THE TRANSIENT ANALYSIS
OF THE SINGLE-PHASE CORE TYPE TRANSFORMER

by

B. K. Sarkar ** and A. K. Mukhopadhyay *

In this paper, Kron's tensorial method has been applied to determine the currents flowing through the different transformer coils from which the transient analysis of the system has been made in terms of design parameters. The theoretically computed transient characteristics have been verified experimentally.

INTRODUCTION

Classical methods have been applied by a number of investigators to study the transient phenomena occurring in a transformer. Thus the transient current phenomena, especially the magnetizing inrush current, in a multiwinding transformer for the different types of interconnections were investigated. Formulae and curves for determining the magnetizing inrush current for a single-phase transformer were developed. The analysis of the transient current for single-phase as well as three-phase transformers was carried out. Moreover the study of transients in a three-phase transformer for open- and short-circuit conditions of the secondary winding was studied. An electromagnetic circuit model of a three-phase transformer has been given. This model was employed in the solution of the transient currents for the three types of transformer cores, viz., three single-phase cores, three-limb core and five-limb core.

All these investigations were mainly confined to open-circuit and short-circuit transients of the transformer under the switching condition. Kron's tensorial method has been applied in this paper to determine the transient currents of a loaded transformer in terms of the design parameters (bucking impedances). However, in this investigation we neglect the magnetizing current.

Starting from the coil currents, the connection tensor of the transformer is developed and with the help of the impedance tensor, expressions for coil currents have been determined from which the steady state and transient characteristics have been evaluated theoretically with the help of an IBM 1130 computer. The theoretical results have been verified experimentally by taking a photograph of the transient current, displayed on the screen of a servoscope.

METHOD

Considering a single-phase core type transformer consisting of two coils 1 and 2 having number of turns $n_p$ and $n_s$ respectively, let $i_1$, $i_2$ and $i'$ be the coil (old) current and $i''$, $i'''$ and $i''''$ be the mesh (new) currents as shown in Fig. 1 and Fig. 2. The connection matrix is formed from the relation between the two sets of currents as given below.

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** Non-member

*** See Editorial of this Issue
Thus

\[
\begin{align*}
l_1 &= l_1' \\
l_2 &= -l_2' \\
l_3 &= l_3'
\end{align*}
\]

Then, considering the mmf producing the core flux to be equal to zero, since the reluctance of the core is negligibly small, the constraint equation is

\[n_p l_1 - n_s l_2 = 0\]

In terms of new mesh currents the constraint equation becomes

\[l_1'' = -\frac{n_p}{n_s} l_1''\]

where \( n = n_s/n_p \)

The transformation tensor \( c_2 \), neglecting the magnetizing current, is given by the relations:

\[l_1'' = l_1''\]

\[l_2'' = -\frac{1}{n} l_2''\]

and thus

\[
c_2 = \begin{pmatrix} 1 & 1' \\ 2 & 2' \end{pmatrix} = -\frac{1}{n} \begin{pmatrix} 1 & 1' \\ 2 & 2' \end{pmatrix}
\]

where the double-dashed currents are the new currents obtained after neglecting the magnetizing currents.
Hence the connection tensor \( C \), neglecting the magnetizing current, and the impedance tensor for the different coils are respectively,

\[
C = C_1 C_3 = \begin{bmatrix}
1 & -n & 0 \\
-2 & -1 & 0 \\
3 & 1 & 0
\end{bmatrix}
\quad \text{and} \quad
Z = \begin{bmatrix}
Z_{1,3} & 0 & 0 \\
0 & Z_{2,3} & 0 \\
0 & 0 & R
\end{bmatrix}
\]

where \( Z_{1,3} \) is the unreferred bucking impedance of the transformer. The result \( Z \) impedance tensor may be developed as

\[
Z' = C_z Z C = \frac{1}{n^2} Z \begin{bmatrix}
1 & 0 & 0 \\
0 & 2nZ_{1,3} + R & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

Let the primary supply voltage be \( e(t) = E \sin(\omega t + \phi) \). Then the impressed voltage tensor of the actual network is

\[
e' = Z' I' = \begin{bmatrix}
I_1' \\
I_2' \\
I_3'
\end{bmatrix}
\]

The new current matrix \( I' \) is given by,

\[
I' = Z'^{-1} e' = \frac{n^2 e}{2nZ_{1,3} + R}
\]

and using the relation \( r = C_i \) (ct., p 56 of Ref 5) the primary and secondary coil relations are obtained as

\[
I_1' = \frac{n^2 e}{2nZ_{1,3} + R} \quad (1)
\]

and

\[
I_2' = -\frac{e}{2nZ_{1,3} + R} \quad (2)
\]

The 'bucking impedance' \( Z_{1,3} \) can be expressed in terms of the 'bucking resistance' and the 'bucking reactance' as

\[
Z_{1,3} = R_{1,3} + jL_{1,3}
\]

where \( p = j\omega \), under steady state conditions.

Then,

\[
2nZ_{1,3} + R = r + pl
\]

where

\[
r = R + 2\pi R_{1,3} \quad \text{and} \quad l = 2\pi L_{1,3}
\]

Then,

\[
I_1' = \frac{n^2 e}{r + pl} = \frac{n^2 e}{l(p + r/l)} \quad (3)
\]
Taking the Laplace transform (cf., Appendix) and simplifying, equation (3) becomes
\[
\mathcal{L}\{i(t)p\} = \frac{n^2E(p)}{1 + \frac{r}{l}}
\]
\[
= \frac{n^2E \cos \theta}{l} \left[ \frac{\omega l^3}{p + \frac{r}{l}} \omega l^3 \frac{1}{p + \frac{r}{l}} \right]
\]
\[
+ \frac{n^2E \sin \theta}{l} \left[ \frac{\omega l^3}{p + \frac{r}{l}} \omega l^3 \frac{1}{p + \frac{r}{l}} \right]
\]
\[
(4)
\]
Taking the inverse Laplace transform (cf., Appendix) the primary current becomes after simplification,
\[
i_p(t) = I \left[ \sin (\omega t + \theta) - \sin (\phi - \theta) e^{-t/(R \cdot L)} \right]
\]
\[
(5)
\]
where,
\[
I \sin \phi = n^2E \omega l/(r^2 + \omega^2 l^2)
\]
\[
I \cos \phi = n^2E r/(r^2 + \omega^2 l^2)
\]
\[
(6)
\]
\[
(7)
\]
\[
\tan \phi = \omega l/r
\]
\[
(8)
\]
and
\[
i_p = n^2E^2/(r^2 + \omega^2 l^2)
\]
\[
(9)
\]
At \( \theta = 0 \), the above expression for current is
\[
i_p(t) = I \left[ \sin \phi e^{-t/(R \cdot L)} + \sin (\omega t - \phi) \right]
\]
\[
(10)
\]
Thus, in terms of design parameters,
\[
i_p(t) = \frac{n^2E}{(R + 2nR_{1,2})^2 + 4n^2\omega^2 L_{1,2}} \left[ \frac{(2n\omega L_{1,2}) e^{\frac{R + 2nR_{1,2}}{2nL_{1,2}}}}{(R + 2nR_{1,2})^2 + 4n^2\omega^2 L_{1,2}} \right]
\]
\[
+ \sin (\omega t - \tan^{-1} \frac{2n\omega L_{1,2}}{R + 2nR_{1,2}})
\]
\[
(11)
\]
The corresponding expression for secondary current will be
\[
i_s(t) = -\frac{nE}{(R + 2nR_{1,2})^2 + 4n^2\omega^2 L_{1,2}} \left[ \frac{(2n\omega L_{1,2}) e^{\frac{R + 2nR_{1,2}}{2nL_{1,2}}}}{(R + 2nR_{1,2})^2 + 4n^2\omega^2 L_{1,2}} \right]
\]
\[
+ \sin (\omega t - \tan^{-1} \frac{2n\omega L_{1,2}}{R + 2nR_{1,2}})
\]
\[
(12)
\]
**SPECIAL CASES**

(1) The short-circuit transient behaviour of the transformer is obtained by putting \( R = 0 \) in the equations (11) and (12). Then the expressions for current in the primary
and secondary windings may be written as

\[ i'(t) = \frac{nE}{(4n^2 R_{1.2}^2 + 4n^2 \omega^2 L_{1.2}^2)^{1/2}} \left[ \frac{(2\omega L_{1.2}) e^{(R_{1.2}+L_{1.2})t}}{(4n^2 R_{1.2}^2 + 4n^2 \omega^2 L_{1.2}^2)^{1/2}} + \sin(\omega t - \tan^{-1} \frac{\omega L_{1.2}}{R_{1.2}}) \right] \] (13)

and

\[ i'(t) = \frac{nE}{(4n^2 R_{1.2}^2 + 4n^2 \omega^2 L_{1.2}^2)^{1/2}} \left[ \frac{(2\omega L_{1.2}) e^{(R_{1.2}/L_{1.2})t}}{(4n^2 R_{1.2}^2 + 4n^2 \omega^2 L_{1.2}^2)^{1/2}} + \sin(\omega t - \tan^{-1} \frac{\omega L_{1.2}}{R_{1.2}}) \right] \] (14)

It is to be noted that the time constant of the exponential decaying curve is constant and it depends on the circuit parameters only.

(2) During short circuit condition \( R = 0 \), the expressions for current in the primary winding is, from equation (5)

\[ i'(t) = \{ \sin(\omega t + \theta - \phi) - \sin(\theta - \phi) e^{-(R_{1.2}/L_{1.2})t} \} \] (15)

where

\[ I = \frac{n^2 E}{Z_{1.2}} \] (16)

\[ I \sin \phi = \frac{nE X_{1.2}}{2Z_{1.2}} \] (17)

\[ \phi = \tan^{-1} \frac{X_{1.2}}{R_{1.2}} \] (18)

and

\[ \sqrt{R_{1.2}^2 + X_{1.2}^2} = Z_{1.2} \] (19)

For the normal transformer the bucking reactance is much greater than the bucking resistance, so that from equations (19) and (18)

\[ Z_{1.2} = X_{1.2} \] (20)

\[ \phi = \pi/2 \] (21)

The value of the transient terms in equations (15) will be zero if

\[ \sin(\theta - \phi) = 0 \]

\[ \theta = \phi \] (22)

Thus, equations (21) and (22)

\[ \theta = \phi = \pi/2 \] (23)

Thus, the transient will disappear if \( \theta = \pi/2 \), when the voltage and current equations become

\[ e = E \cos \omega t \] (24)

and

\[ i'(t) = I \sin \omega t \] (25)

Thus when the switch is closed at an instant when the voltage is maximum, there is no transient current. The current immediately assumes its steady-state value (I) and lags the voltage by \( \pi/2 \).
EXPERIMENTAL RESULTS

The expression for the transient current involves the leakage impedances appearing between primary and secondary windings. So for the theoretical evaluation of the transient currents it is required to measure the leakage impedance. Using the method discussed on the pages of the Quarterly, the design parameters are measured and presented in Table I. The values of the steady-state and short-circuit currents have been predetermined and in order to verify those predetermined values of currents, the transformer is loaded. The results of the test have been presented in Table II. Table III shows the calculated and experimental values of the short-circuit currents for different supply voltages.

The theoretical results, thus obtained, are verified experimentally by taking photographs of the load transient phenomena displayed on the screen of a servoscope which is shown in Fig. 7 corresponding to a load resistance of 24 ohms.

Choosing a proper scale, the experimental curve obtained is compared with the corresponding theoretical curve and shown in Fig. 8, thus verifying the results.

