SURFACE DISPLACEMENT DUE TO DIRECT P AND INTERFACE REFLECTED PP' PHASES IN THE CASE OF A COMPRESSIONAL POINT SOURCE LYING IN THE PERFECTLY ISOTROPIC HOMOGENEOUS LAYER OVERLYING THE SOLID HALF-SPACE

The investigations on the probable relation between the process in the focus and the traces recorded in the seismogram by different seismic waves show that there are two main hypotheses on the nature of disturbances acting at the focus of an earthquake,

(i) A single couple with moment
(ii) A double couple with moment.

Both these models have given results which fit with some aspects of observations. But in some cases their divergence from the results of observation are considerable and the two models referred above have given in some cases contradictory results. This has led Ingram (1963) to suggest three couples without moment along three mutually perpendicular directions as a possible model which represents a explosive type of source.

In the present investigation, it is proposed to solve the problem of a compressional point source in a perfectly isotropic homogeneous solid layer overlying the solid half-space. Such problems were studied by Newland (1952), Pekeris and Karal (1965), Singh (1966), Alterman (1968), Abramovici (1970) and have
already been discussed in (page ). None of them had considered the body forces due to this particular type of source. In the present problem, it is proposed to solve the problem by representing the type of source as represented by body forces in the equations of motion and to compare the result with others.

In the earlier chapter we have solved equations of motion for this particular type of source where the disturbances is harmonic with time.

In the present chapter, we shall consider the time dependent part of the source as \( e^{at} H(t) \) where \( H(t) \) is Heaviside unit function.

The function \( e^{at} H(t) \) has been chosen as according to Cole (1948), the exponentially decaying shock gives a fair representation of the initial disturbance due to an explosion. The actual value of \( a \) depends on various parameters, including the energy of explosive charge in an artificial shock.

The main aspect of this problem is to calculate the displacement due to direct reflected waves and to examine the variations of the amplitudes of different phases with epicentral distance, depth of source and thickness of the layer and frequency.
2.1 Mathematical Analysis

We consider a homogeneous isotropic elastic solid layer \( M_1 \) overlying the semi-infinite homogeneous isotropic elastic medium \( M_2 \). Let \( \alpha_1 \) and \( \beta_1 \) be the compressional and distortional velocity in \( M_1 \) and \( \alpha_2, \beta_2 \) be the same in \( M_2 \) respectively. \( \rho_1, \rho_2 \) are the densities of \( M_1, M_2 \) respectively. Let \( q, w \) be the component of horizontal and vertical displacements, \( Z \) being drawn downwards through the source (Fig. 11). Let \( Z = 0 \) be the free surface and \( Z = H \) be the interface.

Let a compressional point source is at \( Z = h < H \).

In the earlier chapter from equation (1.23) we have the component of displacement \( q(r, z, p) \) due to compressional point source having time variation \( \lambda(t) \) where

\[
q_1(r, z, p) = \frac{P_r(p)}{2\alpha_1^2} \int_{-\infty}^{\infty} \tilde{J}_1(\tau \xi) \frac{x^2}{2\xi} e^{-\alpha_1(h-z)} \frac{d\tilde{\xi}}{d\xi} d\tau
\]

\[
q_2(r, z, p) = \frac{P_r(p)}{2\alpha_1^2} \int_{-\infty}^{\infty} \tilde{J}_1(\tau \xi) \frac{x^2}{2\xi} R_{pp} e^{-\alpha_1(h+z)} d\tau
\]

where \( R_{pp}, R_{pp'}, R_{ps} \) etc. are defined in equation (1.24a and b)

Let us denote the first integral by \( q_1 \), second by \( q_2 \) etc. and so on.

Then we have

\[
q_1(r, z, p) = \frac{P_r(p)}{2\alpha_1^2} \int_{-\infty}^{\infty} \tilde{J}_1(\tau \xi) \frac{x^2}{2\xi} e^{-\alpha_1(h-z)} \frac{d\tilde{\xi}}{d\xi} d\tau
\]

\[
q_2(r, z, p) = \frac{P_r(p)}{2\alpha_1^2} \int_{-\infty}^{\infty} \tilde{J}_1(\tau \xi) \frac{x^2}{2\xi} R_{pp} e^{-\alpha_1(h+z)} d\tau
\]
The integrand of $q_1$ has branch point at $\xi = z^*_1$. The integrands of $q_2$, $q_3$ have branch points at $\xi = z^*_2$, $p^*_1$, and those of $q_4$ to $q_6$ have branch points at $\xi = z^*_3$, $p^*_2$, $p^*_3$, $p^*_4$.

Since the integration along $L_{\alpha_1}$, $L_{\beta_1}$, $L_{\delta_1}$ (Eqn. 1.27) are thoroughly discussed by (Singh 1966); we only give the final results. $q_{1,\alpha_1}$ means the contribution of $L_{\alpha_1}$ to $q_1$.

