CHAPTER 9

Effect of abnormal sex-chromosomes on the succeeding generations (considering X-linked diseases)

Starting from the states of normal sex-chromosomes, if abnormal types are produced in a generation and then stopped, the genetics of X-linked diseases for this case (considering this discontinuity) is discussed in this chapter. Here it is assumed that the types of abnormal sex-chromosomes having fertility non-zero take part at the time of selection.

Suppose the types XXX, XXY, XYY, XXY, XXY, XXX, XXX, XXX, XXX appear in one generation. Among these types only XXX, XXY, XXY, XXX, XXY, XXY have fertility non-zero. Therefore, at the time of selection we will consider the states:

\[ XXX, XXY, XXY, XXX, XXY, XXY, XXY, XXY \]

where \( X \) denotes X-chromosome containing defective gene.

Taking this generation to be the initial generation, let the frequency ratios of these states be \( p_1(0) \), \( p_2(0) \), \( p_3(0) \), \( p_4(0) \), \( p_5(0) \), \( p_6(0) \), \( p_7(0) \), \( p_8(0) \), \( p_9(0) \), \( p_{10}(0) \), \( p_{11}(0) \) respectively where \( p_1(0) + p_2(0) + p_3(0) + p_4(0) = p_5(0) + p_6(0) + p_7(0) + p_8(0) + p_9(0) + p_{10}(0) + p_{11}(0) \).

Then there are many cases having possible selection.

Case 1.

\[ p_1(0) \neq p_5(0) \]

\[ p_2(0) = p_6(0) \]

\[ p_3(0) = p_7(0) \]

\[ p_4(0) = p_8(0) + p_9(0) + p_{10}(0) + p_{11}(0) \]
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**Case 3.**

$p_1(0) = p_5(0)$

$p_2(0) = p_6(0)$

$p_3(0) \triangleq P(0)$

$p_4(0) \geq P(0)$

**Case 6.**

$p_1(0) = p_5(0)$

$p_2(0) = p_6(0)$

$p_3(0) \triangleright P(0)$

$p_3(0) \triangleright P(0) + P(0)$

$p_3(0) \triangleq P(0) + P(0)$

$p_4(0) \geq P(0) + P(0)$

**Case 8.**

$p_1(0) = p_5(0)$

$p_2(0) = p_6(0)$

$p_3(0) \triangleright P(0)$

$p_3(0) \triangleright P(0) + P(0)$

$p_3(0) \triangleright P(0) + P(0) + P(0)$

$p_3(0) \triangleright P(0) + P(0) + P(0)$

**Case 9.**

$p_1(0) = p_5(0)$

$p_2(0) = p_6(0)$

$p_3(0) \triangleright P(0)$

$p_3(0) \triangleright P(0) + P(0)$

$p_3(0) \triangleright P(0) + P(0) + P(0)$

$p_3(0) \triangleright P(0) + P(0) + P(0)$
Case 10.

\[ p_1(0) = p_5(0) \]
\[ p_2(0) = p_6(0) \]
\[ p_3(0) > p_7(0) \]
\[ p_3(0) > p_7(0) + p_3(0) \]
\[ p_3(0) > p_7(0) + p_3(0) + p_9(0) \]
\[ p_3(0) > p_7(0) + p_3(0) + p_9(0) + p_5(0) \]

Similarly for \[ p_1(0) = p_5(0) \]

\[ p_2(0) \text{ being greater than } p_6(0) \text{ there are nine possible cases having selection and} \]

For \[ p_1(0) = p_5(0) \]
\[ p_2(0) < p_6(0) \text{ there are other cases.} \]

Similarly for \[ p_1(0) < p_5(0) \]
and for \[ p_1(0) > p_5(0) \]

there are many other cases having possible selection.

