The classification of hadrons may well be understood from the viewpoint of SU(3) symmetry scheme. Also the mass difference for different multiplets can be interpreted in terms of broken symmetry. However the basic drawback of particle physics SU(3) symmetry lies in the fact that it has no geometrical significance and apparently seems to have no connection with ordinary space time. Moreover, the quarks which seem to comprise the building blocks of hadrons in conformity with SU(3) symmetry have not yet been observed. Thus the SU(3) symmetry scheme as applied to particle physics still remains an ad hoc proposal without any solid foothold.

The recent high energy experiments have helped us to probe the inner structure of hadrons and a new possibility has arisen to study the lepton-hadron relation. The leptons have a positive and decisive role in hadrons physics, which becomes important at sufficiently high energy where the distinction between leptons and hadrons may not be possible and leptons may have strong interaction with quarks as suggested by Pati and Salam\(^\text{(90)}\). This model suggests a particular class of leptons, namely muons \((\mu^+, \nu_\mu, \mu^-)\) can be taken as the basic constituents of hadrons. Some experimental indications to this effect may be cited as follows:

i) The discovery of \(^\mu^-\bar{p}\)-resonance

ii) The emission of a large number of leptons in high energy hadronic collisions.
In fact, the main drawbacks to take leptons as the basic constituents of hadrons lies in the following two aspects:

i) Hadrons are characterised by some internal quantum numbers such as isospin, strangeness and baryon number whereas leptons have no such properties.

ii) All hadrons take part in strong interactions whereas leptons can not take part in it.

In a recent paper (62), it was shown that the internal quantum numbers like isospin, strangeness and baryon number can indeed be related to the internal angular momentum of the constituents. For this, we have to assume that all internal motions are quantised in unit of $\frac{1}{2}\hbar$ instead of $\hbar$ and that the two values of the third component of the orbital momentum represents the two states of matter: particles and antiparticles. In fact it has been shown that although for various reasons, the half-orbital angular momentum is not allowed in atomic or nuclear configurations, the value $l = \frac{1}{2}$ can be allowed in the special case of elementary particle configurations where particles and antiparticles are represented by the two $l_3$ values, (see the Appendix B).

The important point in our assumption is that we have specified the two values of the third component of the orbital momentum for particles and antiparticles. This specification of the third component of the orbital angular momentum evidently specifies a direction in space. Indeed, by defining particles and antiparticles in such a way, we are, in effect, considering the structure of
particles in terms of the intrinsic 'handedness', so that the right and left-handed structures stand to each other in particle antiparticle relation. The charge of the composite system in our configuration is supposed to be due to the charge of the central particle and all other constituents are taken to be neutral and to transform a particle to an antiparticle, we will have to change the direction of the Z-axis as well as to perform the charge conjugation of the central particle. The change in the direction of the Z-axis corresponds to the transformation from right handed co-ordinate system to left-handed co-ordinate system (and vice-versa). The transformation C changes the charge of the central particle and the transformation P (reflection) changes the right handed system to a left-handed system (and vice-versa) by reversing the $l_z$-values of the different constituents. Thus we see that the particle-antiparticle relations as suggested here is consistent with the idea of CP symmetry and that particles-antiparticles are mirror-reflections of each other. Right and left are still indiscernible and the isotropy of space is thus restored. In fact, if we take the conventional definition of particles and antiparticles, the sense of right and left can be determined by any parity violating interaction. On the other hand, if we take the conventional sense of right and left, particles and antiparticles can be distinguished.

So far as strong interactions are concerned, it has been shown\(^{(2)}\) that these processes can indeed be interpreted very nicely when we take that any two constituents (muon-antimuon pair)
form a \(\pi\)-meson cluster in the structure of the hadrons and hadronic processes involving no exchange of hypercharge occur when a \(\pi\)-meson cluster in the structure of the incident hadron interacts with a \(\pi\)-meson in the structure of the target hadron. Since in this scheme, the basic hadronic interaction is the \(\pi\)-\(\pi\) interaction it also satisfies the requirement of duality in the sense that both \(s\)- and \(t\)-channel amplitudes are contributed by the same meson (viz \(\pi\)-meson).

The most interesting aspect of this interpretation of duality is that the inconsistencies which crop up in \(\bar{B}B\) scattering in naive quark model are removed in this scheme\(^{(2)}\). Also for hadronic interactions which are associated with the exchange of hypercharge, a direct interaction mechanism such as stripping or knock-out interaction can be envisaged.

This model allows exactly 8 and 10 members for \(\frac{1}{2}\) and \(\frac{3}{2}\) baryons having the same quantum numbers as predicted by the SU(3) symmetry scheme, but for pseudoscalar and vector mesons, apart from the 8-members allowed by SU(3) the present scheme suggests the existence of three other iso-singlet mesons \(D^*(\phi^*), D^0(\phi^0)\) and \(D^-(\phi^-)\) having strangeness +2, 0 and -2 respectively. In case of vector mesons, the member with \(S = 0\) is identified with the well known \(\phi^0\)-meson. Also this scheme can explain the mass spectrum of all strongly interacting particles in a satisfactory way. In this model of hadrons, it has been shown that, this model helps us to understand the geometrical origin of the SU(3) symmetry of hadrons when we consider the harmonic oscillator potential for the binding force of the constituents. It is also noted that the electromagnetic
mass-difference of the members of an isomultiplet as well as the magnetic moments of baryon can be predicted which are found to be in excellent agreement with experiments.