A. Evaluation of integrals along $L_{\alpha_4}$.

\[
q_{1,\alpha_1}(r, z, p) = \frac{i \mathcal{P} \tilde{\chi}(p)}{2 \alpha_1^2} \left[ A_{p, p} + B_{p} \right] \exp \left\{ -i p \cdot \mathbf{t}_{p} \right\}
\]

\[
q_{2,\alpha_1}(r, z, p) = \frac{i \mathcal{P} \tilde{\chi}(p)}{2 \alpha_1^2} \left[ A_{p, p} + B_{p} \right] \exp \left\{ -i p \cdot \mathbf{t}_{p} \right\}
\]

\[
q_{3,\alpha_1}(r, z, p) = \frac{i \mathcal{P} \tilde{\chi}(p)}{2 \alpha_1^2} \left[ A_{p, p} + B_{p} \right] \exp \left\{ -i p \cdot \mathbf{t}_{p} \right\}
\]

\[
q_{4,\alpha_1}(r, z, p) = \frac{i \mathcal{P} \tilde{\chi}(p)}{2 \alpha_1^2} \left[ A_{p, p} + B_{p} \right] \exp \left\{ -i p \cdot \mathbf{t}_{p} \right\}
\]

\[
q_{5,\alpha_1}(r, z, p) = \frac{i \mathcal{P} \tilde{\chi}(p)}{2 \alpha_1^2} \left[ A_{p, p} + B_{p} \right] \exp \left\{ -i p \cdot \mathbf{t}_{p} \right\}
\]

\[
q_{6,\alpha_1}(r, z, p) = \frac{i \mathcal{P} \tilde{\chi}(p)}{2 \alpha_1^2} \left[ A_{p, p} + B_{p} \right] \exp \left\{ -i p \cdot \mathbf{t}_{p} \right\}
\]
\[ A_p = \frac{1}{r \alpha_1} \quad B_p = \frac{2 \alpha_5}{r \alpha_1 \beta_1} \]

\[ A_{pp} = -\frac{1}{r \alpha_1} \left( \frac{2 c_4^2}{\alpha_1^2 \alpha_1} \right) - \frac{(k + z)^2}{\gamma^2} \quad B_{pp} = \frac{2 \alpha_5}{r \alpha_1 \beta_1 \gamma} \quad \frac{1}{r \alpha_1 \beta_1} \]

\[ A_{ps} = -\frac{2 c_4^2}{\alpha_1 \alpha_1} \quad \frac{h}{\alpha_1^2} \quad B_{ps} = \frac{2 \alpha_5}{r \alpha_1 \beta_1 \gamma} \quad \frac{1}{r \alpha_1 \beta_1} \]

\[ A_{p \delta} = \frac{1}{r \alpha_1} - 2 \frac{\Delta_{12,0}^2}{\Delta_{12,0}^2} \quad B_{p \delta} = -\left( \frac{\Delta_{12,1}^2}{\Delta_{12,0}^2} \right) \frac{1}{r \alpha_1 \beta_1} \]

\[ A_{pp'} = -\left[ \frac{1}{r \alpha_1} \left( \frac{2 - \Delta_{12,1}^2}{\gamma^2 \alpha_1} \right) \left( \frac{2 c_4^2}{\alpha_1^2 \alpha_1} + 2 \frac{\Delta_{12,1}^2}{\Delta_{12,0}^2} \right) \right] \]

\[ B_{pp'} = \frac{1}{r \alpha_1^2} \left[ \frac{2 c_4^2}{\alpha_1 \alpha_1} + \frac{2 c_4^2}{\alpha_1^2 \alpha_1} \right] - \left( \frac{\Delta_{12,1}^2}{\Delta_{12,0}^2} \right) \frac{1}{r \alpha_1 \beta_1} \]

\[ A_{p \delta'} = -\frac{2 c_4^2}{\alpha_1 \alpha_1} - \frac{(2(n - k - 2)^2)}{r^2 \alpha_1} - \frac{c_4^2}{\alpha_1^2 \alpha_1} \left( -2 \frac{\Delta_{12,1}^2}{\Delta_{12,0}^2} - \frac{c_4^2}{\alpha_1^2 \alpha_1} \right) \]

\[ B_{p \delta'} = \frac{2 c_4^2}{\alpha_1 \alpha_1} \left[ \frac{1}{r \alpha_1} \left( -2 \frac{\Delta_{12,1}^2}{\Delta_{12,0}^2} - \frac{c_4^2}{\alpha_1^2 \alpha_1} \right) \right] \]

where

\[ t_p = \frac{r}{\alpha_1} + \frac{(k - z)^2}{2r \alpha_1} \]

\[ t_{pp} = \frac{r}{\alpha_1} + \frac{(k + z)^2}{2r \alpha_1} \]

\[ t_{p \delta} = \frac{r}{\alpha_1} + \frac{4z}{2r \alpha_1} + \sqrt{\frac{4z}{\beta_1^2} - \frac{4}{\alpha_1^2}} \frac{z}{2} \]

\[ t_{p \delta'} = \frac{r}{\alpha_1} + \frac{4z}{2r \alpha_1} + \sqrt{\frac{4z}{\beta_1^2} - \frac{4}{\alpha_1^2}} \frac{z}{2} \]
\[ t_{pp'} = \frac{\gamma}{\sigma_1} + \frac{(2u - h - e)^2}{2\sigma_1} \]
\[ t_{pp''p} = \frac{\gamma}{\sigma_1} + \frac{(2u - h + e)^2}{2\sigma_1} \]
\[ t_{pp''p} = \frac{\gamma}{\sigma_1} + \frac{(2u - h)^2}{2\sigma_1} + \frac{4}{\beta^3} - \frac{1}{\alpha^2} \]

