Chapter III

The Model

3.1.1 A simple two-sector model is formulated to demonstrate the proposed approach (explained in Chapter II) which attempts to be a synthesis of the relevant elements from the dualistic and non-dualistic models. The model formulated though a simple one, attempts to incorporate many of the important aspects relating to the growth process of a developing economy.

In general, under-developed economies are complex in structure and nature. Hence, any model built to represent adequately all the relevant features of such an economy would become a complicated one. It is attempted here to make the proposed model as simple as possible without compromising its basic nature.

3.2.1 The proposed model is formulated for an economy having the following broad features:

(i) The economy is a bi-sectoral one and the sector-division is on the basis of technological levels which are also accompanied by specific organisational characteristic. As an indicator of the technological level, net value added per unit of input cost is used. This is defined as the input productivity of the sector concerned. The logic behind this lies in the fact that for additional production of any product, the economy requires additional inputs. The availability of this additional input depends on the excess of output over requirements. This in value terms is the net value added of a sector. This, however, gives the main impetus to the economic growth of the sector. Input-cost involved in this respect gives the level of technical know-how in the sector. The ratio used to define input productivity may be
modified by bringing the subsistence requirements of the population in the sector into the picture. This is done by subtracting the subsistence requirement of the population in the sector from the net value added obtained. If one does not take account of this factor, the ratio for the subsistence sector may be higher than that for the advanced sector as a great deal of the surplus is used to feed the labourers in the sector contributing nothing or a very small amount to production.

The sector division proposed here is notionally at least different from the sector divisions which have been used earlier, such as the agricultural industrial backward - advanced, traditional - modern, consumption-investment, rural - urban, private - public sector divisions.

Though a simple two-sector model is formulated here, the model may be extended to as many sectors as required applying the same sector division criterion.

(ii) The economy is closed to international trade in the sense that it depends on itself for all the required inputs and labourforce and also for marketing its products. But this restriction may be relaxed and the economy may be opened to international trade in respect of its input requirements, labour requirements and marketing of surplus products.

(iii) The products of both the sectors are treated as final products as well as intermediate products, unlike as in Ozawa's and some other models where the product of the first sector is consumed and that of the second is invested, or as in Lewis, Ranis-Fei and Jorgenson model where the product of the first sector is completely consumed and that of the second sector is partially consumed and partially invested. In this
respect the model has a similarity with Leontief's input-output frame
work or Sraffa's scheme of production of commodities by means of com-
dities, where, the products of all the sectors are both final products
and intermediate products.

(iv) The surplus available in this economy is total commodities
produced minus the total commodities required in the production process
in value terms. This is more or less the same as the physiocratic
'product net' in a different guise. Or, if subsistence requirements of
the population is taken into account, this surplus is the net value
added net of subsistence requirements. Whatever is the procedure of
deriving this surplus, it is directly injected into the economy as inputs.*
This is the same as in Sraffa model. In Lewis, Ranis-Fei and Jorgenson
model the surplus is available in the form of agricultural commodities
and is transformed into wage goods and injected into the economy in the
form of wage goods. The process is an indirect one.

(v) The economy may be analysed in terms of the input-output
relationships. Every commodity is produced by some other commodity or
commodities. This logic is the main concern of Sraffa's production of
commodities by means of commodities. The analysis of an economy in
terms of input-output relations is first found in Quesnay's tableau
economique and latter established by Wassily Leontief in a clearer fashion.

(vi) As in the production of commodities by means of commodi-
ties, the economy has no capital function or savings function or invest-
ment function in the usual sense as capital may be viewed as a bunch of

* Input here implies physical input as well as generalised capital.
commodities which are produced by some other commodities. Instead, it incorporates a new term generalized capital which includes physical capital, human ingenuity and social and technological organizational structure. Contribution to this generalized capital depends on total net value added available in the economy in excess of the subsistence consumption.

(vii) This generalized capital is distributed between the sectors and the distribution is reflected in the allocation parameter, which finds a place specifically in the model. The allocation parameter may be fixed exogeneously as in Mahalanobis' model and in many other planning models or may be determined endogenously as a function of the endogenous variables. By and large the proposed model depends on the second rather than the first alternative and how this is done is explained below.

(viii) The relative bargaining power plays a significant role in this economy. In fact, it is the bargaining power that crucially affects the production environment and the effective allocation of the generalized capital between the two sectors. The higher the bargaining power a sector has, the higher is the generalized capital allocated and consequently, the higher is the rate of growth of the sector.

Though the concept of bargaining power is new in development economies, it is found in labour economics and price determination theories. In this respect, as already noted in the earlier chapters, one may mention the name of Michael Kalecki who used this concept in his 'Economic Dynamics' explicitly.

(ix) The total population and the total labour force is determined by the exogenous forces in this economy. As regards this assumption,
the economy is closer to the views of Lewis, Ranis-Fei, Jorgenson and Sraffa. The sector with high productivity employs labour required in the production process and the rest goes in the residual sector initially. But with economic development both the sectors employ labour according to their requirements and the residual remains as unemployed. The initial stage of this economy is similar to that of Lewis, Ranis-Fei and Jorgenson model.

(x) The wage rate in the high productivity sector is determined by the productivity of labour and the price ratio between the sectors. This implies the existence of trade unions and bargaining power in the high productivity sector. With the progress of the economy, a similar picture may be observed in the low productivity sector also but initially the wage rate in the second sector is taken to be determined by the supply of labour in this sector as in many other dual models.

