CHAPTER 4

REVIEW OF CODAL PROVISIONS

4.1 GENERAL

International standards and codal specifications based on research findings for the design of cold-formed steel members have been in use for many years. This chapter deals with a comparison of the various provisions prescribed in the British Standards (BS), the North American Standards (NAS) and the Indian Standards (IS). Allowable Stress Design (ASD), Load and Resistance Factor Design (LRFD) and Limit State Design (LSD) are discussed to determine the capacities of the cold-formed steel sections. The provisions include the effect of cold-forming, local buckling coefficients for stiffened and unstiffened elements, provision for uniform compression, load carrying capacities of singly symmetric shape with respect to axially and eccentrically applied loads, and provision for combined bending and compression.

Singly symmetric sections like channels and hat sections may fail either by buckling about the plane of symmetry or by a combination of twisting and bending. Doubly symmetric sections like box sections or cruciform sections fail either by buckling about the weak axis or twisting about the shear centre in which the shear center and centroid coincides.
4.2 ALLOWABLE STRESS DESIGN (ASD)

The allowable load carrying capacity of the members in the Allowable Stress Design composed of only stiffened elements is reduced based on the area factor which is the ratio of effective design area to the gross area.

The reduction in the width of the element due to local buckling is taken into account for the effective design area calculation. The stress ratio which is the ratio between the allowable compressive stress in the weakest element of the cross section and the basic design stress is used for the reduction in the load capacity of the unstiffened elements. For members composing of both stiffened and unstiffened elements the form factor is used which is the product of stress factor and area factor.

The required strength of structural members is computed by accepted methods of structural analysis for the specified nominal or working loads for all applicable load combinations in the allowable stress design approach. The required strengths should not exceed the allowable design strengths permitted by the specifications. The allowable design strength is determined by dividing the nominal strength by a factor of safety as given by the inequality relationship in Equation (4.1).

\[ R_a \leq \frac{R_n}{\Omega} \]  

(4.1)

where \( R_a \) - allowable design strength

\( R_n \) - nominal strength

\( \Omega \) - factor of safety
4.3 LOAD AND RESISTANCE FACTOR DESIGN (LRFD)

The probabilistic concept is the basis in the Load and Resistance Factor Design. The use of multiplication factors accounts for the uncertainties in the loads and resistances. The limit state of strength required to resist the extreme loads during the intended life of the structure and the limit state of the ability of the structure to perform its intended function during its life are the two limit states considered in the LRFD.

The required design strength shall be calculated in accordance with Equation (4.2).

\[ R_u \leq \phi R_n \]  \hspace{1cm} (4.2)

where  
\( R_u \) - required design strength
\( R_n \) - nominal strength
\( \phi \) - resistance factor

For a given limit state, the nominal resistance is the strength of the element or member which is computed for nominal section properties and for minimum specified material properties according to the appropriate analytical model which defines the strength. The resistance factor \( \phi \) accounts for the uncertainties and variabilities inherent in \( R_n \) and it is usually less than unity.

4.4 LIMIT STATE DESIGN

Structural members and their connections were designed to have resistance such that the factored resistance equals or exceeds the effect of factored loads and the factored resistance shall be calculated in accordance with Equation (4.3).
\[ \phi R_n \geq R_f \]  

where  
\( \phi \) - Resistance factor  
\( R_n \) - Nominal resistance  
\( \phi R_n \) - Factored resistance  
\( R_f \) - Effect of factored loads

The inclusions of uncertainties and the variabilities of different types of loads and resistances are the two main advantages of LSD which are different and so these are accounted by the use of multiplying factor. Using the probability theory, designs can ideally achieve a more consistent reliability. A more rational and refined design method is provided by LRFD compared to ASD method.

### 4.5 EFFECT OF COLD-FORMING

The provisions for the utilisation of increased yield strength due to cold-forming process presented in BS: 5950(Part5)-2002, IS: 801-2005 and NAS-2007 manual are discussed in this section.

#### 4.5.1 BS : 5950 (Part 5) - 2002

The average yield strength \( Y_{sa} \) may be calculated as given in Equation (4.4).

\[ Y_{sa} = Y_s + \frac{5Nt^2}{A} (U_s - Y_s) \]  

(4.4)
where

\[ Y_{sa} \] - average yield strength

\[ N \] - number of full 90° bends in the section with an internal radius \( \leq 5t \);

\[ t \] - net section thickness of the material in mm

\[ U_s \] - minimum ultimate tensile strength in N/mm\(^2\)

\[ Y_s \] - material yield strength

\[ A \] - gross area of the cross section in mm\(^2\)

The value of yield strength was increased to a limit of 1.25 times that of the yield strength of the virgin steel. For elements of flat width, ‘b’ and thickness ‘t’, the value of \( Y_{sa} \) is modified as follows to provide appropriate compression yield strength \( Y_{sac} \).

For stiffened element

For \( b/t \leq 24 \)

\[
\frac{280}{Y_s} \left( \frac{280}{Y_s} \right)^{1/2} \quad Y_{sac} = Y_{sa} \quad (4.5)
\]

For \( b/t \geq 48 \)

\[
\frac{280}{Y_s}^{1/2} \quad Y_{sac} = Y_s \quad (4.6)
\]

For unstiffened element

For \( b/t \leq 8 \)

\[
\frac{280}{Y_s} \left( \frac{280}{Y_s} \right)^{1/2} \quad Y_{sac} = Y_{sa} \quad (4.7)
\]

For \( b/t \geq 16 \)

\[
\frac{280}{Y_s}^{1/2} \quad Y_{sac} = Y_s \quad (4.8)
\]
For intermediate values of b/t the value of $Y_{sac}$ may be obtained by linear interpolation.

For members which undergo welding, annealing, galvanising or any other heat treatment after forming which may produce softening, the increase in yield strength due to cold working is not allowed.