Mesons

As stated above, we have considered a muonic triplet \((\mu^+, \nu_\mu, \mu^-)\) where \(\nu_\mu\) has been taken as a Majorana spinor, as the basic particles and all the hadrons are then taken to be composed of these constituents. Let us first consider the configuration \((\mu, \nu_\mu)\) where \(\mu\) represents any of the charge states \(\mu^+, \nu_\mu, \mu^-\). For our convenience, we shall call in this configuration \((\mu, \nu_\mu)\), \(\mu\) (i.e., \(\mu^+, \nu_\mu\) or \(\mu^-\)) as the 'central' particle. Of course, this 'central' particle is not at rest, but moves in a harmonic oscillator potential with a certain orbital momentum. However, the main point in distinguishing between the 'central' particle \(\mu\) (\(\mu^+, \nu_\mu\) or \(\mu^-\)) and other particle is that, in the present scheme, we have taken that the 'central' particle can be charged, and other constituents are electrically neutral, and the third component of the orbital momentum of the constituents — apart from the 'central' particle is taken to be specified for particle and antiparticle states. In view of this, the angular momentum of the constituents apart from that of the 'central' particle can be shown to play a significant role in determining the internal quantum numbers like isospin, strangeness and baryon number of a particle.

In the configuration \((\mu \nu_\mu)\), we take that \(\nu_\mu\) is moving with orbital momentum \(\frac{1}{2}\), in a harmonic oscillator potential. Then, by
combining the spin and the orbital angular momentum of \( v_\mu \), we get \( J^\mu = 1 \) or 0. Again, if we take that the 'central' particle \( v_\mu \) is moving with orbital momentum \( \frac{1}{2} \), then by combining the spin and the orbital momentum of the 'central' particle \( v_\mu \), we have \( J^\mu = 1 \) or 0. Thus the total angular momentum \( J \) is given by

\[
D^J = D^{\text{spin}} = \frac{1}{2} + D^{\text{orbit}} = \frac{1}{2} + D^{\text{spin}} = \frac{1}{2} + D^{\text{orbit}} = \frac{1}{2}
\]

and so \( J \) can take the value 2, 1 or 0. This total angular momentum \( J \) is nothing but the spin of the 'particle' represented by the composite system. From this, we see that the composite system \((\mu v_\mu)\) can represent certain type of mesons with spin 0, 1 or 2.

To find the characteristics of these mesons explicitly, let us consider that \( \mu^+ \) and \( \mu^- \) stands to each other as particle and antiparticle. Also we have taken \( v_\mu \) as a Majorana spinor i.e. \( v_\mu = \bar{v}_\mu \). In fact, it is well known that a four component - Majorana spinor is equivalent to a two component weyl spinor in a certain sense. That is, we can say that 'spin down' and 'spin up' states, may be taken as the 'particle' and 'antiparticle' states respectively. So, if we put the restriction that the \((\mu v_\mu)\) - system representing a mesic configuration should have the fermion number zero, \( \mu^+ (\mu^-) \) can then be bound to an antiparticle (particle) only. Furthermore, contending that the configurations \((\mu^+ v_\mu)\) and \((\mu^- v_\mu)\) here stands to each other as particle and antiparticle (or vice versa), we take that the \( l_3 \)-value of \( v_\mu \) in these two cases should be of opposite sign, and we specify this value in the former and latter cases as \( + \frac{1}{2} \) and \( - \frac{1}{2} \) respectively. It is to be noted that, as we have taken as a basic assumption that the two values of \( l_3 \) represent particle and
antiparticle states and have specified \(- I^\nu_3 = \pm \frac{1}{2}\) and \(- \frac{1}{2}\) for the states \((\mu^+ \nu_\mu)\) and \((\mu^- \nu_\mu)\) respectively, the other states \((\mu^+ \nu_\mu)\) with \(l^\nu_3 = - \frac{1}{2}\) and \((\mu^- \nu_\mu)\) with \(l^\nu_3 = + \frac{1}{2}\) are excluded. For according to our contention, the states \((\mu^+ \nu_\mu)\) with \(l^\nu_3 = + \frac{1}{2}\) and \((\mu^+ \nu_\mu)\) with \(l^\nu_3 = - \frac{1}{2}\) should correspond to each other as particle and antiparticle. But this is not possible as they represent same charge state.

Having considered this, we note that in the configuration \((\mu^+ \nu_\mu)\), the different values of \(J^\nu_3\) are related with the different charge stages of the central particle \(\nu\), so that the charge states of the compound system also are completely determined by these values. For then, in the case of \((\mu^+ \nu_\mu)\) - system, we have \(J^\nu_3 = l^\nu_3 + S^\nu_3 = + \frac{1}{2} + \frac{1}{2} = + 1\). Similarly in case of \((\nu_\mu \nu_\mu)\) and \((\mu^- \nu_\mu)\) systems we have \(J^\nu_3 = 0\) and \(- 1\) respectively. So these states \((\mu^+ \nu_\mu)\), \((\nu_\mu \nu_\mu)\), \((\mu^- \nu_\mu)\) can be characterized such that these form a triplet. In view of this, we call this \(J^\nu_3\) as the 'internal' angular momentum of the system \((\mu^\nu_\mu)\). Also we note that according to this formalism, the singlet state having \(J^\nu_3 = 0\) can only correspond to the configuration \((\nu_\mu \nu_\mu)\). In this connection, it may be remarked that as for the configuration \((\nu_\mu \nu_\mu)\), \(l^\nu_3\) value is not specified, that is, it can take both the values \(+ \frac{1}{2}\) and \(- \frac{1}{2}\) here, and so, the particle and antiparticle states in this case can not be distinguished. Considering these aspects, we here identify the three states of the triplet with \(\pi^-\) mesons and the singlet with \(\eta^0\). Also we note that a triplet and a singlet of vector (tensor) mesons, can also be represented by this configuration scheme. We identify these with the triplet of
\[ \rho (A_2)\text{-mesons and the singlet } \omega^0 (\pi^0). \text{ Thus in our scheme} \]

