CHAPTER IV

In this chapter we have dealt with the calculation of rate coefficients for rotational excitation of linear polar molecular ions by electron impact. For such collisions, the charge–dipole interaction is the one having the longest range and dominates over all other anisotropic interactions. Also rotational line spectra can be observed only for molecules with a permanent dipole moment. Hence recent researches have been mainly concentrated on transitions through a charge–dipole interaction potential only.

From the work of Dickinson and Muñoz (1977) and Strekalov (1979) it is clear that in the case of electron–molecular ion collisions, the $\Delta j = 1$ transition cross sections due to charge–dipole interaction can be calculated accurately from a semi–classical first order perturbation theory. A simple approximation like this excludes exchange and polarization phenomena and ignores competing channels such/dissociative recombination, but it is adequate to estimate the effect of rotational excitations. However, when the incident electron penetrates the charge cloud of the molecule, the charge–dipole potential is no longer valid. In this region strongly repulsive central and non–central forces arise. It has not yet been possible to estimate these forces quantitatively. Dickinson and Muñoz (1977) have, however, put forward arguments to show that the net effect of the penetrating trajectories would be an enhancement of the $\Delta j = 1$ cross sections from their purely charge–dipole interaction values.

We have obtained a generalized expression for the dipolar rate coefficients which should be applicable to all systems and which follows naturally from the first–order perturbation theory expression for
probabilities. The results have been compared with the similar formula obtained independently by Dickinson and Flower (1981). We have discussed their results and it has been shown that the two expressions give identical results at low temperatures and even at high temperatures the two expressions give results which are very close.

The possible effect of short-range potential on the $\Delta j = 1$ transition rate has been estimated by following the method of Dickinson and Muñoz (1977). The astrophysical importance of electron-ion collisions in relation to $H_2^+$-ion collisions has been assessed. The difficulties in applying this method to the calculation of $\Delta j = 2$ rates in which the short-range effects are likely to be much more important, have been discussed.

1. Method

Dipole transition rate

The expression for the symmetrized transition probability for the rotational transition $j \rightarrow j'$ in a linear molecular ion, according to the time dependent first-order perturbation theory, is given by eqn. (92) which is

$$P_{j, j'}(E_1, b) = \frac{1}{(2j + 1)\hbar^2} \sum_{m_j} \sum_{m_{j'}} \left| \int_{-\infty}^{\infty} v_{j, j'}(\vec{R}(t)) \exp(i\omega_{j, j'} t/\hbar) \, dt \right|^2 \ldots (92)$$

where the summation is over the magnetic quantum numbers $m_j$ and $m_{j'}$, $v_i(v_{f})$ the initial (final) relative velocity and the potential matrix element is given by eqn. (82):

$$v_{j, j'}(\vec{R}(t)) = \int v_{j, j'}(\hat{s}) \, v'(\vec{R}(t), \hat{s}) Y_{jm_j}(\hat{s}) \, d\hat{s} \ldots (82)$$

The interaction potential $v'$ is a function of the relative coordinate.
$R(t)$ and the orientation of the internuclear axis, $\hat{s}$ and causes the rotational transition. $Y_{jm}$ is a normalized spherical harmonic.

The asymptotic interaction potential between a linear molecular ion and an electron can be expanded as

$$V(R, s) = -\frac{e^2}{R} + \sum_{\lambda} v_\lambda(R) Y_{\lambda0}(\hat{R}, \hat{s})$$

where we have defined

$$v_\lambda(R) = CR^{\lambda+1}$$

and

$$C_\lambda = -e \left( \frac{4\pi}{2\lambda+1} \right)^{1/2} Q_\lambda$$

The term $Q_\lambda$ is the $\lambda$th electric moment of the charge distribution of the molecular ion. Thus, $Q_1 = D$, the dipole moment and $Q_2 = Q^0$, the quadrupole moment of the molecular ion.