It is assumed that for all cases in the next generation all the \( ^X \) (excluding abnormal sex-chromosomes) \( ^X Y, ^X X, ^Y X, ^Y Y \) only will appear. Let the frequency ratios of these states in the next generation be \( p_1(1), p_2(1), p_3(1), p_3'(1), p_3''(1), p_4(1) \) respectively. For all cases it is seen that

\[ p_1(1) + p_2(1) > p_3'(1) + p_3''(1) + p_4(1) \]

But at the time of selection

Number of male \[ = \] Number of female

Therefore, we have to adjust these frequency ratios according to the following seven possibilities:

...
Case 1.

\[ p_1(1) \leq p_3'(1) + p_3''(1) \]

Case 2.

\[ p_1(1) \leq p_3'(1) + p_3''(1) \]
\[ p_1(1) = p_3'(1) \]

Case 3.

\[ p_1(1) \leq p_3'(1) + p_3''(1) \]
\[ p_1(1) \geq p_3'(1) \]

Case 4.

\[ p_1(1) = p_3'(1) + p_3''(1) \]

Case 5.

\[ p_1(1) > p_3'(1) + p_3''(1), \quad p_1(1) \leq p_3'(1) + p_3''(1) + p_4(1) \]

Case 6.

\[ p_1(1) > p_3'(1) + p_3''(1) \]
\[ p_1(1) = p_3'(1) + p_3''(1) + p_4(1) \]

Case 7.

\[ p_1(1) > p_3'(1) + p_3''(1) \]
\[ p_1(1) > p_3'(1) + p_3''(1) + p_4(1) \]

If case 1 happens
Then effectively:

\[ \frac{p_1(1)}{2} \leq p_3'(1) + p_3''(1) + p_4(1) \]

\[ p_3'(1) + p_3''(1) + p_4(1) - p_1(1) \]
\[ 2 \leq p_3'(1) + p_3''(1) + p_4(1) \]

\[ \frac{p_3'(1)}{2} \leq p_3''(1) + p_4(1) \]

\[ \frac{p_4(1)}{2} \leq p_3''(1) + p_4(1) \]
will take part at the time of selection to produce the second generation.

Here
\[ p_1(n + 1) = p_1(n) \]
\[ p_2(n + 1) = p_2(n) \]
\[ p_3'(n + 1) = p_3'(n) \]
\[ p_3''(n + 1) = p_3''(n) \]
\[ p_4(n + 1) = p_4(n) \]

since \[ p_1(0), p_2(0), p_3'(0), p_3''(0), p_4(0) \]
and the relation between them are known therefore

\[ p_1(n), p_2(n), p_3'(n), p_3''(n) \] and \( p_4(n) \) are also known.

If case 2 happens,

Then effectively

\[ \frac{p_3'(1)}{2} \]
\[ \frac{p_3''(1)}{2} \]
\[ \frac{p_4(1)}{2} \]

will take part at the time of selection to produce the second generation.

Here
\[ p_1(n + 1) = p_1(n) \]
\[ p_2(n + 1) = p_2(n) \]
\[ p_3'(n + 1) = p_3'(n) \]
\[ p_3''(n + 1) = p_3''(n) \]
\[ p_4(n + 1) = p_4(n) \]

since \[ p_1(0), p_2(0), p_3'(0), p_3''(0), p_4(0) \]
and the relation between them are known therefore

\[ p_1(n), p_2(n), p_3'(n), p_3''(n) \] and \( p_4(n) \) are also known.
If case 3 happens:  

Then effectively

\[ \frac{p_1(n)}{2\left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

\[ \frac{p_3'(1) + p_3''(1) - p_2(1)}{2\left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

\[ \frac{p_3''(1)}{2\left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

\[ \frac{p_4(1)}{2\left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

\[ \frac{p_4(1)}{2\left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

will take part at the time of selection to produce the second generation.