\[
\begin{pmatrix}
\mu^+ \nu_\mu \\
\nu_\mu \nu_\mu \\
\mu^- \nu_\mu
\end{pmatrix}_{S=0} = \begin{pmatrix}
\pi^+ \\
\pi^0 \\
\pi^-
\end{pmatrix}, \quad \begin{pmatrix}
\rho^+ \\
\rho^0 \\
\rho^-
\end{pmatrix}_{S=1,2} = \begin{pmatrix}
\mu^+ \nu_\mu \\
\nu_\mu \nu_\mu \\
\mu^- \nu_\mu
\end{pmatrix}_{S=2}
\]

\[ (\nu_\mu \nu_\mu)_{S=0} = \eta^0, \quad (\nu_\mu \nu_\mu)_{S=1} = \omega^0, \quad (\nu_\mu \nu_\mu)_{S=2} = \pi^0 \]

Next for \( K \)-mesons, we consider the system \((\nu_\mu, (\nu_\mu \nu_\mu)_{S=0})\) where the centre of mass of the system \((\nu_\mu, \nu_\mu)_{S=0}\) is moving along with \( \nu_\mu \) with orbital angular momentum \( \frac{1}{2} \). For our convenience, we shall represent the configuration as \((\mu, \nu_1, \pi_2)\), the suffixes indicating that \( \nu_\mu \) is the first member and \( \pi \) (i.e., \((\nu_\mu \nu_\mu)_{S=0}\)) is the second member of the configuration apart from the central particle \( \mu \).

To ascertain the possible states of the system \((\mu, \nu_1, \pi_2)\) we consider the the coupling of \( J(\nu_1) \) and \( J(\pi_2) \) where \( J = (1 + s) \) can be denoted here as the total angular momentum of the particular constituent concerned.

We have \[ J(\nu_1) = \frac{1}{2} + \frac{1}{2} = 1 \text{ or } 0 \quad (A.1) \]

\[ J(\pi_2) = 1 (\pi_2) = \frac{1}{2} \quad (A.2) \]

Thus the total angular momentum of the system \((\nu_1, \pi_2)\) is given by

\[ J = J(\nu_1) + J(\pi_2), \quad J(\nu_1) + J(\pi_2) - 1, \quad |J(\nu_1) - J(\pi_2)| \]

\[ = 1 + \frac{1}{2} \neq \frac{3}{2}, 1 - \frac{1}{2} = \frac{1}{2} \text{ or } 0 + \frac{1}{2} = \frac{1}{2} \quad .... \quad (A.3) \]
Now we note that $I^2 = J^2$ can take the values $+\frac{1}{2}$ and $-\frac{1}{2}$.

We contend that one of these values represents the particle state and the other antiparticle state. However, it should be mentioned that just by transforming $I^2 = \frac{1}{2} (-\frac{1}{2})$ to $I^2 = \frac{1}{2} (+\frac{1}{2})$, we do not have the particle antiparticle transformations. In fact, as the charge of the composite system is due to the charge of the central particle $\mu$ and all other constituents are neutral, it can be shown that proper particle-antiparticle relation can only be maintained with the specification of the $I^2$-value of the constituents for particle and antiparticle states. That is, to transform a particle to an antiparticle, we will have to change the direction of the $z$-axis as well as to perform the charge conjugation of transformation of the 'central' particle $\mu$. Otherwise we can not have proper physical states.

To see this let us take in the mesic configuration $(\pi_1^\mu \pi_2^\mu)$ $I^2 = J^2 = -\frac{1}{2}$. As shown above, $J^\mu$ (the total angular momentum of $\pi_\mu$) can take the value 1 (or 0) and $J^\mu = +1, 0$ and $-1$ correspond to the positive, neutral and negative charge state of the central particle respectively. Now, with $I^2 = -\frac{1}{2}$, we have the following $J^\mu$-values for the constituents $(\pi_1^\mu \pi_2^\mu)$.

\[
J^\mu = J^\mu_1 + J^\mu_2 = \begin{array}{l}
\text{i)} \quad +1 - \frac{1}{2} = +\frac{1}{2} \\
\text{ii)} \quad 0 - \frac{1}{2} = -\frac{1}{2} \\
\text{iii)} \quad -1 - \frac{1}{2} = -\frac{3}{2}
\end{array}
\]
From the eqn. (A.3), we know that $J$ may take the value $\frac{3}{2}$ or $\frac{1}{2}$. But from (A.4a), (A.4b) and (A.4c), we see that due to the specification of the $l_2$-value of $\alpha_2$, the $J = \frac{3}{2}$ must be excluded, as we can not have all the four $J_3$-values ($+\frac{3}{2}$, $+\frac{1}{2}$, $-\frac{1}{2}$, $-\frac{3}{2}$) necessary for this. This restricts the value of $J$ to be $\frac{1}{2}$ and from (A.4a) and (A.4b), we can suggest that the two states with $J_3 = \frac{1}{2}$ and $-\frac{1}{2}$ represent here $k^+$ and $k^0$ (as $J_3 \ y_1 = +1$ and 0 correspond to the positive and neutral charge state of the central particle $\pi$). Similarly we can show that with $J_3^\pi = +\frac{1}{2}$, we can get only the antiparticle states $K^-$ and $K^0$. For then we have

