CHAPTER VII

TRANSIENT ANALYSIS OF A THREE-PHASE (THREE-LIMBED) DELTA-DELTA CONNECTED CORE-TYPE TRANSFORMER

I. Introduction:

The most economical arrangement for low voltage 3 Ø transformers having large power output is the delta-delta connected circuit. Large unbalanced three-phase loads may be applied using this connection which only produce unbalanced voltages proportional to the internal impedances of the transformer windings. Delta-delta connected windings of a loaded three-phase core-type transformer have been analysed in this chapter using the unified method of Kron to determine the coil currents in the different branches of transformer windings. The steady state as well as transient currents of the system have been derived and the short circuit characteristics under both the two conditions have been studied. With the help of I.B.M. 1130 computer, different characteristics have been evaluated theoretically and then verified experimentally.
II. METHOD:

From the following current relations:
\[ i_1 = i_1'; \quad i_4 = i_4'; \quad i_7 = i_7'; \]
\[ i_2 = i_2'; \quad i_5 = i_4'i_8'; \quad i_8 = i_8'; \]
\[ i_3 = i_3'; \quad i_6 = -(i_7'i_4'); \quad i_9 = -(i_7'i_8'); \]

where \( i_1', i_2' \) etc. and \( i_1, i_2 \) etc. shown in figure (7.1) are the currents in coil and mesh reference frames, the connection tensor connecting the two reference frames becomes,

\[
\begin{array}{ccccccc}
\text{c1} & 1' & 2' & 3' & 4' & 7' & 8' \\
1 & 1 & & & & & \\
2 & & 1 & & & & \\
3 & & & 1 & & & \\
4 & & & & 1 & & \\
5 & & & & & 1 & \\
6 & & & & & & -1 \\
7 & & & & & & 1 \\
8 & & & & & & 1 \\
9 & & & & & & -1 \\
\end{array}
\]

The equations of constraints in coil reference frame obtained after neglecting the magnetising current of the transformer are

\[ n_p i_1 + n_s i_4 = 0 \]
\[ n_p i_2 + n_s i_5 = 0 \]
\[ n_p i_3 + n_s i_6 = 0 \]
Fig 7.1 Circuit diagram of actual connection
where $n_p$ and $n_s$ are primary and secondary number of turns respectively.

The constraint equations

\[
\begin{align*}
i_{1}' &= -n i_{4}'' \\
i_{2}' &= -n(i_{4}'' + i_{8}'' ) \\
i_{3}' &= n(i_{7}'' - i_{4}'' ) \\
i_{4}' &= i_{4}'' \\
i_{7}' &= i_{7}'' \\
i_{8}' &= i_{8}'' 
\end{align*}
\]

define the connection tensor $c_2$ where

\[
c_2 = \begin{pmatrix}
1' & -n & -n & -n \\
2' & -n & -n & -n \\
3' & -n & n & -n \\
4' & 1 & -n & -n \\
7' & 1 & -n & -n \\
8' & 1 & -n & -n
\end{pmatrix}
\]

and $i_{4}''$, $i_{7}''$ etc. are currents in the new reference frame obtained after neglecting the magnetising currents.

Hence the connection tensor $c$, showing the relation between the currents before and after neglecting the magnetising current, is given by,
The impedance matrix for different coils of transformer is given by,

\[
\begin{array}{cccccccc}
& 4'' & 7'' & 8'' \\
1 & -n & & & \\
2 & -n & -n & & \\
3 & -n & n & & \\
4 & 1 & & & \\
5 & 1 & 1 & & \\
6 & 1 & -1 & & \\
7 & 1 & & & \\
8 & & & & \\
9 & -1 & -1 & & \\
\end{array}
\]
which, when simplified with the following relations among the bucking impedances,

\[ Z_{1-2} = Z_{1-3} = Z_{2-3} = m \]
\[ Z_{1-4} = Z_{2-5} = Z_{3-6} = b \]
\[ Z_{1-5} = Z_{1-6} = Z_{2-4} = Z_{3-4} = Z_{2-6} = Z_{3-5} = h \]
\[ Z_{4-5} = Z_{4-6} = Z_{5-6} = d \]
\[ R_7 = R_8 = R_9 = R \]

assumed after considering the symmetry of the windings of the transformer, becomes,