Table I

<table>
<thead>
<tr>
<th>Secondary short circuit current in amps</th>
<th>Primary input voltage in volts</th>
<th>Input power in watts</th>
<th>Mean leakage impedance ( e_{oa}/I^c )</th>
<th>Mean resistive components in ohms</th>
<th>Mean reactive components in ohms</th>
<th>Leakage impedance in vector ohms</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.65</td>
<td>2.1</td>
<td>5.75</td>
<td>0.457</td>
<td>0.262</td>
<td>0.374</td>
<td>0.131 + j0.187</td>
</tr>
<tr>
<td>6.60</td>
<td>3.0</td>
<td>11.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.00</td>
<td>3.7</td>
<td>16.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.35</td>
<td>4.3</td>
<td>22.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table II

<table>
<thead>
<tr>
<th>Input primary voltage in volts</th>
<th>Resistance of each load in ohms</th>
<th>Primary line current in amps</th>
<th>Secondary line current in amps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>Measured</td>
<td>Calculated</td>
</tr>
<tr>
<td>200</td>
<td>24.00</td>
<td>2.02</td>
<td>2.00</td>
</tr>
<tr>
<td>10.10</td>
<td>4.89</td>
<td>4.75</td>
<td>9.78</td>
</tr>
<tr>
<td>8.98</td>
<td>5.49</td>
<td>5.40</td>
<td>10.98</td>
</tr>
<tr>
<td>7.46</td>
<td>6.58</td>
<td>6.50</td>
<td>13.16</td>
</tr>
<tr>
<td>6.44</td>
<td>7.60</td>
<td>7.50</td>
<td>15.30</td>
</tr>
</tbody>
</table>
Fig. 3 Load transient for load \( R_L = 10 \cdot 10 \, \Omega \)
Fig. 4: Load transient for load \( (R_t) = 7.46 \Omega \)
Fig. 5 Short-circuit transient characteristic for primary voltage 2.1 volts

Fig. 6 Short-circuit transient characteristic for primary voltage 3.7 volts
In the present paper, it is shown how Kron's tensorial method may be applied to determine the transient characteristics of a transformer under switching and short-circuit conditions. The utility of the method lies in the fact that in the present analysis, the transient currents have been obtained in terms of the bucking impedances. Hence the transient characteristics can be predetermined. To ensure these characteristics experimentally is a very difficult task. Thus for a design engineer the present method may well be a useful guide line to predetermine the transient characteristics.
It is worth noting that in the tensorial method both the steady-state and transient analysis may be obtained by using a single procedure and thus separate analysis are not required. This is the essence of Kron's tensorial treatment which has been fully utilized here.

ACKNOWLEDGEMENT

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REFERENCES


APPENDIX

THE LAPLACE TRANSFORM CALCULATIONS FROM FIRST PRINCIPLES

The Laplace transform is one of the major mathematical tools that enables a differential equation to be transformed into relatively simple algebraic equations that can be manipulated until the desired forms are obtained. For complex systems, the method of the Laplace transform has a definite advantage over the classical method and the flow diagram representing the technique of solving a differential equation by Laplace transformation is shown below.
For any function \( i(t) \), whose value is zero for the negative values of the variable, the Laplace transform is given by

\[
I(p) = \int_0^\infty i(t) \, e^{-pt} \, dt
\]

and the inverse transform of \( I(p) \) is given by

\[
I(t) = \frac{1}{2\pi j} \int_{C-j\infty}^{C+j\infty} i(p) \, e^{pt} \, dp
\]

where \( \text{Re} \, p > C > 0 \).

Without actual integration, the inverse Laplace transform may be determined by Heaviside's rule. For example, let

\[
I^1(p) = \frac{\pi^2 e(p)}{L(p + \frac{1}{2})} = L \left[ i(t) \right]
\]

The problem is to determine the nature of \( i^1(t) \). Now if the applied voltage is \( E \sin(\omega t + \theta) \),

\[
e(p) = L \left[ E \sin(\omega t + \theta) \right]
\]

\[
= E \left[ \sin(\omega t) \cos \theta + \cos(\omega t) \sin \theta \right]
\]

\[
= E \cos \theta L(\sin(\omega t)) + E \sin \theta L(\cos(\omega t))
\]

\[
= E \cos \theta \cdot \frac{\omega}{p^2 + \omega^2} + E \sin \theta \cdot \frac{p}{p^2 + \omega^2}
\]  

Thus,

\[
l^1(p) = \frac{\pi^2 E \cos \theta}{L(p + r/l)} \cdot \frac{\omega}{p^2 + \omega^2} + \frac{\pi^2 E \sin \theta}{L(p + r/l)} \cdot \frac{p}{p^2 + \omega^2}
\]

Hence,

\[
i^1(t) = L^{-1} \left[ l^1(p) \right] = \frac{\pi^2 E \cos \theta}{l} L^{-1} \left[ \frac{1}{(p + \frac{r}{l})^2 + \omega^2} \right] + \frac{\pi^2 E \sin \theta}{l} L^{-1} \left[ \frac{p}{(p + \frac{r}{l})^2 + \omega^2} \right]
\]
Now let
\[
\frac{1}{(p + r/l)(p^3 + \omega^3)} = \frac{A_1}{p + r/l} + \frac{A_2 p + A_3}{p^3 + \omega^3}
\]
or
\[
1 = A_1(p^3 + \omega^3) + (A_2 p + A_3)(p + r/l)
\]
Putting \(p = -\frac{r}{l}\) both sides,
\[
1 = A_1(r/l + \omega^3)
\]
or
\[
A_1 = \frac{1}{r/l + \omega^3}.
\]
Equating the coefficient of \(p^3\) from both sides
\[
0 = A_1 + A_3
\]
or
\[
A_3 = -\frac{1}{r/l + \omega^3}.
\]
Also, equating the coefficient of \(p^0\) from both sides
\[
1 = A_1 \omega^3 + A_3 \frac{r}{l}
\]
or
\[
A_1 = \frac{1}{r/l}(1 - A_3 \omega^3)
\]
or
\[
A_3 = -\frac{r/l}{r^3 + \omega^3 l^2}.
\]
Also let
\[
\frac{p}{(p + r/l)(p^3 + \omega^3)} = \frac{B_1}{p + r/l} + \frac{B_2 p + B_3}{p^3 + \omega^3}
\]
or
\[
p = B_1(p^3 + \omega^3) + (B_2 p + B_3)(p + r/l)
\]
Putting \(p = -\frac{r}{l}\) both sides,
\[
-\frac{r}{l} = B_1 \left(\frac{r^3}{l} + \omega^3\right)
\]
or
\[
B_1 = -\frac{r/l}{r^3 + \omega^3 l^2}.
\]
Similarly equating the coefficients from both sides
\[
p^3 : 0 = B_1 + B_2
\]
or
\[
B_1 = -\frac{r/l}{r^3 + \omega^3 l^2}.
\]
\[
p^0 : 0 = B_1 \omega^3 + B_3 \frac{r}{l}
\]
or
\[
B_3 = \frac{\omega^3 l^2}{r^3 + \omega^3 l^2}.
\]
Evaluating the values of the constants \( A_1, A_2, A_3, A_4, B_1 \) and \( B_1 \) equation (c) becomes

\[
I'(p) = \frac{n^2 E \cos \theta}{l} \left[ \frac{\omega l}{r^2 + \omega^2 l^2} + \frac{1}{p + r/l} - \frac{\omega l}{r^2 + \omega^2 l^2} \frac{p}{p^2 + \omega^2} + \frac{rl}{r^2 + \omega^2 l^2} \frac{\omega}{p^2 + \omega^2} \right]
\]

\[
+ \frac{n^2 E \sin \theta}{l} \left[ -\frac{rl}{r^2 + \omega^2 l^2} \frac{p}{p^2 + \omega^2} + \frac{rl}{r^2 + \omega^2 l^2} \frac{\omega l}{p^2 + \omega^2} - \frac{\omega l}{r^2 + \omega^2 l^2} \frac{\omega}{p^2 + \omega^2} \right]
\]

(d)

The inverse Laplace transform will yield the solution of the function in the time domain and thus is given as

\[
I'(t) = \frac{n^2 E \cos \theta}{l} \left[ \frac{\omega l e^{-\frac{t}{l}}}{r^2 + \omega^2 l^2} e^{-\frac{t}{l}} + \frac{\omega l}{r^2 + \omega^2 l^2} \cos \omega t + \frac{rl}{r^2 + \omega^2 l^2} \sin \omega t \right]
\]

\[
+ \frac{n^2 E \sin \theta}{l} \left[ -\frac{rl e^{-\frac{t}{l}}}{r^2 + \omega^2 l^2} e^{-\frac{t}{l}} + \frac{rl}{r^2 + \omega^2 l^2} \cos \omega t + \frac{\omega l}{r^2 + \omega^2 l^2} \sin \omega t \right]
\]

\[
= \frac{n^2 E_0 l}{r^2 + \omega^2 l^2} \cos \theta \cos \omega t e^{-\frac{t}{l}} - \frac{n^2 E_0 l}{r^2 + \omega^2 l^2} \cos \theta \cos \omega t + \frac{n^2 E r}{r^2 + \omega^2 l^2} \cos \theta \sin \omega t
\]

- \frac{n^2 E r}{r^2 + \omega^2 l^2} \sin \theta \cos \omega t + \frac{n^2 E_0 l}{r^2 + \omega^2 l^2} \sin \theta \sin \omega t
eq
\]

Putting, \( \frac{n^2 E_0 l}{r^2 + \omega^2 l^2} = I \sin \phi \) and \( \frac{n^2 E r}{r^2 + \omega^2 l^2} = I \cos \phi \)

where \( I = \frac{1}{r^2 + \omega^2 l^2} \) and \( \tan \phi = \frac{\omega l}{r} \)

(f)

From equations (e) and (f)

\[
I'(t) = I \sin \phi \cos \theta e^{-\frac{t}{l}} - I \sin \phi \cos \theta \cos \omega t + I \cos \phi \cos \theta \sin \omega t
\]

- \( I \cos \phi \sin \theta e^{-\frac{t}{l}} + I \cos \phi \sin \theta \cos \omega t + I \sin \phi \sin \theta \sin \omega t \)

= \( I (\sin \omega t \cos \theta + \cos \omega t \sin \theta) \cos \phi \)

- \( (\cos \omega t \cos \theta - \sin \omega t \sin \theta) \sin \phi \)

- \( (\sin \theta \cos \phi - \cos \theta \sin \phi) e^{-\frac{t}{l}} \)

= \( I (\sin (\omega t + \theta) \cos \phi - \cos (\omega t + \theta) \sin \phi - \sin (\theta - \phi) e^{-\frac{t}{l}} \)

= \( I (\sin (\omega t + \theta - \phi) - \sin (\theta - \phi) e^{-\frac{t}{l}} \)

* * * * * *
The transient analysis of the three-phase core type Y-Y transformer by the Kronian method of approach

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Abstract:
In this paper Kron's tensorial method has been used for determining the currents flowing through the different transformer coils from which the transient analysis of the system has been made in terms of bucking impedences. The theoretically computed characteristics have been verified experimentally.

1. Introduction
The transient phenomena occurring in a transformer has been made by Specht, Blume, Camilh, Tarnham & Peterson, Gururaj, Sonneman, Wagner & Rockefeller, Nakra & Barton, etc. by different classical methods. All the investigations have been mainly confined to the open circuit and short circuit transient phenomena of a transformer under switching condition. In the present paper, Kron's tensorial method has been applied to determine the transient currents of a loaded transformer.

Fig. 1. Winding arrangement in the transformer core.

Fig. 2. Circuit diagram of actual connection.
The connection matrix of the transformer has been developed starting from the coil currents and neglecting the magnetising currents and with the help of the impedance tensor, expressions for coil currents have been determined from which the steady state and transient characteristics have been found out theoretically using an I.B.M. 1130 computer. Some of the theoretically computed characteristics have been verified experimentally.