(xi) The terms of trade in the economy reflect the relative bargaining strengths of the two sectors. It is not just a reflection of the relative demand and supply of the two commodities by the two sectors as in Lewis' model.

(xii) Price in the high productivity sector is determined as an average unit cost plus a mark-up element. It is not determined by the marginal law as in price theory analysis. According to the marginal productivity theory of wages, wage rate is determined by the marginal physical productivity of labour.

3.2.2 The formulation of model for such an economy with an attempt to incorporate, as mentioned earlier, many crucial aspects related to the growth process of a developing economy would result in a highly complex system. Therefore an attempt is made to begin with a model structure that incorporates only the most basic features of the growth process of such an
economy. Later, an attempt is made to extend the model and incorporate different variations. The basic model is enunciated in what follows.

3.3 The following notations are used in the model:

Sector 1 = High Productivity sector
Sector 2 = Low productivity sector
t = t th period of time.

Subscripts 1, 2 denotes sector 1, 2 respectively.

\[ V_r = \text{Net value added in sector } r \]
\[ r = 1, 2 \]
\[ V = V_1 + V_2 = \text{total net value added of the economy}. \]
\[ V' = V - (S_1 + S_2) = \text{total net value added of the economy after deduction of subsistence requirements}. \]
\[ X_r = \text{Output of sector } r \]
\[ r = 1, 2 \]
\[ X = X_1 + X_2 = \text{Total output of the economy} \]
\[ Y_r = \text{Net output of the economy} \]
\[ r = 1, 2 \]
\[ Y = Y_1 + Y_2 = \text{Total net output of the economy} \]
\[ X_{rs} = \text{Product of the } r\text{th sector required in the production of the } s\text{th sector}. \]
\[ r = 1, 2 \quad , \quad s=1,2 \]
\[ C_r = \text{Input cost of sector } r \]
\[ r = 1, 2 \]
\[ S_r = \text{Subsistence allowance in sector } r \]
\[ r = 1, 2 \]
\[ \hat{V}_r = \frac{V_r}{C_r} = \text{Input productivity of sector } r \]
\[ r = 1, 2 \]
\[ v = \frac{v_1}{v_2} \]

= Ratio of input productivities of the two sectors.

\[ v' = \frac{v - s_r}{c_r} \]

= Input productivity of sector \( r \) allowing for subsistence

\[ r = 1, 2 \]

\[ v' = \frac{v'_1}{v'_2} \]

= Ratio of sectoral input productivities when allowance for subsistence is made.

\[ y_r = \frac{y_r}{x_r} \]

= Net value added per unit of output in sector \( r \).

\[ r = 1, 2 \]

\[ y'_r = \frac{y'_r}{x'_r} \]

= Net value added as per unit of output when allowance for subsistence is made.

\[ L_r \]

= Labour required in sector \( r \).

\[ r = 1, 2 \]

\[ a_r \]

= Labour productivity in sector \( r \).

\[ r = 1, 2 \]

\[ N \]

= Total population

\[ N' \]

= Total labour force

\[ w_r \]

= Wage rate in sector \( r \)

\[ r = 1, 2 \]

\[ w = \frac{w_1}{w_2} \]

= Wage ratio between the sectors

\[ p_r \]

= Price of the product of sector \( r \)

\[ r = 1, 2 \]

\[ p \]

= Price level of the economy

\[ m \]

= Mark-up co-efficient

\[ e_r \]

= Generalised capital in sector \( r \)

\[ r = 1, 2 \]

\[ e = e_1 + e_2 \]

= Total generalized capital of the economy

\[ \lambda \]

= Allocation parameter of the economy

\[ b \]

= Relative bargaining strength of the sectors.
\[
\begin{align*}
\phi & = \text{Growth rate of population} \\
\beta & = \text{Growth rate of labour force} \\
\Theta & = \text{Growth rate of the economy} \\
\Theta_r & = \text{Growth rate of the } r\text{th sector} \\
& \quad r = 1, 2 \\
\cdot & = \text{dot over a variable - symbol representing differentiation with respect to time.}
\end{align*}
\]

3.4.1 Model I: The Basic Model

The following relationships represent the basic model:

3.4.1 The most crucial variables of the model are net value added which is more or less the physiocratic 'product net' in a different guise. The net value added is defined in the usual way, by substracting input costs from the total value of the products. The greater the difference, the greater is the absolute net value added. These net value added equations are definitional equations. Hence, the net value added equations are:

\[
\begin{align*}
V_{1t} & = P_{1t} X_{1t} - (P_{1t} X_{11t} + P_{2t} X_{21t}) \quad \cdots \quad (1) \\
V_{2t} & = P_{2t} X_{2t} - (P_{1t} X_{12t} + P_{2t} X_{22t}) \quad \cdots \quad (2) \\
V_t & = V_{1t} + V_{2t} \quad \cdots \quad (3)
\end{align*}
\]

The input costs are:

\[
\begin{align*}
C_{1t} & = P_{1t} X_{11t} + P_{2t} X_{21t} \quad \cdots \quad (4) \\
C_{2t} & = P_{2t} X_{22t} + P_{1t} X_{12t} \quad \cdots \quad (5)
\end{align*}
\]

