CHAPTER II

VARIABLE RESPONSE AND TUNABLE NETWORKS

Summary:

The dual-input technique is capable of shifting the transmission zeros of a network without disturbing its poles or any of the passive components of the subnetwork. In some cases, such zero shift results in the smooth transition of a network transfer function from band-pass, through all-pass, to band attenuation characteristics with selectivity changing all the way. In other cases voltage tunable networks are achieved.

* This chapter is based on the following papers of the author:


1. ZERO SHIFTING BY DUAL INPUTS

1.1. The transfer function of a network can be defined as the ratio of the Laplace transform of the output signal to that of the input signal with zero initial conditions. In terms of roots of the numerator and denominator, a network transfer function can be expressed in the form

\[ \beta = K \frac{(s+a_1)(s+a_2)\cdots(s+a_n)}{(s+b_1)(s+b_2)\cdots(s+b_m)} \]

where a's and b's are the zeros and poles respectively of the network and indicate the points where the network response becomes zero or infinity. K is a constant independent of frequency.

In the design of any dynamic system, the location of poles and zeros in the complex s-plane is crucial to achieving the desired system characteristics. In control engineering feedback technique is mainly employed for shifting/locating the poles of a system without disturbing the zeros. Similarly, feedforward method gives control over the location of zeros. It will be ideal if the zeros and poles could be controlled without any interaction between them.

We shall, in this chapter, outline a method for shifting the zeros of a network without disturbing its poles. Such zero shifting can have many applications. We have discussed in the thesis only a few of them. In this chapter we have dealt with variable response and tunable networks. In the subsequent chapters, realisation of all-pass networks with lumped and distributed elements, various synthesis techniques for realising non-minimum phase transfer functions and those with adjustable complex poles and zeros are discussed.
1.2. For considering the basic scheme let us examine the arrangement of figure 1 where two voltages $e$ and $m e$ are fed through impedances $y_1$ and $y_2$ and the output is obtained across $y_3$.

By Millman's theorem,

$$
\frac{e_{o}}{e} = \frac{e y_1 + m e y_2}{y_1 + y_2 + y_3} \quad \ldots \quad (1)
$$

Here $m$ may be real or complex,

$$
\beta = \frac{e_{o}}{e} = \frac{y_1 + m y_2}{y_1 + y_2 + y_3} \quad \ldots \quad (2)
$$

In general,

$$
\gamma_1 = \frac{N_1(s)}{D_1(s)} \quad \gamma_2 = \frac{N_2(s)}{D_2(s)} \quad \gamma_3 = \frac{N_3(s)}{D_3(s)}
$$

Hence

$$
\beta = \frac{D_3(s) \left[ D_1(s) N_1(s) + m D_1(s) N_2(s) \right]}{N_1(s) D_2(s) D_3(s) + N_2(s) D_3(s) + N_3(s) D_1(s) D_2(s)} \quad \ldots \quad (3)
$$

In cases where $D_1 = D_2 = D_3$ i.e., all the admittance functions have identical poles,

$$
\beta = \frac{N_1(s) + m N_2(s)}{N_1(s) + N_2(s) + N_3(s)} \quad \ldots \quad (4)
$$

From equation (4), it is obvious that the factor $m$ controls the location of zeros of $\beta$ without disturbing the poles. The method applied is clearly akin
to feed-forward technique \([39] - [40]\) employed in control systems. To draw an analogy with simple feedback systems, let us consider the arrangement of figure 2. Here

\[
\frac{\varepsilon_o}{\varepsilon_i} = \frac{G(s)}{1 + mG(s)} \quad (5)
\]

Thus, with feedback one achieves shift of poles by change of \(m\) as against the shift of zeros with feed-forward.

