CHAPTER III

OPTIMISATION OF THE PROGRAMME CONTROLLED SYSTEM FOR WORKING OUT A GIVEN CONTOUR
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3.1. Evaluation of relative importance of different parameters, simplification of the block diagram and mathematical description of the simplified system.

The computational block diagram of the system under consideration is shown in figures 1.8(a) and 1.8(b). Optimisation of this system considering all the multiplicities of its element is quite an involved task. Hence, it is necessary to separate out the factors of primary and secondary importance. For this purpose, it is necessary to determine the relative values of different parameters of a practical system, write down the equations of motion of the individual parts, obtain the interdependence between themselves and on this basis make some approximations and obtain a simplified system. A series of experiments were carried out on this vertical milling machine 6N12P in the Industrial Electric Drives Laboratory.
of Moscow Power Institute by the author as well as by Dr. Zh. Ch. Molonov(34) and A. P. Aphonin(5), the observations of the author are given in the second chapter. The following conclusions follow from the aggregate of these observations:

3.1.1. Simplified model of the stepper motor drive.

(a) The synchronising torque of the stepper motor depends on the currents flowing through the windings and the angular displacement between the rotor axis and the axis of the resultant stator mmf (magnetomotive force). Knowing the form and value of the current it is possible to determine the synchronising torque (e.g., turning moment) at every instant and for every value of angular displacement of the rotor.

Let steady nominal (rated) current flow through one of the windings of the SM (stepper motor) at time $t = t_0$ as shown in figure 3.1. The synchronising torque is determined by the static characteristic for nominal current as shown in figure 3.2 during the interval from $t = t_0$ to $t = t_1$, when commutation of currents starts. The synchronising torque during the subsequent interval from $t_1$ to $t_2$ will be determined by the resultant effect of the currents flowing in the different windings. Duration of this interval will depend on factors like: method of forcing current through the windings, resistance
FIG 3.1 CURRENT WAVEFORMS IN THE WINDINGS OF A FOUR-PHASE STEPPER MOTOR

FIG 3.2 STATIC SYNCHRONISING TORQUE CHARACTERISTIC
and inductance of the windings, rotor construction etc. But the relative value of this duration (rise time) will depend on the repetition frequency of the input pulses. The commutation ends at \( t = t_2 \) and hence from \( t = t_2 \) to \( t_3 \) the synchronising torque will again have only one component, since current is flowing through one winding only. That the synchronising torque at any instant may be determined by the steady state value of current and the angular difference \( \psi(t) - \theta_d \) i.e., by the static characteristic holds good at low and medium frequencies, the rise time being quite small compared to the total time period. This is usually the case with practical arrangements and this assumption will be utilised in subsequent analysis.

(b) The synchronising torque of a SM can be written down in the following way:

\[
T_{syn} = f[i(t), \psi(t) - \theta_d]
\]...(3.1)

where, \( i(t) \) - current in the stator winding;
\( \psi(t) \) - programmed displacement of the stator magnetic field,
\( \theta_d \) - angular displacement of SM shaft in electrical radians.

For steady state current flowing through the windings the corresponding torque may be determined from the static characteristic corresponding to the given value of current.
This characteristic reduces to a periodic function of \( V(t) - \theta_d \) as illustrated in figure (3.2).

While investigating the dynamic performance of a system incorporating a stepper motor different types of approximations on the above mentioned characteristics are employed. For example, in the stable operating region, where operation takes place without loss of information it may be replaced by a straight line from \( V(t) - \theta_d = -\frac{\pi}{2} \) to \( +\frac{\pi}{2} \).

Then,

\[
T_{\text{syn}} = f\left[ V(t) - \theta_d \right]_{i = \text{const.}} \leq C_d \left[ V(t) - \theta_d \right]
\]

...(3.3)

where, \( C_d \) - slope of the approximating straight line for the static synchronising torque characteristic.

When \( \theta_d \) is expressed in mechanical radians \( C_d \) becomes a function of no. of phases, current, no. of commutating intervals and no. of rotor teeth. Thus, it may be written that,

\[
C_d = f_{\perp}(m, i, N, Z_r)
\]

...(3.4)

Here, \( m \) = no. of phases, and \( Z_r \) = no. of rotor teeth.
Consequently, the load has to be considered to be comprised of dry friction and the cutting force.