The average design yield strength ($F_{ya}$) of the full section is considered instead of $F_y$ resulting in the increase in strength due to cold-forming. The average tensile yield point of full section of axially loaded compression members are determined based on either full section tensile tests, or stub column tests, and computed as given in Equation (4.9).

$$F_{ya} = C F_{yc} + (1 - C) F_{yt}$$  \hspace{1cm} (4.9)

where  
\begin{align*}
C & \quad \text{ratio of the total corner area to the total cross-sectional area of the full section} \\
F_{yc} & = B_c F_y / (R/t)^m \\
F_{yt} & = \text{Weighted average yield point of the flat portions} \\

\end{align*}

The above formula does not apply where $F_u/F_y$ is less than 1.2, R/t exceeds 7, and maximum included angle exceeds $120^\circ$

$$B_c = 3.69 \left( F_u/F_y \right) - 0.819 \left( F_u/F_y \right)^2 - 1.79$$

$$m = 0.192 \left( F_u/F_y \right) - 0.068$$
4.5.3 **Discussion**

- BS : 5950 provides a different expression which accounts for the number of bends and limits the increase in the strength to 1.25 times of the virgin yield strength.

- The two codes, IS : 801 and NAS presents similar expressions for the provision of cold-forming. These expressions are the same as proposed by Karren (1967).

4.6 **LOCAL BUCKLING COEFFICIENT**

The plate buckling coefficient $k$, depends on the length to width ratio of the element, the boundary condition along the edge element and the type of stress and its distribution over the width. However, overall, there is only one local buckling load for the overall cross section. Elements which are less critical, offer restraint to the more buckling prone elements; hence increasing the local buckling strength. The local buckling coefficient is nearly independent of the length to width ratio of the element for very long plates. The local buckling is characterized by short waves along the entire length with corresponding buckling wave length to the order of width of the plate elements, while the corner edges remain straight. The buckling strength of plate elements is either explicitly or implicitly prescribed in IS : 801 - 2005, BS : 5950 (Part 5) – 2002 and NAS Manual – 2007. These provisions are compared in this section.

4.6.1 **IS : 801 - 2005 and BS : 5950 (Part 5) - 2002**

Table 4.1 gives the plate buckling coefficients for stiffened and unstiffened elements presented in IS and BS Standards.
Table 4.1 Plate buckling coefficients

<table>
<thead>
<tr>
<th>Element</th>
<th>IS : 801 – 2005</th>
<th>BS : 5950 (Part 5) - 2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffened</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Unstiffened</td>
<td>0.43</td>
<td>0.425</td>
</tr>
</tbody>
</table>

4.6.2 NAS Manual - 2007

(a) Stiffened Element

For stiffened elements with uniform stress gradient, the plate buckling coefficient $k$ is taken as 4. However, when the stiffened element is subjected to stress gradient, the plate buckling coefficient is calculated using expression in Equation (4.11).

\[
k = 4 \quad \text{for Uniform Compression} \tag{4.10}
\]

\[
k = 4 + 2 (1-\psi)^3 + 2(1-\psi) \quad \text{for Stress Gradient} \tag{4.11}
\]

\[
\psi = \frac{f_2}{f_1}
\]

where $f_2$ - minimum edge stress which is either compressive or tensile

$f_1$ - maximum edge stress which is either compressive or tensile

(b) Unstiffened Element

\[
k = 0.43 \tag{4.12}
\]

4.6.3 Discussion

- All the codes disregard conservatively the effect of rotational restraint along the unloaded edges, while calculating the local
buckling strength. This could lead to unnecessary excessive conservatism, especially in the case of unstiffened elements.

- NAS does not account for the effect of compressive stress gradient ($\psi = 1$) on local buckling in the case of unstiffened elements.

- BS does not cover the range of stress gradient, wherein the magnitude of tensile stress is greater than the compressive stress ($\psi < -1$) in the evaluation of local buckling strength of unstiffened elements.

- Stress gradient in plate elements are not accounted in IS: 801-2005. Expressions for local buckling load of stiffened and unstiffened elements under stress gradient are not presented. Design equations implicitly use local buckling coefficient corresponding to the hinged edge condition only.

- The effect of end restraint is not considered directly in the British code, but it calculates the effect of end restraint indirectly by calculating the ratio of the size of two plates, buckling plate and restraining plate. British code does not recommend plate buckling coefficient under stress gradient.

### 4.7 PROVISIONS FOR UNIFORM COMPRESSION

This concept of effective width was introduced by Von Karman et al (1932) and Chajes (1974). According to this approach, instead of considering the non uniform distribution of stress over the entire width of plate, it is assumed that the total load is resisted by an effective width, subject to uniformly distributed stress equal to the edge stress. The width, ‘$b$’, is selected such that the area under the curve of the actual non uniform stress distribution is equal to the sum of the two parts of the effective rectangular
shaded area with total width ‘b’ and an intensity of stress equal to the edge stress. Later, their equation was modified by many investigators based on extensive experimental and analytical investigations to incorporate the effects of imperfections and residual stresses.

Most of the standards adopt the effective width concept to consider the post-buckling reserve capacity for the design of cold-formed steel members. These provisions are compared and discussed in this section.

4.7.1 IS : 801 – 2005

(a) Stiffened Element

The flat width of any stiffened compression element having a flat width to thickness ratio larger than the limiting value is being reduced to an effective design width. For elements without intermediate stiffeners and except for flanges of closed square and rectangular tubes, the effective design width of compression elements is calculated from the following formulae.

(i) For Load Calculation

Flanges are considered to be fully effective,

\[ b_{\text{eff}} = w \]

when \( (w/t)_{\text{lim}} = 1435/\sqrt{f} \) \hspace{1cm} (4.13)

For flanges with b/t larger than \( (w/t)_{\text{lim}} \)

\[ \frac{b_{\text{eff}}}{t} = \frac{2120}{\sqrt{f}} \left[ 1 - \frac{465}{(w/t)\sqrt{f}} \right] \] \hspace{1cm} (4.14)

Where \( b_{\text{eff}} \) - Effective width in cm
(ii) For Deflection Calculation

Flanges are considered to be fully effective,

\[ b_{\text{eff}} = w \]

when \( (w/t)_{\text{lim}} = 1850/\sqrt{f} \) \hspace{1cm} (4.15)

For flanges with w/t larger than \((w/t)_{\text{lim}}\)

\[ \frac{b_{\text{eff}}}{t} = \frac{2710}{\sqrt{f}} \left[ 1 - \frac{600}{(w/t)\sqrt{f}} \right] \] \hspace{1cm} (4.16)

(b) Unstiffened Element

For unstiffened elements the allowable compressive stress, \( F_c \) is deduced based on the flat width to thickness ratio.