In figure 1, if \(y_1\) is an all-pole function with unity magnitude or conductive, equation (4) reduces to

\[
\beta = \frac{\varepsilon_o}{\varepsilon_i} = \frac{1 + mN_2(s)}{1 + N_2(s) + N_3(s)} \quad (6)
\]

Comparison of equations (5) and (6) will make one thing clear viz., the root locus technique for finding out the closed loop poles of a feedback system and developed to a considerable depth \([41]\) can be applied straight away in finding out the locus of the zeros of this feed-forward system.

To extend the discussion to dual-input three-terminal networks, the original dual-input network is redrawn in figure 3.

Let the transfer function of the basic subnetwork be expressed as

\[
\beta_o(s) = \frac{P(s)}{Q(s)} = \frac{V_{23}}{V_{13}} = \frac{\beta e - m e}{e - m e} \quad (7)
\]

Hence

\[
\beta(s) = \beta_o(1 - m) + m \cdot \frac{(1 - m)P(s) + mQ(s)}{Q(s)} \quad (8)
\]
Thus, a change in $m$ changes the location of zeros without disturbing the poles of $\beta(s)$. Further, in this process, there is no change in the components of $\beta_0$-network. For $m = 0$, the zeros of $\beta(s)$ are identical with those of $\beta_0(s)$. Pole-zero cancellation takes place for $m = 1$. With suitable choice of $m$ values, the zeros can be located at the desired point on the s-plane.

Equation (8) is general enough for this zero shifting method to be valid for all types of three-terminal networks, be it active or passive, lossy or lossless, lumped or distributed.

1.3. Let us consider a specific example when

$$\beta_0(s) = \frac{\lambda \omega_0 s}{s^2 + \lambda \omega_0 s + \omega_0^2}. \quad \ldots (9)$$

Here $\lambda$ and $\omega_0$ are real constants. Two networks having their transfer functions defined by equation (9) are shown in figures 4(a) and 4(b).

The overall transfer function $\beta$ is given by [40]

$$\beta(s) = \frac{m(s^2 + \omega_0^2) + \lambda \omega_0 s}{s^2 + \omega_0^2 + \lambda \omega_0 s}. \quad \ldots (10)$$

The zeros of $\beta$ are given by

$$Z_1 = -\frac{\lambda \omega_0}{2m} + \left(\frac{\lambda^2 \omega_0^2}{4m^2} - \omega_0^2\right)^{\frac{1}{2}}. \quad \ldots (11)$$

$$Z_2 = -\frac{\lambda \omega_0}{2m} - \left(\frac{\lambda^2 \omega_0^2}{4m^2} - \omega_0^2\right)^{\frac{1}{2}}. \quad \ldots (12)$$
Or in terms of normalized variable \( \frac{s}{\omega_0} \),

\[ z_1 = -\frac{\lambda}{2m} + (\frac{\lambda^2}{4m^2} - 1)^{-\frac{1}{2}} \] ... (13)

\[ z_2 = -\frac{\lambda}{2m} - (\frac{\lambda^2}{4m^2} - 1)^{-\frac{1}{2}} \] ... (14)

The zeros are complex for \( \lambda < 2m \). In order to find the loci of zeros, we note the following:

(a) \( m \) is negative

\[ z_1 = \frac{\lambda}{2m} + (\frac{\lambda^2}{4m^2} - 1)^{-\frac{1}{2}} \]

\[ z_2 = \frac{\lambda}{2m} - (\frac{\lambda^2}{4m^2} - 1)^{-\frac{1}{2}} \]

For \( m = 0 \),

\[ z_1 = \infty \quad z_2 = 0 \]

\( m = \infty \), \( z_1 = +j \quad z_2 = -j \)

\( m = \frac{\lambda}{2} \), \( z_1 = 1 \quad z_2 = 1 \)

(b) \( m \) is positive

\[ z_1 = -\frac{\lambda}{2m} + (\frac{\lambda^2}{4m^2} - 1)^{-\frac{1}{2}} \]

\[ z_2 = -\frac{\lambda}{2m} - (\frac{\lambda^2}{4m^2} - 1)^{-\frac{1}{2}} \]