2. Method

To form the connection matrix let \( i^1, i^2 \) etc. be the coil (old) currents and \( i'^1, i'^2 \) etc. be the mesh (new) currents as shown in Figures (1) and (2) respectively. The connection matrix is formed from the relation between two sets of currents as given below:

\[
\begin{align*}
   i^1 &= -i'^1 \\
   i^2 &= i'^2 \\
   i^3 &= -i'^3 \\
   i^4 &= (i'^1 + i'^2) \\
   i^5 &= -(i'^1 + i'^2) \\
   i^6 &= -(i'^1 + i'^2)
\end{align*}
\]

Thus,

\[
C_1 = \begin{pmatrix}
    1 & -1 & 1 & 1 & 1 \\
    -1 & 1 & 1 & 1 & 1 \\
    1 & 1 & 1 & 1 & 1 \\
    1 & 1 & 1 & 1 & 1 \\
    1 & 1 & 1 & 1 & 1 \\
    -1 & -1 & -1 & -1 & -1
\end{pmatrix}
\]

Since in a transformer the reluctance of the core is negligibly small, the m.m.f. producing the core flux may be assumed to be zero. Thus, the constraint equations are

\[
\begin{align*}
   n_{p1}i^1 + n_{s1}i^2 - n_{y1}i^3 &= 0 \\
   n_{p2}i^4 + n_{s2}i^5 - n_{y2}i^6 &= 0
\end{align*}
\]

where \( n_p \) and \( n_s \) are primary and secondary number of turns. In terms of new mesh currents, the constraint equations become,

\[
\begin{align*}
   i'^1 &= n_i i^1 \\
   i'^2 &= n_i i^2
\end{align*}
\]
where \( n = n_1/n_2 \)

Now, considering \( I^* \) and \( I'^* \) the new currents after neglecting the magnetising currents, transformation tensor \( C_n \) is given by the relations

\[
\begin{align*}
I^* &= I^1^* \\
I'^* &= I^2'^* \\
I'^* &= \frac{1}{n} I^2'^* \\
I'^* &= \frac{1}{n} I^2'^*
\end{align*}
\]

and thus,

\[
C_n = \begin{bmatrix}
1^* & 2^* \\
1 & 1 \\
2 & 1 \\
4 & \frac{1}{n} \\
5 & \frac{1}{n}
\end{bmatrix}
\]

Therefore, starting with the coil currents and neglecting the magnetising current, the final connection tensor \( C \) is given by,

\[
C = C_1 C_2 = \frac{1}{n} \begin{bmatrix}
1 & -n & 0 \\
2 & 0 & -n \\
3 & n & n \\
4 & 1 & 0 \\
5 & 0 & 1 \\
6 & -1 & -1 \\
7 & 1 & 0 \\
8 & 0 & 1 \\
9 & -1 & -1
\end{bmatrix}
\]
Using the mutual leakage impedance between different coils, the impedance tensor for different coils
\[
Z = \begin{bmatrix}
Z_{p,q} & 0 \\
0 & R_{m,n}
\end{bmatrix}
\]
where \( Z_{p,q} = \begin{cases} Z_{0} & \text{for } m = n \\ Z_{m,n} & \text{for } m \neq n \end{cases} \),

is formed where \( Z_{p,q} = Z_{q,p} \), \( Z_{p,p} = Z_{q,q} = 0 \),

\( R_{m,n} = 0 \) (for \( m > n \)) and \( R_{m,m} = R_m \) (for \( m=n \)) and the unrefereed bucking impedance between two

coils \( a \) and \( b \) is given by, \( Z_{a,b} = -\frac{1}{2} \frac{n_b}{n_a} \left( Z_{a,n} - 2 \frac{n_a}{n_b} Z_{a,b} + \left( \frac{n_a}{n_b} \right)^2 Z_{b,b} \right) \), which is obviously a

negative quantity. Considering the symmetry of the windings of the transformer, the following

relations between the leakage impedances may be assumed.

\[
\begin{align*}
Z_{1,1} &= Z_{2,2} = Z_{3,3} = m \\
Z_{1,2} &= Z_{2,3} = Z_{3,1} = b \\
Z_{1,3} &= Z_{2,1} = Z_{3,2} = c \\
Z_{1,4} &= Z_{2,5} = Z_{3,6} = d \\
R_1 &= R_2 = R_3 = R
\end{align*}
\]

Then the resultant impedance tensor must be developed as

\[
Z' = C_i Z C = \frac{1}{n^3}
\]

Let the primary phase supply voltages for the system be \( e_{pb}, a e_{pb} \) and \( a^2 e_{pb} \) where \( a = e^{j\pi/3} \) is a

cube root of unity. Then, the impressed voltage tensor of the actual network is

\[
\begin{array}{c|c|c}
1^* & 2^* \\
\hline
1^* & -2d + 4n(c-b) - 2n^3 m + 2R & -2d + 2n(c-b) - n^3 m + R \\
2^* & -d + 2n(c-b) - n^3 m + R & -2d + 4n(c-b) - 2n^3 m + 2R
\end{array}
\]

The new current matrix \( i' \) is given by,

\[
i' = Z'^{-1} e' = -\frac{n^3}{y} e_{pb}
\]

where \( y = R - d + 2n(c-b) - n^3 m \)

Then, the current of each coil can be calculated by the relation

\[
i'' = C i'
\]
Thus,

\[ i_1 = -\frac{n}{y} e_{ph}  \]

Hence, the expression for currents in coil 1 and 4 are respectively,

\[ i_1 = \frac{n^2 e_{ph}}{y} = \frac{n^2 e_{ph}}{R - d + 2n (c - b) - n^3 m} \]

and

\[ i_4 = -\frac{n e_{ph}}{y} = -\frac{n e_{ph}}{R - d + 2n (c - b) - n^3 m} \]

Each "bucking impedance" is expressed in terms of "bucking resistance" and "bucking reactance" as

\[
m = R_{1-3} + pL_{1-3} = R_{2-5} + pL_{2-5} = R_{1-3} + pL_{1-3} \\
b = R_{1-4} + pL_{1-4} = R_{2-6} + pL_{2-6} = R_{1-4} + pL_{1-4} \\
c = R_{1-5} + pL_{1-5} = R_{1-4} + pL_{1-4} = R_{1-5} + pL_{1-5} \\
d = R_{1-5} + pL_{1-5} = R_{1-4} + pL_{1-4} = R_{1-5} + pL_{1-5} \\
\]

where \( p = j\omega \) under steady state condition.

Then,

\[ R - d + 2n (c - b) - n^3 m = r + pl \]

where,

\[ r = R - R_{4-3} + 2n (R_{3-5} - R_{2-4}) - n^3 R_{1-2} \]

and

\[ l = -L_{4-5} + 2n (L_{3-5} - L_{2-4}) - n^3 L_{1-2} \]

Thus,

\[ l' = \frac{n^2 e_{ph}}{r + pl} = \frac{n^2 e_{ph}}{i(p + r/l)} \]
Taking the Laplace transform and after simplifying, equation (3) becomes

\[ i'(p) = \frac{n^2 c_{ph}(p)}{(p + r/l)} \]

\[ = \frac{n^2 E \cos \theta}{l} \left[ \frac{\omega l^2}{r^2 + \omega^2 l^2} \cdot \frac{1}{p + r/l} - \frac{\omega l^2}{r^2 + \omega^2 l^2} \cdot \frac{p}{p^2 + \omega^2} \right. \]

\[ \left. + \frac{r l}{r^2 + \omega^2 l^2} \cdot \frac{1}{p + r/l} \right] \]

\[ + \frac{n^2 E \sin \theta}{l} \left[ - \frac{r l}{r^2 + \omega^2 l^2} \cdot \frac{1}{p + r/l} + \frac{r l}{r^2 + \omega^2 l^2} \cdot \frac{p}{p^2 + \omega^2} + \frac{\omega l^2}{r^2 + \omega^2 l^2} \cdot \frac{1}{p^2 + \omega^2} \right] \]

... (4)

Taking the inverse Laplace transform the primary current becomes after simplification,

\[ i'(t) = I [ \sin (\omega t + \theta - \phi) - \sin (\theta - \phi) \ e^{-(r/l)t} ] \]

... (5)

where

\[ I \sin \phi = n^2 E \omega l/r^2 + \omega^2 l^2 \]

... (6)

\[ I \cos \phi = n^2 E \omega r/r^2 + \omega^2 l^2 \]

... (7)

\[ \tan \phi = \omega l/r \]

... (8)

and

\[ I^3 = n^4 E^3/r^2 + \omega^3 l^3 \]

... (9)

At \( \theta = 0 \), the above expression for current is

\[ i^3(t) = I [ \sin \phi \ e^{-(r/l)t} + \sin (\omega t - \phi) ] \]

... (10)

Similarly, the expression for currents in the other coils are represented as

\[ i^1(t) = a_1 I [ \sin \phi \ e^{-(r/l)t} + \sin (\omega t - \phi) ] \]

... (11)

\[ i^2(t) = a_2 I [ \sin \phi \ e^{-(r/l)t} + \sin (\omega t - \phi) ] \]

... (12)

\[ i^3(t) = -\frac{I}{n} [ \sin \phi \ e^{-(r/l)t} + \sin (\omega t - \phi) ] \]

... (13)

\[ i^4(t) = -a_3 \frac{I}{n} [ \sin \phi \ e^{-(r/l)t} + \sin (\omega t - \phi) ] \]

... (14)

\[ i^5(t) = -a_4 \frac{I}{n} [ \sin \phi \ e^{-(r/l)t} + \sin (\omega t - \phi) ] \]

... (15)

\[ i^6(t) = a_1 \frac{I}{n} [ \sin \phi \ e^{-(r/l)t} + \sin (\omega t - \phi) ] \]

... (16)

\[ i^7(t) = a_2 \frac{I}{n} [ \sin \phi \ e^{-(r/l)t} + \sin (\omega t - \phi) ] \]

... (17)

\[ i^8(t) = a_3 \frac{I}{n} [ \sin \phi \ e^{-(r/l)t} + \sin (\omega t - \phi) ] \]

... (18)
Thus, in terms of design parameters the expression for currents in coils I & 4 are evaluated from

\[ i^t(t) = \frac{n^2 E \omega (-L_{4,5} + 2n (L_{3,5} - L_{2,3}) - n^2 L_{1,2}) \epsilon}{(R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2})} t \]

\[ \frac{n^2 E \sin \left( \omega t - \tan^{-1} \frac{\omega (-L_{4,5} + 2n (L_{2,3} - L_{1,2}) - n^2 L_{1,2})}{R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2}} \right)}{R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2}} \]

and

\[ i^t(t) = \frac{n E \omega (-L_{4,5} + 2n (L_{3,5} - L_{2,3}) - n^2 L_{1,2}) \epsilon}{(R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2})} t \]

\[ \frac{n E \sin \left( \omega t - \tan^{-1} \frac{\omega (-L_{4,5} + 2n (L_{2,3} - L_{1,2}) - n^2 L_{1,2})}{R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2}} \right)}{R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2}} \]

Similarly, the expressions for the other coil currents may be determined.

The short circuit transient behaviour of the transformer is obtained by putting \( i_1 = 0 \) in the equations (19) and (20). Then the expressions for current in the primary and secondary windings may be written as

\[ i^t(t) = \frac{n E \omega (-L_{4,5} + 2n (L_{3,5} - L_{2,3}) - n^2 L_{1,2}) \epsilon}{(R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2})} t \]

\[ \frac{n E \sin \left( \omega t - \tan^{-1} \frac{\omega (-L_{4,5} + 2n (L_{2,3} - L_{1,2}) - n^2 L_{1,2})}{R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2}} \right)}{R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2}} \]

and

\[ i^t(t) = \frac{n E \omega (-L_{4,5} + 2n (L_{3,5} - L_{2,3}) - n^2 L_{1,2}) \epsilon}{(R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2})} t \]

\[ \frac{n E \sin \left( \omega t - \tan^{-1} \frac{\omega (-L_{4,5} + 2n (L_{2,3} - L_{1,2}) - n^2 L_{1,2})}{R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2}} \right)}{R - R_{4,5} + 2n (R_{3,5} - R_{2,3}) - n^2 R_{1,2}} \]

3. Experimental results

The theoretical transient characteristics are drawn by taking the following value of the leakage impedances of a three-phase coretype transformer, using a method as suggested by Mukhopadhyay*.