**B. Contribution of \( l \cdot d_5 \)**

\[ q_{4,\alpha_2} (r, z, p) = \frac{\mathcal{P}(p)}{2\sigma_4^2} \mathcal{A}_{pp', \alpha_2} \exp \left\{ -i p \cdot t_{pp', \alpha_2} \right\} \]
\[ q_{5,\alpha_2} (r, z, p) = \frac{\mathcal{P}(p)}{2\sigma_4^2} \mathcal{A}_{pp', \alpha_2} \exp \left\{ -i p \cdot t_{pp', \alpha_2} \right\} \]
\[ q_{6,\alpha_2} (r, z, p) = \frac{\mathcal{P}(p)}{2\sigma_4^2} \mathcal{A}_{pp', \alpha_2} \exp \left\{ -i p \cdot t_{pp', \alpha_2} \right\} \]

\[ \mathcal{A}_{pp', \alpha_2} = \frac{c_3}{\sigma_2^2} \Delta_{23,0}^2 \left( \frac{\Delta_{23,0}^2}{\Delta_{12,0}} + \frac{\Delta_{23,1}}{\Delta_{12,0}} \right) \frac{1}{\gamma^2} \]

\[ \mathcal{A}_{pp', \alpha_2} = \frac{c_3}{\sigma_2^2} \Delta_{23,0}^2 \left( \frac{\Delta_{23,1}}{\Delta_{12,0}} + \frac{\Delta_{23,1}}{\Delta_{12,0}} \right) \frac{1}{\gamma^2} \times \left( \frac{2}{\sigma_3^2} - \frac{1}{\sigma_1^2} \right) \frac{4}{\sigma_2^2} \frac{4}{\sigma_2^2} \frac{4}{\sigma_6} \]

\[ \mathcal{A}_{pp', \alpha_2} = \frac{4}{\sigma_2^2} \left( \frac{2}{\sigma_2^2} - \frac{1}{\sigma_1^2} \right) \Delta_{23,0}^2 \left( \frac{\Delta_{23,1}}{\Delta_{12,0}} + \frac{\Delta_{23,1}}{\Delta_{12,0}} \right) \frac{1}{\gamma^2} \times \left( \frac{2}{\sigma_3^2} - \frac{1}{\sigma_1^2} \right) \frac{4}{\sigma_2^2} \frac{4}{\sigma_6} \frac{4}{\sigma_6} \frac{4}{\sigma_6} \]

\[ \mathcal{A}_{pp', \alpha_2} = \frac{4}{\sigma_2^2} \left( \frac{2}{\sigma_2^2} - \frac{1}{\sigma_1^2} \right) \Delta_{23,0}^2 \left( \frac{\Delta_{23,1}}{\Delta_{12,0}} + \frac{\Delta_{23,1}}{\Delta_{12,0}} \right) \frac{1}{\gamma^2} \times \left( \frac{2}{\sigma_3^2} - \frac{1}{\sigma_1^2} \right) \frac{4}{\sigma_2^2} \frac{4}{\sigma_6} \frac{4}{\sigma_6} \frac{4}{\sigma_6} \]

\[ \mathcal{A}_{pp', \alpha_2} = \frac{4}{\sigma_2^2} \left( \frac{2}{\sigma_2^2} - \frac{1}{\sigma_1^2} \right) \Delta_{23,0}^2 \left( \frac{\Delta_{23,1}}{\Delta_{12,0}} + \frac{\Delta_{23,1}}{\Delta_{12,0}} \right) \frac{1}{\gamma^2} \times \left( \frac{2}{\sigma_3^2} - \frac{1}{\sigma_1^2} \right) \frac{4}{\sigma_2^2} \frac{4}{\sigma_6} \frac{4}{\sigma_6} \frac{4}{\sigma_6} \]
\[ t_{PP', \alpha_{2}} = \frac{\tau}{\alpha_{2}} + \left(2\mu - h - \nu \right) \frac{1}{\alpha_{2}} + \left(2\mu - h + \nu \right) \frac{1}{\alpha_{2}} \]
\[ t_{PP', \alpha_{2}} = \frac{\tau}{\alpha_{2}} + \left(2\mu - h + \nu \right) \frac{1}{\alpha_{2}} + \left(2\mu - h - \nu \right) \frac{1}{\alpha_{2}} \]
\[ t_{PP', \alpha_{2}} = \frac{\tau}{\alpha_{2}} + \left(2\mu - h \right) \frac{1}{\alpha_{2}} - \frac{1}{\alpha_{2}} + \frac{2}{\beta_{1}} \frac{1}{\alpha_{2}} \]

2.8

C. Contribution of \( L_{\beta} \)

\[ q_{2, \beta_{1}} (r, z, p) = \frac{\overline{P}(\rho)}{2\alpha_{2}^{2}} \exp \left\{ -\gamma \frac{\beta_{1}}{\beta_{2}} - \frac{\gamma}{\beta_{2}} \right\} \]
\[ q_{3, \beta_{1}} (r, z, p) = -\frac{\overline{P}(\rho)}{2\alpha_{2}^{2}} \exp \left\{ -\gamma \frac{\beta_{1}}{\beta_{2}} - \frac{\gamma}{\beta_{2}} \right\} \]
\[ q_{4, \beta_{1}} (r, z, p) = \frac{\overline{P}(\rho)}{2\alpha_{2}^{2}} A_{PP', \beta_{1}} \exp \left\{ -\gamma \frac{\beta_{1}}{\beta_{2}} - \frac{\gamma}{\beta_{2}} \right\} \]
\[ q_{5, \beta_{1}} (r, z, p) = \frac{\overline{P}(\rho)}{2\alpha_{2}^{2}} A_{PP', \beta_{1}} \exp \left\{ -\gamma \frac{\beta_{1}}{\beta_{2}} - \frac{\gamma}{\beta_{2}} \right\} \]
\[ q_{6, \beta_{1}} (r, z, p) = \frac{\overline{P}(\rho)}{2\alpha_{2}^{2}} A_{PP', \beta_{1}} \exp \left\{ -\gamma \frac{\beta_{1}}{\beta_{2}} - \frac{\gamma}{\beta_{2}} \right\} \]