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
0 & m & m & b & h & h & & & \\
m & 0 & m & h & b & h & & & \\
m & m & 0 & h & h & b & & & \\
b & h & h & 0 & d & d & & & \\
h & b & h & d & 0 & d & & & \\
h & h & b & d & d & 0 & & & \\
& & & & & & R & & \\
& & & & & & R & & \\
& & & & & & R & & \\
\end{array}
\]

Then the resultant impedance tensor in the new reference frame may be developed as,
\[ Z' = a \pm 2e = 4'' \]

<table>
<thead>
<tr>
<th>4''</th>
<th>7''</th>
<th>8''</th>
</tr>
</thead>
<tbody>
<tr>
<td>6n^2m-6nb</td>
<td>-2n^2m+2nb</td>
<td>2n^2m-2nb</td>
</tr>
<tr>
<td>-12nh+6d</td>
<td>+4nh-2d</td>
<td>-4nh+2d</td>
</tr>
<tr>
<td>-2n^2m+2nb</td>
<td>-2nb+2R</td>
<td>-n^2m+2nh</td>
</tr>
<tr>
<td>+4nh-2d</td>
<td></td>
<td>-d+R</td>
</tr>
<tr>
<td>2n^2m-2nb</td>
<td>-n^2m+2nh</td>
<td>-2nb+2R</td>
</tr>
<tr>
<td>-4nh+2d</td>
<td>-d'+R</td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{array}{cccc}
4'' & 7'' & 8'' \\
6x & -2x & 2x \\
-2x & 2(x+y) & y \\
2x & y & 2(x+y) \\
\end{array}
\]

where \( x = n^2m - nb - 2nh + d \)
and \( y = R - n^2m + 2nh - d \)

and

\[
Z^{-1} = \frac{1}{2x(2x+3y)^2} \cdot \begin{array}{cccc}
4'' & 7'' & 8'' \\
(2x+3y)(2x+y) & 2x(2x+3y) & -2x(2x+3y) \\
2x(2x+3y) & 4x(2x+3y) & -2x(2x+3y) \\
-2x(2x+3y) & -2x(2x+3y) & 4x(2x+3y) \\
\end{array}
\]

\[
= \frac{1}{2x + 3y} \cdot \begin{array}{cccc}
4'' & 7'' & 8'' \\
1+y/2x & 1 & -1 \\
1 & 2 & -1 \\
-1 & -1 & 2 \\
\end{array}
\]
Now the impressed voltage tensor of the actual network is

\[
e' = n e \cdot 4'' \begin{pmatrix} 0 \\ 7'' \ a^2 \\ 8'' \ -a \end{pmatrix}
\]

where 'a' is a cube root of unity and \( e \) be the magnitude of the supply voltage. The new current tensor \( i' \) is given by,

\[
i' = z'^{-1} e' = \frac{ne}{2x + 3y} \cdot 4'' \begin{pmatrix} a^2 + a \\ 2a^2 + a \\ -a^2 - 2a \end{pmatrix} = \frac{ne}{2x + 3y} \cdot 4'' \begin{pmatrix} -1 \\ a^2 - 1 \\ 1 - a \end{pmatrix}
\]

and the current of each coil can be calculated by the relation \( i^c = ci' \)

where,

\[
i^c = \frac{ne}{2x + 3y} \cdot 1 \begin{pmatrix} n \\ na \\ na^2 \\ -1 \\ -a \\ -a^2 \\ a^2 - 1 \\ 1 - a \\ a - a^2 \end{pmatrix}
\]

\[\text{..... (7.1)}\]
Thus the expression for currents flowing through coils 1 and 4 are respectively given by,

\[ i_1 = \frac{n^2e}{2x + 3y} = \frac{n^2e}{3R - n^2m - 2nb + 2nh - d} \quad \ldots \ldots (7.2) \]

\[ i_4 = -\frac{ne}{2x + 3y} = \frac{ne}{3R - n^2m - 2nb + 2nh - d} \quad \ldots \ldots (7.3) \]