For w/t ratio not greater than \( 530/\sqrt{F_y} \)

\[ F_c = 0.60 F_y \] \hspace{1cm} (4.17)

For w/t ratio greater than \( 530/\sqrt{F_y} \) but not greater than \( 1210/\sqrt{F_y} \)

\[ F_c = F_y \left[ 0.767 - \left( 3.15/10^4 \right)(w/t)\sqrt{F_y} \right] \] \hspace{1cm} (4.18)

For w/t ratio greater than \( 1210/\sqrt{F_y} \) but not greater than 25
\[ F_c = \frac{562,000}{(w/t)^2} \]  
(4.19)

For w/t ratio from 25 to 60

\[ F_c = \frac{562,000}{(w/t)^2} \] for angle struts  
(4.20)

\[ F_c = 1390 - 20(w/t) \] for other sections  
(4.21)

When the yield point of steel is less than 2320 kg/cm² then for (w/t) ratio between \( \frac{530}{\sqrt{F_y}} \) and 25

\[ F_c = 0.6F_y - \frac{\left[ \frac{w}{t} - \frac{530}{\sqrt{F_y}} \right] \left(0.6F_y - 900\right)}{25\left[1 - 21.2/\sqrt{F_y}\right]} \]  
(4.22)

4.7.2 BS : 5950 (Part 5) - 2002

(a) Stiffened Element

The effective design width, \( b_{\text{eff(s)}} \) of a stiffened element under uniform compression is obtained from Equations (4.23) and (4.24).

For \( f_c/p_{cr} < 0.123 \)

\[ \frac{b_{\text{eff}}}{b} = 1 \]  
(4.23)

For \( f_c/p_{cr} \geq 0.123 \)

\[ \frac{b_{\text{eff}}}{b} = \left[ 1 + 14 \left( \frac{f_c}{p_{cr}} \right)^{1/2} - 0.35 \right]^{4}^{-0.2} \]  
(4.24)

where \( f_c \) - the compressive stress on the effective element
\( p_{cr} \) - the local buckling stress of the element given

\[ p_{cr} = 0.904 E_k \left( \frac{t}{b} \right)^2 \]

\( k \) - local buckling coefficient usually considered as 4 and the different values are recommended for certain specific geometries.

\( t \) - the element thickness

(b) Unstiffened Element

The effective design width, \( b_{eu} \) of an unstiffened element under uniform compression is obtained using Equation (4.25).

\[ b_{eu} = 0.89 b_{eff} + 0.11 b \quad (4.25) \]

where \( b_{eff} \) - effective width determined using Equations (4.23) and (4.24) with \( k \) value as 0.425

\( b \) - the full flat width

4.7.3 NAS Manual - 2007

(a) Stiffened Element

(i) For Load Calculation

The effective width, for load calculation is determined from Equations (4.26) and (4.27).

\[ b_{eff} = w \quad \text{when} \quad \lambda \leq 0.673 \quad (4.26) \]

\[ b_{eff} = \rho w \quad \text{when} \quad \lambda > 0.673 \]
where \( \rho = (1 - 0.22/\lambda)/\lambda \)

\[ k = 4 \]

\[ \lambda = \frac{f}{\sqrt{f_{cr}}} \] \hspace{1cm} (4.27)

\[ f_{cr} = \frac{k\pi^2E}{12(1-\mu^2)} \left( \frac{t}{w} \right)^2 \]

(ii) For Deflection Calculation

The effective width, used in computing deflection is calculated from the following equations.

\[ b_{\text{eff}} = w \quad \text{when} \quad \lambda \leq 0.673 \] \hspace{1cm} (4.28)

\[ b_{\text{eff}} = \rho w \quad \text{when} \quad \lambda > 0.673 \] \hspace{1cm} (4.29)

An approximate estimate of \( \rho \) is obtained from the Equations (4.26) and (4.27) except that \( f_d \) is substituted for \( f \), where \( f_d \) is the computed compressive stress in the element being considered.

Conservative estimate of \( \rho \) is obtained from the following equations.

\[ \rho = 1 \quad \text{when} \quad \lambda \leq 0.673 \]

\[ \rho = (1.358 - 0.461/\lambda)/\lambda \quad \text{when} \quad 0.673 < \lambda < \lambda_c \]

\[ \rho = \left(0.41 + 0.59 \lambda\sqrt{F_y/F_d} - 0.22/\lambda\right)/\lambda \]

when \( \lambda \geq \lambda_c \)
ρ shall not exceed 1.0 for all cases.

where \( \lambda_c = 0.256 + 0.328(w/t)\sqrt{F_y/E} \)

and \( \lambda \) is defined by Equation (4.27) except that \( f_d \) is substituted for \( f \).

(b) Unstiffened Element

(i) For Load Calculation

The effective width, for load calculation is determined from Equations (4.30) and (4.31).

\[
b_{\text{eff}} = w \quad \text{when} \quad \lambda \leq 0.673 \quad (4.30)
\]

\[
b_{\text{eff}} = \rho w \quad \text{when} \quad \lambda > 0.673 \quad (4.31)
\]

where \( \rho = (1 - 0.22/\lambda)/\lambda \)

\( k = 0.43 \)

\[
\lambda = \sqrt{\frac{f}{f_{cr}}}
\]

\[
f_{cr} = \frac{k\pi^2 E}{12[1-\mu^2]} \left( \frac{t}{w} \right)^2
\]

(ii) For Deflection Calculation

The effective width, used in computing deflection is evaluated from the following equations.

\[
b_{\text{eff}} = w \quad \text{when} \quad \lambda \leq 0.673
\]

\[
b_{\text{eff}} = \rho w \quad \text{when} \quad \lambda > 0.673 \quad (4.32)
\]
\[ \rho \text{ value is computed using Equations (4.26) and (4.27), except that the plate buckling coefficient } k \text{ is taken as 0.43 and } f_d \text{ is substituted for } f_{cr}. \]

4.7.4 Discussion

- IS code allows the use of area factor for stiffened elements based on the effective design width and stress factor for unstiffened elements based on the allowable compressive stress under uniform compression.

