Chapter I

INTRODUCTION

Serious studies of blast wave problems were initiated by G.I. Taylor (1950) and L.I. Sedov (1959).

Blast waves are generated due to an explosion in a medium whose law of variation of density is known beforehand. The physical picture is like this. Some explosives are deposited at a point or in a line. Explosion occurs. Huge amount of energy is released from this idealized point or idealized line in all directions. Pressure rises very rapidly in all directions. As a result the disturbed portion of the fluid is bounded in all sides by a discontinuous surface over which some conservation laws are valid. This discontinuous surface is called the shock surface and the conservation laws give rise to Riemann-Hugoniot relations which act as one set of the boundary conditions in the blast wave problems. Now if we assume spherical symmetry the resulting shock surface is spherical in shape. So we have in this, spherical shock waves moving out in the undisturbed medium just like an expanding sphere at a very fast rate. Of course fluid particles may or may not move in radial directions. Similarly a cylindrical shock surface or a planar shock surface may move according as the explosion is assumed to be
cylindrical or planar. In the case of planar shock wave the disturbed fluid is bounded by a discontinuous plane in one side only. Actual cases might be more complicated. The pictures given above are cases of idealized explosions. Many physical factors crop up. Some of the important factors are gravity, magnetic field, radiation, viscosity, motion of the undisturbed medium, nature of the gas, undisturbed pressure law, conduction of heat etc. Naturally one has to choose some idealized situation and solve the problems.

Even in the most idealized condition the blast wave problems give rise to a set of partial non-linear differential equations with double boundary values. One set of the boundary conditions is given by Hugoniot shock relations and the other is the idealized point or line as the cases may be. By choice of symmetry the blast can always be treated as one dimensional double boundary value problems.

Early workers like Taylor (1950), Sedov (1959), etc. used the method based on similarity transformations. As a result of these transformations the partial differential equations were transformed to a set of ordinary differential equations in a new variable, depending on the space parameter and time. Such solutions, which are known as self-similar solutions were obtained by them.

Kopal et. al. (1951 a,b) applied the blast wave theory to explain the novae and super novae phenomenon in astrophysical models where he took gravity into account. Lucid discussions on the propagation of shock waves in high temperature are also given by Zeldovich and Raizer (1966, 1967).
Elliot (1960) introduced the importance of radiation flux, energy and pressure in the blast wave problems. He considered Taylor's problem with heat flux. For different values of the parameter \( \text{radiation} \), he discussed the effect of radiation heat flux. He, also, like Taylor used similarity transformations and then numerically solved the equations for different values of radiation parameter. He has illustrated some other cases where the similarity method may be used. He cited an example where radiation heat flux, radiation pressure and radiation energy are all equally important.

Parker (1961) has used the hydrodynamic blast wave theory to explain the sudden expansion of solar corona. He has also used the assumptions of strong point explosion. He has used the concept of Roche model for his study. Roche model means that there is a solid spherical central core of the mass which is assumed to be constant whereas the ambient gas at rest obeys the inverse square law for its density. The distance is measured from the centre of the core. He has also used the similarity transformations to study the model. These are some of the examples where hydrodynamic blast wave theory is utilised in astrophysical models.

Using similarity transformations Sedov (1959), and Bhatnagar and Lal (1965) studied in great details the flow variables behind the shock front both in self-gravitating and in non-self gravitating gas for different values of \( \gamma \), the ratio of specific heats.

Using similarity transformations Deb Ray (1965, 1966) gave exact analytic solutions for propagation of shock waves in stellar...
models for $\gamma = 4/3$ and approximate analytic solutions for $\gamma \neq 4/3$. Closed form solutions for flow variable was also obtained by him (1957) in the absence of gravity.

Brinkley and Kirkwood (1947) devised a new technique (B.K. Theory) to study one dimensional blast wave problems in any fluid medium. They have considered the effect of counter pressure in their analysis. They have utilised the equations of motion and continuity of hydrodynamics. But it is a hybrid form of Lagrangian co-ordinate $R$ (shock radius) and $r$ (Eulerian co-ordinate - distance of a fluid particle at a time $t$ from the point of explosion) specialized at the shock front. With these equations they combine the results of differentiation of Rankine Hugoniot relations - the differential operator being such that the shock front remains stationary. Thus they obtain three relations between four partial derivatives which are derivatives of pressure with respect to time and shock radius, and of particle velocity with respect to time and shock radius. So they need a fourth relation to solve the problem. To derive the fourth relation they consider the non-acoustical decay of shock waves of finite amplitude in its passage through the undisturbed gases whose pressure they take into account. They have to put a similarity restraint to the energy-time curve which refers to the observation of the parameter

