CHAPTER I

INTRODUCTION

In the present thesis an attempt has been made to study the steady state characteristics and transient phenomena occurred in transformers under various connections and conditions with the help of tensor method developed by Kron using static and sequence reference frames.

Classical methods being used in solving transformer problems are generally employed to draw the equivalent circuits called equivalent networks for the system followed by the analysis of the same. The mathematical analysis of the equivalent networks is based on Ohm's Law, Kirchhoff's law and Maxwell's Law. Subsequently, a number of circuit theorems namely Superposition theorem, Thevenin's theorem, Reciprocity theorem, Norton's theorem, etc. are evoked which become extremely helpful for the solution of those networks. 'Star-Delta' or 'Delta-Star' transformation is another important development used in complex networks analysis. The method of solving problems by 'Determinants' as applied for the solution of a set of linear equations is advantageous when the number of unknown quantities increases. Afterwards 'Matrix' algebra is more and more being used for systematic manipulation and compact representation of electrical network problems.

Another modern method for the solution of network problems is the signal flow graph technique as developed by Mason and Coates. At present this technique is applied for the solution of many problems of electrical static and rotating networks as well as electronic circuits.
Complex circuit problems are sometimes solved with numerical methods like Relaxation method, graphical method etc.

The linear differential equations associated with the transient solution of various electrical and electronic circuits are now solved with the help of the Laplace transform or the Operational method. The chief advantage of this method of approach is that, it is very direct and does away with the tedious evaluations of arbitrary constants in the usual approach of the differential calculus.

Kron thereafter brought the concepts of tensor analysis from its abstract mathematical status and applied it with great success to the solution of electrical and electronic network problems. Before going into the question of desirability of the application of tensors and the consequent advantages secured in transformer problems, a brief review of the work done on related problems by earlier investigators by classical methods is given.

Bock developed algebraic formulae for determining the unbalance in load of three dissimilar one-phase transformers operating as a three-phase delta-delta bank. Bankus and Gerngross studied the optimum transformer combinations for various combinations of load and showed in graphical form how the magnitude of voltage unbalance varies with different combinations of one and three-phase load, with transformer connection and with transformer sizes. Anderson made a detailed study of the effect of unbalanced or combination of single and three-phase loads supplied from delta-connected transformers and proved that the phase loading is a function of phase impedances, where such transformers have their neutrals brought out. An algebraic analysis
was given by him to support a series of graphs which enable the loading ratios for different impedance conditions to be determined. Stigant studied the behaviour of a three-phase star-star connected transformer using vector method for resistive and inductive loading. Assuming a rating factor Larson developed a relation between the maximum current and the rating of a three-phase delta connected transformer whose voltage regulation for balanced loads had also been determined. In the field of rectifier transformers which are nothing but special types of multi-winding transformers, Maslin studied the interphase transformers used in mercury arc power transformer circuits. Grant made a study regarding the ratings of some rectifier transformer circuits. Mention may be made of Rissik, Sarwate, Teago and Gill, Jungmichl and Eichacker who worked on rectifier transformers by conventional methods.