- The other codes of practice, allows only the effective width concept for both stiffened and unstiffened elements.

4.8 PROVISIONS FOR STRESS GRADIENT

4.8.1 IS : 801 - 2005

IS code does not give explicit equations for evaluating the local buckling of stiffened and unstiffened elements under a general state of non-uniform stress distribution. However, the limiting value of the stress due to the combined effect of axial and bending stresses is covered. As per this standard, the sum of the ratios of actual stress to permissible stress in axial and bending should be less than 1.0.

4.8.2 BS : 5950 (Part 5) - 2002

(a) Stiffened Element

The effective design width of a compression element in which the stress varies linearly from \( f_{c1} \) at one edge to \( f_{c2} \) at the other edge with \( f_{c1} > f_{c2} > 0 \) is determined

For \( f_{cm}/p_{cr} < 0.123 \)
\[ \frac{b_{\text{eff}}}{b} = 1 \quad (4.33) \]

For \( f_{\text{cm}} / p_{\text{cr}} \geq 0.123 \)

\[ \frac{b_{\text{eff}}}{b} = \left[ 1 + 14 \left( \frac{f_{\text{cm}}}{p_{\text{cr}}} \right)^{1/2} - 0.35 \right]^{1/2} \quad (4.34) \]

where \( p_{\text{cr}} \) - the local buckling stress of the element given\(= 0.904 \, E_k \, (t / b)^2 \)

\( k \) - local buckling coefficient usually considered as 4 and the different values are recommended for certain specific geometries.

\( t \) - is the material thickness

\( f_{\text{cm}} \) - mean value of the compressive stress on the element.

In the case of elements in which the stress varies from compression to tension, the moment capacity is determined on the basis of a limiting compressive resistance stress. This stress is used in the evaluation of the effective widths of compression elements and the reduced section properties.

The compressive resistance stress, \( p_0 \), in a stiffened element is minimum of the following equations.

\[ p_0 = \begin{cases} 1.13 - 0.0019 \frac{D_w}{t} \left( \frac{Y_s}{280} \right)^{1/2} p_y \\ p_y \end{cases} \quad (4.35) \]

or

\[ p_0 = p_y \quad (4.36) \]

where \( D_w \) - overall width

\( Y_s \) - material yield strength

\( t \) - thickness

\( p_y \) - design strength
(b) Unstiffened Element

The effective design width of an unstiffened element subjected to stress gradient is obtained as follows.

If the loading is such as to cause compression of the free edge, the effective width is determined in accordance with 4.7.2 (b) with $f_c$ is replaced by the stress at the free edge, $f_{cf}$ and the value of $k$ taken as

$$k = \frac{1.7}{3 + R}$$

(4.37)

where $R$ is the ratio of the stress at the supported edge, $f_{cs}$ to $f_{cf}$ computed on the basis that the element is fully effective and with compressive stresses being taken as positive.

When the loading is such as to cause tension of the free edge the element should be treated as a stiffened element except that the limitation on maximum width to thickness ratio for unstiffened elements is used.

4.8.3 NAS Manual – 2007

(a) Stiffened Element

(i) For Load Calculation

For webs under stress gradient ($f_1$ in compression and $f_2$ in tension as shown in Figure 4.1) the effective widths and plate buckling coefficient shall be calculated as follows:

$$k = 4 + 2(1 + \psi)^3 + 2(1 + \psi)$$

(4.38)

For $h_o / b_o \leq 4$

$$b_1 = b_o / (3 + \psi)$$

(4.39)
\[ b_2 = \frac{b_e}{2} \quad \text{when } \psi > 0.236 \quad (4.40) \]

\[ b_2 = b_e - b_1 \quad \text{when } \psi \leq 0.236 \quad (4.41) \]

(ii) \( b_e \) - Effective width b

Figure 4.1 Webs under stress gradient
In addition, \( b_1 + b_2 \) shall not exceed the compression portion of the web calculated on the basis of effective section.

For \( h_o / b_o > 4 \)

\[
\begin{align*}
  b_1 &= b_v(3 + \psi) \\
  b_2 &= b_v(1 + \psi) - b_1
\end{align*}
\]  

(4.42) \hspace{1cm} (4.43)

For other stiffened elements under stress gradient (\( f_1 \) and \( f_2 \) in compression as shown in Figure 4.2).

\[
\begin{align*}
  k &= 4 + 2(1-\psi)^3 + 2(1-\psi) \\
  b_1 &= b_v(3 - \psi) \\
  b_2 &= b_v - b_1
\end{align*}
\]

Figure 4.2 Stiffened elements under stress gradient
(ii) **For Deflection Calculation**

For webs under stress gradient (\(f_{d1}\) in compression and \(f_{d2}\) in tension) the effective widths and plate buckling coefficient shall be calculated in accordance with 4.8.3 (a) (i) except that \(f_{d1}\) and \(f_{d2}\) are substituted for \(f_1\) and \(f_2\), where \(f_{d1}\) and \(f_{d2}\) are the computed stresses \(f_1\) and \(f_2\) based on the effective section at the load for which the deflection is determined.

(b) **Unstiffened Element**

(i) **For Load Calculation**

The effective width, ‘b’ is evaluated in accordance with section 4.8.3 (a) except that plate buckling coefficient, \(k = 0.43\).