3.4.2 The input productivities are the next important variables. These are defined in the present context as absolute net value added of the sector divided by the input costs of the sector. These variables are important in the present study as the sector division is based on this
criterion. The input productivities are:

\[ v_{1t} = \frac{V_{1t}}{C_{1t}} \quad \cdots \quad (6) \]

\[ v_{2t} = \frac{V_{2t}}{C_{2t}} \quad \cdots \quad (7) \]

and the intersectoral productivity ratio is:

\[ v_{t} = \frac{v_{1t}}{v_{2t}} \quad \cdots \quad (8) \]

Alternatively, if subsistence requirement of the population in the sector is subtracted from the net value added in both the sectors, the input productivity relations are modified as follows:

\[ v'_{1t} = \frac{V_{1t} - S_{1t}}{C_{1t}} \quad \cdots \quad (6') \]

\[ v'_{2t} = \frac{V_{2t} - S_{2t}}{C_{2t}} \quad \cdots \quad (7') \]

\[ v'_{t} = \frac{v'_{1t}}{v'_{2t}} \quad \cdots \quad (8') \]

3.4.3 Another set of new variables are introduced here, the net value added per unit of output in the sector. They are ratios of the net value added in a sector and output produced in that sector. These are given by:

\[ \gamma_{1t} = \frac{V_{1t}}{X_{1t}} \quad \cdots \quad (9) \]

\[ \gamma_{2t} = \frac{V_{2t}}{X_{2t}} \quad \cdots \quad (10) \]

\[ \gamma_{t} = \frac{\gamma_{1t}}{\gamma_{2t}} \quad \cdots \quad (11) \]
Alternatively, if subsistence requirements of the population in the sector is subtracted from the net value added in both the sectors, the above relations reduces to

\[ \gamma'_{it} = \frac{V_{it} - S_{1t}}{x_{1t}} \quad \ldots \quad (9') \]
\[ \gamma'_{2t} = \frac{V_{2t} - S_{2t}}{x_{2t}} \quad \ldots \quad (10') \]
\[ \gamma'_{t} = \frac{\gamma'_{1t}}{\gamma'_{2t}} \quad \ldots \quad (11') \]

3.4.4 The model has a new set of variables for generalized capital which includes along with the stock of physical capital, the human ingenuity factor and the techno-social organizational factor. All net value added (apart from the subsistence fund in the modification) is taken to contribute to the formation of the generalized capital. Hence, the total generalized capital is taken to be determined by the total net value added in terms by the relationship

\[ \dot{E}_t = f_1 \left( \frac{V_t}{s_t} \right) \quad \ldots \quad (12) \]

where

\[ \dot{E}_t = \frac{dE}{dt} \]

and \( f_1(z) = 0 \)

when \( z = 0 \)

and \( \frac{df_1(z)}{dz} \geq 0 \).

Here, \( E_t = E_{1t} + E_{2t} \) \quad \ldots \quad (13)

When subsistence allowance is made, relation (12) reduces to

\[ \dot{E}_t = f_1 \left( \frac{V_t}{s_t} \right) \quad \ldots \quad (12') \]
This is distributed between the sectors by the allocation parameter \( \lambda_t \). The allocation parameter \( \lambda_t \) is taken to be a function of the relative bargaining power of the two sectors in respect of generalized capital. The relative bargaining power in turn is taken to be a function alternatively of (i) sectoral productivity ratio (ii) sectoral labour productivity ratio, (iii) the sectoral generalized capital ratio. The allocation parameter may alternatively, be exogenously fixed and may be a controlled parameter as in the case of Mahalanobis' model.

The sectoral generalized capital extensions and allocation parameter relationships are taken as:

\[
\begin{align*}
\dot{E}_{1t} & = \lambda_t \dot{E}_t \quad \ldots \quad (14) \\
\dot{E}_{2t} & = (1 - \lambda_t) \dot{E}_t \quad \ldots \quad (15) \\
\text{where, } \lambda_t & = f_2 (b_t) \quad \ldots \quad (16)
\end{align*}
\]

3.4.5 Relative bargaining power of the two sectors depends on the market situation, their (sectors') relative production abilities, organisational power, relative surpluses generated in the sectors, relative productivities of labours in the two sectors etc. Since no independent estimate of relative bargaining power is available, sectoral input productivity-ratio or sectoral labour productivity-ratio or sectoral generalized capital ratio may be taken as alternative proxies for the relative bargaining strength.

Thus as the first alternative we may take \( b_t \) as the ratio between the sectoral input productivities

\[
b_t = f \left( \frac{v_{1t}}{v_{2t}} \right) = f (v_t) \quad \ldots \quad (17a)
\]
Taking $f$ as an identity function (17a) reduces to -

$$b_t = v_t \quad \ldots \quad (17'a)$$

One may alternatively take net value added per unit of output as a proxy for the sectoral productivity and hence, relative bargaining strength between the two sectors may be taken to be a function of a ratio between these.