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The steady state and short circuit
\[ Z_{1-5} = Z_{5-1} = Z \]
\[ Z_{1-4} = Z_{4-1} = Z_{2-4} = Z_{4-2} = Z_{3-4} = Z_{4-3} = Z \]
\[ Z_{1-3} = Z_{3-1} = Z \]

 currents have been predetermined and in order to verify those predetermined values of currents the transformer is loaded. The results have been presented in table I and table II shows the calculated and experimental values of the short circuit currents.

<table>
<thead>
<tr>
<th>Primary Input voltage in volts</th>
<th>Resistance of each load in ohms</th>
<th>Primary line current in amps</th>
<th>Secondary line current in amps.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Calculated</td>
<td>Measured</td>
</tr>
<tr>
<td>400</td>
<td>32.0</td>
<td>2.40</td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td>25.8</td>
<td>2.97</td>
<td>3.05</td>
</tr>
<tr>
<td></td>
<td>21.5</td>
<td>3.55</td>
<td>3.65</td>
</tr>
<tr>
<td></td>
<td>18.3</td>
<td>4.16</td>
<td>4.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Primary Input voltage in volts</th>
<th>Primary short circuit current in amps.</th>
<th>Secondary short circuit current in amps.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>Measured</td>
</tr>
<tr>
<td>8.4</td>
<td>2.04</td>
<td>2.00</td>
</tr>
<tr>
<td>12.6</td>
<td>3.06</td>
<td>3.00</td>
</tr>
<tr>
<td>16.8</td>
<td>4.08</td>
<td>4.10</td>
</tr>
<tr>
<td>20.0</td>
<td>4.86</td>
<td>4.80</td>
</tr>
</tbody>
</table>
Fig. 3. Load transient for load ($R_L$) = 32 ohm.
Fig. 4. Load transient for load ($R_L = 25$ ohm)

Fig. 5. Short circuit transient characteristics for primary voltage 8.4 volts.
Fig 6. Short circuit transient characteristic for primary voltage 16.8 volts

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The transient currents have been evaluated with the help of IBM 1130 computer. The load transient curves have been shown in Figures (3) & (4) and Figures (5) & (6) represent the short circuit transient curves.

4. Conclusion
In the present paper Kron's tensorial method has been applied to determine the transient characteristics of a transformer. The transient characteristics that are obtained in terms of bucking impedances can be predetermined and hence for a design engineer the present method will be a guide line to predetermine the transient characteristics. Again both the steady state and transient analysis may be obtained by this method by using a unique approach.

The transient part of current is developed by proper mathematical manipulation from the expression (1) for currents from which the steady state part has been obtained after simplification. So the transient part is expected to be valid as the steady state is verified.

6. Acknowledgement
The authors are grateful to Prof. S.P. Bhattacharyya, Professor of the Department of Applied Physics, Calcutta University, for his valuable discussion.

7. References
THE STEADY STATE AND TRANSIENT ANALYSIS OF TWO SINGLE-PHASE TRANSFORMERS CONNECTED IN PARALLEL

by

B K Sarkar * and A K Mukhopadhyay *

In the field of electrical technology, specially in the generating plant parallel operation of transformers are required for supplying a load in excess of the rating of an existing transformer. In the present paper, Kron's tensorial approach has been adopted to determine the currents flowing through the different branches of two transformers connected in parallel, from which the transient analysis of the system has been developed in terms of design parameters. The theoretically computed characteristics have been verified experimentally.

LIST OF SYMBOLS

- $e(t)$ = applied voltage
- $E$ = peak value of voltage
- $i^m$ = coil current in the $m$th coil
- $i^m'$ = mesh current in the $m$th mesh
- $i^m''$ = mesh current after neglecting magnetizing current, $m = 3, 4$
- $C_k$ = connection tensor, $k = 1, 2$
- $(C_k)^*$ = transpose of $C_k$
- $Z$ = impedance tensor
- $Z'$ = transformed impedance tensor
- $\theta$ = phase angle
- $f$ = frequency
- $w = 2\pi f$
- $\phi$ = differential operator
- $Z_{m-n}$ = bucking impedance between coils $m$ and $n$
- $R_{m-n}$ = bucking resistance between coils $m$ and $n$
- $L_{m-n}$ = bucking inductance between coils $m$ and $n$
- $n_{pr}$ = primary number of turns, $r = 1, 2$
- $n_{aq}$ = secondary number of turns, $q = 3, 4$
- $n_1 = n_{a1}/n_{p1}$
- $n_2 = n_{a2}/n_{p2}$
- $z_{aa} = $ self impedance of the coil $a$
- $z_{bb} = $ self impedance of the coil $b$
- $z_{ab} = $ mutual impedance between coils $a$ and $b$

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INTRODUCTION

Parallel operation of transformers is an extremely important operation of the generating station and with the help of this operation energy may be supplied to loads in excess of the capacity of a single transformer. Hence the transient analysis of the system of transformers connected in parallel is important. An attempt has been made in this paper to study the performance of the system under steady state and transient conditions considering different conditions of transformers from the design point of view.

Mathematical analysis of the transformer for the determination of transient phenomena have been made by a number of workers using traditional methods. The equivalent circuit of different types of transformers have been developed and the transient voltages in the three-phase core type transformer have been determined. Study has been further extended to determine the magnetizing inrush current for a single-phase transformer. Expressions for the first as well as the subsequent peaks of inrush currents in terms of line voltage and pertinent factors of design have been found. An electromagnetic circuit model of three-phase transformer has been given. The short circuit and open circuit transient phenomena under switching condition were the main object of all the above investigations.

Kron’s tensorial method has been applied to determine currents flowing through different transformer coils from which the transient analysis of the system has been made in terms of design parameters. In the present paper transient analysis of two loaded transformers connected in parallel has been made for different switching conditions. Kron’s method of approach assuming the transformer circuits as mesh networks has been applied to develop the expressions for coil currents from which the steady state and transient characteristics have been found out. The validity of theoretically computed characteristics has been established experimentally.

METHOD

Considering two single-phase core type transformers each consisting of two coils 1, 3 and 2, 4, let \( i_1,\ i_3 \), etc., be the coil currents and \( i_1',\ i_3', \) etc., be the mesh currents as shown in Fig. 1. The current relations from which the connection tensor \( C_1 \) is formed, are as follows:

\[
\begin{align*}
    i_1 &= i_1' \\
    i_2 &= i_3' \\
    i_3 &= i_1' \\
    i_4 &= i_3' \\
    i_5 &= (i_1' + i_3')
\end{align*}
\]

and thus...
Since, the reluctance of the core in a transformer is negligibly small, the m.m.f. producing the core flux may be assumed to be zero. Then the equations of constraints in terms of the coil currents may be written as
\[ \pi_{P1} I_1 + \pi_{A1} I_{1'} = 0 \\
\pi_{P2} I_2 + \pi_{A2} I_{2'} = 0 \]
In terms of new mesh currents these constraint equations become

\[ i'_1 - n_1 i''_1 \]

and

\[ i'_2 - n_2 i''_2 \]

If after neglecting the magnetizing current, the currents be denoted by \( i'' \) and \( i' \), the transformation tensor \( C_2 \) is formed from the following relations

\[ i'_1 = -n_1 i''_1 \]

\[ i'_2 = -n_2 i''_2 \]

\[ i'_3 = i''_3 \]

\[ i'_4 = i''_4 \]

and thus

\[
C_2 = \begin{pmatrix}
3'' & 4'' \\
1' & \pi_1 \\
2' & -\pi_2 \\
3' & 1 \\
4' & 1
\end{pmatrix}
\]

Hence, the final connection tensor \( C \), neglecting the magnetizing current is given by

\[
C = C_1 C_2 = \begin{pmatrix}
3'' & 4'' \\
1 & -\pi_1 & 0 \\
2 & 0 & -\pi_2 \\
3 & 1 & 0 \\
4 & 0 & 1 \\
5 & 1 & 1
\end{pmatrix}
\]

Using unreferred bucking impedances the impedance tensor for different coils may be constructed as
where the unreferred bucking impedance between two coils \( a \) and \( b \) is given by\(^a\),

\[
Z_{a-b} = -\frac{n_b}{n_a} \left[ z_{a-a} - 2 \frac{n_a}{n_b} z_{a-b} + \left( \frac{n_a}{n_b} \right)^2 z_{b-b} \right],
\]

which is obviously a negative quantity.

Then the resultant impedance tensor may be developed as

\[
Z' = G_z ZC = \begin{bmatrix}
3' & 4'' \\
3'' & 4'
\end{bmatrix}
= \begin{bmatrix}
3' & 4'' \\
3'' & 4'
\end{bmatrix}
= \begin{bmatrix}
A & R \\
R & B
\end{bmatrix}
\]

where \( A = -2n_z z_{1-3} + R \), \( B = -2n_z z_{2-4} + R \).

Then the impressed voltage tensor of the actual network is

\[
e'(t) = \begin{bmatrix}
3'' \\
4''
\end{bmatrix}
= \begin{bmatrix}
-n_1 \\
-n_1
\end{bmatrix}
e(t)
\]

where \( e(t) = E \sin(\omega t + \theta) \).

The new current matrix \( i' \) is given by

\[
i'_t = Z'^{-1} e' = \frac{e}{AB - R^2} \begin{bmatrix}
3'' \\
4''
\end{bmatrix}
= \begin{bmatrix}
-n_1 B + n_1 R \\
-n_1 A + n_1 R
\end{bmatrix}
\]

Then the current of each coil can be calculated by the relation \( i^o = c i' \), thus
Hence the expressions for currents in coils 1, 2, 3, 4 and 5 are

\[
\begin{align*}
t_1 &= \frac{-n_1 e}{AB - R^2} (n_1 B - n_2 R) \\
 t_2 &= -\frac{n_2 e}{AB - R^2} (n_2 A - n_1 R) \\
 t_3 &= -\frac{e}{AB - R^2} (n_1 B - n_2 R) \\
 t_4 &= -\frac{e}{AB - R^2} (n_2 A - n_1 R) \\
 t_5 &= -\frac{e}{AB - R^2} \left\{(-n_1 B + n_2 R) + (-n_2 A + n_1 R)\right\}
\end{align*}
\]

respectively. The primary and the secondary circuits of a transformer are linked by mutual flux and therefore any switching process results in the development of transient currents. Different cases will now be studied corresponding to different conditions of the two transformers.