$$\psi (R) = \int_0^\infty f(R, \xi) d\xi$$

where

$$f(R, \xi) = \frac{\alpha \beta |u|^\gamma}{R^\alpha \beta \cdot u}$$
is the energy-time integrand. \( \mathcal{J} \) is observed to be very slowly varying function of \( R \). Therefore, one can take sufficiently accurate estimate of \( \mathcal{J} \) without performing the actual integration. After the introduction of \( \mathcal{J} \) they obtain the desired fourth relation from the normalized energy time integral and ultimately they obtain two basic first order differential equations in shock overpressure and available mechanical energy against shock distance \( R \) which may be solved with suitable initial conditions.

Sachdev (1971) has modified the above method to suit the cases of strong point explosion i.e. when the pressure in the undisturbed medium is negligibly small. In doing so, he allows the shock strength to be infinite. As a result the differential expression connecting the available mechanical energy with the shock radius tend to zero so that the solution for homogeneous medium assumes the same form as Taylor-Sedov solution. He thus chooses the parameter \( \mathcal{J} \) in such a manner that the solution for different \( \mathcal{J} \) and \( \alpha \) \(( = 2, 1 \) for spherical and cylindrical symmetry respectively) coincides exactly with the corresponding Taylor-Sedov solution. Then he assumes that this parameter \( \mathcal{J} \) \(( \mathcal{J}, \alpha \) served the purpose of inhomogeneous medium also. He observes that the results obtained by him for cases in exponentially increasing and decreasing mediums compares favourably with the results obtained by Laumbach and Probstein (1969) and other workers in the field.

Bhatnagar and Kushwaha (1961) modifies the B.K. Theory to include radiation pressure and radiation energy to study the propa-
gation of intense shock waves in stellar envelopes. By the method so obtained they compute the radial velocity of BW Vulpeculae star. The computed velocity and the observed velocity are compared by them. Results are better than the one obtained by them previously without taking consideration of radiation pressure and radiation energy. B.K. Theory is also applied to study the explosions under water.

Most of these workers consider either plane, cylindrical or spherical symmetry for their works. Particle isentropy is considered in most of the cases. The gas is assumed to be perfect.

Recently Laumbach and Probstein (1969, 1970) studied the cases of strong point explosion under different physical conditions. They (1969) developed a new method which helps the study of an axi-symmetric medium. Sakashita (1971) has applied the Laumbach and Probstein method to study the behaviour of shock velocity and shock envelope in a model which is of great importance in astrophysics.

Laumbach and Probstein’s (1969) method is based on the following assumptions. The shock is very intense so that the counter pressure is taken to be zero. So they assume the ambient medium to be cold. The gas is perfect. Viscosity, conductivity, radiation effects are neglected. The explosion is so strong that flow remains locally radial. Density in the undisturbed medium is taken to vary exponentially with the atmospheric height. As a result of this assumption the density in the undisturbed medium becomes a function of radial distance from the source of explosion and the polar angle
They devise their technique in such a way that dependence on $\Theta$ is eliminated by the introduction of reduced distance and reduced shock velocities. It is exactly this point which encourages one to study the flow variables in axi-symmetric or spheroidal models with this technique.

Their main assumption was that the entire mass remain in a thin shell behind the shock front after the explosion. This thin shell expands with the front. Ultimately it breaks down when shock front attains infinite velocity in a finite time (blow out time). Their results are good for downward propagating shock waves but for upward propagating shock it is not so good.

They further assume that the total energy of explosions is constant. These energy integrals are conserved. They are transformed into a first order differential equation whose variables are square of the reduced velocity and reduced distance. These differential equation have to be solved numerically. They study the other flow variables and compare the results with earlier well known results.

For transformation of the energy integrals they have to expand Bubrian distance in Taylor's series and retain terms upto second order. This result were differentiated twice with respect to time and specialized at the shock front. With the repeated assumption of migration of mass near the front they are able to transform the energy integrals into first order differential equation.

They (1970) in a second paper used the above technique to study the same medium but with radiation heat flux important in
comparison with radiation pressure and radiation energy. Here they are able to transform the energy integrals again to the first order differential equation, with the assumption of a particular law for radiation meanfree path following Elliot (1960). For this particular law they obtain differential equations independent of polar angle. For other viable forms of the law of radiation mean free path they cannot obtain differential equation independent of polar angle. Under the above assumption they have obtained the analogous value of radiation parameter as that of Elliot (1960). Here also they made a very careful analysis of the flow variables for different values of the radiation parameter.