$$b_t = f \left( \frac{y_{1t}}{y_{2t}} \right) \quad \ldots \quad (17'b)$$

Taking $f$ as an identity function (17b) reduces to -

$$b_t = \frac{y_{1t}}{y_{2t}} = y_t \quad \ldots \quad (17'b)$$

When allowance for subsistence requirements are made (17a) and (17b) and consequently, (17'a) and (17'b) reduces to

$$b_t = f \left( v_t^' \right) \quad \ldots \quad (17'c)$$

when $f$ is taken as an identity function reduces to -

$$b_t = v_t^' \quad \ldots \quad (17'c)$$

and

$$b_t = f \left( \frac{y_{1t}}{y_{2t}} \right) = f \left( y_t^' \right) \quad \ldots \quad (17'd)$$

when $f$ is an identity function reduces to -

$$b_t = \frac{y_{1t}}{y_{2t}} = y_t^' \quad \ldots \quad (17'd)$$

Alternatively, one could take the relative bargaining power to be a function of ratio between labour productivities of the two sectors,

$$b_t = f \left( \frac{\alpha_{1t}}{\alpha_{2t}} \right) \quad \ldots \quad (17'e)$$
which when $f$ is taken as an identity function reduces to -

$$b_t = \frac{a_{1t}}{a_{2t}} \quad \ldots \quad (17'e)$$

It may also be expressed as a function of the generalized capital ratios between the two sectors

$$b_t = f\left(\frac{E_{1t}}{E_{2t}}\right) \quad \ldots \quad (17'f)$$

which when $f$ is taken as an identity function reduces to -

$$b_t = \frac{E_{1t}}{E_{2t}} \quad \ldots \quad (17'f)$$

Whether the relation $(17a)$, $(17b)$, $(17c)$, $(17d)$, $(17e)$ or $(17f)$ will be taken as a proxy for $b_t$ depends on the nature of the economy concerned and purpose of the study.

3.4 Input requirements in this model are functions of outputs and the generalized capital in the sectors, incorporating both scale economies or diseconomies and technical progress dependent on the generalized capital. This is a generalization of Leontief's input coefficients which are fixed. In most of the economic growth and production models inputs are the functions of outputs only and in general they are fixed.

The functional relationships between the input requirements are such that as the output of a sector increases, input requirements increase and as generalized capital increases in a sector, the input requirements are expected to be reduced as more of generalized capital implies higher level of technical progress, higher level of organization and higher level of human ingenuity.
\[ x_{1lt} = \alpha_{11} (x_{1t}, E_{1t}) \]
\[ x_{2lt} = \alpha_{21} (x_{1t}, E_{1t}) \]
\[ x_{12t} = \alpha_{12} (x_{2t}, E_{2t}) \]
\[ x_{22t} = \alpha_{22} (x_{2t}, E_{2t}) \]

where,
\[ \frac{\delta x_{1lt}}{\delta x_{1t}} > 0, \quad \frac{\delta x_{1lt}}{\delta E_{1t}} < 0 \]
\[ \frac{\delta x_{2lt}}{\delta x_{1t}} > 0, \quad \frac{\delta x_{2lt}}{\delta E_{1t}} < 0 \]
\[ \frac{\delta^2 x_{1lt}}{\delta x_{1t}^2} < 0, \quad \frac{\delta^2 x_{1lt}}{\delta E_{1t}^2} < 0 \]
\[ \frac{\delta^2 x_{2lt}}{\delta x_{2t}^2} < 0, \quad \frac{\delta^2 x_{2lt}}{\delta E_{2t}^2} < 0 \]

3.4.7 Growth of output of any sector depends on the growth of the generalized capital extension of that sector. This reflects the contribution of generalized capital to output. This takes the place of output-capital relationship of Harrod-Domar and many other growth models. Though this relationship the impact of human ingenuity, technical progress and

* For typing difficulties \( \delta \) is used for partial derivatives in place of \( \partial \).
organizational power is also brought in in this structure. The output-
generalized-capital-relations are given by -

\[ \dot{x}_{1t} = g_1 \left( \dot{e}_{1t} \right) \]  \hspace{1cm} (22)

where \( g_1 \geq 0 \) and \( g_1(z) = 0 \)

when \( z = 0 \)

\[ \dot{x}_{2t} = g_2 \left( \dot{e}_{2t} \right) \]  \hspace{1cm} (23)

where \( g_2 \geq 0 \) and \( g_2(z) = 0 \)

when \( z = 0 \)

3.4.8 Labour productivity in this model is defined as the ratio of sectoral net value added and labour employed in the sector which gives additional net value added per unit of labour employed in the sector. In most of the models gross labour productivity is defined as the ratio of the total output and labour employed in the sector is used for labour productivity.

The labour productivity relations are -

\[ a_{1t} = \frac{v_{1t}}{l_{1t}} \]  \hspace{1cm} (24)

\[ a_{2t} = \frac{v_{2t}}{l_{2t}} \]  \hspace{1cm} (25)

3.4.9 Population growth and hence labour force growth depends on a number of non-economic factors along with economic factors and it is not possible to control these growth rates by economic measures alone. Moreover, the control of population growth is a long-run process. Hence, in this model, population growth and supply of labour force are taken to be determined exogenously as in Sraffa's analysis and in many other economic development theories. This exogeneity of labour supply is also consistent with the Marxian concept of the 'industrial reserve army' of labour.
Total population of the economy and labour supply $N$ are taken to be an exogenous function of time. One would expect both $N_t$ and $N_t'$ to be growing. Therefore, $\beta$ and $\beta'$ to be $> 0$. The actual population and labour force relations assuming exponential growth may be taken to be given by:

$$N_t = N_0 e^{\beta t} \quad \ldots \quad (26)$$

$$\beta > 0$$

$$N_t' = N_0' e^{\beta' t} \quad \ldots \quad (27)$$

$$\beta' > 0$$

3.4.10 In this model, the high productivity sector employs labour which is required in the production process and the rest of the labour force goes to the residual sector initially. With economic development the situation changes and both the sectors begin to employ according to their requirements and the rest goes to the pool of unemployment. The labour requirements in their turn depend on the total outputs and total generalized capital employed in the sector. This replaces the fixed labour coefficients of many growth and development models. Economies or diseconomies of scale and technical progress are brought into these through the generalized capital.