**CASE 1**

When the turn ratios of two transformers be equal (i.e., \(n_1 = n_2 = n\)), the expression for coil current in the primary winding from equation (2) becomes

\[
\begin{align*}
t_1 &= -\frac{n_1 e}{AB - R^2} (n_1 B - n_2 R) \\
 t_2 &= -\frac{n_2 e}{AB - R^2} (n_2 A - n_1 R) \\
 t_3 &= -\frac{e}{AB - R^2} (n_1 B - n_2 R) \\
 t_4 &= -\frac{e}{AB - R^2} (n_2 A - n_1 R) \\
 t_5 &= -\frac{e}{AB - R^2} \left\{(-n_1 B + n_2 R) + (-n_2 A + n_1 R)\right\}
\end{align*}
\]

Now expressing 'bucking impedances' in terms of 'bucking resistances' and 'bucking reactances' as

\[
- z_{1-1} = R_{1-1} + p L_{1-1}
\]

and

\[
- z_{2-4} = R_{2-4} + p L_{2-4}
\]

the following simplifications

\[
AB - R^2 = 4n^4 \left\{(R_{1-2}R_{2-4} + p (L_{1-2}R_{2-4} + R_{1-2}L_{2-4}) + p^2 (L_{1-2}L_{2-4}))\right\}
\]

\[+ 2nR \left\{(R_{1-2}R_{2-4} + p (L_{1-2}R_{2-4} + R_{1-2}L_{2-4})\right\}
\]
\[ AB - R^2 = a_1 (p^2 + a_4 p + a_5) \quad (10) \]

and

\[ n^4 (B - R) = e 2 n^4 (R_{1-4} + p L_{2-4}) \]

\[ = e a_5 (p + a_6) \quad (11) \]

may be made where

\[ a_1 = 4n^3 (L_{1-3} L_{1-4}) \quad (12) \]

\[ a_4 = \frac{4n^2 (L_{1-3} R_{1-4} + R_{1-4} L_{2-4}) + 2nR (L_{1-3} L_{2-4})}{4n^2 (L_{1-3} L_{1-4})} \quad (13) \]

\[ a_5 = \frac{4n^2 (R_{1-4} R_{2-4}) + 2nR (R_{1-4} + R_{2-4})}{4n^2 (L_{1-3} L_{1-4})} \quad (14) \]

\[ a_7 = 2n^4 L_{2-4} \quad (15) \]

and

\[ a_8 = \frac{R_{2-4}}{L_{1-4}} \quad (16) \]

Thus

\[ i^1 = \frac{a_5 (p + a_6)}{a_1 (p^2 + a_4 p + a_5)} \quad (17) \]

Taking Laplace transforms and simplifying equation (17),

\[ i^1(p) = \frac{a_5 (p + a_6)}{a_1 (p^2 + a_4 p + a_5)} e(p) \quad (18) \]

\[ = \frac{a_5 E}{a_1} \cos \theta \left[ \frac{A_4 (p + \frac{a_6}{2})}{(p + \frac{a_2}{2}) - a_6^2} + \frac{A_4 (A_1 - \frac{a_4}{2})}{(p + \frac{a_2}{2}) - a_6^2} + \frac{A_5}{p^2 + w^2} + \frac{A_6}{p^3 + w^2} \right] \]

\[ + \frac{a_5 E}{a_1} \sin \theta \left[ \frac{B_4 (p + \frac{a_6}{2})}{(p + \frac{a_2}{2}) - a_6^2} + \frac{B_4 (B_1 - \frac{a_5}{2})}{(p + \frac{a_2}{2}) - a_6^2} + \frac{B_5}{p^2 + w^2} + \frac{B_6}{p^3 + w^2} \right] \quad (19) \]

where

\[ a_6 = \pm \sqrt{\frac{a_2^2}{4} - a_4} \quad (20) \]

Again taking the inverse Laplace transforms the expression for currents after simplification becomes

\[ i^1(t) = \left[ I_1 \sin(wt + \phi) + I_3 e^{-u t} \sinh(a_4 t + \beta) \right] \cos \theta \]

\[ + \left[ I_1 \sin(wt + \phi) + I_3 e^{-u t} \sinh(a_4 t + \gamma) \right] \sin \theta \quad (21) \]
The constants $C_1, C_2, \ldots$, and $d_1, d_2, \ldots$, arising from Laplace transformation are given by

$$C_1 = \frac{a_2 E^2}{a_1} \cdot \frac{a_3 - a_4 a_6 - w^3}{\Delta}$$
$$C_2 = \frac{a_2 E^2}{a_1} \cdot \frac{a_4 a_3 + a_2^2 a_6 - a_4 a_5 + a_3 w^2}{\Delta}$$
$$C_3 = \frac{a_2 E^2}{a_1} \cdot \frac{a_5 a_6 - a_6 w^3}{\Delta}$$
$$C_4 = \frac{a_2 E^2}{a_1} \cdot \frac{a_6 a_6 w^3 + a_4 w^3}{\Delta}$$
$$d_1 = \frac{a_3 E}{a_1} \cdot \frac{a_5 a_3 + a_6 w^2 - a_6 w^3}{\Delta}$$
$$d_2 = \frac{a_3 E}{a_1} \cdot \frac{a_4 - a_4 w^3 - a_3 a_6}{\Delta}$$
$$d_3 = \frac{a_3 E}{a_1} \cdot \frac{a_5 a_5 + a_6 w^3 - a_6 w^3}{\Delta}$$
$$d_4 = \frac{a_3 E}{a_1} \cdot \frac{a_5 a_6 w^3 - a_3 w^3 + w^3}{\Delta}$$

and

$$\Delta = (a_1 - w^3)^3 + a_1^2 w^3$$

At $\theta = 0$, the expression for current from equation (21) becomes

$$I(t) = I_0 \sin (\omega t + \phi) + I_s e^{2a_4 t} \sinh (a_4 t + \beta)$$

Now, if the constant $C_3$ is less than $C_1$ (i.e., $C_3 < C_1$), the current expression reduces to

$$I(t) = I_0 \sin (\omega t + \phi) + I_s e^{2a_4 t} \cosh (a_4 t - \beta)$$
where

\[ I_1^2 = C_1^4 \cdot C_4 \cdot \tan \phi = \frac{C_2}{C_4} \quad (31) \]

\[ I_2^2 = C_1^4 \cdot C_1 \cdot \tanh \beta = \frac{C_3}{C_1} \quad (32) \]

The expression of current for the short-circuited secondary (when \( R = 0 \)) in coil 1 from equation (7) is

\[ i^1 = \frac{ne}{-2x_{1,3}} = \frac{nE \sin (\omega t + \theta)}{2x(p + y)} \quad (33) \]

where

\[ x = L_{1,3} \quad ; \quad y = \frac{R_{1,3}}{L_{1,3}} \quad (34) \]

Taking Laplace transforms and simplifying, equation (33) becomes

\[ i^1(p) = \frac{ne(p)}{2x(p + y)} \quad (35) \]

\[ = \frac{nE \cos \theta}{2x} \left[ \frac{1}{w^2 + y^2} \cdot \frac{1}{p + y} + \frac{w}{w(w+y)} \cdot \frac{w}{p} \cdot \frac{1}{p + w^2} \cdot \frac{p}{p^2 + w^2} \right] \]

\[ + \frac{nE \sin \theta}{2x} \left[ \frac{1}{w^2 + y^2} \cdot \frac{1}{p + y} + \frac{w}{w(w+y)} \cdot \frac{w}{p} \cdot \frac{1}{p + w^2} \cdot \frac{p}{p^2 + w^2} \right] \quad (36) \]

Again taking inverse Laplace transforms, the expressions for short-circuit transient, after simplification, becomes

\[ i^1(t) = I \left[ \sin(\omega t - \phi) + \sin(\theta - \phi)e^{-y^2} \right] \quad (37) \]

where

\[ I \sin \phi = \frac{nE \omega}{2x(w^2 + y^2)} \quad (38) \]

\[ I \cos \phi = \frac{nE y}{2x(w^2 + y^2)} \quad (39) \]

\[ \tan \phi = \frac{w}{y} \quad (40) \]

\[ I = \frac{nE}{2x(w^2 + y^2)^{1/2}} \quad (41) \]

At \( \theta = 0 \), the above expressions for current become

\[ i^1(t) = I \sin(\omega t - \phi) + I \sin \phi e^{-y^2} \quad (42) \]

Thus, the short circuit transient currents of coils 1 and 2 in terms of design parameters become
respectively. Similarly the expressions for the secondary currents may be determined.

CASE II

When the turn ratios of two transformers having the same bucking impedances (\(i.e., n_{1,1} = n_{2,1} = 2\)) are equal \(n_1 = n_2 = n\), the expression for coil current in the primary winding from equation (2) becomes

\[
i_1 = \frac{n^2 E}{A + R} \left( \frac{\sin(wt + \theta)}{p + \frac{r}{l}} \right)
\]

where \(r = 2R + 2n^2r_{1,1}\), \(l = 2nL_{1,1}\), and \(p = \omega n\) under steady state condition.

Taking Laplace transforms and simplifying, equation (45) becomes

\[
i^*(p) = \frac{n^2 e(p)}{l(p + \frac{r}{l})}
\]

\[
= \frac{n^4 E \cos \theta}{l} \left[ \frac{\omega l^2}{p + \frac{r}{l}} - \frac{\omega l^2}{p^2 + \omega^2 l^2} + \frac{p}{p^2 + \omega^2 l^2} + \frac{\omega l^2}{p^2 + \omega^2 l^2} \right]
+ \frac{n^4 E \sin \theta}{l} \left[ \frac{r l}{p + \frac{r}{l}} - \frac{\omega l^2}{p^2 + \omega^2 l^2} + \frac{p}{p^2 + \omega^2 l^2} - \frac{\omega l^2}{p^2 + \omega^2 l^2} \right]
\]

(47)

Again taking the inverse Laplace transforms the expression for currents in coil 1 becomes after simplification,

\[
i^*(t) = \frac{1}{l} \left[ \sin(wt + \phi) - \sin(\theta - \phi) e^{-\frac{t}{l}} \right]
\]

(48)

where

\[
I \sin \phi = \frac{n^2 E l}{p^2 + \omega^2 l^2}
\]

\[
I \cos \phi = \frac{n^2 E l}{r^2 + \omega^2 l^2}
\]

\[
\tan \phi = \frac{\omega l}{r}
\]

(49)

(50)

(51)
and

\[ I = \frac{n^2 E}{(R + \omega L_1)^2} \]  

(52)

At \( \theta = 0 \), the above expression for current becomes

\[ i^1(t) = I \sin(\omega t - \phi) + I \sin \phi e^{-\frac{t}{\tau}} \]  

(53)

Similarly, the expression of currents for the other coils are given by

\[ i^2(t) = \frac{I}{n} \sin(\omega t - \phi) + \frac{I}{n} \sin \phi e^{-\frac{t}{\tau}} \]  

(54)

\[ i^3(t) = \frac{I}{n} \sin(\omega t - \phi) + \frac{I}{n} \sin \phi e^{-\frac{t}{\tau}} \]  

(55)

\[ i^4(t) = \frac{I}{n} \sin(\omega t - \phi) - \frac{I}{n} \sin \phi e^{-\frac{t}{\tau}} \]  

(56)

\[ i^5(t) = -\frac{2I}{n} \sin(\omega t - \phi) - \frac{2I}{n} \sin \phi e^{-\frac{t}{\tau}} \]  

(57)

Thus, the expression of currents in coils 1, 3 and 5 in terms of design parameters are

\[ i^1(t) = \frac{n^2 E \sin(\omega t - \tan^{-1} \frac{2nL_1}{2R + 2nR_{1,3}})}{(2nR_{1,3})^2 + 4n^2 \omega^2 L_{1,3}^2} + \frac{2n^3 E \omega L_{1,3} e^{-\frac{t}{\tau}}}{3nL_{1,3}} \]  

(58)

\[ i^3(t) = \frac{n^2 E \sin(\omega t - \tan^{-1} \frac{2nL_1}{2R + 2nR_{1,3}})}{(2nR_{1,3})^2 + 4n^2 \omega^2 L_{1,3}^2} + \frac{2n^3 E \omega L_{1,3} e^{-\frac{t}{\tau}}}{3nL_{1,3}} \]  

(59)

and

\[ i^5(t) = -\frac{2n^2 E \sin(\omega t - \tan^{-1} \frac{wL_{1,3}}{R_{1,3}})}{(2nR_{1,3})^2 + 4n^2 \omega^2 L_{1,3}^2} + \frac{2n^3 E \omega L_{1,3} e^{-\frac{t}{\tau}}}{3nL_{1,3}} \]  

(60)

respectively. Similarly, the expressions for the currents of other coils may be determined.