$$J_3 = J_3^\mu 1 + J_3^\pi 2$$

$$= i) +1 + \frac{1}{2} = + \frac{3}{2} \quad \ldots \quad (A.5a)$$

$$ii) 0 + \frac{1}{2} = + \frac{1}{2} \quad \ldots \quad (A.5b)$$

$$iii) -1 + \frac{1}{2} = - \frac{1}{2} \quad \ldots \quad (A.5c)$$

With $J = \frac{1}{2}$, we can have only the two states $J_3 = +\frac{1}{2}$ and $-\frac{1}{2}$ and from (A.5b) and (A.5c), we see that these two states correspond to $K^0$ and $K^-$, since $J_3 \ y_1 = 0$ and $-1$ represent the neutral and negative charge state of the central particle $\pi$. From a comparison of (A.4a) and (A.4b) with (A.5b) and (A.5c), it is evident that the antiparticle state is not achieved just by changing the direction of the Z-axis. In fact this must be accompanied by the charge conjugation transformation of the central particle $\pi$. Otherwise we do not get any proper physical state.
We have found that in the configuration \((\mu \nu_{\mu_1} \pi_2)\) with the constraint \(l_3^2 = -\frac{1}{2}(+ \frac{1}{2})\) for particle (antiparticle), the total angular momentum of the constituents \(\nu_{\mu_1}\) and \(\pi_2\) can take only the value \(\frac{1}{2}\). We define this as the 'internal' angular momentum of the system representing K-mesons and shall denote it as \(J^k\). However, we realise that the \(J\)-value here represents, in effect the isospin of the system. In fact, as according to the present scheme, the charge of a composite system describing a particle is due to the charge of the central particle, and \(l_3\)-values of other orbiting constituents are specified for particles and antiparticles, the projection of the total angular momentum of the constituents on to the \(Z\)-axis is found to be related with the electric charge of the system. Indeed, we shall see that isospin, strangeness and baryon number, all these internal quantum numbers of a particle, can be directly related with the angular momentum of the constituents.

It is to be noted that just like - pseudoscalar K-mesons, we can also construct vector (tensor) mesons doublets \((K^{*+}, K^{*0})\), \((K^{*+}, K^{*-0})\) and \((K^{*0}, K^{*-})\), \((K^{*0}, K^{*-})\) with the same configuration \((\mu \nu_{\mu_1} \pi_2)\) with the constraint that the coupling of the angular momenta of all the constituents lead to a \(S = 1\) (2) state. Thus we write:

\[
\begin{pmatrix}
\mu^+ & \nu_{\mu_1} & \pi_2 \\
\nu_\mu & \nu_{\mu_1} & \pi_2
\end{pmatrix}_{s=0}^{l_3^2 = -\frac{1}{2}} = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}, \quad \begin{pmatrix}
\nu_\mu & \nu_{\mu_1} & \pi_2 \\
\mu^- & \nu_{\mu_1} & \pi_2
\end{pmatrix}_{s=0}^{l_3^2 = +\frac{1}{2}} = \begin{pmatrix} K^- \\ \bar{K}^0 \end{pmatrix}
\]
To represent the isosinglet $-\pi$ meson, we propose the configuration (H taking $1^+$ as usual.

\[
\begin{pmatrix}
\mu^+ \nu_1 \pi_2 \\
\nu_\mu \nu_1 \pi_2
\end{pmatrix}
= \begin{pmatrix}
K^{*+} \\
K^{*0}
\end{pmatrix}, \quad \begin{pmatrix}
\nu_\mu \nu_1 \pi_2 \\
\mu \nu_1 \pi_2
\end{pmatrix}
= \begin{pmatrix}
K^{*-0} \\
K^{*-+}
\end{pmatrix}
\]

\[
S=1 \\
1_{S}^2 = -1/2
\]

\[
\begin{pmatrix}
\mu^+ \nu_1 \pi_2 \\
\nu_\mu \nu_1 \pi_2
\end{pmatrix}
= \begin{pmatrix}
K^{*++} \\
K^{*0+}
\end{pmatrix}, \quad \begin{pmatrix}
\nu_\mu \nu_1 \pi_2 \\
\mu \nu_1 \pi_2
\end{pmatrix}
= \begin{pmatrix}
K^{*0-} \\
K^{*0-}
\end{pmatrix}
\]

\[
S=2 \\
1_{S}^2 = -1/2
\]

To represent the isosinglet $\phi^0$ meson, we propose the configuration $(\mu \nu_1 \pi_2 \pi_3)$ taking $1_{\pi}^2 = J_3^\pi = 1/2$ and $1_{\pi}^2 = J_3^\pi = -1/2$ as usual.

To find out the internal angular momentum $J^\phi$, we note that $J^\nu_1 = 1$, $J^\pi_2 = 1/2$, $J^\pi_3 = 1/2$. Now the consideration of symmetry under permutation of $\pi_2$ and $\pi_3$ allows the value $J^\phi = 2, 1, 0$.

This is evident from the fact that to have a symmetric tensor to be formed from the product of the wave function of $\pi_2$ and $\pi_3$, we must have an even value for $J^\pi_2 - J^\pi_3$ where $J^\pi_2 = J^\pi_3 = 1/2$, and $J^\pi_2$ is the resultant of $J^\pi_2$ and $J^\pi_3$. In our case we see that this is satisfied only when $J^\pi_2 = 1$. Thus $J^\phi$ resulting from the combination of $J^\nu_1$ and $J^\pi_2$ is either $1 + 1 = 2$, $1 + 1 - 1 = 1$.
or \( l - 1 = 0 \). The value of \( J^\circ = 2 \) is excluded in our scheme as it is not compatible with the \( l_3 \) values specified for \( \pi_2 \) and \( \pi_3 \). The configuration \((\mu \nu_1 \pi_2 \pi_3)\) can also lead us to two other iso-singlet mesons having strangeness \( +2 \) and \(-2 \) and electric charge positive and negative respectively.

