 + 228.95 X_1 + 9773 X_2 + 8240 X_3 + 56454 X_4 \]
\[ (51.027) \quad (1.2914) \quad (1.1664) \quad (1.4826) \]
\[ + (-3.7887 X_5) \]
\[ (4.0520) \quad (5.114) \quad (258.55) \]

(2) \[ \log Y = 1.5235 + .3966 \log X_1 + .2384 \log X_2 + .2566 \log X_3 + .1934 \log X_4 + .0049 \log X_5 \]
\[ (.0742) \quad (.0960) \quad (.0673) \]
\[ R^2 = .9055 \]
\[ F(5,114) = 109.09 \]
(3) \[ Y = -2732.3 + 2230.1 \log X_1 + 4266.7 \log X_2 + 4266.1 \log X_3 \\
\quad (1477.6) (1914.8) (1340.4) \\
+ 2899.1 \log X_4 + 398.38 \log X_5 \quad R^2 = .8216 \\
\quad (1015.2) (279.44) \quad F(5,114) = 50.121
\]

In every stage equation No. (1) is linear; equation No. (2) loglinear and equation No. (3) is semilogarithmic form. The figures within the parentheses are the standard errors of the estimates. Single asterisk indicates 5% level of significance. Double asterisks indicate 1% level of significance.

It is clear from the estimates that a loglinear form (Cobb-Douglas form) comes out best in all the stages according to our criteria (i.e. $R^2$ + other considerations such as the level of significance and the correctness of the signs associated with the variables). It is interesting to note that a linear form comes out best in the fourth stage according to $R^2$ and $F$ values alone. But the coefficients associated with irrigation water of the linear equation in the fourth stage come out with the wrong sign which was unexpected from an economic analysis. Thus, if we consider the correctness of the signs and the level of significance, a log-linear form comes out best in the fourth stage also.
Almost the same picture (i.e. loglinear as a best fitting form) was obtained in other cases when we examined these three forms in the different stages in times of preliminary investigation. On the basis of the preliminary investigation, we accepted log-linear form and excluded other two forms such as linear and semi-log forms. It should be mentioned that the loglinear form with land, labour, seeds, manures and fertilizers, and irrigation water was accepted in the present study.