2.9
D. Contribution of \( L_{\beta_z} \)

\[
q_{i, \beta_z}(r, z, p) = \frac{P_\beta(p)}{2\alpha_i^2} \exp \left\{ -\frac{i\pi \beta_z}{\alpha_i} \right\} \left( \frac{1}{\alpha_i} - \frac{i\pi \beta_z}{\alpha_i} \right) (2\alpha_i \cdot \mathbf{h} \cdot \mathbf{z} \cdot \mathbf{p})
\]

\[
q_{5, \beta_z}(r, z, p) = \frac{P_\beta(p)}{2\alpha_i^2} \exp \left\{ -\frac{i\pi \beta_z}{\alpha_i} \right\} \left( \frac{1}{\alpha_i} - \frac{i\pi \beta_z}{\alpha_i} \right) (2\alpha_i \cdot \mathbf{h} \cdot \mathbf{z} \cdot \mathbf{p})
\]

\[
q_{6, \beta_z}(r, z, p) = \frac{P_\beta(p)}{2\alpha_i^2} \exp \left\{ -\frac{i\pi \beta_z}{\alpha_i} \right\} \left( \frac{1}{\alpha_i} - \frac{i\pi \beta_z}{\alpha_i} \right) (2\alpha_i \cdot \mathbf{h} \cdot \mathbf{z} \cdot \mathbf{p})
\]

\[
\frac{1}{c_2} = \left( \frac{1}{\alpha_i^2} - \frac{1}{\alpha_z^2} \right)
\]

\[
\frac{1}{c_3} = \left( \frac{1}{\alpha_i^2} - \frac{1}{\alpha_z^2} \right)
\]

\[
\frac{1}{c_4} = \left( \frac{1}{\alpha_i^2} - \frac{1}{\alpha_z^2} \right)
\]

\[
\frac{1}{c_5} = \left( \frac{1}{\alpha_i^2} - \frac{1}{\alpha_z^2} \right)
\]

\[
\frac{1}{c_6} = \left( \frac{1}{\alpha_i^2} - \frac{1}{\alpha_z^2} \right)
\]

\[
\Delta_{a_i} = -4 \left[ \frac{1}{c_{4, 6}} \left( \frac{1}{\alpha_i^2} - \frac{i}{\alpha_i \alpha_z} + \frac{1}{\alpha_i^2} \right) \left( \frac{1}{\alpha_i^2} - \frac{i}{\alpha_i \alpha_z} \right) \right]
\]

\[
-2 \frac{\mu_n}{\mu_i} \left( \frac{1}{\alpha_i^2} \right) \left( \frac{1}{c_{4, 6}} \right) - \frac{1}{\alpha_i^2} \left( \frac{\mu_n}{\mu_i} \right) \left( \frac{1}{\alpha_i^2} \right) \left( \frac{1}{c_{4, 6}} \right)
\]
\[
\Delta_{12,1}^{\alpha_1} = -41 \left[ -\frac{1}{\alpha_1^{a_1^2}} \left( \frac{i}{\alpha_1^2} - \frac{i}{\alpha_1^3} \right) + \frac{\mu^2}{\mu^3} \left( \frac{1}{\alpha_1^2} - \frac{i}{\alpha_1^3} \right) \right] \\
+ 2 \frac{\mu^2}{\mu_3} \left( \frac{i}{\alpha_1^2} - \frac{i}{\alpha_2^3} \right) \left( \frac{1}{\alpha_1^2} - \frac{i}{\alpha_1^3} \right) - \frac{1}{4} \frac{\mu^2}{\mu_3} \left( \frac{i}{\alpha_1^2} - \frac{i}{\alpha_1^3} \right) \\
\Delta_{12,0}^{\alpha_1} = -\Delta_{12,0}^{\alpha_1} \\
\Delta_{23,0}^{\alpha_2} = -4 \left[ \frac{1}{\alpha_2^2} \left( \frac{i}{\alpha_2^2} - \frac{i}{\alpha_2^3} \right) + \frac{\mu^2}{\mu_4} \left( \frac{i}{\alpha_2^2} - \frac{i}{\alpha_2^3} \right) \right] \\
- 2 \frac{\mu^2}{\mu_3} \frac{1}{\alpha_2^2} \left( \frac{i}{\alpha_2^2} - \frac{i}{\alpha_2^3} \right) + \frac{1}{4} \frac{\mu^2}{\mu_3} \left( \frac{i}{\alpha_2^2} - \frac{i}{\alpha_2^3} \right) \left( \frac{i}{\alpha_2^2} - \frac{i}{\alpha_2^3} \right) \\
\Delta_{23,1}^{\alpha_2} = -4 \left[ -\frac{1}{\alpha_3^2} \left( \frac{i}{\alpha_3^2} - \frac{i}{\alpha_3^3} \right) - \frac{\mu^2}{\mu_5} \left( \frac{i}{\alpha_3^2} - \frac{i}{\alpha_3^3} \right) \right] \\
+ 2 \frac{\mu^2}{\mu_3} \frac{1}{\alpha_3^2} \left( \frac{i}{\alpha_3^2} - \frac{i}{\alpha_3^3} \right) - \frac{1}{4} \frac{\mu^2}{\mu_3} \left( \frac{i}{\alpha_3^2} - \frac{i}{\alpha_3^3} \right) \left( \frac{i}{\alpha_3^2} - \frac{i}{\alpha_3^3} \right) \\
\Delta_{13,0}^{\beta_1} = -4 \left[ -\frac{1}{\beta_1^2} \left( \frac{i}{\beta_1^2} - \frac{i}{\beta_1^3} \right) + \frac{\mu^2}{\mu_6} \left( \frac{i}{\beta_1^2} - \frac{i}{\beta_1^3} \right) \right] \\
- 2 \frac{\mu^2}{\mu_3} \frac{1}{\beta_1^2} \left( \frac{i}{\beta_1^2} - \frac{i}{\beta_1^3} \right) \left( \frac{i}{\beta_1^2} - \frac{i}{\beta_1^3} \right) \left( \frac{i}{\beta_1^2} - \frac{i}{\beta_1^3} \right)
\[
\Delta_{12,11} = -4 \left[ \frac{1}{\mu_1} \left( \frac{\chi}{\beta_1^2} - \frac{\chi}{\beta_2^2} \alpha_5 \gamma \right) - \frac{\mu_{12}}{\mu_1} \frac{1}{\beta_1^2} \beta_1 \left( \frac{\chi}{\beta_2^2} - \frac{\chi}{\beta_5} \gamma \right) \right] + 2 \frac{\mu_{12}}{\mu_1} \frac{1}{\beta_2^2} \alpha_5 \gamma \left[ -\frac{1}{\alpha_5^2} \left( \frac{\chi}{\beta_2^2} - \frac{\chi}{\beta_5} \gamma \right) \right].
\]