The labour relations for the initial period may be expressed as:

$$L_{1t} = L_1 \left( X_{1t}, E_{1t} \right) \quad \ldots \quad (28)$$

where, $\frac{\delta L_{1t}}{\delta X_{1t}} \geq 0, \quad \frac{\delta L_{1t}}{\delta E_{1t}} \leq 0 \quad \ldots \quad (29)$

$$L_{2t} = N_t' - L_{1t}$$

Alternatively, with economic development when the second sector is also organized and it employs labour according to its production—
requirement only the labour relation for the second sector changes to -

\[ L_{2t} = L_2(X_{2t}, E_{2t}) \]

where,

\[ \frac{\partial L_{2t}}{\partial X_{2t}} \geq 0, \quad \frac{\partial L_{2t}}{\partial E_{2t}} \leq 0 \]  \hspace{1cm} (29)

and an unemployment relation \( U_t = N_t - (L_{1t} + L_{2t}) \) \hspace{1cm} (29's)
comes in along with (28) and (29').

The implications behind the relations (28) and (29) are that as the output of a sector increases, the labour requirement of that sector increases while with additional generalized capital in a sector the level of the technique of production used goes up higher and the resulting techniques may be more labour-saving so that with the greater availability of the generalized capital, labour requirement may decline.

These labour equations may be replaced by alternative formulations reflecting different setups. These will be discussed in a later part of this thesis.

3.4.11 The first sector in this model is the organized sector and has the power to keep the extra labour force outside this sector and the burden goes to the second sector and initially the second sector bears this burden of extra labour force. Since the labourers are also organised in the first sector and due to the activities of the trade unions prevalent in an organized sector, the labourers in this sector have bargaining power to some extent and hence the wage rate determined in this sector is a function of the price-ratio and labour productivity, while the wage rate in the second sector at the initial stage, 'when the sector is suffering from extra burden of labour force', is determined as a function
of labour available and labour productivity in that sector. When the
second sector becomes organised during the process of economic develop­
ment and is able to keep the burden of labour force outside the sector,
the labourers also, however, become organised and the wage rate is deter­
mined in a similar fashion as in the first sector.

In the determination of wage rates, the present model makes a
significant departure from other models. For example, in Lewis' model
and in Ranis-Fei model, the wage rates are institutionally fixed and in
the model of Jorgenson, they are equal to the marginal physical produc­
tivity of labour.

The wage relations used here are:

\[ w_{1t} = \phi_{1} \left( \frac{p_{1t}}{p_{2t}}, a_{1t} \right) \]  \hspace{1cm} (30)

where, \( \frac{\delta w_{1t}}{\delta \left( \frac{p_{1t}}{p_{2t}} \right)} \geq 0, \frac{\delta w_{1t}}{\delta a_{1t}} \geq 0 \)

\[ w_{2t} = \phi_{2} \left( a_{2t}, L_{2t} \right) \]  \hspace{1cm} (31)

where, \( \frac{\delta w_{2t}}{\delta a_{2t}} \geq 0, \frac{\delta w_{2t}}{\delta L_{2t}} \leq 0 \)

and in the later stages

\[ w_{2t} = \phi_{2} \left( \frac{p_{1t}}{p_{2t}}, a_{2t} \right) \]  \hspace{1cm} (32)

where, \( \frac{\delta w_{2t}}{\delta \left( \frac{p_{1t}}{p_{2t}} \right)} \geq 0, \frac{\delta w_{2t}}{\delta a_{2t}} \geq 0 \)
3.4.12 Terms of trade between two sectors as reflected by the ratio of prices of outputs of the two sectors has an important role in economic development literature. Terms of trade in the Lewis and Ranis-Fei model is determined by the relative demand and supply of the products of the two sectors. But in the present model terms of trade is taken to be determined as a function of the relative bargaining power between the sectors and the relative bargaining power may in turn be represented by a function of the different individual ratios, as mentioned in the relations (17). Hence the terms of trade may be assumed to be expressed as a function of

\[
T = \frac{P_{1t}}{P_{2t}} = f_3 \left( \frac{E_{1t}}{E_{2t}}, \frac{a_{1t}}{a_{2t}}, \frac{v_{1t}}{v_{2t}}, \frac{\gamma_{1t}}{\gamma_{2t}} \right)
\]  

(32)

3.4.13 The overall price level of the economy is determined by the usual formula for the price index with either base year weights or current year weights.

\[
P_t = \frac{P_{1t} X_{10} + P_{2t} X_{20}}{X_{10} + X_{20}}
\]  

or

\[
P_t = \frac{P_{1t} X_{1t} + P_{2t} X_{2t}}{X_{1t} + X_{2t}}
\]  

(33')

The price of the product of the first sector as explained in the last chapter is taken to be determined by the average unit cost of the product and a mark-up. In a perfect market situation prices are
determined by marginal productivity and supply and demand equilibrium considerations. Here the monopoly character of the first sector is taken to dominate the determination of the price of the first output and the price of the product in the first sector is given by the relation -

\[ p_{1t} = \left\{ \frac{p_{1t} x_{11t} + p_{2t} x_{21t} + \omega_{1t} l_{1t}}{x_{1t}} \right\} (1 + m) \] ...