The first term in the expression of the interaction potential (eqn. (99)) is the Coulomb interaction (isotropic) which, in a semiclassical framework, determines the trajectory of the electron to be a hyperbola. The second term is the anisotropic part (dipole, quadrupole, etc.) responsible for rotational transitions. Thus

$$V'(R(t), \hat{s}) = \sum_{\lambda} v_\lambda(R) Y_{\lambda0}(\hat{R}, \hat{s})$$

$$= \sum_{\lambda} \sum_\mu \left( \frac{4\pi}{2\lambda+1} \right)^{1/2} \frac{v_\lambda(R) Y_{\lambda\mu}(\hat{R}) Y_{\lambda\mu}^*(\hat{s})}{Y_{\lambda\mu}}$$

Using the relation

$$\int Y_{l_1 m_1}(\hat{s}) Y_{l_2 m_2}(\hat{s}) Y_{l_3 m_3}(\hat{s}) d\hat{s} = \left[ \frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi} \right]^{1/2} x\left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{array} \right) \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right),$$

...
we obtain the potential matrix element
\[ V_{j'j}(\vec{r}(t)) = \sum_{\lambda, \mu} (-)^{m_j' + \mu} \left[ (2j + 1)(2j' + 1) \right]^{1/2} \nu_\lambda(\vec{r}) \nu_{\lambda\mu}(\vec{r}) \]

Substituting (104) in eqn. (92) and using the symmetry properties of 3-j coefficients we obtain the expression for the symmetrized transition probability as
\[ P_{jj'} = (2j' + 1) \frac{V_{j'j}}{V_j} \sum_{\lambda, \mu} \frac{1}{2\lambda + 1} \left| \begin{pmatrix} j & j' & \lambda \\ 0 & 0 & 0 \end{pmatrix} \right| V_{\lambda\mu}^2 \]

where \( V_{\lambda\mu} \) is given by
\[ V_{\lambda\mu} = \frac{1}{\pi} \int_{-\infty}^{+\infty} \nu_\lambda(\vec{r}) \nu_{\lambda\mu}(\vec{r}) \exp(i\omega_j t) dt \]

with the conditions:
\[ |j - j'| \leq \lambda \leq j + j' \]
and
\[ j + j' + \lambda = \text{even}. \]

The integral in equation (103) is to be evaluated for the classical hyperbolic trajectory of the relative motion. Let us choose the plane of the motion to be the xy - plane and the axis Ox to be directed along the axis of symmetry of the hyperbolic trajectory. In this choice
\[ \vec{r} = (x, y, 0) = (R, \pi/2, \theta) \]

moreover,
\[ \nu_{\lambda\mu}(\vec{r}) = \nu_{\lambda\mu}(\pi/2, 0) e^{i\mu\theta} \]
\[ = \nu_{\lambda\mu}(\pi/2, 0) \left( \frac{x + i y}{R} \right)^\mu \]

At \( t = 0, \theta = y = 0 \).
The equation of motion of the electron in parametric representation is of the form

\[
R = a(\xi \cosh \xi - 1), \quad x = a(\xi - \cosh \xi) \\
y = a(\xi^2 - 1)^{1/2} \sinh \xi, \quad t = \frac{3}{v} (\xi \sinh \xi - \xi) \tag{110}
\]

where \(v\) is the geometric mean of the initial and final relative velocities:

\[
v = (v_1 v_f)^{1/2} \tag{111}
\]

\[
a = e^2/2E \tag{112}
\]

with

\[
E = \frac{1}{2} m v^2 \tag{113}
\]

Also, \(\xi\) is given by

\[
\xi = \frac{e^2|\Delta E|}{\hbar m v^3} \tag{114}
\]

and \(\xi\), the eccentricity of the hyperbola, is

\[
\xi = (1 + b^2/a^2)^{1/2} \tag{115}
\]

\(b\) being the impact parameter.

With eqns. (109) and (110), we obtain

\[
V_{\lambda \mu} = \left[ C_{\lambda \mu} Y_{\lambda \mu}(\pi/2, 0)/\hbar \omega^\lambda \right] I_{\lambda \mu}(\xi, \beta) \tag{116}
\]

where

\[
I_{\lambda \mu}(\xi, \beta) = \int_{-\infty}^{\infty} \exp[i\beta(\xi \sinh \xi - \xi)] \frac{[\xi - \cosh \xi + i(\xi^2 - 1)^{1/2} \sinh \xi]^\mu}{(\xi \cosh \xi - 1)^{\lambda+\mu}} \, d\xi \tag{117}
\]

with \(\beta\) defined by

\[
\beta = a\omega_{j,j}/v \tag{118}
\]

The integral \(I_{\lambda \mu}(\xi, \beta)\) is similar to that arising in the problem of Coulomb excitation of nuclei (Alder et al 1956).
We are interested on transitions caused by charge-dipole interaction potential (corresponding to $\lambda = 1$). Thus, putting $\lambda = 1$ we get from eqn. (105) the symmetrized transition probability (in au.) for the transition $|\Delta j| = 1$

$$P_{j,j+1} = \frac{2j'+1}{3} \left( \begin{array}{ccc} j & j' & \lambda \\ 0 & 0 & 0 \end{array} \right)^2 \frac{\nu_f}{\nu_i} \frac{D^2}{E} \left[ I_{111}^2 + I_{1-1}^2 \right]$$

...(119).