**Baryons**

We now extend the above scheme to baryons also and consider the following considerations for \( N, Y (\pi^\pi \pi^\pi) \) and \( \Xi \) particles.

\[
(\mu \nu_1 \pi_2 \nu_\mu_3) = N
\]

\[
(\mu \nu_1 \pi_2 \nu_\mu_3 \pi_4) = Y (\pi^\pi \pi^\pi)
\]

\[
(\mu \nu_1 \pi_2 \nu_\mu_3 \pi_4 \pi_5) = \Xi
\]

As before, \( \nu_{\mu_1} \) and \( \pi_1 \) represents the ith member of the configuration and by \( \pi \) we here mean the centre of mass of the system \((\nu_\mu \nu_\mu)^{S=0}\). Here also, the orbital angular momenta of each constituent is taken to be \( l/2 \). In case of mesons we have considered that for particles (antiparticles) states \( l_3^2 = \frac{\lambda^2}{2} \pm \frac{1}{2} (\pm \frac{1}{2}) \). In this case also we shall specify the \( l_3 \)-values of the constituents \( \nu_\mu_3, \pi_4 \) and \( \pi_5 \) so that the two different values represent particle and antiparticle states. Here we shall take \( l_3^\mu_3 = l_3^\pi_4 = l_3^\pi_5 = \pm \frac{1}{2} (\pm \frac{1}{2}) \) for particles (antiparticles). We note that \( J^\mu_3 \) here may be 1 or 0.
It is found that \( J^3 = 0 \) and 1 gives us spin \( \frac{1}{2} \) and spin \( \frac{3}{2} \) baryons respectively.

These considerations now help us to calculate the internal angular momentum of baryons according to the following scheme. We discuss the configuration of nucleons which is according to the present model

\[
N = (\mu_\mu_1 \pi_2 \nu_\mu_3)
\]

For simplicity we denote the internal angular momentum by \( J^N \).

Then \( J^N = J (\nu_\mu_1 \pi_2 \nu_\mu_3) \)

where \( J (\nu_\mu_1 \pi_2 \nu_\mu_3) \) means the total angular momentum of \( \nu_\mu_1 \pi_2 \) and \( \nu_\mu_3 \). Thus we have

\[
J^N = J^\mu_1 + J^\pi_2 + J^\nu_\mu_3
\]

\[
= 1 + \frac{1}{2} + 0 = \frac{3}{2}
\]

or \( 1 - \frac{1}{2} + 0 = \frac{1}{2} \)
To ascertain the exact value, we consider the values of the third component of $J^N$ as $J_3^N$-values of $\pi_2$ and $\nu_{\mu_3}$ are specified. We have

$$J_3^N = J_3^{\nu_{\mu_1}} + J_3^{\pi_2} + J_3^{\nu_{\mu_3}}$$

$$= 1 - \frac{1}{2} + 0 = \frac{1}{2} \quad \ldots \quad (A.7a)$$

$$\text{or} \quad = 0 - \frac{1}{2} + 0 = -\frac{1}{2} \quad \ldots \quad (A.7b)$$

$$\text{or} \quad = -1 - \frac{1}{2} + 0 = -\frac{3}{2} \quad \ldots \quad (A.7c)$$

Obviously $J^N$ can take only the value $\frac{1}{2}$, and recalling that $J_3^{\nu_{\mu_1}} = 1$ and 0 correspond to the positive and neutral charge states of the central particle, we have for $J_3^N = \frac{1}{2}$ proton and $J_3^N = -\frac{1}{2}$ for neutron. Similar consideration applies for the other baryons $Y(\Lambda, \Sigma)$ and $\Xi$.

In an analogous way, we can construct the spin $\frac{3}{2}$ baryonic states noting that in this case $J_3^{\nu_{\mu_3}} = 1$.

It is here noted that in case of spin $\frac{1}{2}$ baryons, the heaviest element $\Xi$ was taken to have the configuration ($\nu_{\mu_1} \pi_2 \nu_{\mu_3} \pi_4 \pi_5$) whereas for spin $\frac{3}{2}$ baryons, we have considered the configuration of $\Xi$ as ($\nu_{\mu_1} \pi_2 \nu_{\mu_3} \pi_4 \pi_5 \pi_6$). In fact it was shown that for reasons of symmetry under permutation of different $\pi$-mesons in the structure and also for the specification of $l_3$-values, we can not add another $\pi$-meson in the former case. Similarly, no other new $\pi$-meson can be added to the configuration of $\Xi$. 
 Iso-spin, Strangeness and Baryon Number