3.1(5). There may be a coupling between the two axes of movement through the load. This may, for example, arise due to the fact that the direction of motion may not coincide with the direction of cutting force. But the mutual dependence is of deterministic type. Therefore, the co-ordinates X and Y may be examined separately though not independently.

3.1(6) In addition to the type of coupling mentioned above, there may be coupling between the co-ordinates when they are not exactly at right angles to each other. But, the effect of this type of coupling in a well fabricated machine is almost imperceptible and may be disregarded for all practical purposes.

3.1(7) The combination of inertia with dry friction of the drive or the table, viscous friction of the drive and the elasticities may be represented by a non-system and the mathematical tools of analysis for non-linear systems may be applied to this problem.

3.2. Minimisation of error.

In this article the problem of minimising error for a deterministic system is being examined.

Let,
The coefficient of elasticity of the link joining the stepper motor shaft with the table; 

$\alpha$ - value of elementary step;

$T_d$ - dry frictional torque on the stepper motor shaft;

$T_L$ - load torque due to the cutting operation;

$T_c$ - dry frictional torque on the table;

$\theta_c$ - equivalent angular displacement of the table.

Then the equation of motion of the system under consideration may be written in the following way:

$$J_d \ddot{\theta}_d + B \dot{\theta}_d = C_d[y(t) - \theta_d] - C_{dc}(\theta_d - \theta_c) - \text{Sign}(\dot{\theta}_d)T_d$$

$$J_c \ddot{\theta}_c = C_{dc}(\theta_d - \theta_c) - T_K$$

and

$$T_K = T_L + \text{Sign}(\dot{\theta}_c)T_c$$

Here, $T_L$ is a function of the cutting force and in general the direction of this force may not coincide with the direction of cutting.

The foregoing equations in conjunction with the knowledge of general set up of the machine enable one to
represent the block diagram of the system as shown in figure 3.3.

Next, it is assumed that a given contour has to be worked out. This contour may be a closed or an open one. The illustrative one shown in figure 3.4 is chosen to be a closed contour. However, the analytical approach to the problem will not very much depend on the nature of the contour it being equally applicable to a closed or open contour.

As is the usual practice, the speed of the instrument along the contour is assumed to be constant, or to be more precise, the frequency of the input impulse fed to X and Y axes give a constant value along the tangential direction. In other words, if \( f_x \) and \( f_y \) be the frequencies for X and Y axes, then \( f_x^2 + f_y^2 = \text{constant} \). The value of this constant has to be determined from certain technical considerations, such as toughness of the tool, of the material being cut, the available power for driving the tool.

Apart from this, a contour is usually divided into several linear segments, the number of such segments depending on factors like the desired degree of accuracy, nature of the contour etc. The contour then may be broken up into two curves: \( X(t) \) and \( Y(t) \) as shown in figures 3.5(a) and 3.5(b).
FIG 3.3 SIMPLIFIED BLOCK DIAGRAM OF THE AUTOMATIC CONTROL SYSTEM FOR ONE CO-ORDINATE

FIG 3.4 AN ILLUSTRATIVE CONTOUR
FIG 3.5 $X(t)$ AND $Y(t)$ PLOTS FOR THE ILLUSTRATIVE CONTOUR

FIG 3.6 MOTION ALONG ONE CO-ORDINATE AROUND A TURNING POINT
With the object of minimising the error, attention is drawn to the fact that, the significant contribution to error arises due to non-linearity, inertia and elasticity present in the system (even with the simplications mentioned above). The influence of non-linearity does not necessitate particular attention, since it can be taken into consideration through the amplitude - phase (i.e. through the frequency response) characteristics of the combination: elastic link + viscous friction + dry friction + inertial element. But the starting and finishing of the contour as well as the turning points in it calls for special attention, since the motion of the instrument at these points is considerably influenced by elasticity and inertia of the system.

At the beginning and end of any contour the speed of the instrument is zero. Elastic deformation does not exist at the beginning and it must not be present when the contour is worked out.