\[
\Delta_{23,0}^{\beta_1} = -4 \left[ -\frac{1}{\mu_1} \left( \frac{\chi}{\beta_1^2} - \frac{\chi}{\beta_3} \gamma \right) - \frac{\mu_{12}}{\mu_1} \frac{1}{\beta_1^2} \beta_1 \left( \frac{\chi}{\beta_3} - \frac{\chi}{\beta_5} \gamma \right) \right] + 2 \frac{\mu_{12}}{\mu_1} \frac{1}{\beta_1^2} \left[ -\frac{1}{\alpha_5^2} \left( \frac{\chi}{\beta_1^2} - \frac{\chi}{\beta_3} \gamma \right) \right].
\]

\[
\Delta_{23,1}^{\beta_1} = -4 \left[ \frac{1}{\mu_1} \left( \frac{\chi}{\beta_1^2} - \frac{\chi}{\beta_3} \gamma \right) + \frac{\mu_{12}}{\mu_1} \frac{1}{\beta_1^2} \beta_1 \left( \frac{\chi}{\beta_3} - \frac{\chi}{\beta_5} \gamma \right) \right] + 2 \frac{\mu_{12}}{\mu_1} \frac{1}{\beta_1^2} \left[ -\frac{1}{\alpha_5^2} \left( \frac{\chi}{\beta_1^2} - \frac{\chi}{\beta_3} \gamma \right) \right].
\]

\[
\Delta_{12,0}^{\beta_1} = -4 \left[ \frac{1}{\mu_1} \left( \frac{\chi}{\beta_1^2} - \frac{\chi}{\beta_2^2} \alpha_5 \gamma \right) + \frac{\mu_{12}}{\mu_1} \frac{1}{\beta_1^2} \beta_1 \left( \frac{\chi}{\beta_2^2} - \frac{\chi}{\beta_5} \gamma \right) - \frac{1}{\alpha_5^2} \beta_1^2 \left( \frac{\chi}{\beta_2^2} - \frac{\chi}{\beta_5} \gamma \right) \right].
\]

\[
\Delta_{23,0} = -4 \left[ \frac{1}{\mu_1} \left( \frac{\chi}{\beta_1^2} + \frac{1}{\alpha_5^2} \gamma \right) + \frac{\mu_{12}}{\mu_1} \frac{1}{\beta_1^2} \beta_1 \left( \frac{\chi}{\beta_2^2} - \frac{\chi}{\beta_5} \gamma \right) \right] + 2 \frac{\mu_{12}}{\mu_1} \frac{1}{\beta_1^2} \left[ \frac{1}{\alpha_5^2} \left( \frac{\chi}{\beta_1^2} + \frac{1}{\alpha_5^2} \gamma \right) \right].
\]

\[
\Delta_{23,1} = -4 \left[ \frac{1}{\mu_1} \left( \frac{\chi}{\beta_1^2} + \frac{1}{\alpha_5^2} \gamma \right) - \frac{\mu_{12}}{\mu_1} \frac{1}{\beta_1^2} \beta_1 \left( \frac{\chi}{\beta_2^2} - \frac{\chi}{\beta_5} \gamma \right) \right] + 2 \frac{\mu_{12}}{\mu_1} \frac{1}{\beta_1^2} \left[ \frac{1}{\alpha_5^2} \left( \frac{\chi}{\beta_1^2} + \frac{1}{\alpha_5^2} \gamma \right) \right].
\]
2.2 **Displacement due to decaying shock:** 
\[ \chi(t) = e^{-at} \mu(t) \]

From the equations in (2.3) and (2.6) we have found that evaluation of the integrals along \( L_{\alpha_1} \) and \( L_{\alpha_2} \) are of the forms: \( I_1 \) and \( I_2 \) respectively, where,

(a) \[ I_1 (r, z, p) = \overline{\chi}(p) \left[ A_1 e^{-i\beta_1} + B_1 e^{+i\beta_1} \right] \]

\( A_1, B_1 \) are independent of transform variable \( p \) and \( t \) represents the arrival time for different phases. \( A_1 \) and \( B_1 \) are the functions of elastic constants of the medium, \( \gamma, \lambda, \eta, \xi \) and independent of \( p \).