The total cross section for the transition $j \to j' = j \pm 1$ is given by

$$Q_{jj'} = \int P_{jj'} d\sigma_R$$

...(120)

where

$$d\sigma_R = \frac{1}{4} \frac{a^2}{2} \sin^{-4}(\theta/2) d\Omega$$

...(121)

is the Rutherford cross section for scattering into the solid angle $d\Omega$.

Thus, the cross section for $\Delta j = \pm 1$ can be written as

$$Q = C J(\xi)$$

...(122)

where

$$C = \frac{4D^2}{24E} \frac{\nu_f}{\nu_i} \frac{j}{2j+1}$$

...(123)

and

$$J(\xi) = \int \left[ I_{111}^2 + I_{1-1}^2 \right] \sin^{-4} \frac{\theta}{2} \sin \theta d\theta$$

...(124).

Here $\theta$ is the angle of scattering and $j_0$ is the greater of the initial and final rotational quantum numbers.

The function $J(\xi)$ can be obtained by numerically performing the $I$ integration and then integrating over $\theta$. However, comparing the integrals with similar integrals $I_{1\mu}$ arising in a repulsive potential it is seen that $I_{1\mu} = e^{\pi \xi} I_{1\mu}$. The $I_{1\mu}$ integrals have been tabulated by Alder and Winther (1956) for low values of $\xi$ and can be used in our calculations. For high values of $\xi$ we have calculated the corresponding functions $J(\xi)$ from the table of cross section of...
excitation of CH⁺ given by Dickinson and Muñoz (1977). These are very close to the Coulomb-Born cross section obtained by Chu and Dalgarnd (1974).

We have made a least square fit to the function \( J(\xi) \). The functional form was taken as

\[
J(\xi) = A + \frac{A_1}{\xi} + \frac{A_2}{\xi^2}, \quad \ldots (125)
\]

It was found that the fit reproduced all the CH⁺ cross sections calculated from first order perturbation theory to within 2.5%. Also the Coulomb Born cross section at 1.5 times the threshold energy for CH⁺ was reproduced to within 10%. The constant \( A \) is determined from the low energy limit \( (\xi \to \infty) \) of the cross section obtained from the limiting form of the complex Hankel function

\[
Q_{jj'} \approx \frac{4\pi^2}{3\sqrt{3}} \frac{D^2}{E_1} (2j'+1) \left( \frac{j}{0} \frac{j'}{0} \frac{1}{0} \right)^2. \quad \ldots (126)
\]

When all quantities are measured in a.u., the constants are \( A = 58.04 \), \( A_1 = 13.10 \), \( A_2 = -3.3048 \).

The rate constant at temperature \( T \) is \( \langle Q v_i \rangle \), the average being over a Maxwellian distribution of velocities of the incident electrons. The integration can be performed by expressing the incident energy as \( E \pm \Delta E/2 \), or in other words, by replacing the geometric mean of the incident and final energies by the arithmetic mean. The expression for the rate constant using (125) is

\[
K(\mathcal{J} \to \mathcal{J} \pm 1, T) = \frac{7.194 \times 10^{-7}}{\sqrt{T}} D^2 \left( \frac{j}{2j+1} \right) \exp(-\Delta E/2kT) \left[ 58.04 - \frac{1.521 \times 10^{-3}}{|\Delta E|^{1/2}} (T)^{0.75} - \frac{6.47 \times 10^{-9}}{|\Delta E|} (T)^{1.5} \right] \quad \ldots (127)
\]
k being the Boltzmann constant.
Here $\Delta E$ is in atomic units and the final rate coefficient is obtained in cm$^3$/sec. This formula automatically satisfies the principle of detailed balance.

**Contribution of the short-range potential**

Dickinson and Muñoz (1977) derived an expression for the contribution of trajectories penetrating the electron charge cloud to the cross section from an assumed empirical form of the probability vs impact parameter curve in this region. The expression contains a parameter $\eta$, which is the maximum value of the unsymmetrized probability of transition attained for penetrating collisions.