In the above discussion we have noted that the 'internal'
angular momentum of a system (J-value) generated by the spin and
the orbital angular momentum of the constituents apart from the
central particle represents, in effect, the iso-spin of the particle
concerned. To this end, we have taken that the particle μ can
exhibit itself in three charge states. Moreover, in case of π, ρ
and Λ2 mesons, we have considered that the total angular momentum
of νμ generated by the spin and the orbital momentum is 1, and
due to some constraints on the formation of the bound states, the
configurations (μ+νμ), (νμ νμ), (μ−νμ) are characterised by the
Jνμ values of + 1, 0 and −1 respectively. That is, Jνμ here
is completely dependent on the charge state of the central particle.
But for other particles, we have considered that the Jνμ−value of
any constituent is completely specified, and is independent of the
charge state of the central particle. In view of this, we have
noted that the projection quantum number of the 'internal' angular
momentum relates, in effect, the state of the particular configura-
tion of the charge state of the central particle, and thus represent
the different charge states of the system. We may note here other
specific characteristics of the angular momentum of the constituents
of a particle. It has been pointed out that excepting νμ1, the
Lνμ−value of any other constituent is completely specified, and is
independent of the charge state of the central particle. So the
summation of these Lνμ−values of the different constituents (excepting
that of νμ1) in a contain configuration will provide us a good
quantum number which will be preserved in certain interactions.
To be more specific, we recall here the specific values of $l_3$ of the constituents, and find their summation for different configurations. We have for the 'Particle' states

$$l_3^2 = l_2', \quad l_3' = l_3^4 = l_3^5 = l_3^6 = + l_2'$$

So for different particles, we write the summed value $l_3$ as follows:

$$K = (\mu, v_{\mu_1}, \pi_2) ; \quad l_3^K = l_3^2 = - l_2' \quad \ldots \quad (A.8a)$$

$$N = (\mu, v_{\mu_1}, \pi_2, v_{\mu_3}) ; \quad l_3^N = l_3^2 + l_3^4 =$$

$$= - \frac{1}{2} + \frac{1}{2} = 0 \quad \ldots \quad (A.8b)$$

$$Y = (\pi, e) = (\mu, v_{\mu_1}, \pi_2, v_{\mu_3}, \pi_4)$$

$$l_3^Y = l_3^2 + l_3^4$$

$$= - \frac{1}{2} + \frac{1}{2} = \frac{1}{2} \quad \ldots \quad (A.8c)$$

Similarly for vector mesons, and spin $\frac{3}{2}$ baryons we have

$$l_3^K = \frac{1}{2}, \quad l_3^N = 0 ; \quad l_3^Y = \frac{1}{2} \quad \ldots \quad (A.9)$$
Evidently these $l_3$-values represent, in effect, $(-\frac{1}{2})S$ where $S$ is the strangeness of the particle concerned. That is, strangeness is here related with the third component of the orbital angular momentum of these constituents which have their $l_3$-values fixed irrespective of the charge state of the system.

Now, in our scheme, we note that $\nu_3, \pi_4, \pi_5$ and $\pi_6$, all have $l_3 = +\frac{1}{2}$ for particle states and $-\frac{1}{2}$ for antiparticle states. Again, all these constituents are concerned with baryon. So the $l_3$-value of any of these constituents can be identified as $\frac{1}{2}N$ where $N$ is the baryon number of the particle.

We now discuss few other interesting aspects of the model in brief.

**Mass-Spectrum of Mesons and Baryons**

It is well known that, a certain regularity in mass rule (equal spacing rule) is the most interesting aspect of the mass spectrum of hadrons. Here we can show that the configuration scheme considered above can nicely interpret this and the masses of the strongly interacting particles can be correctly predicted. For this we will have to neglect the interaction among different $\pi$-mesons (i.e., $(\nu_\mu, \nu_\mu)$ system) in a certain configuration, and take that the binding energies for all such $\pi$-mesons are equal. That is, in case of mesons, for $\pi_2$ and $\pi_3$ and in case of baryons for $\pi_4$, $\pi_5$ and $\pi_6$, we have equal binding energy.
Considering this, we now recall the configuration of vector mesons

\[ \rho = (\mu \nu_{\mu})_{s=1} \quad \omega^0 = (\nu_\mu \nu_\mu)_{s=1} \]

\[ k = (\mu \nu_{\mu})_{s=1} ~ \pi_2 \quad \phi^0 = (\nu_\mu \nu_\mu)_{s=1} ~ \pi_2 \pi_3 \]

So the mass relation can be written as follows

\[ m_k = m_\rho + m_2 - E_B \quad \ldots \quad (A.10) \]

\[ m_\phi = m_\rho + m_2 - E_B + m_3 - E_B \quad \ldots \quad (A.11) \]

where \( m_\rho \) and \( m_\pi \) stands for the average value \( \frac{3m_\rho + m_\omega}{4} \) and \( \frac{3m_\pi + m_\eta}{4} \) respectively and \( E_B \) is the binding energy. From these two equations, we have

\[ m_k = \frac{m_\rho + m_\phi}{2} \quad \ldots \quad (A.12) \]

Putting the experimental values, we have

\[ \frac{m_\rho + m_\phi}{2} = 6.375 \quad \text{(unit } m_\pi^+ = 1) \]

and \( m_k^* = 6.38 \)
Classification of Mesons and Baryons

We know that the number of elementary particles of a certain kind seems to be fixed by a certain rule. For example, we have groups like 8 mesons, 8 spin 1/2 baryons and 10 spin 3/2 baryons. In fact this very aspect was emphasized by many physicists and unitary symmetry scheme had played a major role in predicting the exact number of members having the same spin and parity. It is interesting that the model considered here also exactly fixes the number of particles of a certain class so that the scheme can be taken as fully compatible one.

Geometrical Origin of the SU(3) Symmetry of Hadrons

The model of hadrons discussed above helps us to understand the geometrical origin of SU(3) symmetry in particle physics, when we take into account the harmonic oscillator potential for the binding force of the constituents. An interesting consequence of this geometrical origin of SU(3) symmetry is that the mass difference between various iso-spin multiplets does not appear here as a broken symmetry but is a manifestation of the different number of oscillator quanta for hadrons having different hypercharge.