Expressing each bucking impedance in terms of resistance and reactance, viz.

\[

t_1 = \begin{cases} 
R_{1-2} + L_{1-2}p = R_{1-3} + L_{1-3}p = R_{2-3} + L_{2-3}p \\
R_{1-4} + L_{1-4}p = R_{2-5} + L_{2-5}p = R_{3-6} + L_{3-6}p \\
R_{1-5} + L_{1-5}p = R_{1-6} + L_{1-6}p = R_{2-4} + L_{2-4}p \\
R_{3-5} + L_{3-5}p = R_{2-6} + L_{2-6}p = R_{3-4} + L_{3-4}p \\
R_{4-5} + L_{4-5}p = R_{5-6} + L_{5-6}p = R_{4-6} + L_{4-6}p 
\end{cases}
\]

the following relation may be obtained:

\[
-n^2m - 2nb + 2nh - d = n^2R_{1-2} + 2n(R_{3-5} - R_{2-5}) - R_{4-5} + p(-n^2L_{1-2} + 2n(L_{3-5} - L_{2-5}) - L_{4-5})
\]

\[
= R_1 + pl
\]

where \( R_1 = -n^2R_{1-2} + 2n(R_{3-5} - R_{2-5}) - R_{4-5} \)

and \( l = -n^2L_{1-2} + 2n(L_{3-5} - L_{2-5}) - L_{4-5} \)

Thus,

\[ i_1 = \frac{n^2e}{3R + R_1 + lp} = \frac{n^2e}{r + lp} = \frac{n^2e}{l(p + r/l)} \quad \ldots \ldots (7.4) \]

where \( r = 3R + R_1 \)

\[ \ldots \ldots (7.5) \]

After proper simplification as shown in Chapter III the equation (7.4) reduce to
\[ i^1(t) = I \left[ \sin \phi e^{- \frac{(r/l) \cdot t}{(r/l)} + \sin (wt - \phi)} \right] \] \hspace{1cm} \cdots (7.6)

where, \[ I \sin \phi = \frac{n E \cdot w l}{r^2 + w^2l^2} \] \hspace{1cm} \cdots (7.7)

\[ I \cos \phi = \frac{n E \cdot r}{r^2 + w^2l^2} \] \hspace{1cm} \cdots (7.8)

\[ \tan \phi = \frac{w l}{r} \] \hspace{1cm} \cdots (7.9)

and \[ I = \frac{n E}{(r^2 + w^2l^2)^{1/2}} \] \hspace{1cm} \cdots (7.10)

Similarly the expressions for currents of other coils are given by,

\[ i^2(t) = I \cdot a \left[ \sin \phi e^{- \frac{(r/l) \cdot t}{(r/l)} + \sin (wt - \phi)} \right] \] \hspace{1cm} \cdots (7.11)

\[ i^3(t) = I \cdot a^2 \left[ \sin \phi e^{- \frac{(r/l) \cdot t}{(r/l)} + \sin (wt - \phi)} \right] \] \hspace{1cm} \cdots (7.12)

\[ i^4(t) = - \frac{I}{n} \left[ \sin \phi e^{- \frac{(r/l) \cdot t}{(r/l)} + \sin (wt - \phi)} \right] \] \hspace{1cm} \cdots (7.13)

\[ i^5(t) = - \frac{I}{n} \cdot a \left[ \sin \phi e^{- \frac{(r/l) \cdot t}{(r/l)} + \sin (wt - \phi)} \right] \] \hspace{1cm} \cdots (7.14)

\[ i^6(t) = - \frac{I}{n} \cdot a^2 \left[ \sin \phi e^{- \frac{(r/l) \cdot t}{(r/l)} + \sin (wt - \phi)} \right] \] \hspace{1cm} \cdots (7.15)

\[ i^7(t) = \frac{I}{n} (a^2 - 1) \left[ \sin \phi e^{- \frac{(r/l) \cdot t}{(r/l)} + \sin (wt - \phi)} \right] \] \hspace{1cm} \cdots (7.16)