(b) \[ I_2 (r, z, p) = \overline{\chi}(p) B_2 e^{-i\beta_2} ; \quad B_2 \] is independent of \( p \).

**Inversion of \( I_1 (r, z, p) \) and \( I_2 (r, z, p) \)**

For \( \chi(t) = e^{-at} \mu(t) \quad a > 0 \)

\[ I_1 (r, z, p) = \frac{A_1}{i} \delta(t - t_1) + \frac{A_1}{i} + B_1 \exp\left\{ -a(t - t_1) \right\} \mu(t - t_1) \quad ..2.12a \]

Similarly \[ I_2 (r, z, t) = B_2 e^{-a(t - t_2)} \mu(t - t_2) \quad ..2.12b \]
Fig. 2.1; P-Phase

Fig. 2.2; PP-Phase

Fig. 2.3; PS-Phase

Fig. 2.4; PP'-Phase

Fig. 2.5; PP'P-Phase

Fig. 2.6; PP'S-Phase

Fig. 2.7; PP'P-New

(\gamma_1)
2.3 **Displacement due to direct P-wave**

For $X(t) = e^{-at}H(t)$, $a > 0$.

From 2.3 and 2.12a:

$$q_{1,1}(r, z, t) = \frac{A_p}{i} \delta(t - t_p) + \left( -\frac{aA_p}{t} \right) e^{-a(t - t_p)} H(t - t_p)$$

$A_p$, $B_p$, and $t_p = r + \frac{(1 - z)^2}{2\pi\alpha}$ are defined in equation (2.4 & 2.5).

Therefore; displacement due to direct P-wave is:

$$q_{pp}(r, z, t) = \frac{A_p}{i} \delta(t - t_p) + \left( -\frac{aA_p}{t} \right) e^{-a(t - t_p)} H(t - t_p)$$

(ii) **Displacement due to reflected waves at the free surface** ($PP$, $PS$) (Fig. 2.1); (2.3)

Considering the inverse Laplace transform of $q_{2,1}(r, z, p)$ and $q_{3,1}(r, z, p)$ in equation (2.3) and studying the travel time, we have displacements due to $PP$ and $PS$ phase:

$$q_{pp}(r, z, t) = \frac{A_{pp}}{i} \delta(t - t_{pp}) + \left( -\frac{aA_{pp}}{t} + B_{pp} \right) e^{-a(t - t_{pp})} H(t - t_{pp})$$

$$q_{ps}(r, z, t) = \frac{A_{ps}}{i} \delta(t - t_{ps}) + \left( -\frac{aA_{ps}}{t} + B_{ps} \right) e^{-a(t - t_{ps})} H(t - t_{ps})$$
$A_{pg}$, $B_{ps}$, $t_{ps}$ are defined in equations (2.4 & 2.5).

(iii) Displacement due to once reflected $pp'$ phase (Fig. 2.4)

Inverse Laplace transform of $q_{4, 5, 6} (r, z, \rho)$ gives the displacement due to reflected $pp'$ phase, which is reflected at the interface.

$$q_{pp'}(r, z, t) = \frac{A_{pp'}}{t} \delta (t - t_{pp'}) + \left( -\frac{aA_{pp'}}{t} + B_{pp'} \right) e^{-a(t-t_{pp'})} H(t-t_{pp'})$$

...(2.15)

where $A_{pp'}$, $B_{pp'}$, $t_{pp'}$ are defined in equation (2.4 & 2.5)

(iv) Displacement due to $pp''p$ phase (Fig. 2.5)

Inverse Laplace transform of $q_{5, 6} (r, z, \rho)$ gives the displacement due to $pp''p$ phase.

$$q_{pp''p}(r, z, t) = \frac{A_{pp''p}}{t} \delta (t - t_{pp''p}) + \left( -\frac{aA_{pp''p}}{t} + B_{pp''p} \right) e^{-a(t-t_{pp''p})} H(t-t_{pp''p})$$

...(2.16)

$A_{pp''p}$, $B_{pp''p}$, $t_{pp''p}$ are given in equation (2.4 & 2.5)

(v) Displacement due to $pp's$ phase (Fig. 2.6)

Inverse Laplace transform of $q_{6, 6} (r, z, \rho)$ gives the displacement due to $pp's$ phase.

...(2.17)
\( q_{pp'}(r, z, t) = \frac{A_{pp'}}{t} \delta(t - t_{pp'}) + \left(\frac{B_{pp'}}{t} + C_{pp'}\right) e^{-\alpha(t - t_{pp'})} \eta(t - t_{pp'}) \)

(2.17)

\( q_{pp'} \), \( B_{pp'} \), \( C_{pp'} \) are defined in equation (2.14-15).

(vi) Displacement due to head wave \( PP' \) (Fig. 2.7).

Inverse Laplace transform of \( q_{pp', \alpha} r_z \) gives the displacement due to \( PP' \) phase,

\( q_{pp'}(r, z, t) = \frac{A_{pp'}}{t} \frac{\delta(t - t_{pp'})}{t} \eta(t - t_{pp'}) \)

(2.18)

where \( t_{pp'} = \frac{t}{\alpha_z^2} + \sqrt{\frac{1}{\alpha_1^2} - \frac{1}{\alpha_z^2}} \) \((2.12 - h - z)\).