(34)

It may, however, be noted that, in a centrally controlled economy the mark-up rate could be used as a control parameter along with the allocation parameter \( \lambda_t \).

3.4.14 The structure of the growth model of an under-developed economy is now complete. This model consists of 31 equations (equations (26) and (27) are not included as population growth and labour force growth are exogenous to the system and since equation (15) follows from equations (13) and (14), it may also be excluded) and 31 variables. Among these equations (1) to (8), (13), (24), (25) are definitional equations and other equations are technical or behavioural equations or a mixture of both. The 31 endogenous variables are \( X_{1t}, X_{2t}, Y_{1t}, Y_{2t}, V_{t}, U_t, E_{1t}, E_{2t}, E_{t}, \dot{L}_{1t}, \dot{L}_{2t}, \omega_{1t}, \omega_{2t}, \dot{W}_{1t}, \dot{W}_{2t}, \dot{E}_{1t}, \dot{E}_{2t}, \dot{E}_{t}, \dot{P}_{it}, \dot{P}_{2t}, \dot{P}_{t}, \dot{V}_{1t}, \dot{V}_{2t}, \dot{V}_{t}, \dot{\lambda}_{1t}, \dot{\lambda}_{2t}, \dot{\lambda}_{t}, \dot{\tau}_{1t}, \dot{\tau}_{2t}, \dot{\tau}_{t}, \dot{b}_{t} \).

Alternatively, one could include (26) and (27) in the system with the assumption that the population growth rate and the labour force growth rate are exogenously determined. Then \( N_t \) and \( N'_t \) are endogenous variables to the modified system. In this case, the model consists of 33 equations with 33 endogenous variables. In a more general model where population and labour force are taken to be influenced by the economic factors, equations (26) and (27) could be replaced by other suitable population and labour force determining equations.
3.5.1 The model has two parts which may be termed as the (1) Static part of the model, consisting of the relations which describe the one point behaviour of the economy and (2) the Dynamic part of the model, consisting of the relations which describe the multipoint behaviour of the economy.

3.5.2 Model IA - The Static Part

The static behaviour of the model is summarized by the relationships (1), (2), (3), (4), (5), (6) or (6'), (7), (7') or (8), (8') or (9), (9') or (9'), (10), (10'), (11) or (11') or (11'), (17a) or (17'a), (17b) or (17'b), (17c) or (17'c), (17d) or (17'd), (17e) or (17'e), (17f) or (17'f), (18), (19), (20), (21), (24), (25), (28), (29) or (29'), (30), (31) or (31'), (32), (33) or (33'), (34).

3.5.3 Model IB - The Dynamic Part

The dynamic behaviour of the model is summarized by the relationships (12) or (12'), (14), (15), (22), (23), (26), (27).

The first five equations when \( V_t \) is used to determine the ensuing points in the growth paths of \( X_{1t} \), \( X_{2t} \), \( E_{1t} \), \( E_{2t} \), \( E_t \) while when \( V_t^\prime \) is used, \( N_t \), \( N_t^\prime \) and \( S_{1t} \), \( S_{2t} \) would also be relevant for determining the paths. These in turn determine as per the static part \( V_t \), \( P_t \) and \( \lambda_t \) and the process continues. If the relative prices remain constant which can be regarded as an equilibrium version of the model when \( V_t/P_t \) and \( \lambda_t \) are simple functions of \( X_{1t} \), \( X_{2t} \), \( E_{1t} \), \( E_{2t} \) only and the growth paths can then be relatively easily worked out.

The explicit paths would depend on the specific functional forms involved in the static and dynamic parts and would be quite complicated.
even in the simplest of cases, including that of the case of constant relative prices. These paths are traced in some simple versions of the model later in Chapter VI of the thesis. Here some obvious characteristics of the growth paths are noted in section 3.7.

3.6.1 It is obvious from the model that all the relations in the static part of the model may be expressed in terms of $X_{1t}$, $X_{2t}$, $E_{1t}$, $E_{2t}$. The analytic solutions in the general case however are difficult to obtain. In the discrete time case the time derivatives are to be replaced by time differences and iterative methods can be used to construct the growth paths. In the continuous time case the time derivatives may be approximated by their time-differentials choosing a small enough time-interval between two consecutive points of time and the iterative method used to approximate the actual growth paths. Given the initial values of $X_{1t}$, $X_{2t}$, $E_{1t}$, $E_{2t}$ all the variables for the initial point of time are determined. With all the variables known for the initial point of time (period) $X_{1t}$, $X_{2t}$, $E_{1t}$, $E_{2t}$ for the instantaneously next point of time or for the next period are determined. Again, with these, all other variables for the next instant (period) are determined and the iterative process can be carried on further in a similar fashion.

As mentioned earlier with some simple specifications of the equilibrium (dynamic part with relative prices constant) specific solutions for the growth paths of the productions, net outputs, total net value added etc. are attempted in chapter VI.

Schematically, the solution to this model may be written as follows —
Where, the last subscript $i$, 0, 1 etc. denote the suitably chosen consecutive instants in continuous analysis or the actual periods in discrete analysis and $\Delta$ denotes the forward differences with respect to time.

3.7.1 From the dynamic part of the model, mentioned in section 3.5.3, some obvious characteristics of the growth paths of the model relating to the nature and behaviour of total net value added, allocation parameter, population growth rate etc. may be noted.