Here

\[ p_1(n + 1) = p_1(n) \]

\[ p_2(n + 1) = p_2(n) \]

\[ p_3'(n + 1) = p_3'(n) \]

\[ p_3''(n + 1) = p_3''(n) \]

\[ p_4(n + 1) = p_4(n) \]

If case 4 happens:  

Then effectively

\[ \frac{p_4(1)}{2\left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

\[ \frac{p_4(1)}{2\left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

\[ \frac{p_4(1)}{2\left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

\[ \frac{p_4(1)}{2\left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

\[ \frac{p_4(1)}{2\left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

Since \( p_1(0), p_2(0), p_3'(0), p_3''(0), p_4(0) \) and the relation between them are known, therefore \( p_1(n), p_2(n), p_3'(n), p_3''(n) \) and \( p_4(n) \) are also known.

\[ \frac{p_1(0)}{p_3''(0)} \]

\[ \frac{p_2(0)}{p_3''(0)} \]

\[ \frac{p_3'(0)}{p_3''(0)} \]

\[ \frac{p_4(0)}{p_3''(0)} \]

where \( P_1(0) + P_2(0) = P_3'(0) + P_3''(0) + P_4(0) \) 

and \( p_1(t) = p_3'(t) + p_3''(t) \)
will take part at the time of selection to produce the second generation.

Here also \( p_{x}(n + 1) = p_{x}(n) \) \((x = 1, 2, 3, 3', 4)\) and they are known since the initial frequency ratios are known.

**If Case 5 happens:**

Then effectively

\[
\begin{align*}
\frac{p_{x}(1)}{2} & \left\{ p_{3}^{1}(1) + p_{3}^{n}(1) + p_{4}(1) \right\} \\
p_{3}^{1}(1) + p_{3}^{n}(1) + p_{4}(1) - p_{1}(1) & \left\{ p_{3}^{1}(1) + p_{3}^{n}(1) + p_{4}(1) \right\} \\
p_{3}^{1}(1) & \left\{ p_{3}^{1}(1) + p_{3}^{n}(1) + p_{4}(1) \right\} \\
p_{3}^{n}(1) & \left\{ p_{3}^{1}(1) + p_{3}^{n}(1) + p_{4}(1) \right\} \\
p_{4}(1) & \left\{ p_{3}^{1}(1) + p_{3}^{n}(1) + p_{4}(1) \right\}
\end{align*}
\]

will take part at the time of selection to produce the second generation.

Here also \( p_{x}(n + 1) = p_{x}(n) \) \((x = 1, 2, 3, 3', 4)\) and they are known since the initial frequency ratios are known.

**If Case 6 happens:**

Then effectively

\[
\begin{align*}
\frac{p_{x}(1)}{2} & \left\{ p_{3}^{1}(1) + p_{3}^{n}(1) + p_{4}(1) \right\} \\
p_{3}^{1}(1) & \left\{ p_{3}^{1}(1) + p_{3}^{n}(1) + p_{4}(1) \right\} \\
p_{3}^{n}(1) & \left\{ p_{3}^{1}(1) + p_{3}^{n}(1) + p_{4}(1) \right\} \\
p_{4}(1) & \left\{ p_{3}^{1}(1) + p_{3}^{n}(1) + p_{4}(1) \right\}
\end{align*}
\]

will take part at the time of selection to produce the second generation.

Here also \( p_{x}(n + 1) = p_{x}(n) \) \((x = 1, 2, 3, 3', 4)\) and they are known since the initial frequency ratios are known.
If case 7 happens:

Then effectively

\[ \frac{p_2'(1)}{2 \left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

\[ \frac{p_2''(1)}{2 \left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

\[ \frac{p_4(1)}{2 \left( p_3'(1) + p_3''(1) + p_4(1) \right)} \]

i.e.,

\[ p_1(0) \text{ } XY \]

\[ p_2(0) \text{ } XX \]

\[ p_3'(0) \text{ } XX \]

\[ p_3''(0) \text{ } XX \]

\[ p_4(0) \text{ } X \overline{X} \]

where \( p_1(0) + p_2(0) = p_3'(0) + p_3''(0) + p_4(0) \)

and \( p_1(0) > p_3'(0) \) and \( p_3''(0) \).

will take part at the time of selection to produce the second generation.

Here also

\[ p_r(n + 1) = p_r(n) \]

\( (r = 1, 2, 3', 3'', 4) \)

and they are known since the initial frequency ratios are known.