For \( m = 0 \),

\[ z_1 = 0 \quad z_2 = -\infty \]

\( m = \infty \), \( z_1 = +j \quad z_2 = -j \)

\( m = \frac{\lambda}{2} \), \( z_1 = -1 \quad z_2 = -1 \)
(c) For the loci of the complex part, we may write the zeros as

\[ Z_1, Z_2 = \frac{\lambda}{2m} \pm j \left(1 - \frac{\lambda^2}{4m^2}\right)^{\frac{1}{2}} \]

Hence

\[ Z^2 = \alpha^2 + \beta^2 = 1 \]

Thus, the locus of zeros on \( S/\omega_0 \)-plane is a circle with unity radius having its centre at the origin. The negative values of \( m \) locate the zeros on the right half plane and positive \( m \) values locate the zeros on the left half of the complex \( S/\omega_0 \)-plane. Further, when the zeros are real, the positive and negative values of \( m \) cover the entire real axis. This is shown in figure 5.

2. VARIATION OF NETWORK RESPONSE

2.1. To continue examining the same network we note, for sinusoidal excitation of frequency \( s = j \omega \), the transfer function becomes

\[ \beta(j \omega) = \frac{m (\omega^2 - \omega_0^2) + j \lambda \omega_0 \omega}{\omega^2 - \omega_0^2 + j \lambda \omega_0 \omega} \]

By dividing numerator and denominator by \( -j \omega_0 \omega \) and writing \( x = \frac{\omega}{\omega_0} \) and \( u = x - \frac{1}{x} \),

\[ \beta(j \omega) = \frac{\lambda + j m (x - \frac{1}{x})}{\lambda + j (x - \frac{1}{x})} \]

\[ = \frac{\lambda + j m u}{\lambda + j u} \] \hspace{1cm} (15)
\[ |\beta(j\omega)| = \left[ \frac{\lambda^2 + m^2 u^2}{\lambda^2 + u^2} \right]^{1/2} \] ... \hspace{1cm} (16)

and
\[ \angle \beta = \phi = \tan^{-1} \frac{m u}{\lambda} - \tan^{-1} \frac{u}{\lambda} \] ... \hspace{1cm} (17)

Confining our attention to negative values of \( m \) only, it is obvious from equations (15) and (16) that \( \beta \) gives

- **band-pass for** \[ 0 < |m| < 1 \]
- **all-pass for** \[ |m| = 1 \]
- **band attenuation for** \[ 1 < |m| \]

In this change, the centre frequency \( \omega_c \) remains undisturbed. Since, \( u^2 \) has the same value for \( x \) and \( \frac{1}{x} \), the curves will have logarithmic symmetry about \( x = 1 \). Further, it will be seen that the selectivity varies over a wider range in the band attenuation case than in the band-pass case. While in the former case, the steepness of the response curve can be quite high, being limited by the value of \( m \) only, in the latter case, it is rather low, though this can be improved by a suitable choice of \( \beta \). These characteristics are illustrated in figure 6 which is plotted for \( \lambda = 5 \). It may be seen that the positive values of \( m \) also produce similar transition from band-pass to band-attenuation filters with selectivity undergoing smooth change. However, here the all-pass stage is replaced by a zero phase shift condition. The change in selectivity is obviously achieved in a more elegant fashion than in some other recent methods e.g., [37]. It can have important bio-medical applications [38] besides uses in transducers and automatic control systems.
It may be noted here that \( \beta_0 \) as defined by equation (9) is not unique for realising such variable response networks. For example, the networks \([2]\) tabulated in Chapter III whose transfer function is \( 1 - \beta_0 \) can also produce similar variation in response. Evidently, the method can be extended to higher order networks by either cascading or starting with a suitable function.

3. VOLTAGE TUNABLE FILTERS

In the following sections we shall describe a few voltage tunable networks which are realised by shifting the transmission zeros by feeding two inputs derived from the same source. The main interest is in attaining null or shifting the frequency of zero phase shift so as to achieve tunability.