During short-circuit condition \( (R = 0) \), the short-circuit transients of coils 1 and 3 are respectively

\[ i^1(t) = \frac{n^2 E \sin(\omega t - \tan^{-1} \frac{wL_{1,3}}{R_{1,3}})}{4n^2 R_{1,3}^2 + 4n^2 \omega^2 L_{1,3}^2} + \frac{2n^3 E \omega L_{1,3} e^{-\frac{t}{\tau}}}{4n^2 R_{1,3}^2 + 4n^2 \omega^2 L_{1,3}^2} \]  

(61)

and

\[ i^3(t) = \frac{n^2 E \sin(\omega t - \tan^{-1} \frac{wL_{1,3}}{R_{1,3}})}{4n^2 R_{1,3}^2 + 4n^2 \omega^2 L_{1,3}^2} + \frac{2n^3 E \omega L_{1,3} e^{-\frac{t}{\tau}}}{4n^2 R_{1,3}^2 + 4n^2 \omega^2 L_{1,3}^2} \]  

(62)
Similar expressions of currents for the coils 2 and 4 may be obtained.

**EXPERIMENTAL RESULTS**

The theoretical transient characteristics are drawn by measuring the leakage impedances between primary and secondary windings of the two single-phase core type transformers, using the method as suggested by Mukhopadhyay* and the results have been presented in Table I. The values of the steady state currents have been predetermined and in order to verify those predetermined values of currents the transformers are loaded and the results have been presented in Table II. Figs. 2, 3, 4 and 5 represent the load transient curves.

![TRANSFORMER NO.1](image)

Fig. 2: Load transient for load \(R_L = 4.32 \Omega\)

Fig. 3. Load transient for load \( (R_L) = 5.48 \Omega \)
Fig. 4. Load transient for load \( R_L = 4.32 \Omega \)
Fig. 5 Load transient for load ($R_L = 5.48 \Omega$)
<table>
<thead>
<tr>
<th>Transformer number</th>
<th>Primary input voltage</th>
<th>Secondary short-circuit current</th>
<th>Input power</th>
<th>Mean leakage impedance in ohms</th>
<th>Mean resistive components in ohms</th>
<th>Mean reactive components in ohms</th>
<th>Transformer number</th>
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<tr>
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<td>20-00</td>
<td>14-3</td>
<td>4-60</td>
<td>2-10</td>
<td>1</td>
</tr>
<tr>
<td>6-70</td>
<td>14-3</td>
<td>0-00</td>
<td>20-00</td>
<td>14-3</td>
<td>4-60</td>
<td>2-10</td>
<td>1</td>
</tr>
<tr>
<td>6-00</td>
<td>14-3</td>
<td>0-00</td>
<td>20-00</td>
<td>14-3</td>
<td>4-60</td>
<td>2-10</td>
<td>1</td>
</tr>
<tr>
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<td>14-3</td>
<td>0-00</td>
<td>20-00</td>
<td>14-3</td>
<td>4-60</td>
<td>2-10</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table I**

**Transformer Number:**
- 7-05
- 6-00
- 6-70
- 6-00
- 4-00
- 2-00
- 1-00

**Primary Input Voltage:**
- 6-00
- 2-00
- 1-00

**Secondary Short-Circuit Current:**
- 0-00
- 0-00
- 0-00
- 0-00
- 0-00
- 0-00
- 0-00

**Input Power:**
- 20-00
- 20-00
- 20-00
- 20-00
- 20-00
- 20-00
- 20-00

**Mean Leakage Impedance in Ohms:**
- 14-3
- 14-3
- 14-3
- 14-3
- 14-3
- 14-3
- 14-3

**Mean Resistive Components in Ohms:**
- 4-60
- 4-60
- 4-60
- 4-60
- 4-60
- 4-60
- 4-60

**Mean Reactive Components in Ohms:**
- 2-10
- 2-10
- 2-10
- 2-10
- 2-10
- 2-10
- 2-10
<table>
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<th>Primary input voltage (m volts)</th>
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<th>Transformer II</th>
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<td>50.00</td>
<td>12.10</td>
<td>13.10</td>
</tr>
<tr>
<td>60.00</td>
<td>22.90</td>
<td>23.90</td>
</tr>
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<td>70.00</td>
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<td>34.70</td>
</tr>
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<td>80.00</td>
<td>44.50</td>
<td>45.50</td>
</tr>
<tr>
<td>90.00</td>
<td>55.30</td>
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<th>Measured</th>
<th>Calculated</th>
<th>Measured</th>
<th>Calculated</th>
<th>Measured</th>
<th>Calculated</th>
<th>Measured</th>
<th>Calculated</th>
<th>Measured</th>
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</thead>
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<tr>
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<td>12.10</td>
<td>13.10</td>
<td>12.10</td>
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<td>12.10</td>
<td>13.10</td>
<td>12.10</td>
<td>13.10</td>
<td>12.10</td>
</tr>
<tr>
<td>Secondary currents in amps</td>
<td>7.40</td>
<td>8.40</td>
<td>7.40</td>
<td>8.40</td>
<td>7.40</td>
<td>8.40</td>
<td>7.40</td>
<td>8.40</td>
<td>7.40</td>
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<tr>
<td>Common load current in amps</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table II

390 THF AM TKIX AND TNSOK QUARTERLY
CONCLUSIONS

The Kronian method of approach is a generalized tool of solving electrical machines. In this paper this tool has been successfully applied to determine the transient characteristics of a loaded transformer in terms of the bucking impedances and thus the transient characteristics can be predetermined. Hence the present method is advantageous for a design engineer to predetermine the transient characteristics.

Again in Kron's tensorial method both the steady state and transient analysis may be obtained by using a unique approach and no separate analysis to determine them is necessary.

The behaviour of transients occurred in the transformer for suddenly applied loads are studied by performing some mathematical manipulations on the steady state expressions. The experimental verification of the steady state expressions proves the validity of the expressions for the transient currents since the latter characteristics have been developed from the same expressions, (by using some mathematical operation, viz., Laplace transform operation) from which the steady state characteristics have been determined.

ACKNOWLEDGEMENT

The authors wish to acknowledge the helpful discussions received from Professor S P Bhattacharyya, Department of Applied Physics, University of Calcutta.

REFERENCES


(Continued on p. 83)

THE STEADY STATE AND TRANSIENT ANALYSIS OF TWO SINGLE-PHASE TRANSFORMERS CONNECTED IN PARALLEL

(Continued from p. 80)

THE TRANSIENT LOAD SHARING OF THREE-PHASE TRANSFORMERS

by

B. K. Sarkar * and A. K. Mukhopadhyay *

The paper includes the investigation of transient phenomena of two three-phase transformers not necessarily identical, connected in parallel. Expressions for coil currents in different branches of two transformers have been analysed from which the transient characteristics of the system have been obtained in terms of design parameters. The validity of the computed characteristics have been verified experimentally.

LIST OF SYMBOLS

\( e(t) \) = applied voltage
\( E \) = peak value of voltage
\( a \) = cube root of unity
\( i_m \) = coil current in the mth coil
\( i_m' \) = mesh current in the mth mesh
\( i_m'' \) = mesh current after neglecting magnetizing current
\( c_k \) = connection tensor, \( k = 1, 2 \)
\( (c_k)^t \) = transpose of \( c_k \)
\( Z \) = impedance tensor
\( Z' \) = transformed impedance tensor
\( \theta \) = phase angle
\( f \) = frequency
\( \omega \) = \( 2\pi f \) = differential operator
\( z_{m-n} \) = bucking impedance between coils \( m \) and \( n \)
\( R_{m-n} \) = bucking resistance between coils \( m \) and \( n \)
\( L_{m-n} \) = bucking inductance between coils \( m \) and \( n \)
\( n_{Pr} \) = primary number of turns, \( r = 1, 2, \ldots, 6 \)
\( n_{Sc} \) = secondary number of turns, \( q = 7, 8, \ldots, 12 \)
\( n_k \) = \( n_{Sc}/n_{Pr} \) \( k = 7, 8, 9 \); \( m = 1, 2, 3 \)
\( n_l \) = \( n_{Sc}/n_{Py} \) \( l = 10, 11', 12' \); \( q = 4, 5, 6 \)
\( z_{aa} \) = self impedance of the coil \( a \)
\( z_{bb} \) = self impedance of the coil \( b \)
\( z_{ab} \) = mutual impedance between coils \( a \) and \( b \)

* Department of Applied Physics, 92 Acharya Pratul Chandra Road, Calcutta 9, India.
The analysis of the transient current for a single-phase as well as three-phase transformers has been made by a number of investigators following the usual classical method. A method of calculating the transient current in transformers, allowing for mutual coupling between phases, has been developed. Most of the previous works involving the study of transients have been confined to the open-circuit and short-circuit transient phenomena.

A generalized method has been applied to study the transients of a single-phase transformer. In the present paper the same method has been applied in order to study the behavior of two three-phase transformers connected in parallel and not identical, feeding a common load. The analysis has been divided into two parts, viz., the evaluation of the steady-state currents in different branches of the circuit and development of the expression for transient current, under load and short-circuit conditions, in terms of design parameters. The validity of theoretically computed transient characteristics has been established experimentally.

CONNECTION TENSOR

The first analytical step is to determine a relation between the coil currents and the hypothetical mesh current as shown in Fig. 1 and thence to develop a connection tensor $C'$ relating currents in old (coil) and new (mesh) reference frames. The current relations are as follows:

$$
t^1 = t^1' \\
1^2 = i^2' \\
1^3 = i^3' \\
1^4 = i^4' \\
n_1 = -n_1 - 1^0 = 1^0' \quad \text{(1)}
$$

Since in a transformer the reluctance of the core is negligibly small, another connection tensor $C''$ neglecting the magnetizing current may be developed from the following relations:

$$
t^1 = -n_1 i^0'' \\
t^2 = -n_1 i^0'' \\
t^3 = n_1 (i^0'' + i^0'') \\
t^4 = -n_2 (i^0'' + i^0'') \\
t^4'' = -n_2 i_1 i^0'' \\
t^4' = 1^0'' \quad \text{(2)}
$$

where the double-dashed currents are the new currents obtained after neglecting the m.m.f. due to the core flux.

Thus, starting from the coil currents and neglecting the magnetizing current, the final connection tensor $C$ is given by
A. C Supply

![Diagram of A.C Supply network](image)

**Fig. 1 Circuit diagram**

---

**Members' Activities**

(Continued from p. 30)

Dr J. Myslik has reported that 8,400 copies of the book "Tensor Analysis of Networks", by Gabriel Kron, have been published in Russian.

Professors P. Onodera and S. Okada et al., have submitted a paper on a 'Symmetrically-mixed Analysis of Electrical Networks with Switching Elements'. We hope to publish it in a forthcoming issue of the Quarterly.

Dr Ake Tellquist has submitted a 'Preamble to Residential Planning in the Eighties' (excerpt from a prize-winning book). We hope to publish it in due course.
Using the mutual leakage impedance between different coils and considering the external load, the impedance tensor for the system, viz.,

\[
Z = \begin{bmatrix}
Z_{p,q} \\
R_{r,s}
\end{bmatrix}
\]

\((p, q = 1, 2, \ldots, 12)\)
\((r, s = 13, 14, 15)\)

is constructed where

\[
R_{r,s} = 0 \quad \text{if} \quad r \neq s
\]

\[
= R_r \quad \text{if} \quad r = s
\]

\[
z_{p,q} = z_{p,q} \quad \text{if} \quad p \neq q
\]

\[
= 0 \quad \text{if} \quad p = q
\]

\(p = 1, 2, 3\)

\(q = 4, 5, 6, 10, 11, 12\)

\(p = 4, 5, 6\)

\(q = 1, 2, 3, 7, 8, 9\)
in which the unreflected bucking impedance between two coils \( a \) and \( b \) as suggested by Kron * is given by
\[
z_{a-b} = -\frac{\eta_b}{2\eta_a} \left[ z_{aa} - 2\frac{\eta_a}{\eta_b} z_{ab} + \left(\frac{\eta_a}{\eta_b}\right)^2 z_{bb} \right]
\]
which is essentially a negative quantity.