Appendix B

Validity of the Half-Integral Orbital Angular Momentum in Elementary Particle Configuration.

Here we show the validity of the assumption that the orbital angular momentum can take the value $\frac{1}{2} (k = 1)$ in the special case of
elementary particle configurations where the two values of the third component of the orbital angular momentum represent the two states of matter: particles and antiparticles. The argument generally put forward to eliminate the half-integral orbital angular momentum are as follows:

1) The wave function \( \psi \) is not single valued when the orbital angular momentum \( \ell \) takes any half integer value.

2) The eigen function of \( L_z \) and \( \ell^2 \) which are of the form

\[
f_{\ell,m} (\cos\theta) e^{im\phi}
\]

with

\[
\frac{d}{dz} \left[(1-z^2) \frac{d}{dz} f_{\ell,m} \right] + \left[\ell(\ell-1) - \frac{m^2}{1-z^2}\right] f_{\ell,m} = 0 \quad \text{... (B.1)}
\]

where \( z = \cos \theta \), suggest that the function \( f_{\ell,m} \) becomes singular when \( \ell \) is not an integer.

3) Pauli argued that the half-integral \( \ell \) eigen-functions are not orthogonal and hence the Hamiltonian is not hermitian on the space spanned by them.

4) Half-integral orbital angular momentum cases are representations not of the rotation group, out of its covering group SU(2).

5) Whippman\(^{(92)}\) pointed out that the state \( \psi_{\ell, \pm \frac{1}{2}} \) describes a physical situation where particles are being created at any point on the positive \( Z \)-axis arbitrarily far from the origin and are similarly being destroyed on the negative \( Z \)-axis. Though the total number of particles present over all space is still conserved and it does not violate any fundamental principle such as the conservation
of probability, the situation is not likely to occur in nature.

6) Schwinger\(^{(93)}\) showed that the half-integral orbital angular momentum is not possible on the basis of the infinitesimal generators of the rotation group.

7) From the group theoretical considerations, Van Winter\(^{(94)}\) has shown that for half integral orbital angular momentum, the eigen values of \(L_x\) and \(L_y\) are not equal to the eigen values of \(L_z\), and that, though the operator \(L^2\) still commutes with \(L_z\), it does not commute with \(L_x\) and \(L_y\).

So far as the arguments (1), (2), (3), (4) are concerned, these are not beyond criticism and Whippman\(^{(95)}\) has discussed in details the invalidity or the incorrectness of these arguments. So far as the arguments (5), (6) and (7) are concerned, we shall show here that these are not applicable in the elementary particle configurations as we have considered. For completeness, we shall recapitulate here the criticisms for (1), (2), (3) and (4) and then demonstrate that arguments (5), (6) and (7) are not applicable in our case.

It is generally recognised that the argument for the single valuedness of the wave function \(\psi\) is unsatisfactory. Indeed, the state function \(\psi\) attains its physical meaning only through \(|\psi|^2\) and this remains unaffected, when the sign of \(\psi\) is altered. The usual physical conditions require that all physically observable quantities such as the expectation values of hermitian operators should be single valued functions of position, but this does not impose the restriction that the wave function \(\psi\) should be single-valued as well.
The argument that the function $f_{l,m}$ as defined by equation (2) becomes singular when $l$ is not an integer, is also not satisfactory. Indeed, Pauli has demonstrated that the regular eigen-functions of $L^2$ and $L_z$ for all $l$ can be constructed and these are given by

$$Y_{l,m} = f_{l,|m|}^{1m}\phi$$

with

$$f_{l,|m|} = (1-z^2)^{|m|/2}e^{\frac{2-|m|}{dz}(z^2-l^2)}$$

which are well behaved for $l, m$ with $l-|m|$ integral. The ladder operators $L_\pm$ applied to $f_{l,m}$ give $f_{l,m+1}$ for all $m$ except $\pm l/2$, and $L_- f_{l,l/2}$ or $L_+ f_{l,-l/2}$ give functions which are eigenfunction $L_z$ and $L^2$ with the appropriate eigenvalues and which are square integrable though not regular.

The argument that the half-integral $l$ eigen functions are not orthogonal and hence the Hamiltonian is not Hermitian on the space spanned by them is also not satisfactory. Indeed Pauli proved that if half-integral values are permitted, one of the eigen-functions for angular momentum $l$ is not orthogonal to the eigen-functions for angular momentum $1/2$ if $l-1/2$ is odd. This only rules out half the eigen-functions - the eigen-functions corresponding to $l = 0, 1, 2, \ldots$ and $l = 1/2, 3/2, 5/2, \ldots$ are orthogonal as are those corresponding to $l = 0, 1, 2 \ldots$ and $l = 3/2, 7/2, 11/2, \ldots$.

The argument that the half-integral orbital momentum cases are representations not of the rotation group but of its covering group $SU(2)$, is not valid, as can be seen from the fact that half-
integral spin representations are representations not for the Lorentz group but of its covering group. So, if half-integral spin value is allowed, we have no reason to forbid the half-integral orbital angular momentum on this ground. Moreover, it may be cited that in the problem of orbital angular momentum in four dimension, only some of the representations of the four-dimensional rotation group are used.

Now we show that the argument (5) as put forward by Whippman is not applicable to the elementary particle configurations as proposed here. To this end, we first give the details of Whippman's arguments.