\[ i^8(t) = \frac{I}{n} (1 - a) \left[ \sin \phi e^{- \frac{(r/l) \cdot t}{(r/l)} + \sin (wt - \phi)} \right] \] \hspace{1cm} \cdots (7.17)

\[ i^9(t) = \frac{I}{n} (a - a^2) \left[ \sin \phi e^{- \frac{(r/l) \cdot t}{(r/l)} + \sin (wt - \phi)} \right] \] \hspace{1cm} \cdots (7.18)
Now,
\[ \text{Isin} \theta = \frac{n^2 E \sin (wt - \tan^{-1} \frac{\omega}{3R - n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5})}{\left[ \left( 3R - n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5 \right)^2 + w^2 \left( -n^2 L_1 - 2 + 2n(L_3 - 5 - L_2 - 5) - L_4 - 5 \right)^2 \right]^{\frac{1}{2}}} \]

\[ \ldots \ldots (7.19) \]

and
\[ I = \frac{n^2 E}{\left[ \left( 3R - n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5 \right)^2 + w^2 \left( -n^2 L_1 - 2 + 2n(L_3 - 5 - L_2 - 5) - L_4 - 5 \right)^2 \right]^{\frac{1}{2}}} \]

\[ \ldots \ldots (7.20) \]

Then,
\[ i^1(t) = \frac{n^2 E \sin (wt - \tan^{-1} \frac{\omega}{3R - n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5})}{\left[ \left( 3R - n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5 \right)^2 + w^2 \left( -n^2 L_1 - 2 + 2n(L_3 - 5 - L_2 - 5) - L_4 - 5 \right)^2 \right]^{\frac{1}{2}}} \]

\[ i^4(t) = \frac{n^2 E \sin (wt - \tan^{-1} \frac{\omega}{3R - n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5})}{\left[ \left( 3R - n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5 \right)^2 + w^2 \left( -n^2 L_1 - 2 + 2n(L_3 - 5 - L_2 - 5) - L_4 - 5 \right)^2 \right]^{\frac{1}{2}}} \]

\[ \ldots \ldots (7.21) \]

\[ \ldots \ldots (7.22) \]
Special Case: Short Circuit Condition.

The short-circuit transient behaviour of the transformer is obtained by putting $R = 0$ in the equations (7.21), (7.22) and (7.23). The expressions for currents in coils 1 and 4 are respectively,

$$i_1 = \frac{n^2 E \sin (wt - \tan^{-1} \frac{w(-n^2 L_1 - 2n(L_3 - 5 - L_2 - 5) - L_4 - 5)}{3R - n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5}} + \frac{n^2 E \sin (wt - \tan^{-1} \frac{w(-n^2 L_1 - 2n(L_3 - 5 - L_2 - 5) - L_4 - 5)}{3R - n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5}})}{[(-n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5)^2 + w^2(-n^2 L_1 - 2 + 2n(L_3 - 5 - L_2 - 5) - L_4 - 5)^2]^\frac{1}{2}} \quad \ldots (7.24)$$

and

$$i_4 = -\frac{n^2 E \sin (wt - \tan^{-1} \frac{w(-n^2 L_1 - 2n(L_3 - 5 - L_2 - 5) - L_4 - 5)}{3R - n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5}} - \frac{n^2 E \sin (wt - \tan^{-1} \frac{w(-n^2 L_1 - 2n(L_3 - 5 - L_2 - 5) - L_4 - 5)}{3R - n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5}})}{[(-n^2 R_1 - 2 + 2n(R_3 - 5 - R_2 - 5) - R_4 - 5)^2 + w^2(-n^2 L_1 - 2 + 2n(L_3 - 5 - L_2 - 5) - L_4 - 5)^2]^\frac{1}{2}} \quad \ldots (7.25)$$
Experimental Results:

The bucking impedances between any pair of coils in a three phase core type transformer, appearing in the expressions for currents, are measured by the method as discussed in Chapters III and VI and the results have been shown in table (6.1) and (6.2). With the help of those measured parameters as obtained in Chapter VI, the steady-state and the short circuit currents have been predetermined. In order to verify those predetermined values of currents, the load test as well as the short circuit test have been performed.

