INTRODUCTION
Graph theory is one of the most active fields of research area in the subject of discrete mathematics on account of its wide applications to different branches like electronics, computer science, combinatorial, computational chemistry, networks, communications, electrical engineering, psychology sociology and so on. Graph theory came into existence during the first half of 18th century when in 1736, famous Swiss mathematician Leonard Euler (1707-1782) settled a famous unsolved problem of his days ‘Konigsberg Bridge Problem’. Since then, graph theory was rediscovered independently by many mathematicians like G.Kirchhoff, A.Cayley, F.Guthrie, P.J.Heawood and W.R.Hamilton over the past decades. In 1847, G.Kirchhoff (1824-1887) developed the theory of trees in order to study the applications dealing with electrical networks. Later in 1857, A.Cayley (1821-1895) discovered the important class of graphs called trees in order to count the distinct isomers of saturated hydrocarbons $C_nH_{2n+2}$, with a given $n$ number of carbon atoms. During these periods, two new ideas came into light in the field of graph theory. One of them is the most celebrated ‘Four Color Conjecture’, which was first investigated by F.Guthrie (1831-1899). This problem was settled in the year 1976 by K.Appel and W.Haken and their proof employs a very intricate computer analysis of reducible configurations. The second major idea was due to W.R.Hamilton (1805-1865), who used the idea of spanning cycle (in graph theoretic terminology) in 1859, for an intriguing puzzle that used
the edges on a regular dodecahedron. Following these, no much activity was found on this area till 19th century. This area of mathematics didn’t start to develop into an organized branch of mathematics until the second half of 19th century and there was not even a single book on graph theory until 1936.

In 1936, König [40], the Hungarian mathematician wrote the first book on graph theory. Later, books on graph theory are also written by Berge [8], Ore [50] and Harary [31]. Applications of graph theory in different branches like engineering, technology, biological sciences, archeology, ecology, planning etc. can be found in the book of Roberts [54]. The connection between graph theory and other branches of mathematics were discussed in the book of Beineke and Wilson [7].

The topic of domination was the fastest expanding area within the graph theory because of its wide variety of applications in fields like social sciences, networks, algorithm designs etc. In 1958, Berge [8] introduced the concept of external stability and co-efficient of external stability in his book *Theory of graphs and its applications*. Later in 1962, Ore [50] published a famous book *Theory of graphs*, in which he introduced the word domination. The domination concept was in inhibition till 1975, as there was no much work was done till 1977. Cockayne and Hedetniemi [20] published a paper, *Towards a theory of domination in graphs* in the year 1977. This paper has thrown opened the wide variety of new ideas and applications to many areas. A reason
for the high level of research activity in this area is the fact that there are many real world applications. Towards the end of 1997, Haynes, Hedetniemi and Slater [32, 33] brought out a comprehensive two volume text book, *Fundamentals of domination in graphs* and *Domination in graphs: Advanced topics*, which contain more than 1200 bibliographical entries. For more detailed survey on domination theory, see [35, 36, 37, 56].

Graph theory also has applications in combinatorics. In combinatorial mathematics, a block design is a particular kind of hyper graph or set system which has applications to finite geometry, cryptography and algebraic geometry. In the class of incomplete block design, the balanced incomplete block design given by Yates is the simplest one. A balanced incomplete block design is one among the many variations that have been studied in block designs and it is a set of $v$ vertices arranged in $b$ blocks of $k$ vertices each in such a way that each vertex occurs in exactly $r$ blocks and every pair of unordered vertices occurs in $\lambda$ blocks and it is denoted by the representation, $(v,b,r,k,\lambda)$-design. The connection between graph theory and designs were first observed by Berge [8]. Motivated by the works of Berge, Paola [51] has given a link between graphs and balanced incomplete block designs. As the class of balanced incomplete block designs do not fit for many experimental situations as this design requires large number of replications. To overcome this, Bose and Nair [13] introduced a class of binary, equireplicate and proper
designs called partially balanced incomplete block designs which was included as a special case of the balanced incomplete block designs. Bose and Shimamoto [14] are first to introduce the concept of association schemes in partially balanced incomplete block designs. More about association schemes can be found in Bannai and Ito [6], Godsil [25] and Bailey [3] and a catalogue of different partially balanced incomplete block designs on two associate classes can be found in Clatworthy [16]. Bose [12] in his pioneering paper, used the graph theoretic method for the study of association schemes of partially balanced incomplete block designs and also shown that the concept of strongly regular graphs is isomorphic with the association schemes of partially balanced incomplete block designs (with two associate classes).