3.7.2 One of the major factors determining the growth paths in the economy according to this model is the total net value added of the economy. This, in turn, may be zero, positive or negative at any point
of time depending on the input-output and other relations involved. Each of these cases has a different kind of impact on the growth paths of the economy.

Case (i) - Total net value added is zero at the $t$ th point of time.

i.e. $V_t = 0$

$\Rightarrow \dot{E}_t = 0$

$\Rightarrow \dot{E}_{1t} = 0$

and $\dot{E}_{2t} = 0$

which in turn implies $\dot{X}_{1t} = 0$

and $\dot{X}_{2t} = 0$.

Therefore, it is obvious that if the total net value added of the economy is nil at any point of time, the economy is stagnant.

Case (ii) - Total net value added is positive at the $t$ th point of time.

i.e. $V_t > 0$

$\Rightarrow \dot{E}_t > 0$.

(a) Both $\dot{E}_{1t}$ and $\dot{E}_{2t} > 0$, if $\lambda_t \neq 0$ or 1

$\Rightarrow \dot{X}_{1t}$ and $\dot{X}_{2t} > 0$.

(b) $\Rightarrow \dot{E}_{1t} = 0$ but $\dot{E}_{2t} > 0$ if $\lambda_t = 0$

$\Rightarrow \dot{X}_{1t} = 0$ and $\dot{X}_{2t} > 0$.

(c) $\Rightarrow \dot{E}_{2t} = 0$ but $\dot{E}_{1t} > 0$, if $\lambda_t = 1$

$\Rightarrow \dot{X}_{2t} = 0$, $\dot{X}_{1t} > 0$. 
In all the three cases (a), (b), (c) the economy is growing with different sectoral growth rates. Actually, however, the growth process may be interrupted in the cases (b) and (c) due to shortage of required inputs from the stagnant sectors. In general if the gap between the growth rates of the two sectors is large, the growth rate of the growing sector in particular and the growth rate of the economy in general may be hampered by the shortage of required inputs from the lagging sector. But if the gap between the two sectors is narrower, the growth may be more even and unhindered and hence in the long run greater. All these aspects will be discussed in the later part of the dissertation in greater detail.

Case (iii) - Total net value added may be negative at the t th point of time.

\[ V_t < 0 \]

and hence \( \dot{E}_t < 0 \).

(a) Both \( \dot{E}_{1t} < 0 \) and \( \dot{E}_{2t} < 0 \)

and \( \dot{X}_{1t} \leq 0, \text{ if } \lambda_t \neq 0 \text{ or } 1 \)

and \( \lambda_t > 0 \).

(b) \( \Rightarrow \dot{E}_{1t} = 0, \dot{E}_{2t} < 0 \)

\( \Rightarrow \dot{X}_{1t} = 0, \text{ and } \dot{X}_{2t} < 0, \text{ if } \lambda_t = 0 \).

(c) \( \Rightarrow \dot{E}_{1t} < 0, \dot{E}_{2t} = 0 \)

\( \Rightarrow \dot{X}_{1t} < 0, \text{ and } \dot{X}_{2t} = 0, \text{ if } \lambda_t = 1 \).

In all the three cases (a), (b), (c) the economy is declining with different sectoral growth rates. In case (a) both the sectors are declining while in cases (b) and (c) one sector is declining and the
other is stagnating.

Whatever may be the situation, (a), (b) or (c), it cannot con-continue for a long time. If some pressures in the opposite direction do not develop to reverse the direction of growth rate, some pressure must operate to make the economy stagnant.

Normally one expects, as history has borne out, that at least after some considerable lapse of time, pressures to reverse the decline develop and the precipitating decline is arrested and perhaps is later turned into positive growth.

3.7.3 The total generalized capital extended in the economy at any point of time is distributed between the sectors according to the value of the allocation parameter which may be determined endogenously or exogenously or may be treated as a controlled variable fixed by the central authority. However the allocation parameter is determined, the growth process will be determined by the different values assumed by the allocation parameter.

Case (i) - As noted earlier, if total generalized capital extended is allocated entirely to the first sector and the entire growth of the economy comes from the first sector.

But this may sooner or later affect the growth of the first sector as the input requirements of the first sector in the form of the output of the second sector falls short of the supply as the second sector is stagnant. Hence attempts may be made either to change the allocation parameter in favour of the second sector or to make the first sector use some input substitutes for the product of the second sector. This problem will be taken up in the later part of this thesis again and will be discussed in greater detail.
Case (ii) On the contrary if total generalized capital extended is allocated entirely to the second sector, the entire growth of the economy comes through the second sector only.

The problem of input shortage in this situation may crop up sooner or later and the growth of the second sector and the economy is limited by this shortage problem.

The first situation is more probable than the second one as by assumption the first sector is characterized by a high level of technology, higher organizational capacity, monopoly strength etc. and hence the bargaining strength of this sector is relatively higher than the second one. Consequently, in an uncontrolled economy this sector may pull the allocation parameter in its favour and hence obtain the major share of the generalized capital.

The second situation is probable in a controlled economy where the central authority lays great emphasis on the development of the second sector.

3.7.4 In the present model, population growth rate and labour force growth rate are taken as exogenous to the economic system. It is, however, interesting to note the impact of the equality or inequality relation between the population and/or labour force growth rate and economic and/or sectoral growth rates of the economy on the growth process of the economy. In this respect one may distinguish between four different situations.