At the free surface, \( P, PP, PS \) phases come together making one composite pulse \( PP' \), \( PP' \), \( PP' \), \( PP' \) come together making one composite pulse \( PP' \).

2.4 Numerical Results for the Surface Motion for decaying Shock

We have taken the following values of elastic constants

\( \alpha_1 = 6.2 \text{ Km/sec} \)
\( \beta_1 = 3.75 \text{ Km/sec} \)
\( \alpha_2 = 8.2 \text{ Km/sec} \)
\( \beta_2 = 4.65 \text{ Km/sec} \)

\( \frac{\beta_2}{\beta_1} = 1.345 \) which have been taken in the Chapter I.

First to observe how the amplitude decreases with reflection, we shall show in Fig. 2.10 the ratio of amplitude of radial displacement due to reflected \( PP' \), interface reflected
with direct $P$-wave

$PP'$ and head wave $PP_2P'$, we shall denote these ratios as

$L_{pp}$, $L_{pp'}$, $L_{pp_2p'}$ respectively. From the earlier
calculations, we have:

$$|L_{pp}| = 1 - \frac{2\zeta_3^j}{\alpha_3^j \zeta_3^j} \left( \frac{b + \xi}{\gamma} \right)$$

$$|L_{pp'}| = \left\{ \left[ \frac{2,4 - 1 - 2}{\gamma \alpha_1} \left( - \frac{2i \Delta_{12,1}}{\Delta_{12,0}} \right) \right] \right\}$$

$$|L_{pp_2p'}| = \frac{C_3 \alpha_1}{\alpha_2^2} \frac{\Delta_{23,0}}{\Delta_{12,0}^2} \left( \frac{\Delta_{23,1}}{\Delta_{23,0}} - \frac{\Delta_{12,1}}{\Delta_{12,0}} \right) \frac{1}{\gamma}$$

$$\frac{1}{C_2^2} = \left( \frac{1}{2\beta_1^2} - \frac{1}{\alpha_1^2} \right)$$

$$\frac{1}{C_1} = \left( \frac{1}{\beta_1^2} - \frac{1}{\alpha_1^2} \right)^{1/2}$$

$$\frac{1}{C_3} = \left( \frac{1}{\alpha_1^2} - \frac{1}{\alpha_2^2} \right)^{1/2}$$
Table 1

Variation of Amplitude ratio with epicentral distance.

\[ \frac{\eta}{\xi} = 30 \text{ K.M.} \]

<table>
<thead>
<tr>
<th>( \eta/\xi )</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \left</td>
<td>\frac{H_{PP}}{H} \right</td>
<td>)</td>
<td>0.80</td>
<td>0.90</td>
<td>0.92</td>
</tr>
<tr>
<td>( \left</td>
<td>\frac{H_{PP'}}{H} \right</td>
<td>)</td>
<td>0.20</td>
<td>0.24</td>
<td>0.33</td>
</tr>
<tr>
<td>( \left</td>
<td>\frac{H_{P'P'}}{H} \right</td>
<td>)</td>
<td>0.034</td>
<td>0.035</td>
<td>0.037</td>
</tr>
</tbody>
</table>

Remark

From this table it is clear that amplitude of displacement due to head wave \( PP_p \) and are smaller than these of direct \( P \), reflected \( PP \) and \( PP' \) phases at large distance.

Table 2

Variation of Amplitude ratio with depth of source for \( q(r, z, t) \), shown in (Fig.2.9).

\[ H = 30 \text{ K.M.} \]

<table>
<thead>
<tr>
<th>( \frac{H}{r} \times 10^{-3} )</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \left</td>
<td>\frac{H_{PP}}{H} \right</td>
<td>)</td>
<td>0.31</td>
<td>0.63</td>
<td>0.81</td>
</tr>
<tr>
<td>( \left</td>
<td>\frac{H_{PP'}}{H} \right</td>
<td>)</td>
<td>0.03</td>
<td>0.03</td>
<td>0.043</td>
</tr>
</tbody>
</table>
Table 3

Variation of Amplitude ratio with thickness of the layer

\[ h = 5 \text{ K.M.} \]

<table>
<thead>
<tr>
<th>( \frac{H}{T} \times 10^{-3} )</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>L_{pp'}</td>
<td>)</td>
<td>0.60</td>
<td>0.51</td>
<td>0.46</td>
</tr>
</tbody>
</table>

2.5 Surface displacement \( q(r, z, t) \) due to \( P, PP' \) phases

At the free surface i.e. at \( z = 0 \), the \( P, PP, PS \) phases come together making one composite pulse \( P \) and \( PP' \), \( PP'P, PP'S \) also come together at \( z = 0 \) making one composite pulse \( PP' \). Therefore to present surface displacement due to \( P \) and \( PP' \) phases, we need to add the contribution of \( P, PP, PS \) phases at \( z = 0 \) and that of \( PP', PP'P, PP'S \) at \( z = 0 \).