Transition from one segment to the next one associated with a change in the direction of motion and consequent change of speed along the co-ordinate axes for a given contour speed. This phenomenon, in its turn, brings in the undesired effect of inertial element, which always opposes any sudden change of speed. As a result error will develop.
The change of direction of motion is associated with changes in the resultant effect of factors like: components of cutting forces along the co-ordinate axes, dry friction, and deformation of the elastic link, which may act against or along the direction of movement and may be the reason for the appearance of error.

For realising minimum error, the control strategy must be so chosen that at the end of each segment the input command compels the instrument to start moving along the direction of the next segment. Hence, in this sense, working out of a particular segment of the contour has to be viewed along with the adjacent segments.

At this point it will be of interest to get an insight into the operation through the equations of motion of the system. The equations are:

\[ J_d \ddot{\theta}_d + B \dot{\theta}_d + (C_d + C_{dc}) \dot{\theta}_d + \text{Sign}(\dot{\theta}_d)T_d = C_d \gamma(t) + C_{dc} \theta_c \]

\[ J_c \ddot{\theta}_c + C_{dc} \dot{\theta}_c + \text{Sign}(\dot{\theta}_c)T_c + T_L = C_{dc} \theta_d \]

For constant speed along the contour,

\[ \dot{\theta}_d = \dot{\theta}_c = \text{ Const.} = a \ (\text{say}) \]

\[ \ddot{\theta}_d = \ddot{\theta}_c = 0 \]

Substituting equation (3.9) into (3.8),

\[ \theta_d = \theta_c + \left[ \text{Sign}(\dot{\theta}_c)T_c + T_L \right] / C_{dc} \]
Again, from (3.10) and (3.7) :-
\[ v(t) = \theta_c + \left[ T_L + \text{sign}(\dot{\theta}_c)T_c + \text{sign}(\dot{\theta}_d)T_d + B\dot{\theta}_d \right]/C_d + \left[ T_L + \text{sign}(\dot{\theta}_c)T_c \right]/C_d \]
\[ \text{ ...(3.11)} \]

Let there be a change of direction of motion along the contour at the point B. This will correspond to a change of speed along both the co-ordinate axes.

The motion along one particular axis may be as shown in figure 3.6. With the tool moving along the contour at a constant speed, the expressions for \( \theta_d \) and \( v(t) \) can be obtained from equations (3.10) and (3.11). The variations of \( \theta_d \) and \( v(t) \) are represented in figure 3.6. From the nature of \( v(t) \) it is evident that, to keep the tool moving along the contour at a fixed speed it becomes necessary to apply a number of input impulses all on a sudden at the point B (i.e., at the turning point). But it is evident that such a thing cannot be physically realised.

Zusman and Tikhomirov\(^{(51)}\) has suggested a method by which error is reduced. This method consists in continuously reducing the frequency of the input signal from some point A. The input frequency gets linearly reduced with time. The frequency then starts increasing and at point C it corresponds to the given contour speed. But the instant at which speed/frequency reduction should start has been chosen empirically for a particular desired degree of accuracy. But there is no guarantee that this gives minimum possible error.
The problem of minimising error at the turning points in a contour is tackled in a somewhat different way in the present dissertation.

At the point A deceleration of the contour speed is started and at the end of the interval (figure 3.6) the displacement, $\theta_c$, reaches the turning point value. Different values of $t_1$ (in other words, different starting points for deceleration) should give the same value of $\theta_c$ at the end of the interval. But in each case the point reached in the state space of the system will be different giving rise to different values of $v(t)$ and $\theta_d$ for the same value of $\theta_c$.

Similarly, acceleration period may end at different instants if the initial values are different. This may also be stated as different positions of the point C will correspond to different starting points in the state space with different values of $v(t)$ and $\theta_d$ at the point B. The initial conditions at B while working out the portion BC may be considered as the final conditions if it is assumed that deceleration starts from point C and the trajectory is followed in the reverse direction for an interval $t_2$. Working in such a way, the final value of $\theta_c$ should always be the turning-point value, but the values of $v(t)$ and $\theta_d$ will be different for different values of $t_2$. 
The input signal may be calculated from the differential equations. Steady speed operation of the segments reveals a break in the continuity of $v(t)$ and $\theta_d$. If the error is to be minimised the continuity of these two terms must be restored. That is, the values of $v(t)$ and $\theta_d$ at the point B must be the same whether approaching it from A or C.