The resultant impedance tensor in the new reference frame may be calculated by the relation \( Z' = C^T Z C \), \( C \) being the transpose of \( C \) where

\[
Z' =
\begin{array}{cccc}
T' & 8' & 10' & 11' \\
2A & A & 2R & R \\
A & 2A & R & 2R \\
2R & R & 2B & B \\
R & 2R & B & 2B \\
\end{array}
\]

(5)

\[
A = -\eta_1^2 b_1 + 2\eta_1 (d_4 - d_1) + b_1 + R \\
B = -\eta_2^2 d_1 + 2\eta_2 (d_4 - d_1) + d_1 + R \\
b_1 = z_{1,3} = z_{2,3} = z_{3,3} \\
b_2 = z_{1,4} = z_{2,4} = z_{3,4} \\
b_3 = z_{1,7} = z_{2,7} = z_{3,7} \\
b_4 = z_{1,6} = z_{2,6} = z_{3,6} \\
A = z_{4,1} = z_{5,1} = z_{6,1} \\
d_1 = z_{4,3} = z_{5,3} = z_{6,3} \\
d_2 = z_{4,10} = z_{5,10} = z_{6,10} \\
d_3 = z_{4,11} = z_{5,11} = z_{6,11} \\
d_4 = z_{4,12} = z_{5,12} = z_{6,12} \\
R = R_{1,3} = R_{1,4} = R_{1,5} \\
\]

(6)

**COIL CURRENTS**

The impressed voltage tensor in the new reference frame may be taken to be
where \( e' = C_e(t) \) and \( e(t) = E \sin(\omega t + \theta) \). The coil currents maybe calculated from the relation \( i^c = C i' \), where \( i^c = Z^{-1} e^c \) and hence

\[
\begin{align*}
7^c &= n_1 (a^c - 1) \\
8^c &= n_1 (a^c - a) \\
9^c &= n_2 (a^c - a) \\
10^c &= n_1 (a^c - l) \\
11^c &= n_2 (a^c - a) \\
\end{align*}
\]

\( i^c = \frac{1}{AB - R^2} \)

\( 1 = n_1 e (n_1 B - n_2 R) \)
\( 2 = a . n_1 e (n_1 B - n_2 R) \)
\( 3 = a^2 . n_1 e (n_1 B - n_2 R) \)
\( 4 = n_2 e (n_2 A - n_1 R) \)
\( 5 = a . n_2 e (n_2 A - n_1 R) \)
\( 6 = a^2 . n_2 e (n_2 A - n_1 R) \)
\( 7 = - e (n_1 B - n_2 R) \)
\( 8 = - a . e (n_1 B - n_2 R) \)
\( 9 = - a^2 . e (n_1 B - n_2 R) \)
\( 10 = - e (n_2 A - n_1 R) \)
\( 11 = - a . e (n_2 A - n_1 R) \)
\( 12 = - a^2 . e (n_2 A - n_1 R) \)
\( 13 = - e n_2 (B - R) + n_2 (A - R) \)
\( 14 = - a . e n_2 (B - R) + n_2 (A - R) \)
\( 15 = - a^2 . e n_2 (B - R) + n_2 (A - R) \)

**Primary Current**

The primary and the secondary circuits of a transformer are linked by mutual flux and therefore any switching process develops the transient phenomenon in the circuit. The primary current in coil 1 from equation (8) is

\[
i^1 = \frac{n_1 e (n_1 B - n_2 L)}{AB - R^2}
\]
which is to be analysed for studying the performance of two three-phase transformers connected in parallel under steady state and transient conditions. Each bucking impedance of equation (6) is expressed in terms of bucking resistance and bucking reactance and the following simplification may be made

\[ A = R + R_1 + pL_1 \]  
\[ B = R + R_2 + pL_2 \]  

(10)  
(11)

where

\[ R_1 = -n_1^2R_{1,1} + 2n_1(R_{1,4} - R_{1,7}) - R_{1,8} \]
\[ L_1 = -n_1^2L_{1,1} + 2n_1(L_{1,4} - L_{1,7}) - L_{1,8} \]
\[ R_2 = -n_2^2R_{1,4} + 2n_2(R_{4,11} - R_{4,16}) - R_{10-11} \]
\[ L_2 = -n_2^2L_{1,4} + 2n_2(L_{4,11} - L_{4,16}) - L_{10-11} \]

\[ p\theta = \rho \omega \quad \text{under steady state condition} \]

Simplification of equation (9) with the help of equations (10), (11) and (12) yields

\[ i^t = \frac{n_1^2e(p + \gamma)}{L_1(p^2 + ap + \beta)} \]

(13)

where

\[ a = R \left(\frac{1}{L_1} + \frac{1}{L_2}\right) + \frac{R_1 + R_2}{L_1L_2} \]
\[ \beta = \frac{R_1R_2}{L_1L_2} \left(R_1 + R_2\right) \]
\[ \gamma = \frac{R}{L_2} + \frac{R}{n_1L_1} (n_1 - n_2) \]

Taking Laplace transforms, equation (13) becomes

\[ i^t(p) = \frac{n_1^2E_0}{L_1} \cos \theta \left[ \frac{a_1(p + \frac{a_1}{2})}{(p + \frac{a_1}{2})^2 - a_1^2} + \frac{(a_1 - a_1\alpha_1)}{a_1^2} + \frac{a_1p}{(p + \frac{a_1}{2})^2 - a_1^2} + \frac{a_1\omega}{p^2 + \omega^2} + \frac{a_1\omega}{p^2 + \omega^2} \right] \]

\[ + \frac{n_1^2E}{L_1} \sin \theta \left[ \frac{a_2(p + \frac{a_2}{2})}{(p + \frac{a_2}{2})^2 - a_2^2} + \frac{(a_2 - a_2\alpha_2)}{a_2^2} + \frac{a_2p}{(p + \frac{a_2}{2})^2 - a_2^2} + \frac{a_2\omega}{p^2 + \omega^2} + \frac{a_2\omega}{p^2 + \omega^2} \right] \]

(17)

where

\[ a_2 = \sqrt{\left(\frac{\beta}{4}\right) - \beta} \]

(18)
The expression for currents in the time domain becomes after simplification

\[ I(t) = \cos \theta \left[ I_1 \sin(\omega t + \phi) + I_2 \exp(-a \cdot \frac{t}{2}) \sin \beta(a_t + \delta) \right] 
+ \sin \theta \left[ I_1 \sin(\omega t + \psi) + I_2 \exp(-a \cdot \frac{t}{2}) \sin \alpha(a_t + \epsilon) \right] \]

where

\[ I_1 = \sqrt{C_1 + C_2}, \quad \phi = \tan^{-1} \left( \frac{C_2}{C_1} \right) \]

\[ I_2 = \sqrt{C_3 + C_4} \quad \text{if} \quad C_4 > C_3 \quad \tan \delta = \frac{C_4}{C_3} \]

\[ I_3 = \sqrt{d_1 + d_2}, \quad \psi = \tan^{-1} \left( \frac{d_2}{d_1} \right) \]

\[ I_4 = \sqrt{d_3 - d_4} \quad \tan \epsilon = \frac{d_4}{d_3} \]

\[ C_1 = -\frac{n_1^2 E\omega \beta - a\gamma - \omega^2}{L_1} \]

\[ C_2 = \frac{n_2^2 E\omega - aR + c^2 \beta - \beta \gamma + \gamma \omega^2}{L_1} \]

\[ C_3 = \frac{n_2^2 E\omega \beta - a\gamma - \omega^2}{L_1} \]

\[ C_4 = \frac{n_2^2 E\omega \beta \gamma - \gamma \omega^2 + a\omega^2}{L_1} \]

\[ C_5 = C_4 - \frac{(a/2)C_1}{a_4} \]

\[ C_6 = \frac{C_4}{\omega} \]

\[ d_1 = -\frac{n_2^2 E \beta \gamma + a\omega^2 - \gamma \omega^2}{L_1} \]

\[ d_2 = \frac{n_2^2 E \beta^2 - \beta \omega^2 - a\beta \gamma}{L_1} \]

\[ d_3 = \frac{n_2^2 E \beta \gamma + a\omega^2 - \gamma \omega^2}{L_1} \]

\[ d_4 = \frac{n_2^2 E a\omega^2 - \beta \omega^2 + \omega^2}{L_1} \]

\[ d_5 = \frac{d_4 - (a/2)d_1}{a_4} \]

\[ d_6 = \frac{d_4}{\omega} \]
Equation (19) gives the generalized expression for the current shared by the primary coil 1 of one transformer.

If at the time of switching, the impressed voltage has no phase lag, i.e., if \( \beta = 0 \), the expression for current from equation (19) reduces to

\[
I^1(t) = I_1 \sin(\omega t + \phi) + I_1 \exp(-\alpha \frac{t}{2}) \sin(h(a t + \delta)) \quad \text{if} \quad C_1 < C_4 \tag{25}
\]

\[
I^1(t) = I_1 \sin(\omega t + \phi) + I_1 \exp(-\alpha \frac{t}{2}) \cos(h(a t - \beta)) \quad \text{if} \quad C_1 > C_4 \tag{26}
\]

assuming the relations

\[
I_1 = \sqrt{C_1^2 + C_2^2} ; \quad \tan \phi = \frac{C_2}{C_4}; \tag{27}
\]

and

\[
I_2 = \sqrt{C_1^2 - C_2^2}; \quad \tan \beta = \frac{C_2}{C_4} \tag{28}
\]

As the process develops, the exponential terms of equations (25) or (26) gradually die down giving ultimately the steady state response.

**Primary Short Circuit Current**

The short circuit transient behaviour of primary current may be evaluated by substituting \( R = 0 \) in equation (9) which yields

\[
I^1(t) = \frac{\mathbf{R}^2 \mathbf{E} \mathbf{0}}{1} \left( \frac{p}{p^2 + \omega^2 I^2} \frac{1}{p + \frac{r}{l}} + \frac{r}{p^2 + \omega^2 I^2} \frac{1}{p^2 + \omega^2 I^2} \right) \tag{29}
\]

where

\[
r = -\mathbf{R}^2 \mathbf{R}_1 + 2 \mathbf{R}_1 (\mathbf{R}_1 - \mathbf{R}_4) - \mathbf{R}_4 \tag{30}
\]

Taking Laplace transforms and simplifying, equation (29) becomes

\[
I^1(\mathbf{s}) = \frac{\mathbf{R}^2 \mathbf{E} \mathbf{0}}{1} \left( \frac{p}{p^2 + \omega^2 I^2} \frac{1}{p + \frac{r}{l}} + \frac{r}{p^2 + \omega^2 I^2} \frac{1}{p^2 + \omega^2 I^2} \right) \tag{31}
\]

Taking the inverse Laplace transform, the expression for short circuit primary current under transient condition becomes

\[
i^1(t) = I_1 \sin(\omega t - \omega - \phi) - I_1 \sin(\theta - \phi) \exp\left(\frac{-r t}{l}\right) \tag{32}
\]
where

\[
I = \frac{m_i^2 E}{\sqrt{r^2 + \omega^2 I}} \quad \phi = \tan^{-1} \left( \frac{\omega I}{r} \right) \quad (32)
\]

At \( \theta = 0 \), the above expression for current reduces to

\[
i^s(t) = I \sin(\omega t - \phi) + I \sin \phi \exp \left( -\frac{rt}{l} \right) \quad (33)
\]

**Secondary Current**

The load transient current in the secondary coil for coil \( \tau \) is developed from equation (3) where

\[
i^s = e \frac{(m_2 B - m_2 R)}{AB} \quad (34)
\]

Equation (34) is similar to equation (9) and thus for \( \theta = 0 \) the solution is given by

\[
i^s(t) = \frac{I_s}{m_2} \sin(\omega t + \phi) + \frac{I_f}{m_2} \exp \left( -\frac{t}{2} \right) \sinh(\alpha t - \delta) \quad \text{if } C_2 > C_1 \quad (35)
\]

\[
i^s(t) = \frac{I_s}{m_2} \sin(\omega t + \phi) + \frac{I_f}{m_2} \exp \left( -\frac{t}{2} \right) \cosh(\alpha t - \delta) \quad \text{if } C_2 < C_1 \quad (35)
\]

The steady state and transient currents passing through any one of the coils under load or short circuit conditions may be determined in a similar fashion.