Calculation: Let us denote \( Q(r, t) \) by the following eqn.

\[
Q(r, t) = \left[ q_p(r, z, t) + q_{pp}(r, z, t) + q_{ps}(r, z, t) \right]_{z=0}
\]

\[
+ \left[ q_{pp'}(r, z, t) + q_{pp'p}(r, z, t) + q_{pp's}(r, z, t) \right]_{z=0}
\]
### Table 4

<table>
<thead>
<tr>
<th>( a^2 )</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.8</th>
<th>1.2</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2a^2}{\rho^2} \times 10^3 )</td>
<td>-0.54</td>
<td>-1.20</td>
<td>-3.97</td>
<td>3.53</td>
<td>-1.5</td>
<td>-1.0</td>
</tr>
<tr>
<td>( \frac{2a^2}{\rho^2} \times 10^3 )</td>
<td>-0.134</td>
<td>-1.20</td>
<td>-1.05</td>
<td>-0.49</td>
<td>0.00</td>
<td>( \rho = 4.00 \text{ km} )</td>
</tr>
<tr>
<td>( \frac{2a^2}{\rho^2} \times 10^3 )</td>
<td>-0.026</td>
<td>-0.085</td>
<td>-0.04</td>
<td>0.00</td>
<td>( \rho = 10.00 \text{ km} )</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5

<table>
<thead>
<tr>
<th>( \rho = 1000 \text{ km} )</th>
<th>( h = 5 \text{ km} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^2 )</td>
<td>0.0</td>
</tr>
<tr>
<td>( \frac{2a^2}{\rho^2} \times 10^3 )</td>
<td>-0.221</td>
</tr>
<tr>
<td>( \frac{2a^2}{\rho^2} \times 10^3 )</td>
<td>-0.192</td>
</tr>
</tbody>
</table>

### Table 6

<table>
<thead>
<tr>
<th>( \rho = 1000 \text{ km} )</th>
<th>( h = 30 \text{ km} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^2 )</td>
<td>0.0</td>
</tr>
<tr>
<td>( \frac{2a^2}{\rho^2} \times 10^3 )</td>
<td>-0.221</td>
</tr>
<tr>
<td>( \frac{2a^2}{\rho^2} \times 10^3 )</td>
<td>-0.189</td>
</tr>
</tbody>
</table>
2.6 Discussion of the Results

It is shown in Fig. (2.8) that number of reflection causes decrease in amplitude of displacement. The amplitude ratio of reflected PP phase with that of P-phase is around .8 to .9 and the amplitude ratio of reflected PP' phase with that of P phase is around .2 to .4 where PP is reflected at the free surface and PP' is reflected at the interface. Therefore it is observed that reflection at the interface causes much decrease in amplitude than the reflection at the free surface. We also observe that among the body waves, the amplitude of P wave is the largest.

For the time dependent, body forces as \( e^{-at}h(t) \) we have from the table 1, that contribution of headwave and diffracted waves are smaller than those of direct and reflected waves which is also shown in the case of Lamb-pulse by Singh (1969) for large distance.

For \( \chi(t) = e^{-at}h(t) \) the superposition of direct P and reflected P and S waves at the free surface and interface would give rise to a train of waves, as observed in actual seismograms. From the Fig. (2.11a) it is shown that at the model ( \( \theta = 1000 \text{ K.M}, h = 5 \text{ K.M}, H = 30 \text{ K.M} \) the arrival time at the direct P and reflected PP' wave differ by a small quantity, they make only one peak. But for the model ( \( \theta = 200 \text{ K.M}, h = 5 \text{ K.M}, H = 30 \text{ K.M} \) ) two peaks due to the arrival of two phases
Fig. 2. Variation of amplitude ratio with the epicentral distance.

Fig. 3. Variation of amplitude ratio with the depth of source.
Fig. 2.10. Variation of amplitude ratio, $\frac{L_{pp}}{L}$ (4)
with the thickness of the layer.
Fig. 2.11a.

$N_0 q \times 10^2 \left[ \frac{m^2}{s^2} \right]$

$\alpha \tau$

$P$

$P'$

Fig. 2.11b.

$N_0 q \times 10^2 \left[ \frac{m^2}{s^2} \right]$

$\alpha \tau$

$P$

$P'$

Surface displacement $q(\gamma, z, t^{'})$ due to the P and PP'-phases.
Fig. 2.11c. Surface displacement $q(y, z, t)$ due to the P and PP' phases.
Surface displacement $q(r, z, \kappa)$ due to P and PP-phases.
Surface displacement $q(\gamma, z, t)$ due to $P$ and $PP'$-phases.

Fig. 2.13a. $h = 5 \text{ km}$

Fig. 2.13b. $h = 10 \text{ km}$
are seen clearly, as there exists some delay between the arrival time of two phases. Therefore it is seen that at large distance; the direct and reflected waves superposed on each other. Such features were also discussed by Alterman (1968).

From the Figs (2.12a), (2.14b) it is observed that the thickness of the layer affects the displacement due to PP' phase. As the thickness of the layer increases, the difference between the arrival time of direct and reflected wave PP' increases, therefore they make separate peak. For $H = 40 \text{K.M}$, the interface reflected phase PP' is clearly separated from the direct P wave.

From the Figs (2.14a), (2.13b) it is shown that the depth of the source below the interface do not affect so much like thickness of the layer on the direct P, reflected BP' phase. The form of Figs (2.13a) for $h = 5 \text{K.M}$ $h = 10 \text{K.M}$ are almost same which were also discussed by Singh (1966).

2.7 Conclusion

It is concluded that the present method of solving the problem of wave propagation due to compressional point source considering the appropriate body forces due to this source give the satisfactory results which are in agreement with others who solved the problem by usual technique considering the wave potential and introducing the source effect by source-potential or discontinuity of stress across the source-level.
But this present method is more direct and has more physical significance in the case of

(i) any type of source

(ii) finite source

(iii) various time dependence of the source function.