Assuming $\frac{1}{E} \gg r_c$, the dimension of the charge cloud, in atomic units (an approximation which is valid for all energies of interest even at the highest temperature) we can derive an expression for the contribution of the penetrating trajectories to the rate coefficient

$$K^{\text{SR}}(j \rightarrow j+1,T) = 1.44 \times 10^{-6} \frac{e^{-\Delta E/kt}}{\sqrt{T}} \eta \left[ R_a(1 + 3R_a) - 6(1+R_a)^2 \right] \ldots \text{eq. (128)}$$

Here $R_a = \sqrt{2} r_c$, $r_c$ being taken as roughly twice the equilibrium internuclear distance $R_c$ for a diatomic molecular ion. For a linear polyatomic molecular ion $R_c$ is the distance between two extreme nuclei.

Estimation of $\eta$ is not possible without further quantitative information about the nature and strength of short range interactions. However, some general conclusions about $\eta$ can be made in the following manner. For low impact parameters where the trajectories penetrate the molecular charge cloud all the collisions are sudden. For an anisotropic potential with only $P_1(\hat{R},\hat{S})$ angular dependence, the maximum allowed value of $|\Delta j|$ does not exceed $V_i = \int_0^{\infty} v^i(t') dt$. The maximum
transition probability thus should be inversely proportional to $V_1$ measured in a.u. We have assumed that the maximum is attained for an impact parameter $b_2/2$ where $b_2$ is the impact parameter for which trajectories just enter the charge cloud. For sudden collisions, $V_1$ can be roughly approximated as $V' \frac{b}{v}$, $V'$ being the magnitude of the anisotropic potential at the turning point at an impact parameter $b$. If we assume that the isotropic and anisotropic potentials are of the same order of magnitude at the turning point then $V_1 \sim \frac{E_1 b_2}{2v}$. This does not exceed unity by a large amount and thus even at low impact parameters the higher order transitions from a pure $P_1$ potential will not be important. However, inside the molecular charge cloud the anisotropic potential deviates considerably from the pure $P_1$ form. Because of the coupling of different terms of the Legendre expansion of the anisotropic potential $\eta$ decreases. If terms upto $P_n$ are significant in the region $b = 0$ to $b_2$, giving transitions upto $\Delta j = n$, from statistical considerations, the average of the transition probabilities may be assumed to be only $1/n$ for sudden collisions. Thus for $\Delta j = \pm 1$ we take $\eta = \frac{1}{n} \frac{\sqrt{3}}{2j+1}$. The $j$ dependent factor arises from the dynamics of sudden collisions in which the $\Delta j = + 1$ and $\Delta j = -1$ transition probabilities are related through the corresponding 3-$j$ coefficients for a pure $P_1$ potential.

If we choose a rigid ellipsoid model for the short range interaction having $a$ and $b$ as the semi-major and semi-minor axes respectively, then only terms upto $P_n$ will be significant, $n$ being given by $a/b$. For the molecular ions we make the rough estimate $n \approx r(\text{atom})/R_c$, where $r(\text{atom})$ is the mean atomic radius of the isolated atoms comprising...
the molecular ion. These values for different ions have been obtained from Fraga et al (1976). This rough estimate gives \( n = 2 \) for \( \text{CH}^+ \), \( 4 \) for \( \text{N}_2\text{H}^+ \) and \( 5 \) for \( \text{HCO}^+ \). For the polyatomic ions, internuclear separations calculated by Haese and Woods (1979) were used.

The pure dipole cross sections are estimated by integrating over all impact parameters. However, since the dipolar contribution from the region \( b = 0 \) to \( b_2 \) is generally a small part of the total dipole cross section and also in view of the fact that there are many other sources of uncertainty in the evaluation of short range cross sections, the two rates may be simply added to yield the total \( \Delta j = \pm 1 \) transition rate. The downward rate may be obtained from the principle of detailed balance.

\[ \Delta j = 2 \text{ transitions} \]

For molecular ions with finite quadrupole moments there will be a long range charge-quadrupole interaction causing \( |\Delta j| = 2 \) transitions. This has a much shorter range than the charge dipole interaction and the first order transition probability exceeds unity for close collisions. At this range of impact parameters higher order processes will start contributing. Though the quadrupole moments of the relevant molecular ions are not known, it is expected that the strong coupling region will be dominant over the first order perturbation region as far as \( |\Delta j| = 2 \) transitions are concerned. The short range interactions also start becoming important in this region. Under the circumstances, the best that can be done is to assume a contribution of the strong coupling region to the \( |\Delta j| = 2 \) transition similar to the \( |\Delta j| = 1 \) transition. This contribution will generally dominate over the first...
order cross section obtained from the weak coupling region. Hence we may use the same eq. (128) for the $|\Delta j| = 2$ rate. In this the j dependent factor in $\eta$ should be given by $3 (j + 1) (j + 2)/2(2j + 3) (2j + 1)$. This factor follows from the geometrical factor in sudden collision dynamics.