A number of investigators applied classical method to study the transient phenomena occurring in a transformer. Among them, mention may be made of Blume, Camilli, Farnham and Peterson who studied the current transient phenomena, especially the magnetising inrush current, in a multi-winding transformer for the different types of interconnections and they showed its effect on system operation. They also extended their work to show how the inrush current can be reduced in magnitude. Schmidt derived formulae for determining first the inrush current and then the short-circuit current of a single-phase transformer under the assumption that the magnetising characteristics consists of two straight lines. Voltage and current oscillograms had also been given for no-load and rated load during the switching-on process. A basic method had been described by Nelson and Benko to determine the crest value of
the transient inrush currents flowing in power transformer windings formed either from imperfect synchronising procedures or from erroneous switching operations. Pender and Kirkland investigated the magnetising inrush phenomena characterised by a large exponentially decaying direct component of current in the neutral of a three-phase three-limb transformer. The considerations associated with the design and demonstration of short-circuit with-stand capability in power transformers were discussed by McPhutt, Johnson, Nelson and Ayers. Without considering the load, Specht studied the magnetising inrush current of a three-phase star connected transformer and determined some characteristics for calculating approximately the inrush current. He also developed formulas and curves for determining the magnetising inrush current for a single-phase transformer. He also showed that the same method may be applied to certain types of phase controlled rectifiers supplying a series resistance and inductive load. Huber developed a mathematical formula by means of which the magnetising inrush current of an unloaded transformer can be calculated. The various factors determining the inrush current were explained and finally he showed how the inrush current and its decay time varied with the transformer rating. A mathematical analysis was made by Walker to find the relation between the current, excitation and branch flux of a three-phase core type transformer. Sonneman, Wagner and Rockefeller analysed the transient current for single-phase as well as three-phase transformers. He also determined the most suitable proportioning of harmonic restraint to fundamental-frequency operating current. Committee, AIEEE gave a table of the crest inrush current to crest full load current for different ratings of single-phase and
three-phase transformers. They showed that the transformer built up of cold-rolled steel, have larger inrush currents than those built up of hot-rolled steel. Bogush\(^{13}\) drew attention to the fact that the maximum amplitude of the periodic throw-on current of three-phase three-limbed transformers may almost reach the amplitude of the rated current of transformers. Jones and Reed\(^{43}\) calculated the inrush current flowing through the star windings of an unloaded star-delta transformer on three-leg cores. Nakra and Barton\(^{69}\) gave an electromagnetic circuit model of three-phase transformer which includes effects of hysteresis and the non-linear characteristics of ferromagnetism of the core material. The model is suitable for any number of windings of transformers with a variety of inter-phase connections. They applied this model to obtain the transient currents for different types of transformers. Wiszniewski\(^{98}\) described a simple method of determining the values of currents due to faults between turns in power transformers. Gururaj\(^{29,30}\) studied the transient of a three-phase transformer with open and short circuit condition of the secondary windings. This analysis is related specifically to three-phase core type transformers with single winding on the three limbs. He also developed an equivalent circuit to represent several sets of transformer terminal conditions and from these equivalent circuits, the transient voltages in the three-phase core-type transformer had been studied. The expression of transient voltage in a system consisting of a transmission line and a transformer winding in series was deduced by Lovass-Nagy\(^{57}\) using matrix algebra. Harner and Rodriguez\(^{32}\) analysed the resonant
frequencies of three-phase power transformers under faulted secondary and related to the transient-recovery voltage appearing across a fault-interrupting device. Macfadyen, Simpson, Slater and Wood\textsuperscript{60} described a method for analysing the transient behaviour of a three-phase three-limb transformer taking into account the mutual coupling between phases and non-linearity of the magnetic material. Among other investigators in this field mention may be made of Karmakar and Gupta\textsuperscript{46}, Ratnam and Sarma\textsuperscript{77}, Johnson and Schultz\textsuperscript{41}, Corlateanu and Murgu\textsuperscript{20} who analysed the transient behaviour of various multi-winding transformers. Finzi and Mutschler\textsuperscript{22} developed a comparatively simple formula that expresses the first as well as subsequent peaks of inrush currents in terms of line voltage and pertinent factors of design. Roife\textsuperscript{79} and Grossner\textsuperscript{28} studied the transient phenomena occurring in pulse transformer and electronic transformer respectively. Pendlebury\textsuperscript{75} suggested a computerised method of determining the inrush current both for single-phase and three-phase transformers. A simple mathematical expression relating the applied voltage to the maximum inrush current in the no-load switching of a transformer had been developed by Malyshev\textsuperscript{61}.

Using the theory of symmetrical components and alpha, beta, and zero components of three-phase systems, Pandey\textsuperscript{73} presented a method for studying sequential faults on the opposite sides of a three-phase distribution transformer. The problem had been solved in three-stages, namely, for a two line to ground fault, an open conductor and the resulting double fault conditions with grounded as well as with the ungrounded transformers when the grounding wire of the transformer burns out. Hawkes\textsuperscript{33} gave an equivalent zero-sequence circuit of star connected auto-transformers commonly found
on the British transmission system. The concept of equivalent circuit for determining zero-sequence impedance of a three-phase core type transformer had been introduced by Oels. Based on the use of a zero-sequence voltage filter, Kisel and Chernopystov suggested a simple method to find the zero-sequence parameters of a transformer operating on unsymmetrical loads. Wesolowski and Schaupp derived formulas for calculating the reactances of the high-voltage and the low-voltage side and the exciting impedance of a transformer. They also suggested methods of calculating the zero-sequence impedance as well as the ratio between the zero sequence impedance and positive sequence impedance of a transformer with the help of an equivalent circuit. Assuming large ratio of zero-sequence impedances of transformers operating in parallel, Salzmann determined the load shearing of each unit using symmetrical components.