3.1 Null networks:

To illustrate the basic principle let us consider the simple circuit arrangement \([32]\) of figure 7.

By Millman's theorem,

\[
\mathcal{E}_0 = \frac{\mathcal{E}_1 e_1}{\gamma_1 + \gamma_2 + \gamma_3} + \frac{\mathcal{E}_2 e_2}{\gamma_1 + \gamma_2 + \gamma_3} \quad \ldots \quad (18)
\]

Let

\[
e_1 = m e_i, \quad e_2 = e_i
\]

Hence

\[
\frac{\mathcal{E}_0}{e_i} = \frac{m \gamma_1 + \gamma_2}{\gamma_1 + \gamma_2 + \gamma_3} \quad \ldots \quad (19)
\]

Thus for null realisation,

\[
m \gamma_1 + \gamma_2 = 0
\]
If \( y_1 \) and \( y_2 \) axe of opposite nature i.e., capacitive and inductive, \( m \) is positive. Otherwise it is negative. It will be seen that the null condition is independent of \( y_3 \) which, however, has important effect on the selectivity of response. We shall now consider a few specific networks based on this principle.

3.1.1. Consider the twin-T 8C network of figure 8. By considering \( e_1 \) alone the output is

\[
\ell_{o1} = \frac{z_2 (z_3 z_3' + z_3 z_4 + z_2' z_4')}{D} e_1 \quad \cdots \quad (21)
\]

where

\[
D = (z_1 + z_2) (z_2 z_3' + z_3 z_4 + z_3' z_4') + (z_3 + z_4) (z_1 z_3' + z_1 z_2 + z_1' z_2)
\]

\[
\cdots \quad (22)
\]

Similarly for \( e_{o2} \), considering \( e_2 \) alone,

\[
\ell_{o2} = \frac{z_4 (z_1 z_1' + z_1 z_2 + z_1' z_2)}{D} e_2 \quad \cdots \quad (23)
\]

Hence output for both the inputs is given by application of superposition theorem

\[
\ell_o = \ell_{o1} + \ell_{o2}
\]

\[
= 0 \text{ for null.} \quad \cdots \quad (24)
\]

Let

\[
\ell_i = e_i = \frac{e_i}{m}
\]
Hence,

\[ z_2 \left( z_3 z_3' + z_3 z_4 + z_4 z_4' \right) + z_4 (z_1 z_1' + z_1 z_2 + z_2 z_2') m_1 = 0 \]  

(25)
giving the condition for a null.

To apply the results to a symmetrical twin-Tee network, let

\[ z_1 = z_1' = R \]
\[ z_2 = \frac{1}{j \omega C} \]
\[ z_3 = z_3' = \frac{1}{j \omega C} \]
\[ z_4 = \frac{R}{n} \]

Equation (25) then reduces to

\[ \frac{1}{j \omega C} \left[ -\frac{1}{\omega^2 C^2} + \frac{2R}{j \omega C} \right] + \frac{m}{n} \left[ \frac{R^2}{\omega^2 C^2} + \frac{2R}{j \omega C} \right] \]

(26)

Equating real and imaginary parts,

\[ \frac{m}{n} \frac{R^3}{C^3} - \frac{2R}{n \omega^2 C^2} = 0 \]

and

\[ \frac{m}{n} \frac{2R^2}{n \omega C} - \frac{1}{n \omega^3 C^3} = 0 \]

which then reduce to

\[ \omega^2 = \frac{m}{n} \left[ -\frac{1}{m R^2 C^2} \right] \]

(27)

and

\[ \omega^2 = \frac{n}{2} \left[ -\frac{1}{m R^2 C^2} \right] \]

(28)
For equations (27) and (28) to be simultaneously satisfied,

\[ n = 2 \]  \hspace{1cm} (29)

Hence

\[ \omega_0 = \frac{1}{\sqrt{n} R C} \]  \hspace{1cm} (30)

Thus the null frequency can be changed by changing \( n \) alone leaving the passive components of the network undisturbed.