Load test is performed by connecting the delta-connected primary windings of the transformer (3 K.V*A, 230 volts, 7.5 amps/12.9 amps) to a three phase supply and the delta connected secondary windings are connected with star connected loads. Ammeters and voltmeters are connected, after proper calibration. The schematic diagram of the load test is shown in figure (7.2) and the results have been presented in table (7.1). Table (7.2) shows the calculated and experimental values of the short circuit currents for different supply voltages.

The theoretical load transient and the short circuit transient currents have been calculated with the help of I.B.M. 1130 computer with necessary programme (7.1). The load transient curves have been shown in figures (7.3) & (7.4) and figures (7.5) & (7.6) represent the short circuit transient characteristics.
Fig. 7.2: Load test
### TABLE - 7.1

**Load Test**

<table>
<thead>
<tr>
<th>Primary input voltage in volts</th>
<th>Resistance of each load in ohms</th>
<th>Primary line current in amps</th>
<th>Secondary line current in amps</th>
<th>Load current in amps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Calculated</td>
<td>Measured</td>
<td>Calculated</td>
</tr>
<tr>
<td>230.00</td>
<td>10.86</td>
<td>2.35</td>
<td>2.45</td>
<td>4.04</td>
</tr>
<tr>
<td></td>
<td>8.94</td>
<td>2.84</td>
<td>2.90</td>
<td>4.90</td>
</tr>
<tr>
<td></td>
<td>7.35</td>
<td>3.45</td>
<td>3.60</td>
<td>5.95</td>
</tr>
<tr>
<td></td>
<td>6.03</td>
<td>4.19</td>
<td>4.10</td>
<td>7.22</td>
</tr>
</tbody>
</table>

### TABLE - 7.2

**Short-circuit Test**

<table>
<thead>
<tr>
<th>Primary input voltage in volts</th>
<th>Primary short circuit current in amps.</th>
<th>Secondary short circuit current in amps.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>Measured</td>
</tr>
<tr>
<td>6.00</td>
<td>2.55</td>
<td>2.60</td>
</tr>
<tr>
<td>7.00</td>
<td>2.95</td>
<td>2.85</td>
</tr>
<tr>
<td>9.00</td>
<td>3.79</td>
<td>3.65</td>
</tr>
<tr>
<td>10.00</td>
<td>4.21</td>
<td>4.10</td>
</tr>
</tbody>
</table>
PROGRAMME - 7.1

// JOB T

LOG DRIVE CART SPEC CART AVAIL PHY DRIVE
0000 0006 0006 0000

V2 MOB ACTUAL 16K CONFIG 16K

// FOR

*EXTENDED PRECISION

*LST SOURCE PROGRAM

*ONE WORK INTEGERS

*IOCS (CARD, DISK, 1132 PRINTER)

C PROJECT PROGRAM

C BHANU KUMAR SARKAR

DIMENSION T(7), CT(7), CS(7), CR(7)

READ (2,5) N, M

5 FORMAT (215)

DO 20 J = 1, M

READ (2,4) (T(I), I = 1, N)

4 FORMAT (7E11.4)

DO 10 I = 1, N

PI = 3.14286

CT(I) = 2.345* SIN(1.21*PI/180.)/EXP(14810.*T(I))

CS(I) = 2.345* SIN(((18000.*T(I))-1.21)*(PI/180.))

CR(I) = CT(I) + CS(I)

10 CONTINUE

WRITE (3,6) (CT(I), CS(I), CR(I), I = 1, N)

6 FORMAT (1X, 9F10.6)

20 CONTINUE

CALL EXIT

END
IV DISCUSSIONS:

The Kronian method of approach has been successfully applied in this chapter to determine both the steady-state and transient characteristics of a 3Ω, delta-delta transformer with load in terms of its bucking impedances as presented in equations (7.21), (7.22) etc. with the help of which the short-circuit transient currents have been evaluated without going through separate analysis.

The transient part of current is developed by applying Laplace transform method on the same expression (7.2) for currents from which the steady state part has been obtained. So the experimental verification of the later proves the validity of the former expressions.