2. Results and Discussion

For pure dipole transitions, the trajectory integrals $I_{1\mu}$ in the attractive and repulsive Coulomb fields are related to each other. Indeed, the ratio of cross sections for rotational excitation of HD$^+$ in collision with electrons and positrons should be $e^{2\pi^2 \xi}$. This ratio is satisfied in the calculations of Ray and Barua (1975) at all energies greater than 0.1 eV. The discrepancy of 5 to 12% as observed is due to the short range cut-off used by them.

The values of molecular constants used in the calculations are shown in Table 1. The rate coefficients for CH$^+$ obtained by us are in good agreement with the calculations of Dickinson and Flower (1980) at low energies, which is expected since in both cases the fit is made by considering the low energy limit of the dipole cross section. However, at high energies also the fit is satisfactory, and for CH$^+$ the agreement is within 3% at all temperatures. For N$_2$H$^+$, the maximum discrepancy is 5% at 1000 K when the same value of dipole moment is used.

The values of rate coefficients at a few representative temperatures are given in Table 2 for $0-1$ and $1-2$ transitions. It is seen that the maximum occurs near a temperature $T$ such that $kT \approx 2\Delta E$ and the peak is flatter for higher $\Delta E$ transition. The ratio of the short range contribution to the pure dipole rate will have a weak temperature
dependence. This ratio is higher for molecules of larger size and lower dipole moment. Thus for 0—1 transition in HCO⁺ the ratio varies from 13.5% to 8.5% while for the same transition in CH⁺ the ratio lies between 98% and 83%. For ΔE ≤ kT the ratio increases with increasing ΔE.

For j = 2 or 3, Δj = 1 the condition ΔE ≤ kT is well satisfied for N₂H⁺ and HCO⁺ near or above 30 K and thus the relative short range contribution to these transitions is somewhat greater compared to 0—1 transition. This argument is not valid for larger j and at very low temperatures and the relative contribution of the short range interactions to the rate coefficient may decrease with increasing j.

Since the dipole moment and internuclear distances of N₂H⁺ are similar to those of the neutral molecule HCN, it is interesting to compare the effect of electron collisions on ionic and neutral molecular species by comparing the rate coefficients for neutral molecules and molecular ions. The rate coefficients for e-HCN collisions have been calculated in Chapter III. The HCN rates are lower than those of N₂H⁺ by only about 10 – 15% at 100 K but are smaller by a factor of 5 to 6 at low temperatures (5K to 10K). This is due to the fact that at low energies the effect of the attractive Coulomb interaction for electron-ion collisions is more important. The effect of abiaticity of the collisions is decreased by the attractive field and therefore the effect will be greater for more adiabatic collisions i.e. for higher energy transfers. Thus the difference between the HCN and the N₂H⁺ rates increases with increasing initial j.
Now we compare the effectiveness of electrons and molecular hydrogen in exciting the rotational levels of molecular ions. This will enable us to determine the range of electron concentrations in which electron-ion collisions should be considered in addition to H₂-ion collisions to determine the population distribution of the rotational levels of the ion in the interstellar cloud. We consider the case of N₂H⁺ since a close-coupling calculation has been done for the rate coefficient of H₂—N₂H⁺ collisions (Green, 1975) from an ab-initio potential energy surface. It is seen that the ratio of the two rates changes from $1.4 \times 10^5$ at 5K to $6.7 \times 10^4$ at 40K for the 0—1 transition in N₂H⁺. For the 1—2 transition the ratios are $9.2 \times 10^4$ and $4.4 \times 10^4$ respectively. Thus the electron-ion collisions will be significant compared to that of H₂-ion collisions for electron concentrations of $10^{-5}$ times that of H₂ or larger. Similar values for neutral HCN molecule has been obtained for optically thin clouds by Dickinson et al (1977) and for optically thick clouds in Part II (Chapter III). The transition rates for HCO⁺ are somewhat higher than for N₂H⁺ but the relative importance of the electron and hydrogen collisions should be of the same order of magnitude.