3.1.2. To consider another specific example let us analyse the circuits of figure 9. These RC networks utilize two Tee networks in tandem and hence are, in a way, generalization of the bridged-Tee network described earlier.

For figure 7(a), nodal analysis shows [34] that

\[ \varepsilon_0 = \frac{D}{\Delta} \]  \hspace{1cm} (31)

where

\[ D = \frac{m \varepsilon}{Z_0} a \left( q^2 + g^2 + a^2 g^2 + 2 a g q \right) \\
+ 2 g y + 2 a^2 g^2 - \frac{m \varepsilon}{Z_0} a^2 g^2 + a^3 g e \]  \hspace{1cm} (32)

\[ \Delta = a \left( g + g + a g \right)^2 \left( a^2 g + \frac{1}{Z_0} + \frac{1}{Z_0} \right) \\
- a^4 g^2 \left( g + g + a g \right) \\
- a^2 g^2 \left( a^2 g + \frac{1}{Z_0} + \frac{1}{Z_0} \right) \]  \hspace{1cm} (33)

\[ \gamma = \frac{1}{\nu R} \]  \hspace{1cm} \[ \gamma = j \omega C \]  \hspace{1cm} (34)

For null \( D = 0 \).
Or
\[ m \left[ \omega_0^2 \frac{C^2}{R^3} \left( 1 + a + a^2 \right) \right] - \frac{j 2 m \omega_0}{R} (1 + a) = \frac{\alpha^2 Z_0}{R_3} \] ... (34)

where \( \omega_0 \) is the null frequency. Let \( Z_0 \) be a series combination of \( R_0 \) and \( C_0 \) so that
\[ Z_0 = R_0 + \frac{j}{\omega C_0} \] ... (35)

Equating real and imaginary parts in equation (34)

\[ \omega_0^2 = \frac{\alpha^2 R_0}{C^2 R^3 m} + \frac{1}{R^2 C^2} \left( 1 + a + a^2 \right) \] ... (36)

and
\[ \omega_0^2 = \frac{\alpha^2}{R^2 C C_0} \frac{1}{Z_m (1 + a)} \] ... (37)

If
\[ \frac{\alpha^2 R_0}{C^2 R^3 m} \gg \frac{1}{R^2 C^2} \left( 1 + a + a^2 \right) \]
i.e.,
\[ \frac{R_0}{R} \gg \frac{1}{m} \left( 1 + a + a^2 \right) \] ... (38)

then equation (36) reduces to
\[ \omega_0^2 = \frac{\alpha^2 \frac{R_0}{C}}{C^2 R^3 m} \] ... (39)

For equations (37) and (39) to be simultaneously satisfied,
\[ R C = 2 \left( 1 + a \right) \frac{R_0}{C_0} \] ... (40)
giving the null condition for any value of \( m \). The null frequency can obviously be varied widely by varying \( m \) alone. Incidentally, it will be seen that the null conditions do not depend on \( Z \) which includes load impedance as well. But \( Z \) will affect the selectivity and transmission symmetry about null.

For the circuit of figure 7(b) similar analysis [or by interchanging \( g \) and \( y \) in equation (31)] shows that

\[
D = \frac{m e a}{Z_o} \left( \gamma^2 + \gamma^2 + a^2 \gamma^2 + 2 \gamma \gamma + 2 a \gamma + 2 a \gamma \gamma \right)
\]

\[
- \frac{m e}{Z_o} \frac{\alpha^2 \gamma + a^3 \gamma e}{e} = 0 \text{ for null.} \quad \ldots (41)
\]

As before, let \( Z \) be a series combination of resistance and capacitance so that

\[
Z_o = R_o - \frac{j}{\omega C_o} \quad \ldots (42)
\]