In another paper (1970), they considered the radiation effects in the far flow field through the assumption of \( \frac{\partial T}{\partial r} = 0 \) \( T \) being temperature at a distance \( r \) from the source of explosion measured radially outwards. Here they conclude that an ascending Hugoniot shock followed by an isothermal flow propagates at a considerable higher speed (for a particular location) than the one followed by an adiabatic flow.

The method of Laumbach and Probstein (1969) is useful in studies of axi-symmetric exponential media because a transformation is available which eliminates dependance on the polar angle. The modified B.K. Theory can also be similarly used for studies of axi-symmetric models (Sachdev, 1971, 1972).

Oppenheim et.al (1975) has developed another technique to study the flow variables in an exponential medium. For an ascending
shock they have improved the results obtained by Laumbach and Probstein, as their results are valid upto 9-10 scale heights. They have obtained exact analytic solution for the near field (i.e. short time after initiations) by means of a perturbation technique. They have further obtained a similarity solutions for the far flow field. They have found their results to agree very well with the known results.

Next, we recount below, a brief survey of the works contained in the thesis.

Sakashita (1971) has chosen a model which is very important in astrophysics. It is a model with spheroidal symmetry. Details of the models is given in the introduction of Chapter II. Sakashita has obtained shock velocity, shock envelope and calculated the blow out time with Laumbach and Probstein technique. In chapter II we have considered the same model as that of Sakashita and have tried to study it under different physical circumstances or by different methods.

In section 1, the B.K. Theory (1947) as modified by Sachdev (1971) is applied to obtain shock velocity. We have obtained shock velocity in two different ways. Firstly by the introduction of reduced time we have eliminated the dependence on the similarity parameter $\lambda$ of B.K. Theory and have obtained the shock velocity. Secondly, as suggested by Sachdev (1971), we have chosen an appropriate value of $\lambda$ and have obtained the shock velocity. All results are compared with the Sakashita's result in different graphs.
In section 2, we have chosen the same model. The technique applied here is that of Laumbach and Probstein. We have assumed further that to a first approximation the density variation in the radial direction in the thin shell behind the front is negligible. With this modification we have Laumbach and Probstein's technique to obtain the shock velocities under three circumstances, namely

a) when radiation heat flux is much more important than radiation pressure and radiation energy,

b) when the particle follows an isentropic path,

c) when the temperature gradient is flat behind the shock front.

We give the results graphically.

In section 3, we have assumed that the temperature gradient is flat behind the shock front but withdraw the assumption that the density variation with respect to radial distance from the source is negligible. We have obtained the shock velocity, shock envelope for different values of the eccentricity of the model. The results are presented graphically and compared with Sakashita's (1971) works who has obtained the same variables under the assumption of particle-isentropy. The technique followed here is that of Laumbach and Probstein.

It is already mentioned that Elliot (1960), Bhatnagar and Kushwa (1961) considered the effects of radiation in the explosion.
problems. Sedov (1959), Kopal et el. (1951b), Bhatnagar and Lal (1965) and Deb Ray (1965) among others have considered the self-gravitating gaseous medium and have studied the nature of flow variables behind the shock front. Here in chapter III we have used similarity transformations to solve the problem of point explosion in a self-gravitating gaseous medium where radiation heat flux is assumed to be more important than radiation pressure and energy. Results of the study of flow variables for different parameters are shown in tabular forms.

Roche model was considered by Kopal et el. (1951a), Deb Ray (1966) and others. Parker (1961) has studied the expansion of solar corona with the help of hydrodynamic blast wave theory using the concept of Roche model. He did not take into account of solar gravity and radiation. Here in chapter IV we present the study of the same model as that of Parker (1961) but with the effect of solar gravity and radiation heat flux taken into consideration. The results here are presented in graphical forms.

Propagation of planar shock waves in a medium whose density follows an exponential law, has been studied by Raizer (1964), Grover and Hardy (1966) and Deb Ray (1974) and others.

The importance of study of blast waves in spherical and cylindrical exponential mediums to astrophysical problem has been brought out by Grover and Hardy (1966). In the fifth chapter we have found out the solutions for a strong point or line explosions in
increasing exponential medium. We present our results in tabular form.

In the last chapter we have developed a technique to consider the explosions in a medium where density in the undisturbed medium is a function of the product of radial distance from the source and a function of polar angle measured from the axis of symmetry using Eulerian equations. In the first part of this chapter we have presented an extension of our results obtained in chapter five to the axi-symmetric mediums. In the second part of the chapter six we have shown how the similarity solutions can be made useful in a medium whose density satisfies some axi-symmetric power law by extending the exact analytic solution obtained by Deb-Ray (1957).