All the above workers have either applied the vectorial or ordinary algebraic method to study the behaviour of multi-winding transformers. Since various kinds of interconnections between coils are possible and as the number of windings increases, number of equations developed by the conventional method multiply and the analysis becomes more tedious.

To minimise the difficulty of solving a large number of equations and to solve widely divergent types of complex network problems, the most powerful method, the "tensor method" is extremely useful. Tensor analysis is introduced as a new mathematical tool by mathematicians during the nineteenth century and since then it has been employed with increasing frequency in the physical and technological problems by physicists and technologists. The application of tensors
as a unified method of approach to the solution of electrical and electronic engineering problems was first introduced by Kron\textsuperscript{51,52,53} and extended later on by Bewley\textsuperscript{6,7}, Boyajian\textsuperscript{14}, Gibbs\textsuperscript{24,25}, Higgins\textsuperscript{35,59}, Happ\textsuperscript{31}, Hoffman\textsuperscript{36,37,38}, Lynn\textsuperscript{58} and others.

Tensor analysis may be considered as an extension and generalisation of the vector analysis from three-dimensional to n-dimensional spaces. It differs from the conventional method of analysis in its versatility of application and organisation.

A tensor can be considered to be a particular kind of entity, geometrical or physical, represented by the aggregate of a set of components, the aggregate being represented by a single symbol, with a suffix which can identify the components by taking a range of values. In some cases the components will have physical existence and the aggregate may be fictitious, in other cases the aggregate will have existence and the components will be fictitious. Operations can be carried out on the aggregate tensor which will give information about the new components in different axes of reference, or about quantities in a given system which will remain invariant when the reference axes are changed (transformed).

From an engineering point of view one important postulate resulting from the theory of relativity is the principle of covariance. This states that, mathematically, the laws of nature are covariant (tensor) differential equations. In other words to express the laws of nature mathematically one must free them from particular reference frames and write them as tensor equations.

In engineering, an extensive field of analysis of macroscopic behaviour can be generalised by removing the effects of particular reference frames and co-ordinates. The results of the developments in geometry of generalised spaces for over a century are available.
for direct application to the generalised or configuration spaces represented by the many engineering variables concerned, when these are treated as co-ordinates. Recently in Japan, Professor Kondo has published reports on the application of tensor analysis to the quantum theory of turbulence. Applications of tensors to quantised and statistical effects in electronics and information theory are still in process of development.

Kron used tensors in electrical networks and machines in an unusual manner. In electrical networks he considered the voltages in the column voltage-matrix as components of a single network-entity, 'voltage', and every current to be a component of a total entity 'current'. From this point of view the network equations are not expressed initially in a form giving the equations of solution of a particular network but as the general equations applying to all networks.

Tensor analysis does not in general described a particular system. It drops the definite article and describes voltage, flux, current etc. as articles. When any given configuration is to be analysed, the tensor equation is expressed in that particular form. This principle was extended by Kron to deal with machines, treated as systems of moving coils, and a generalised form of Ohm's law has been derived for all machines. Kron generalised the equations by writing them in tensorial form and used the existing techniques of transformation to obtain equations of solution.

Following the tensorial method Kron gave the solution of a few multi-winding transformer problems developing their transformation tensors under different connections.

Bewley studied the multi-winding transformer using tensor algebra and made a comparative study with the methods used more
commonly for the solution of such circuits. Stigant\textsuperscript{91,92} used the tensor algebra to obtain the impedances of multi-winding transformers in terms of the basic self and mutual impedances. To study the behaviour of single phase two-winding and multi-winding transformers Bhattacharyya\textsuperscript{8} applied the tensor method in transformer circuits. He also gave an idea of solving multi-winding transformer problems in the three-phase system by the application of tensor method. Mukherjee\textsuperscript{64} analysed scott and Le-blanc connections of transformers tensorially and obtained their characteristics. Mukhopadhyay\textsuperscript{65,66,67} applied the tensor method to obtain the behaviour of the multi-winding transformers as well as rectifier transformers under different connections. He also suggested different methods for the determination of the bucking impedance between any two coils of a single-phase as well as a three-phase core-type transformer. His analysis has thrown a light on the way to study the transient phenomena of a transformer tensorially.

The author in his present work has shown how the Kronian method can be successfully applied to study the steady state as well as transient phenomena of a single-phase and a three-phase core-type transformers under different types of connections and loads in static and sequence reference frames.