The continuity equations are obtained in the following way:

Let $\theta_{c1}$, $\dot{\theta}_{c1}$ be the position and velocity for the x-coordinate at A.

Let $\theta_{c2}$, $\dot{\theta}_{c2}$ be the position and velocity for the x-coordinate at C.

$$\ddot{\theta}_c = -\alpha_{x1} \quad \text{constant deceleration from A to B.}$$

$$\ddot{\theta}_c = -\alpha_{x2} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{C to B.}$$

The equations of motion of the system are:

$$J_d \ddot{\theta}_d + B \dot{\theta}_d + (c_d + c_{dc}) \theta_d + \text{sign}(\dot{\theta}_d) T_d = c_d v(t) + c_{dc} \theta_c \quad \ldots(3.12)$$

$$J_c \ddot{\theta}_c + C_{dc} \theta_c + \text{sign}(\dot{\theta}_c) T_c + T_L = c_{dc} \theta_d \quad \ldots(3.13)$$

From A to B:

$$\ddot{\theta}_c = -\alpha_{x1}$$

$$\dot{\theta}_c = \dot{\theta}_{c1} - \alpha_{x1} t$$

and

$$\theta_c = \theta_{c1} + \dot{\theta}_{c1} t - \frac{1}{2} \alpha_{x1} t^2 \quad \ldots(3.14)$$
Combining equations (3.14) and (3.15)
\[ \theta_d = \left[ -J_c a_{x1} + C_{dc} (\theta_{c1} + \dot{\theta}_{c1} t - \frac{1}{2} a_{x1} t^2) + \right. \]
\[ \left. + \text{Sign}(\dot{\theta}_c) T_c + T_{l1} \right] / C_{dc} \] ...(3.15)

Differentiating equation (3.15) with respect to time

\[ \dot{\theta}_d = \dot{\theta}_{c1} - a_{x1} t \] ...(3.16)

and

\[ \ddot{\theta}_d = -a_{x1} \] ...(3.17)

Putting the values of \( \theta_d \), \( \dot{\theta}_d \) and \( \ddot{\theta}_d \) into equation (3.12) :-

\[ C_{d} \gamma(t) = -J_d a_{x1} + B (\dot{\theta}_{c1} - a_{x1} t) + \frac{C_d + C_{dc}}{C_{dc}} \left[ -J_c a_{x1} + C_{dc} \right. \]
\[ \left. \left( \theta_{c1} + \dot{\theta}_{c1} t - \frac{1}{2} a_{x1} t^2 \right) + \text{Sign}(\dot{\theta}_c) T_c + T_{l1} \right] + \text{Sign}(\dot{\theta}_d) T_d \]
\[ - C_{dc} \left( \theta_{c1} + \dot{\theta}_{c1} t - \frac{1}{2} a_{x1} t^2 \right) \]
\[ = \cdots (3.18) \]

Similarly, for region C to B :-

\[ \theta_d = \left[ -J_c a_{x2} + C_{dc} (\theta_{c2} + \dot{\theta}_{c2} t - \frac{1}{2} a_{x2} t^2) + \text{Sign}(\dot{\theta}_c) T_c + T_{l2} \right] / C_{dc} \]
\[ \cdots (3.19) \]

\[ C_{d} \gamma(t) = -J_d a_{x2} + B (\dot{\theta}_{c2} - a_{x2} t) + \frac{C_d + C_{dc}}{C_{dc}} \left[ -J_c a_{x2} + C_{dc} \right. \]
\[ \left. \left( \theta_{c2} + \dot{\theta}_{c2} t - \frac{1}{2} a_{x2} t^2 \right) + \text{Sign}(\dot{\theta}_c) T_c + T_{l2} \right] + \text{Sign}(\dot{\theta}_d) T_d \]
\[ - C_{dc} \left( \theta_{c2} + \dot{\theta}_{c2} t - \frac{1}{2} a_{x2} t^2 \right) \]
\[ = \cdots (3.20) \]