An estimate of the ratio of electron to H₂ abundance in some interstellar clouds has been given by Wootten et al (1979) from a model of gas phase ion-molecule reactions. Their method can deal with both dense cool clouds and the warmer ones. On the basis of the observational data available on H₂, HCO⁺ and CO densities they deduce that the relative electron densities are less than $10^{-7}$ for a large number of clouds where these ions are observed. These values are consistent with those obtained from a study of deuteration of HCO⁺ (Watson et al 1978),
applicable only to cold clouds. Thus for those clouds listed the
effect of electron collisions on excitation of ionic rotational lines
will not be important. However, at the centre of Orion A where these
molecular lines have been observed (Turner and Thaddeus 1977), the
electron concentration was estimated to be much higher. Near the
centre a mean value of $8 \times 10^{-5}$ for the electron to $H_2$ ratio has been
estimated in Part II (Chapter III) from the consideration of HCN and
$^{13}CO$ excitation. A value greater than $5 \times 10^{-4}$ for Orion clouds was also
suggested from a similar argument by Philips and Huggins (1977). Thus
at the centre of Orion A cloud where both HCO$^+$ and $N_2H^+$ have relatively
high brightness temperatures, the effect of electron ion collisions
seems to be important.

For $H_2 - N_2H^+$ close-coupling calculations the $|\Delta j| = 2$ rates are
almost as important as the $|\Delta j| = 1$ rates. They differ by a factor of
2 to 3. Our estimate of the $|\Delta j| = 2$ rates are very approximate and
numerical values cannot be taken too seriously. However, in our model
the $\Delta j = 2$ rates come out to be lower than the $\Delta j = 1$ rates by a
factor 6 to 7 for initial $j = 0$. 

Table - 6.

The values of dipole moment $\mu$, rotational constant $B$ and $R_c$ (the internuclear distance) of $\text{CH}_2^+$, $\text{N}_2\text{H}^+$ and $\text{HCO}^+$.  

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\text{CH}_2^+$</th>
<th>$\text{N}_2\text{H}^+$</th>
<th>$\text{HCO}^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_c$ (Å)</td>
<td>$1.23^a$</td>
<td>$2.13^b$</td>
<td>$2.20^b$</td>
</tr>
<tr>
<td>$\mu$ (Debye)</td>
<td>$1.70^c$</td>
<td>$3.50^b$</td>
<td>$4.07^b$</td>
</tr>
<tr>
<td>$B$ (cm$^{-1}$)</td>
<td>$13.9^c$</td>
<td>$1.55^d$</td>
<td>$1.49^d$</td>
</tr>
</tbody>
</table>

a) Huber and Herzberg (1979)  
b) Haese and Woods (1979)  
c) Chu and Dalgarno (1974)  
d) Turner and Thaddeus (1977)
Table - 7.

Rate Coefficients (dipolar, short-range and total) in $10^{-7} \text{ cm}^3 \text{ sec}^{-1}$ of CH$^+$, N$_2$H$^+$ and HCO$^+$ at some representative temperatures in collision with electrons. The upper value is the rate coefficient for 0 - 1 transition and the lower value is that for 1-2 transition.

<table>
<thead>
<tr>
<th>T(K)</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>500</th>
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<tbody>
<tr>
<td>Dipolar</td>
<td>1.24</td>
<td>6.17</td>
<td>13.0</td>
<td>14.1</td>
<td>10.4</td>
</tr>
<tr>
<td></td>
<td>.02</td>
<td>.57</td>
<td>3.82</td>
<td>6.05</td>
<td>5.64</td>
</tr>
<tr>
<td>Short Range</td>
<td>1.22</td>
<td>6.33</td>
<td>13.3</td>
<td>14.0</td>
<td>8.62</td>
</tr>
<tr>
<td>CH$^+$</td>
<td>.02</td>
<td>.57</td>
<td>.39</td>
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<td>5.25</td>
</tr>
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<td>26.3</td>
<td>28.1</td>
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<td></td>
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<td>23.6</td>
<td>17.4</td>
<td>8.09</td>
</tr>
<tr>
<td>N$_2$H$^+$</td>
<td>16.0</td>
<td>17.7</td>
<td>14.7</td>
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<td>HCO$^+$</td>
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<td>4.32</td>
</tr>
<tr>
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