Walikar et. al. [57] introduced the designs called \((v, \beta, \mu)\)-designs, whose blocks are maximum independent sets in regular graphs on \(v\) vertices. Walikar et. al. [59] has also established the relation between dominating sets of a graph with the blocks of partially balanced incomplete block designs. Also it is known that, it is possible to construct the strongly regular graph \(G\) with the parameters \((v, n_1, p_{11}^1, p_{11}^2)\) from a given partially balanced incomplete block design with two association schemes having parameters \((v, b, r, k, \lambda_1, \lambda_2)\).

Applications of graph theory can also be seen in chemistry. As we know that, the adjacency matrix represents the non-pictorial form of
corresponding graph and also many researchers came to know that eigenvalues of a graph are precisely the eigenvalues of the adjacency matrix of the corresponding graph. A chemical application of graph theory was initiated by Hückel [39] in his molecular orbital theory (HMO) during 1930’s. In quantum chemistry, skeletons of certain non-saturated hydrocarbons were represented by graphs. The energy levels of electrons in molecules of such non-saturated hydrocarbons are nothing but the eigenvalues of the corresponding graph. From the pioneering work of Coulson [22], there exists a continuous interest towards the general mathematical properties of the total $\pi$–electron energy. In 1978, Gutman [27] introduced the concept of graph energy as the sum of the absolute value of the eigenvalues the graph. In view of the great success of graph energy concept few analogous quantities such as Laplacian energy, Incidence energy, Distance Energy, Skew energy and so on were also conceived. For more detailed survey, see [1, 5, 10 30, 40].

AN OUTLINE OF THE PRESENT WORK

The thesis entitled “A STUDY ON SOME CONCEPTS IN GRAPH THEORY AND APPLICATIONS” mainly consists of five chapters.
In Chapter-1, basic definitions, terminologies and results which are needed for the subsequent chapters were collected. All the graphs considered in the thesis are finite, undirected graph without loops or multiple edges. For graph theoretical terminology, refer to Harary [31] and Parthasarathy [52].

In Chapter-2, a new class of graphs called “Equitable dominating graph” were introduced and studied. The equitable dominating graph, $ED(G)$ of a graph $G$ is a graph with $V(ED(G)) = V(G) \cup D(G)$, where $D(G)$ is the set of all minimal equitable dominating sets of $G$ and two vertices $u$ and $v$ of $V(ED(G))$ are adjacent to each other if $u \in V(G)$ and $v$ is a minimal equitable dominating set of $G$ containing $u$. This chapter begins with the condition for $ED(G)$ to be connected. Followed by this, results related to completeness, bounds on order and size, equitable domatic partition, and equitable independence number of $ED(G)$ were discussed. This chapter is concluded on obtaining the conditions under which $ED(G)$ is Eulerian and Hamiltonian.

Chapter-3 deals with “The Maximal equitable domination number of a graph”. An equitable dominating set $D$ of a graph $G$ is said to be maximal if $V - D$ is not an equitable dominating set. The maximal equitable domination number $\gamma^e_m(G)$ of $G$ is the minimum cardinality of a maximal equitable dominating set. In this chapter, the properties of maximal equitable
dominating number of a graph were investigated and many bounds on this new parameter are obtained.

In Chapter-4, the “Partially balanced incomplete block designs associated with graphs” was discussed. In this chapter, discussion begins by obtaining the perfect domination number of Petersen graph and Clebsch graph. Also, shown that the minimum perfect dominating set of Petersen graph and Clebsch graph induces $K_{1,3}$ and $C_4$ respectively. This chapter is concluded by defining the two class association scheme and also by obtaining blocks of partially balanced incomplete block designs with parameters of first kind and second kind of Clebsch graph.

In Chapter-5, a study on “Neighborhood energy and equitable energy of a graph” was given. In the first section of this chapter, a discussion about neighborhood energy of a graph was given. The characteristic polynomial of a graph, by considering the open neighborhood matrix of that graph was given. Also, neighborhood spectrum and neighborhood energy of these graphs were obtained. In the second section of this chapter, equitable energy of a graph was discussed. The characteristic polynomial of graph was obtained by considering the corresponding equitable adjacency matrix of that graph. This section is concluded by obtaining equitable spectrum and equitable energy of these graphs.