Equating real and imaginary parts to zero in equation (41),

\[
\frac{\alpha^3 \gamma^3}{j \omega C_o} = m a \left( \gamma^2 + \gamma^2 + a^2 \gamma^2 + a \gamma \right) \quad \ldots (43)
\]

and

\[
- \alpha^3 \gamma^3 R_o = 2 \gamma \gamma m a \gamma (1 + a) \quad \ldots (44)
\]

On simplification:

\[
\omega_o^2 = \frac{m}{R^2 C^2 \left[ \frac{\alpha^2}{C_o} + m (1 + a + a^2) \right]} \quad \ldots (45)
\]

and

\[
\omega_o^2 = \frac{Z \cdot m (1 + a)}{\alpha^2 C^2 \cdot R \cdot R_o} \quad \ldots (46)
\]

If \( \frac{\alpha^2}{C_o} \gg \gamma m (1 + a + a^2) \)
then equation (45) reduces to
\[ \omega_0^2 = \frac{C_0 \cdot m}{\alpha^2 R^3 C^3} \]...

(48)

For equations (46) and (48) to be satisfied simultaneously
\[ \frac{2m(1+\alpha)}{\alpha^2 C^2 R R_0} = \frac{C_0 \cdot m}{\alpha^2 R^2 C^3} \]
or
\[ R_0 C_0 = 2(1+\alpha) RC \]...

(49)

which gives the conditions for null variability. Conditions for equations (40) and (49) are easily attained if \( \alpha \gg 1 \) and \( m \ll 1 \). However, if they are not adequately satisfied, exact zero transmission is obtained only at one value of \( m \) only. This is similar to the case of a bridged-Tee or a twin-Tee network with appreciable source impedance [35].

It will be seen that the variation of \( m \) changes the notch frequency in opposite directions in two cases. In figure 9(a) if \( R_0 = 0 \) and in figure 9(b) if \( C_0 = \infty \) null is still obtainable for one value of \( m \) only.

3.2. Tunable band-pass filter:

For this purpose we shall use a modified HLC bridged-Tee network (figure 10) incorporating a single lossy-inductor and giving band-pass characteristics in place of the usual band-stop or notch response curve. Here \( C \) represents the stray capacitance and will not be considered for the present.
3.2.1. Transfer function:

Driven by an ideal voltage source and working into an open circuit load, simple nodal analysis shows [36] that the network has a voltage transfer function given by

\[ \beta = \frac{E_o}{e} = \frac{-m - \omega^2 L c_2 + j \omega \omega_0 R (c_1 + c_2) + j \omega^2 c_2}{1 - \omega^2 L c_2 R \tau - \omega^2 L c_2 + j \omega R (c_1 + c_2) + j \omega^2 c_2 + j \omega^2 R c_1 c_2 L} \]  

... (50)

To find out the frequency of, and condition for, unity transmission gain, the real and imaginary parts of the numerator and denominator respectively are equated giving

\[ m - \omega^2 L c_2 = 1 - \omega_0 c_1 c_2 R \tau - \omega^2 L c_2 \]

and

\[ \omega_0 m R (c_1 + c_2) + \omega_0 c_2 = \omega_0 R (c_1 + c_2) + \omega_0 c_2 - \omega_0^2 R c_1 c_2 L \]

or

\[ \omega_0^2 = \frac{1 - m}{c_1 c_2 R \tau} = \frac{(1 - m) (c_1 + c_2)}{L c_1 c_2} \]  

... (51)

Hence

\[ \frac{L}{\tau} = (c_1 + c_2) R \]  

... (52)

Since equation (52) does not contain \( m \), the frequency of zero phase shift can be varied smoothly by varying \( m \) alone with all other components in the network unchanged. It is obvious that the value of \( m \) is permissible between 0 and one.
Writing

\[ x = \omega \left( \frac{C_1 C_2 R_\gamma}{C_1 R} \right)^{\frac{1}{2}} \quad b = \left( \frac{C_2 \gamma}{C_1 R} \right)^{\frac{1}{2}} \]