(ii) **For Deflection Evaluated**

The effective width \(b_d\), used in computing deflection is evaluated from the following equations.

\[
\begin{align*}
    b_d &= w \quad \text{when} \quad \lambda \leq 0.673 \\
    b_d &= \rho w \quad \text{when} \quad \lambda > 0.673
\end{align*}
\]

An approximate estimate of \(\rho\) is obtained from the Equations (4.26) and (4.27) except that \(F_d\) is substituted for \(F_n\), where \(F_d\) is the computed compressive stress in the element being considered.

Conservative estimate of \(\rho\) is obtained from the following equations.

\[
\rho = 1 \quad \text{when} \quad \lambda \leq 0.673
\]
\[ \rho = \frac{(1.358 - 0.461/\lambda)\lambda}{\lambda} \text{ when } 0.673 < \lambda < \lambda_c \quad (4.44) \]

\[ \rho = \frac{0.41 + 0.59\sqrt{F_y/F_d} - 0.22/\lambda}{\lambda} \quad (4.45) \]

when \( \lambda \geq \lambda_c \)

\[ \rho \text{ shall not exceed 1.0 for all cases.} \]

where \( \lambda_c = 0.256 + 0.328(w/t)\sqrt{F_y/E} \quad (4.46) \]

and \( \lambda \) is defined by Equation (4.27) except that \( F_1 \) is substituted for \( F_n \).

\[ k = 0.43. \]

4.8.4 Discussion

- IS code does not consider the local buckling of stiffened and unstiffened elements under a general state of non-uniform stress distribution.

- Most of the other codes use the same equation proposed by Winter (1948) for calculating the ratio of effective width to total width.

- In the case of unstiffened elements the NAS conservatively disregards the stress gradient and assumes uniform compression of intensity equal to the maximum compressive stress, over the entire width of the plate element.
4.9 LOAD CARRYING CAPACITY IN COMPRESSION MEMBERS

4.9.1 IS: 801 - 2005

(a) Doubly Symmetric Sections

For doubly symmetric sections of rectangular, square or circular shapes and other shapes not subjected to torsional-flexural buckling, and for members braced against twisting:

For \( KL / r < \frac{C_c}{\sqrt{Q}} \)

\[
F_{all(2)} = \frac{12}{23} Q F_{y} - \frac{3(QF_{y})^2}{23\pi^2E}\left(\frac{KL}{r}\right)^2
\]  \hspace{1cm} (4.47)

For \( KL / r \geq \frac{C_c}{\sqrt{Q}} \)

\[
F_{all(2)} = \frac{12\pi^2E}{23(KL/r)^2}
\]  \hspace{1cm} (4.48)

where \( C_c = \sqrt{\left(2\pi^2E/F_{y}\right)} \) \hspace{1cm} (4.49)

\( Q \) - a factor determined as follows

- For members composed entirely of stiffened elements, \( Q_a \) is the ratio between the effective design area as determined from the effective design width of such elements and the gross area of cross-section.

- For members composed entirely of unstiffened elements \( Q_s \) is the ratio between the allowable compression stress \( F_c \) for the
weakest element of the cross-section and the basic design stress $F$.

- For members composed of both stiffened and unstiffened elements the factor $Q$ is the product of an area factor $Q_a$ calculated as given above and stress factor $Q_s$ calculated as given above.

**b) Singly Symmetric Sections**

A typical singly symmetric cold-formed steel member subjected to axial and eccentric loading in the plane of symmetry may fail (i) by yielding with large deformation in the plane of symmetry (ii) by flexural buckling about the asymmetric axis and material yielding or (iii) by Torsional flexural buckling mode. Hence a member has to be checked with respect to all the above failure modes. The allowable stress equations recommended by various standards are discussed in the following sections.

IS 801:2005 is based on Allowable Stress Design (ASD). The post buckling strength is accounted by considering the concept of effective width in the case of stiffened members and reduced stress in the case of unstiffened members. The allowable stress is obtained by using the form factor and a factor of safety of 1.92.

For singly symmetric or non symmetric shapes of open cross section or intermittently fastened singly symmetric components of built-up shapes having $Q = 1.0$ which is subjected to torsional-flexural buckling and which are not braced against twisting, the allowable stress is the least of $F_{all(1)}$ determined as follows and $F_{all(2)}$ calculated as per section 4.9.1 (a).

For $\sigma_{TFO} > 0.5 F_y$
\[ F_{all(1)} = 0.522F_y - \frac{F_y^2}{7.67\sigma_{TFO}} \]  

(4.50)

For \( \sigma_{TFO} \leq 0.5 \ F_y \)

\[ F_{all(1)} = 0.522\sigma_{TFO} \]  

(4.51)

where

\[ \sigma_{TFO} = \frac{1}{2\beta} \left[ \left( \sigma_a + \sigma_t \right) - \sqrt{\left( \left( \sigma_a + \sigma_t \right)^2 - 4\beta \sigma_a \sigma_t \right)} \right] \]  

(4.52)

\[ \sigma_{ex} = \frac{\pi^2E}{(KL/r_x)^2} \]  

(4.53)

\[ \sigma_t = \frac{1}{Ar_0^2} \left[ GJ + \frac{\pi^2EC_w}{(KL)^2} \right] \]  

(4.54)

\[ \beta = 1 - \left( \frac{x_0}{r_0} \right)^2 \]  

(4.55)

\[ r_0 = \sqrt{r_x^2 + r_y^2 + x_0^2} \]  

(4.56)

4.9.2 BS : 5950 (Part 5) - 2002

(a) Doubly Symmetric Sections

For sections symmetrical about both principal axes and closed cross sections which are not subjected to torsional-flexural buckling or braced against twisting, the buckling resistance under axial load \( P_c \) is obtained from the following equations.

\[ P_c = \frac{P_E P_{CS}}{\phi + \sqrt{\phi^2 - P_E P_{CS}}} \]  

(4.57)
where \( \phi = \frac{P_{cs}(1 + \eta)P_E}{2} \)

where \( P_{cs} \) - short strut capacity = \( A_{eff} p_y \)

\( A_{eff} \) - effective cross-sectional area

\( p_y \) - the design strength

\( P_E = \frac{\pi^2 EI}{L_E^2} \)

\( \eta \) - Perry coefficient, such that

for \( L_E / r \leq 20 \) \( \eta = 0 \)

for \( L_E / r > 20 \) \( \eta = 0.002 (L_E / r - 20) \)

\( r \) - radius of gyration corresponding to \( P_E \)

(b) **Singly Symmetric Sections**

For members which have at least one axis of symmetry and which are subjected to torsional-flexural buckling, the ultimate loads are calculated in accordance with section 4.9.2 (a) except that effective length \( L_E \) is substituted by a factored effective length \( \alpha L_E \), where values of \( \alpha \) are determined from the following.