**RESULTS**

Figs. 2(a), 2(b), 3(a) and 3(b) show computed current-transient characteristics for two 400 V, 3 kVA, 3-phase, 3 limb, core-type transformers connected in parallel with two loads 8.27 \( \Omega \) and 9.54 \( \Omega \) respectively with the assumption that the number of turns of both the transformers are the same. The theoretical characteristics were obtained by using parameters which were measured by a method as suggested by Mukhopadhyay. Values of the steady state currents have been predetermined and in order to verify those predetermined values of currents, a load test has been performed. The results have been presented in Tables 1, 2 and 3.

<table>
<thead>
<tr>
<th>Value of ( R ) in ohms</th>
<th>Current ( i^t ) in amps</th>
<th>Current ( i^s ) in amps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>Measured</td>
</tr>
<tr>
<td>21.67</td>
<td>2.29</td>
<td>2.30</td>
</tr>
<tr>
<td>14.22</td>
<td>3.47</td>
<td>3.55</td>
</tr>
<tr>
<td>12.40</td>
<td>3.97</td>
<td>4.00</td>
</tr>
<tr>
<td>9.54</td>
<td>5.12</td>
<td>5.15</td>
</tr>
<tr>
<td>8.27</td>
<td>5.28</td>
<td>6.00</td>
</tr>
</tbody>
</table>

Table 1. Calculated and measured values of primary steady state current \( e(t) = 250 \) V.
Fig. 2(a) Load transient of transformer 1 with load $R_L = 8.27 \Omega$

<table>
<thead>
<tr>
<th>Value of $R$ in ohms</th>
<th>Current $i^1$ in amps</th>
<th>Current $i^{10}$ in amps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>Measured</td>
</tr>
<tr>
<td>21.67</td>
<td>3.95</td>
<td>4.00</td>
</tr>
<tr>
<td>15.22</td>
<td>5.98</td>
<td>6.05</td>
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<tr>
<td>12.60</td>
<td>6.84</td>
<td>6.90</td>
</tr>
<tr>
<td>9.56</td>
<td>8.23</td>
<td>8.90</td>
</tr>
<tr>
<td>3.27</td>
<td>10.14</td>
<td>10.30</td>
</tr>
</tbody>
</table>

Table 2: Calculated and measured values of secondary steady state current $e(t) = 220 \, \text{V}$
Fig. 2(b): Load transient of transformer Z with load $R_L = 8.27 \Omega$
THE TRANSIENT LOAD SHARING OF THREE-PHASE TRANSFORMERS

by

B. K. Sarkar* and A. K. Mukhopadhyay*

The paper includes the investigation of transient phenomena of two three-phase transformers not necessarily identical, connected in parallel. Expressions for coil currents in different branches of two transformers have been analyzed from which the transient characteristics of the system have been obtained in terms of design parameters. The validity of the computed characteristics have been verified experimentally.

(Continued from the Matrix and Tensor Quarterly, September 1979.)

Fig. 3(b). Load transient of transformer 2 with load $R_L = 9.54 \, \Omega$

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Fig. 3(d): Load transient of transformer 1 with load $R_L = 9.54 \Omega$
<table>
<thead>
<tr>
<th>Value of R in ohms</th>
<th>Current i in amp</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Calculated</td>
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<tr>
<td>21.67</td>
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<td>9.54</td>
<td>12.98</td>
</tr>
<tr>
<td>8.27</td>
<td>14.91</td>
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</tbody>
</table>

Table 3 Calculated and measured values of load steady state current $e(t) = 220$ V

CONCLUSIONS

In this paper, Kron's mesh approach has been successfully applied to study the steady state and transient characteristics of two parallel-connected transformers (with load) in terms of the bucking impedances under different conditions. The method is unique and useful in the sense that one may get both the steady state and transient characteristics under various conditions from the same study. Hence the present method is advantageous for a design engineer to predetermine the transient current.

The transient characteristics have been developed by using some mathematical operations on the same expression from which the steady state currents have been deter mined. Hence the experimental verification of the latter proves the validity of the former expressions.

ACKNOWLEDGEMENT

The authors are grateful to Professor S. P. Bhattacharyya of the department of Applied Physics, University of Calcutta.

REFERENCES

Determination of A and B

The value of $A$ from equation (6) is

$$A = - n_1 b_1 + 2n_1 (b_n - b_1) - b_2 + R$$

$$= - n_1 z_{1,-1} + 2n_1 (z_{1,-1} - z_{1,1}) - z_{2,1} + R$$

$$= [R - n_1^2 z_{1,-1} + 2n_1 (z_{1,-1} - z_{1,1}) - z_{2,1}] + p [2n_1 (z_{1,1} - L_{1,1}) - L_{1,1}]$$

$$= R + R_1 + p L_1$$

Also the value of $B$ from equation (6) is

$$B = - n_1^2 d_1 + 2n_1 (d_n - d_1) - d_2 + R$$

$$= - n_1^2 z_{2,-1} + 2n_1 (z_{2,-1} - z_{2,1}) - z_{3,1} + R$$

$$= [R - n_1^2 z_{2,-1} + 2n_1 (z_{2,-1} - z_{2,1}) - z_{3,1}] + p [2n_1 (z_{2,1} - L_{1,1}) - L_{1,1}]$$

$$= R + R_2 + p L_2$$

Here all the bucking impedances are expressed items of bucking resistances and bucking reactances which are measured easily.

Evaluation of Transfer Function

If the applied voltage be $e(t) = E \sin(\omega t + \theta)$, its Laplace transform is given by

$$e(p) = \mathcal{L}[E \sin(\omega t + \theta)]$$

$$= E \cos \theta \frac{p}{p^2 + \omega^2} + E \sin \theta \frac{\omega}{p^2 + \omega^2}$$

(37)

Thus combining equations (13) and (37)

$$i^1(p) = \frac{n_1^2 E \omega}{L_1} \cos \theta \left[ \frac{(p + \gamma)}{(p + ap + \beta)(p^2 + \omega^2)} \right] + \frac{n_1^2 E \omega}{L_1} \sin \theta \left[ \frac{p(p + \gamma)}{(p^2 + ap + \beta)(p^2 + \omega^2)} \right]$$

Hence

$$i^1(t) = \mathcal{L}^{-1} [i^1(p)]$$

$$= \frac{n_1^2 E \omega \cos \theta}{L_1} \mathcal{L}^{-1} \left[ \frac{(p + \gamma)}{(p + ap + \beta)(p^2 + \omega^2)} \right] + \frac{n_1^2 E \omega \sin \theta}{L_1} \mathcal{L}^{-1} \left[ \frac{p(p + \gamma)}{(p^2 + ap + \beta)(p^2 + \omega^2)} \right]$$
Now let
\[ \frac{p + y}{(p^2 + ap + \beta)(p^2 + \omega^2)} = \frac{a_1 p + a_2}{p^2 + ap + \beta} + \frac{a_3 p + a_4}{p^2 + \omega^2} \]
or
\[ p + y = (a_1 p + a_2)(p^2 + \omega^2) + (a_3 p + a_4)(p^2 + ap + \beta) \quad (38) \]

Equating the coefficients of \( p^4, p^3 \) and \( p^2 \) from both sides of the above identity and rearranging we get the following three equations

\[ a_1 + a_2 + a_3 = 0 \quad (39) \]
\[ (\beta - \omega^3) a + a_4 = 1 \quad (40) \]
\[ \omega^3 a_3 + \beta a_4 = \gamma \quad (41) \]

Hence using Cramer's rule

\[
\begin{vmatrix}
0 & a & 1 \\
1 & (\beta - \omega^3) & a \\
y & 0 & \beta \\
\end{vmatrix} = \frac{1}{\Delta} \begin{vmatrix}
1 & a & 0 \\
0 & 1 & a \\
\omega^3 & \gamma & \beta \\
\end{vmatrix} ; \quad \begin{vmatrix}
a_1 \\
\end{vmatrix} = \frac{1}{\Delta} 
\]

where

\[ \Delta = \begin{vmatrix}
1 & a & 1 \\
0 & (\beta - \omega^3) & a \\
\omega^3 & 0 & \beta \\
\end{vmatrix} \]

Similarly assuming,

\[ \frac{p \omega^2 + y}{(p^2 + ap + \beta)(p^2 + \omega^2)} = \frac{b_1 p + b_2}{p^2 + ap + \beta} + \frac{b_3 p + b_4}{p^2 + \omega^2} \]
or
\[ p (p + y) = (b_1 p + b_2)(p^2 + \omega^2) + (b_3 p + b_4)(p^2 + ap + \beta) \]

and equating the coefficients of \( p^4, p^3 \) and \( p^2 \) from both sides and after simplification we get

\[
\begin{vmatrix}
1 & a & 1 \\
\beta - \omega^3 & a \\
0 & \beta \\
\end{vmatrix} = \frac{1}{\Delta} \begin{vmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
0 & \gamma & \omega^3 \\
0 & \gamma & \omega^3 \\
\end{vmatrix} ; \quad \begin{vmatrix}
b_1 \\
\end{vmatrix} = \frac{1}{\Delta} 
\]

and

\[
\begin{vmatrix}
1 & 1 & 1 \\
\beta - \omega^3 & a \\
0 & \beta \\
\end{vmatrix} = \frac{1}{\Delta} \begin{vmatrix}
1 & a & 1 \\
0 & \gamma & \omega^3 \\
0 & \gamma & \omega^3 \\
0 & \gamma & \omega^3 \\
\end{vmatrix} ; \quad \begin{vmatrix}
b_3 \\
\end{vmatrix} = \frac{1}{\Delta} 
\]
where

$$\Delta = \begin{bmatrix} a & 1 \\ 0 & \beta - \omega^2 \\ \omega^2 & 0 \end{bmatrix}$$

Details of the Test Transformers

\[ n_1 = n_2 = 0.58 \]
\[ z_{1,2} = 1.045 + j4.602 \quad z_{1,4} = 0.613 + j2.865 \]
\[ z_{1,7} = 0.365 + j0.653 \quad z_{7,4} = 0.325 + j1.717 \]
\[ z_{2,5} = 2.691 + j14.280 \quad z_{4,11} = 1.435 + j8.553 \]
\[ z_{4,10} = 0.768 + j1.223 \quad z_{10,11} = 0.794 + j5.123 \]
\[ R = 8.27 \text{ ohm} \]
\[ R = 9.54 \text{ ohm} \]

**SYMMETRICALLY-MIXED ANALYSIS**

**OF ELECTRICAL NETWORKS WITH SWITCHING ELEMENTS**

(Continued from p 47.)

**REFERENCES**