Equation (50) simplifies to

\[ \beta = \frac{m + j b \omega}{1 - \omega^2 + j b \omega} \]

and

\[ |\beta|^2 = \frac{m^2 + \omega^2 \omega^2}{(1 - \omega^2)^2 + \omega^2 \omega^2} \]

A plot of \(|\beta|\) in figures 11 and 12 for different values of \(m\) and \(b\) reveals certain interesting features. The maximum value of \(|\beta|\) exceeds unity and increases with the increase of \(m\) and decreases with \(b\). There is no transmission symmetry for \(m \neq 0\). Though the frequency of unity gain changes with \(m\), the peak transmission always takes place at \(x \approx 1\). The exact value of peak transmission may be found as follows:

\[ \frac{d}{dx} |\beta| = 0 \]

On simplification:

\[ x_p = \left[ \left( \left( 1 + \frac{m^2}{b^2} \right)^2 - m^2 \right)^{\frac{1}{2}} - \frac{m^2}{b^2} \right]^{\frac{1}{2}} \]

It is seen that \(x_p \approx 1\) so long as \((1 + \frac{m^2}{b^2})^2 \gg m^2\). This is the case for normal values of \(m \ (< 1)\) and \(b \ (< 1)\). However, if selectivity can be sacrificed, \(b\) can be increased by proper adjustments of resistance and capacitance values so that tuning of the central frequency \(x_p\) can be achieved.
The value of $|\beta|$ at $x = 1$ is given by

$$|\beta|_{x=1} = \left(1 + \frac{m^2}{b^2}\right)^{\frac{1}{2}}$$

which always exceeds unity. In case $(m^2/b^2) << 1$,

$$\kappa_P \sim \left(1 - \frac{m^2}{b^2}\right)^{\frac{1}{4}}$$

Thus, wide tuning is possible by varying $m$ alone. As $m$ approaches unity, $x_p$ changes rapidly giving a broader tuning. However, since selectivity is extremely poor for high value of $b$, much of the effect of tuning is lost.

As indicated earlier and seen from figures 11 and 12, selectivity worsens as $b$ increases. The exact expression for bandwidth may be found by equating $|\beta|$ to $\frac{1}{2}$ of its peak value. But the resultant expression is too complicated for much practical use. However, when $x \approx 1$ an expression for bandwidth can be obtained in the following way. The 3-db points are given by

$$|\beta|^2 = \frac{1}{2} \left(1 + \frac{m^2}{b^2}\right) = \frac{1}{K}, \text{ say.}$$

Hence from equation (55)

$$\gamma^2 + \gamma^2 (b - 2 - b K) - K m^2 + 1 = 0$$

where $y = x^2$.

If $y_1$ and $y_2$ are the roots of equation (61)

$$y_1 + y_2 = 2 - b K - b^2$$

$$y_1 y_2 = 1 - K m^2$$

Hence bandwidth

$$\Omega_b = \sqrt{y_1} - \sqrt{y_2}$$
or
\[ \Omega_b = \frac{\omega_1 - \omega_2}{\omega_p} = \left[ 2 + \frac{2}{b} k - \frac{2}{b} (1 - k m^2)^{1/2} \right]^{1/2} \] ... (62)

where \( \omega_1 \) and \( \omega_2 \) are the 3-db frequencies. Equation (62) confirms that the selectivity worsens with the increase of \( b \).

3.2.2. Output impedance

It will be of interest to find out the output impedance \( Z_0 \) which is obtained by computing the impedance at the output terminals when there is no load and when the driving voltage sources are replaced by their internal impedances (zero in the present case). Simple analysis shows that

\[ Z_0 = \frac{\gamma \left( 1 - \frac{a^2}{b^2} \right) \chi^2 + j \frac{2a}{b} \chi}{1 - (1+a) \chi^2 + j \frac{\chi}{b} \left( \frac{b^2}{b^2 + a^2 - a \chi^2} \right)} \] ... (63)

where

\[ \alpha = \frac{C_1 + C_2}{C_1}. \]

It will be seen that \( Z_0 \) is independent of \( \lambda \). For \( \chi = 1 \), which, for all practical purposes, is the centre frequency,

\[ Z_0 = \frac{\gamma}{b^2} \left( a - j \frac{1}{b} \right) \] ... (64)

Thus at the centre frequency, \( Z \) is complex having a capacitive component.