For \( P_E \leq P_{TF} \) \( \alpha = 1 \)

For \( P_E > P_{TF} \) \( \alpha = \sqrt{\frac{P_{Ey}}{P_{TF}}} \) (4.58)

where

\( P_{Ey} = \frac{\pi^2 EI_y}{L_E^2} \) (4.59)
\[ P_{TP} = \left( \frac{1}{2\beta} \right) \left( (P_{EX} + P_T) - \sqrt{(P_{EX} + P_T)^2 - 4\beta P_{EX} P_T} \right) \]  

(4.60)

\[ P_{EX} = \frac{\pi^2 EI_x}{L_E^2} \]  

(4.61)

\[ P_T = \frac{1}{r_0^2} \left[ GJ + \frac{2\pi^2 EC_w}{L_E^2} \right] \]  

(4.62)

\( \beta \) and \( r_0 \) are calculated using Equations (4.55) and (4.56) respectively.

### 4.9.3 NAS Manual - 2007

The nominal axial strength \( P_n \) is calculated as follows.

\[ P_n = A_e F_n \]

Design Strength \( = P_n/1.80 \) by Allowable Stress Design

\( = 0.85 \times P_n \) by Load and Resistance Factor Design

\( F_n \) is determined as follows.

For \( \lambda_c \leq 1.5 \) \( F_n = \left[ 0.658 \lambda_c^{-2} \right] F_y \)  

(4.63)

For \( \lambda_c > 1.5 \) \( F_n = \left[ \frac{0.877}{\lambda_c^2} \right] F_y \)  

(4.64)
where

$$\lambda_c = \sqrt{\frac{F_y}{F_e}}$$

and $F_e$ is the least of the elastic flexural, torsional and torsional-flexural buckling stress determined appropriately as follows.

(a) **Doubly Symmetric Sections**

(i) For doubly symmetric sections, closed cross sections and other sections which are not subjected to torsional or torsional-flexural buckling, the elastic flexural buckling stress, $F_e$ is determined as follows.

$$F_e = \frac{\pi^2 E}{(KL/r)^2}$$  \hspace{1cm} (4.65)

(ii) For doubly symmetric sections subjected to torsional buckling, $F_e$ is taken as the smaller of $F_e$ calculated above and $F_e = \sigma_t$.

where

$$\sigma_t = \frac{1}{A r_0^2} \left[ G J + \frac{\pi^2 E C_w}{(K_t L_t)^2} \right]$$  \hspace{1cm} (4.66)

- $K_t$ . Effective length factor for torsion
- $L_t$ . Effective length for torsion

(b) **Singly Symmetric Sections**

For singly symmetric sections subjected to torsional-flexural buckling, $F_e$ is taken as the smaller of $F_e$ calculated below and $F_e$ determined according to section 4.8.3 (a) (i).
\[
F_e = \frac{1}{2\beta} \left[ (\sigma_{ex} + \sigma_i) - \sqrt{\left( (\sigma_{ex} + \sigma_i)^2 - 4\beta \sigma_{ex} \sigma_i \right)} \right] \quad (4.67)
\]

Alternatively, a conservative estimate of \( F_e \) is obtained using the following equation.

\[
F_e = \frac{\sigma_i \sigma_{ex}}{\sigma_i + \sigma_{ex}} \quad (4.68)
\]

where \( \sigma_{ex} = \frac{\pi^2 E}{(K_L \lambda / r_s)^2} \quad (4.69) \)

\( \beta \) and \( r_0 \) are calculated using Equations (4.55) and (4.56) respectively and \( \sigma_i \) is calculated using Equation (4.66).

### 4.9.4 Discussion

The load carrying capacities of all the specimens tested are calculated based on the three codes of practice. The results are presented, compared with the experimental and analytical results and discussed in Chapter 6.

### 4.10 COMBINED BENDING AND COMPRESSION

#### 4.10.1 NAS Manual - 2007

For laterally unbraced segments of singly-doubly symmetric sections subject to lateral torsional buckling, the nominal flexural strength \( M_n \) shall be calculated as follows:

\[
M_n = Z_c F_c \quad (4.70)
\]
where

\[ Z_c = \text{Elastic section modulus of effective section calculated relative to extreme compression fibre at } F_c \]

\[ F_c = \text{Shall be determined as follows} \]

For \( F_e \geq 2.78F_y \)

The member segment is not considered subject to lateral-torsional buckling at bending moments less than or equal to \( M_y \).

For \( 2.78F_y > F_e > 0.56F_y \)

\[ F_e = \frac{10}{9}F_y \left(1 - \frac{10F_y}{36F_e}\right) \quad (4.71) \]

For \( F_e \leq 0.56F_y \)

\[ F_e = F_e \]

where

\[ F_y = \text{Design yield stress} \]

\[ F_e = \text{Elastic critical lateral-torsional buckling stress calculated as below} \]

(a) For singly, doubly and point symmetric sections

(i) For bending about the symmetry axis

\[ F_e = \frac{C_b t A}{S_f} \sqrt{\frac{\sigma_y}{\sigma_i}} \quad (4.72) \]
for singly and doubly-symmetric sections

\[ F_e = \frac{C_b E A}{2 S_t \sqrt{\sigma_{e_2} \sigma_{i_1}}} \]  \hspace{1cm} (4.73)

and for point symmetric sections

\[ C_b \] shall be permitted to be conservatively taken as unity for all cases.