Let us first consider the function \( \gamma_{1,m} \) defined by equations (B.3) and (B.4). Again, it is noted that when \( 1 \) is half integral, we may also define the irregular solution of equation (B.2), namely

\[
Z_{1,m} = f_{1,-|m|} e^{im\phi}
\]

with

\[
f_{1,-|m|} = (1 - z^2)^{|m|/2} \frac{1+|m|}{dz|1+|m|} (z^2-1^2)
\]

which is an eigen function of \( L^2 \) and \( L_z \), but is not square integrable unless \( m = \frac{1}{2} \). In the latter case, however, \( Z_{1,\frac{1}{2}} \) is a perfectly acceptable eigen-function of \( L^2 \) and \( L_z \). In fact, if \( Y_{1,\frac{1}{2}} \) is an eigen function, then so is \( Z_{1,\frac{1}{2}} \) since

\[
L^+ Y_{1,-\frac{1}{2}} \sim Z_{1,\frac{1}{2}} \quad \cdots
\]
as can be proved using the explicit forms of \( L_+ \). It then follows that the states generated from eigen-states by the action of \( L_+ \) are themselves eigen states. But then, by the superposition principle

\[
\psi_{1, \pm \frac{1}{2}} = Y_{1, \pm \frac{1}{2}} + a Z_{1, \pm \frac{1}{2}} \quad \ldots \quad (B.9)
\]

is an acceptable wave function where \( a \) is a constant.

Now, the equations (B.3), (B.4), (B.5) and (B.6) suggest that as

\[
\begin{align*}
Z & \to \pm 1 \\
Y_{1, \pm \frac{1}{2}} & \sim (1 - z^2)^{1/4} \\
Z_{1, \pm \frac{1}{2}} & \sim (1 - z^2)^{-1/4}
\end{align*}
\quad \ldots \quad (B.10) \quad (B.11)
\]

and using this, we see that

\[
\lim_{z \to 1} (1 - z^2) \left[ \frac{\psi^*}{\partial z} \psi - \frac{\psi}{\partial z} \psi^* \right] Y_{1, \pm \frac{1}{2}} \quad \ldots \quad (B.12)
\]

is a non-zero constant. But this expression is just

\[
\lim_{Z \to 1} \int_0^{2\pi} J_\theta Y \sin \theta \, d\phi
\]
to within a factor where \( J_{\theta} \) is the component of probability current in the \( \theta \)-direction for the state \( \psi_{1, l_\theta} \). Thus this represents the total flux of particles away from the positive Z-axis at a distance \( r \) from the origin.

Demonstrating this result, Whippmann points out that, the state \( \psi_{l, \frac{1}{2}} \) describes a physical situation where particles are being created at any point on the positive Z-axis arbitrarily far from the origin, and similarly are being destroyed on the negative Z-axis. However, the conservation of probability is not violated by this. But this situation does not seem to occur.

Here we note that, in our elementary particle configurations, where we have suggested that \( l_z = \frac{1}{2} \) and \( -\frac{1}{2} \) represent particle and antiparticle states (or vice versa), the situation described above does not arise. For, if particles are created at any point on the positive Z-axis arbitrarily far from the origin, then to maintain the conservation of probability (and hence the conservation of the total number of particles present over all space), equal number of particles are to be destroyed at any point on the negative Z-axis. However, because, of the specification of \( l_z \)-values for particle and antiparticle states, positive and negative Z-axis are not on the same footing in such a scheme. For, in this configuration scheme, the Z-axis in the positive and negative directions correspond to CP inverted states. Thus in such a configuration, creation of particles arbitrarily far from the origin is not allowed unless the conservation of probability is violated. So, Whippman's argument for the elimination of the half-integral orbital momentum is not applicable in the present case.
The assumption that the two values of \( l_z = \frac{1}{2} \) and \(-\frac{1}{2}\) correspond to particles and antiparticles (or vice-versa) clearly specifies a direction in space and hence rotational invariance is not maintained. Thus, Schwinger's argument for the elimination of the half-integral \( l \)-value on the basis of the infinitesimal generators of the rotation group is also not applicable in the present case. The same is the case with Van Winter's arguments too. For, here \( Z \)-direction is specified by the angular momentum vector and hence excludes the possibility of arbitrary choice in contrast to the \( X \) and \( Y \) directions. However, it must be emphasised that the specification of \( l_z \)-values for particles and antiparticles does not destroy the isotropy of space. In fact, as mentioned earlier, by defining particles and antiparticles in such a way, we are in effect, considering the structure of particles in terms of the intrinsic 'handedness' so that the right and left handed structures stand to each other in particle-antiparticle relation. However, right and left are still indiscernible and thus the isotropy of space is restored. Indeed, if we take the conventional definition of particles and antiparticles, the sense of right and left can be determined by any parity violating interaction. On the other hand, if we take conventional sense of right and left, particles and antiparticles can be distinguished.

It may be added here that in a recent paper\(^{(95)}\), Roy has shown that in a flat Finsler space or a locally anisotropic space, a suitable basis can be constructed for the representation of the Lie algebra associated with the generators of the rotation group \( O(3) \). But this representation of the Lie algebra does not give
rise to the representation of the group $O(3)$. Thus indicates that though $l = \frac{1}{2}$ is allowed in an anisotropic or Finsler space, it cannot be observed in an isotropic space. This points out to the fact that $l = \frac{1}{2}$ can never be observed in the external space of elementary particles.

From the above arguments, we thus conclude that though the half-integral orbital angular momentum is not allowed in atomic or nuclear configurations, the value $l = \frac{1}{2}$ can be allowed in the special case of elementary particle configurations where particles and antiparticles are represented by the two different $l_z$-values.