3.2.3. Effect of stray capacitance

We shall now consider the effect of stray capacitance \( \beta \) that may be present across the inductor as is shown in figure 10. Nodal analysis shows
As before, for unity gain,

\[
\omega_0 = \frac{(1-m)[(c_1+c_2)R + C_R]}{C (1-m) LC (c_1 + c_2) R + L C c_2 R} \]

Thus the condition for unity gain becomes a function of m. However, if

\[
\frac{C_1 C_2}{C} \gg \frac{(1-m) L}{R} + (c_1 + c_2) \]

and

\[
\frac{C_1 C_2}{C} \gg (1-m)(c_1 + c_2) \]

then (66) reduces to

\[
\frac{1}{\tau} = (c_1 + c_2) R + C_R \]

Thus the tuning condition becomes independent of m.

### 3.3 Voltage Tunability

All the networks described under Section 3 are tunable by means of the variation of gain of the auxiliary amplifier or, in effect, by controlling the amount of signal voltage reaching the tunable network through the second channel. Thus, if the amplitude of a signal from a transducer carries the
information regarding the measurand, this information can be retrieved by
feeding a reference voltage and this signal from the transducer, after suitable
phase correction in relation to the reference signal, to a dual-input network
of the type discussed above. The arrangement is shown by a block diagram in
figure 13. Phase correction is necessary when the signal and reference voltages
are not related by a proper multiplying constant i.e., when \( m \) is not real or
of proper sign. Once voltage tunability of a filter is achieved, realisation
of voltage variable oscillators is only a simple additional step involving
feedback and a voltage tunable filter.

4. CONCLUSION

We have presented a technique for single element control of the position
of the zeros of a network without disturbing its poles. The method is
applicable to any type of three-terminal network. A few effects of such zero
shifting on the network transfer function have been considered. The transfer
functions chosen for this purpose are illustrative and not exclusive.
Voltage tunability of a network can have important uses in transducer
applications, variable frequency oscillators, automatic process control system
and bio-medical applications requiring variable selectivity.
5. Legends to the figures:

Fig. 1: Basic scheme for zero shifting.
Fig. 2: Basic feedback system.
Fig. 3: Dual-input network.
Fig. 4: Two networks with transfer functions defined by equation (9).
Fig. 5: Loci of zeros on \( \frac{s}{\omega_0} \) plane.
Fig. 6: Response variation for change of \( m \).
Fig. 7: Null realisation.
Fig. 8: Twin-Tee network for null realisation.
Fig. 9: RC ladders giving null response.
Fig. 10: Modified RLC bridged-Tee network.
Fig. 11: Network response for \( b = 0.5 \).
Fig. 12: Network response for \( b = 0.1 \).
Fig. 13: Schematic diagram of a voltage tunable network.
Fig. 1: Basic scheme for zero shifting.

Fig. 2: Basic feedback configuration.

Fig. 3: The dual-input network.

Fig. 4: Two networks having transfer function defined by equation 9.
Fig. 5: Loci of zeros on s/\textomega \ plane.
Fig. 6: Variable response diagram.
Fig. 7: Principle of null realization.

Fig. 8: Twin-Tee null network.

Fig. 9: Null realization with RC ladders.

Fig. 10: Modified HLC bridged-Tee network.
Fig. 11: Variation of network response for $b=0.1$. 

$\beta$ vs. $x$ for different $m$ values.
Fig. 12: Transmission characteristics for $b=0.5$.

Fig. 13: Schematic diagram of a voltage tunable filter.