\[ r_o = \sqrt{r_x^2 + r_y^2 + x_o^2} \]

where

\[ r_x, r_y = \text{Radii of gyration of cross section about centroidal principal axes} \]

\[ r_o = \text{Distance from shear centre to centroid along principal x-axis, taken as negative} \]

\[ A = \text{Full unreduced cross sectional area} \]

\[ S_t = \text{Elastic section modulus of full unreduced section relative to extreme compression fibre} \]

\[ \sigma_{e_2} = \frac{\pi^2 E}{(K_y L_y / r_y)^2} \] \hspace{1cm} (4.74)

for twisting

\[ \sigma_i = \frac{1}{Ar_o^2} \left[ \frac{GJ}{2} + \frac{\pi^2 E C_w}{(K_y L_y)^2} \right] \] \hspace{1cm} (4.75)
(ii) Bending about perpendicular to the axis of symmetry

\[ F_e = \frac{C_s A \sigma_{ex}}{C_{TF} S_f} \left[ j + C_s \frac{j^2 + r_o^2}{r_o} \frac{\sigma_t}{\sigma_{ex}} \right] \quad (4.76) \]

\[ C_s = +1 \text{ for moment causing compression on shear centre side of centroid} \]
\[ = -1 \text{ for moment causing tension on shear centre side of centroid} \]

\[ \sigma_{ex} = \frac{\pi^2 E}{(K_x L_x / r_x)^2} \quad (4.77) \]

\[ C_{TF} = 1. \]

\[ j = \frac{1}{2I_y} \left[ \int_A x^2 dA + \int_A xy \, dA \right] - x_o \quad (4.78) \]

(b) For I sections, singly symmetric C-sections or Z-sections bent about the centroidal axis perpendicular to the web (x-axis) the following equations shall be permitted to be used in lieu of (a) to calculate \( F_e \)

\[ F_e = \frac{C_b \pi^2 E dI_{xc}}{S_f (K_y L_y)^2} \quad (4.79) \]

for doubly-symmetric I-sections and singly-symmetric C-sections

\[ F_e = \frac{C_b \pi^2 E dI_{xc}}{2S_f (K_y L_y)^2} \quad (4.80) \]

for point symmetric Z-sections
where

\[ d = \text{Depth of section} \]

\[ I_{yc} = \text{Moment of inertia of compression portion of section about centroidal axis of entire section parallel to web, using full unreduced section.} \]

For members subjected to lateral buckling the following equations are used to find the design strength

\[
\frac{W_c P}{P_n} + \frac{W_c C_{mx} M_x}{M_{nx} a_x} + \frac{W_c C_{my} M_y}{M_{ny} a_y} \leq 1.0 \quad (4.81)
\]

\[
\frac{W_c P}{P_{no}} + \frac{W_h M_x}{M_{nx}} + \frac{W_b M_y}{M_{ny}} \leq 1.0 \quad (4.82)
\]

when \( \Omega_c P / P_n \leq 1.0 \) the following equation shall be permitted to be used in lieu of the above two equations:

\[
\frac{W_c P}{P_n} + \frac{W_h M_x}{M_{nx}} + \frac{W_b M_y}{M_{ny}} \leq 1.0 \quad (4.83)
\]

where

\[ W_c = 1.80 \]

\[ P = \text{Required compressive axial strength} \]

\[ P_n = \text{Nominal axial strength} \]

\[ P_{no} = \text{Nominal axial strength, with } F_n = F_y \]

\[ W_b = 1.67 \]
\( M_x, M_y = \) required flexural strengths with respect to centroidal axes of effective section determined for required compressive axial strength alone.

\[
a_x = 1 - \frac{WP}{P_{Ex}} > 0
\]

\[
a_y = 1 - \frac{WP}{P_{Ey}} > 0
\]

where

\[
P_{Ex} = \frac{p^2EI}{(K_xL_x)^2}
\]

\[
P_{Ey} = \frac{\pi^2EI_y}{(K_yL_y)^2}
\]

4.10.2 BS : 5950 (Part 5) - 2002

Compression members which are also subjected to bending should be checked for local capacity at the points of greatest bending moment and axial load (usually at the ends). These members should also be checked for overall buckling.

The checks given below in 4.10.2.1 and 4.10.2.2 apply to members which have at least one axis of symmetry and which are not subjected to torsional or torsional flexural buckling.

4.10.2.1 Local Capacity Check

The member should satisfy the following relationship:
\[
\frac{F_c}{P_{cs}} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} \leq 1
\]  (4.86)

\(F_c\) - Applied axial load

\(P_{cs}\) - Short strut capacity

\(M_x\) - Applied bending moment about the x axis

\(M_{cx}\) - Moment capacity in bending about the x axis in the absence of \(F_c\) and \(M_y\)

\(M_y\) - Applied bending moment about the y axis

\(M_{cy}\) - Moment capacity in bending about the y axis in the absence of \(F_c\) and \(M_x\)

4.10.2.2 Overall Buckling Check

For beams not subject to lateral buckling the following relationship should be satisfied.

\[
\frac{F_c}{P_c} + \frac{M_x}{C_{bx} M_{cx} \left(1 - \frac{F_c}{P_{Ex}}\right)} + \frac{M_y}{C_{by} M_{cy} \left(1 - \frac{F_c}{P_{Ey}}\right)} \leq 1
\]  (4.87)

For beams subject to lateral buckling the following relationship should be satisfied.

\[
\frac{F_c}{P_c} + \frac{M_x}{M_b} + \frac{M_y}{C_{by} M_{cy} \left(1 - \frac{F_c}{P_{Ey}}\right)} \leq 1
\]  (4.88)

where

\(P_c\) - Axial buckling resistance in the absence of moments
\( P_{Ex} \) - Flexural buckling load in compression for bending about the x axis

\( P_{Ey} \) - Flexural buckling load in compression for bending about the y axis

\( C_{bx}, C_{by} \) - \( C_b \) factors with regard to moment variation about the x and y axes

\( M_b \) - lateral buckling resistance moment about the x (major) axis

\( F_c, M_x, M_{cx}, M_y \) and \( M_{cy} \) are as defined

The magnitudes of moments \( M_x \) and \( M_y \) should take into account any moment induced by the change in neutral axis position of the effective cross-section caused by the axial load. In the determination of \( C_{bx} \) and \( C_{by} \) the effects of change in the neutral axis position on the moment variation may be neglected.