Case (i) - The population growth rate is higher than the overall economic growth rate i.e.

\[ \frac{\dot{x}_t}{x_t} > \delta \]
This has two possible outcomes - The total net value added available for the formation of generalized capital is reduced as more is required as subsistence allowance, where \( V'_{t} \) and not \( V_{t} \) is taken to contribute to the formation of generalized capital. This evidently pulls down the growth rate of the economy. The other possible outcome of such outstripping population growth is the increasing burden of unemployment and disguised unemployment in the economy.

Case (ii) - Labour force may grow more rapidly than at the economic growth rate. This usually is a concomitant of population growth. There may, however, be some short-term situations in which population may grow without labour force growing substantially. This may happen where the age-distribution changes for some reason or the other. Labour force growing more rapidly than at the economic growth rate would imply that labour force grows more than required. This leads to a great burden of unemployment (disguised or explicit) on the economy. A major portion of the net value added may get used up in feeding the unemployed labour force. Hence less is available for the formation of generalized capital. Hence the growth rate of the economy deteriorates.

In terms of the symbols of the model, this situation may be expressed as follows -

\[
\frac{\dot{x}_t}{x_t} > \frac{\dot{V}_t}{V_t}
\]

\[
\Rightarrow \quad \frac{\dot{N}_t}{N_t} > \frac{L_{1t} + L_{2t}}{L_t} \quad (L_t \text{ includes the disguisedly unemployed also})
\]

\[
\Rightarrow \quad \frac{\dot{V}_t}{V_t} > 0
\]

\[
\Rightarrow \quad \frac{\dot{L_t}}{L_t} < 0 \quad \text{after a stage}
\]

\[
\Rightarrow \quad \frac{\dot{E}_t}{E_t} < 0
\]

\[
\Rightarrow \quad \frac{\dot{X}_t}{X_t} < 0
\]
Case (iii) - Labour force may grow more rapidly than the low productivity sector and less rapidly than the high productivity sector, i.e.

\[
\frac{\dot{x}_{1t}}{x_{1t}} > \phi' > \frac{\dot{x}_{2t}}{x_{2t}}
\]

If labour is completely immobile between the sectors the first sector would suffer with the shortage of labour and the second sector may face some problem of unemployment or disguised unemployment.

If labour is a completely mobile factor, the first sector may not suffer from shortage of labour and the burden of unemployed or disguisedly unemployed in the second sector may be reduced.

If extra labour force is completely absorbed by the first sector in the process of economic growth, the burden gets completely eliminated.

In this situation, the possibility of the total labour requirement of the economy being greater than or equal to the total labour available may also arise.

Case (iv) - The fourth possible situation may be one where labour force growth rate is lower than the growth rate of both the sectors or the growth rate of the economy as a whole i.e.

\[
\phi' < \frac{\dot{x}_{1t}}{x_{1t}} < \frac{\dot{x}_{2t}}{x_{2t}}
\]

or, \( \phi' < \frac{\dot{x}_{t}}{x_{t}} \).

In this situation the demand for labour force may not ultimately be met by the supply of the labour force. Hence, economic growth is limited by the shortage of labour force and the growth rate may come to
be circumscribed by the growth rate of the labour force. This may build up pressures to evolve labour-saving technology and the economic growth rate may be altered.

These different situations relating to population-growth may in actuality be the ones faced by different developing economies. This aspect will be taken up again in the later part of this thesis.

3.8 Some of the points to be noted about the model may be summarized as follows -

(i) An immediate advantage of this model over many other sector division models is the flexibility in the number of sectors as it is a condensed form of Leontief's input-output system. It can be extended to any number of sector divisions to incorporate the pluralistic structure of an economy. It can also be treated as an aggregative one sector model. An attempt made to formulate one sector and multisector models in the later parts of this dissertation.

(ii) This model may be used to analyse the growth paths of different sectors of either a developing or a developed economy (with price and wage-relations suitably altered), unlike some of the other growth models which are intended either to analyse the growth paths of a developing economy or a developed economy only through in the present form it is built for the former as a synthesis of both, incorporating the elements of both dualistic and nondualistic models.

(iii) The model may be transformed into a controlled of planning model treating the allocation parameter and the mark-up coefficient as controlled variables.
(iv) The model may be turned into an econometric model and estimated.

(v) This model may be applied to analyse the inter-country growth paths of two or more countries differing in their bargaining strengths.

(vi) In most of the growth models 'capital' plays a crucial role, but the distinction between transitory reproducible goods and consumption goods is not necessary in many cases. The proposed model does not enter into the controversial concept of capital. Instead it incorporates a new concept of generalized capital which enables it to incorporate human ingenuity factor and social, technological, organisational structure of the economy alongwith physical capital into the growth model.

(vii) Labour inputs and commodity inputs being functions of output and generalized capital of the relevant sector allows for the impact of technological progress and of varying returns to scale on input requirements, as the development process proceeds. This is not possible when one has fixed input coefficients.

(viii) The role of relative bargaining power in the divergence of the growth paths of the different sectors of a developing economy is taken note of through its impact on the allocation of the generalized capital to the two sectors and the terms of trade between the two sectors. It is also brought in in the formation of the prices and wages in the first sector.

3.9 In the next two chapters some variants of the model will be considered and in chapter VI the growth paths of the model in some of its simple version will be traced.