### 4.10.3 IS : 801 - 2005

Singly Symmetric Shapes or Intermittently Fastened Singly-Symmetric components of Built-Up Shapes Having \( Q = 1.0 \) which may be subject to Torsional Flexural Buckling – Singly symmetric shapes subject to both axis compression and bending applied in the plane of symmetry shall be proportioned to meet the following four requirements as applicable:

\[
a) \quad \frac{F_a}{F_{a1}} + \frac{f_{bl} C_m}{F_{bl} \left( 1 - \frac{F_a}{F_c} \right)} \leq 1 \tag{4.89}
\]

\[
\frac{F_a}{F_{a0}} + \frac{f_{bl}}{F_{bl}} \leq 1
\]
when

\[ \frac{f_a}{F_{al}} \leq 0.15, \]
the following formula may be used in lieu of the above two formulae

\[ \frac{F_a}{F_{al}} + \frac{f_b}{F_{bl}} \leq 1.0 \]  \hspace{1cm} (4.90)

b) If the point of application of the eccentric load is located on the side of the centroid opposite from that of the shear centre, that is, if \( e \) is positive, then the average compression stress \( f_a \) shall also not exceed \( F_a \) given below

For \( \sigma_{TF} > 0.5 F_y \)

\[ F_a = 0.522 F_y - \frac{F_y^2}{7.67\sigma_{TF}} \]  \hspace{1cm} (4.91)

For \( \sigma_{TF} \leq 0.5 F_y \)

\[ F_a = 0.522\sigma_{TF} \]  \hspace{1cm} (4.92)

where

\( \sigma_{TF} \) shall be determined according to the formula:

\[ \frac{\sigma_{TF}}{\sigma_{TF0}} + \frac{C_{TF} \sigma_{bl}}{\sigma_{bT} \left( 1 - \frac{\sigma_{TF}}{\sigma_e} \right)} = 1.0 \]  \hspace{1cm} (4.93)

c) Except for T sections-or unsymmetric I-sections, if the point of application of the eccentric load is between the shear centre and the centroid, that is, if \( e \) is negative and if \( F_{al} \) is larger than \( F_{a2} \) then the average compressive stress \( f_a \) shall also not exceed \( F_a \) given below:
\[ F_a = F_{a2} + \frac{e}{x_0} (F_{aE} - F_{a2}) \]  

(4.94)

where

- \( F_a \) - Maximum average compressive stress
- \( f_{b1} \) - Bending stress
- \( F_{ao} \) - Allowable compressive stress under concentric loading for \( L = 0 \)
- \( F_{ac} \) - Average allowable compressive stress, the calculated values of \( f_a \) and \( F_a \) for \( e = x_o \)
- \( F_{aE} \) - Average allowable compressive stress the calculated values of \( f_a \) for \( e = x_o \)
- \( F_{a1} \) - Allowable compressive stress under concentric loading for buckling in the plane of symmetry
- \( F_{a2} \) - Allowable compressive stress under concentric loading
- \( F_b \) - Maximum bending stress in compression
- \( F_{b1} \) - Maximum bending stress in compression and the possibility of lateral buckling is excluded

\[ f_e = \frac{12\pi^2E}{23\left(\frac{KL_p}{r_p}\right)^2} \]

- \( f_a \) - Axial stress
- \( f_b \) - Maximum bending stress based on effective design widths
- \( \sigma_{bt} \) - Maximum compression bending stress caused bur to tension on the shear centre side
- \( \sigma_{bc} \) - Maximum compression bending stress caused bur to compression on the shear centre side
\( \sigma_{bi} \) - Maximum compression bending stress in the section caused by \( \sigma_{TF} \)

\( \sigma_{TF} \) - Average elastic torsional-flexural buckling stress

\( \sigma_{TFO} \) - Elastic torsional-flexural buckling stress under concentric loading

\[
\sigma_e = \frac{\pi^2 E}{(K \frac{L_e}{r_e})^2}
\]

4.10.4 Summary

The following points may be considered when the design strength calculations are made using these three codes.

- NAS does not take into account the nonlinearity in the case of eccentric loading except in the moment amplification effects.

- In the calculation of axial load, only the effective section corresponding to uniform compression is taken in the NAS and while evaluating the moment capacity only under plane of symmetry, bending stress on the gross section is taken, whereas the actual state of stress at failure due to eccentric load would be very different from both assumed stress and hence actual effective section at failure could be different.

- NAS considers the effect of reduction in the effective section only in the calculation of effective area and effective section modulus, which are used to reduce the corresponding fully effective section strength. This may be a conservative or un conservative assumption.
• The BS code presents an interaction equation for failure by yielding and equations for failure by buckling. The effective section properties under compression and bending are calculated at the yield stress stage.

• BS deals with a high degree nonlinear interactive problem after local buckling as a combination of axial load and bending problem; whereas the only non-linear interaction considered, is moment magnification due to axial load.

• The design strength in BS is governed by the overall instability, accounting for the effect of local buckling only on the reduction in area or section modulus and the elastic buckling strength calculations are based only on the gross section properties.

• The ultimate load calculated by IS recommendation is very conservative compared to the experimental results.

• The conservative value of the local buckling strength results in the conservative estimation of the post-buckling strength.

• The IS does not allow unstiffened elements to be stressed beyond the local buckling strength under uniform compression, thus disregarding the appreciable post-local buckling strength. This is the reason for the greater conservatism in the results of specimens with unstiffened elements having a larger flat width to thickness ratio.

• The recommendations to calculate the local buckling strength and the post buckling behaviour do not account for stress gradient. Hence, the ultimate strength results are conservative particularly when the eccentricities cause larger compression at the supported edge of the unstiffened elements.