Writing \( t_1 \) for \( t \) in equation (3.15) and \( t_2 \) for \( t \) in equation (3.19) and then equating the two values of \( \theta_d \) gives the continuity equation for \( \theta_d \) . This gives,

\[ (\frac{1}{2} a_{x1}) t_1^2 - \dot{\theta}_{c1} t_1 - k_3 = (\frac{1}{2} a_{x2}) t_2^2 - \dot{\theta}_{c2} t_2 - k_4 \]
\[ \cdots (3.21) \]
where, \( K_3 = \theta_{c1} + \left[ \text{sign}(\dot{\theta}_{c1}) T_c + T_{L1} - J_c \alpha x_1 \right] / C_d c \) \hspace{1cm} \ldots (3.22)

and \( K_4 = \theta_{c2} + \left[ \text{sign}(\dot{\theta}_{c2}) T_c + T_{L2} - J_c \alpha x_2 \right] / C_d c \) \hspace{1cm} \ldots (3.23)

Similarly, writing \( t_1 \) for \( t \) in equation (3.18) and \( t_2 \) for \( t \) in equation (3.20) and then equating the two values of \( C_d V(t) \) gives the continuity equation for \( V(t) \).

This may be expressed as:

\[
\left( \frac{1}{2} \alpha x_1 \right) t_1^2 + (\alpha x_1 B / C_d - \dot{\theta}_{c1}) t_1 - K_1
\]

\[
= \left( \frac{1}{2} \alpha x_2 \right) t_2^2 + (\alpha x_2 B / C_d - \dot{\theta}_{c2}) t_2 - K_2 \hspace{1cm} \ldots (3.24)
\]

where,

\[
K_1 = \left[ -J_d \alpha x_1 - J_c \frac{C_d + C_{dc}}{C_d c} \alpha x_1 + C_{dc} \theta_{c1} + B \theta_{c1} + \text{sign}(\dot{\theta}_{c1}) T_c \right. \\
+ T_{L1} + \text{sign}(\dot{\theta}_{d1}) T_d \] / C_d 
\]

\[
K_2 = \left[ -J_d \alpha x_2 - J_c \frac{C_d + C_{dc}}{C_d c} \alpha x_2 + C_{dc} \theta_{c2} + B \theta_{c2} + \text{sign}(\dot{\theta}_{c2}) T_c \right. \\
+ T_{L2} + \text{sign}(\dot{\theta}_{d2}) T_d \] / C_d 
\]

Subtracting equation (3.21) from (3.24) gives the following equation:

\[
(B \alpha x_1 / C_d) t_1 - K_1 + K_3 = (B \alpha x_2 / C_d) t_2 - K_2 + K_4
\]

or

\[
t_1 = (\alpha x_2 / \alpha x_1) t_2 + (K_1 - K_2 - K_3 + K_4) C_d / (B \alpha x_1) \hspace{1cm} \ldots (3.27)
\]

This equation when combined with (3.21) or (3.24) yields a quadratic equation in \( t_2 \) or \( t_1 \). One of the coefficients of this equation contains an unknown quantity \( \Delta \theta_c (\equiv \theta_{c2} - \theta_{c1}) \). The evaluation of \( \Delta \theta_c \) is achieved through minimisation of \((t_1 + t_2)\) by varying \( \Delta \theta_c \) within practically possible limits. But by virtue of equation (3.27)
minimisation of $t_1$ or $t_2$ leads automatically to minimisation of $(t_1 + t_2)$. Hence, $\Delta \theta_c$ is to be so chosen that the quadratic equation gives one positive real root of practically minimum possible value, and

$t_1$ and $t_2$ may be evaluated in this way. But it is to be noted that equations similar to equations (3.21), (3.24) and (3.22) will be obtained for the $y$-axis also and they will give another set of values for $t_1$ and $t_2$. Since these two requirements may not be satisfied simultaneously a compromise is inevitable. This is done by taking the value of $(\alpha_x/\alpha_x)$ equal to unity and the second term on the right-hand side of equation (3.27) is taken for the axis which is liable to have larger error i.e. speed change through larger value. Writing $T_x(\gamma)$ for this term equation (3.27) may be rewritten as

$$t_1 = t_2 + T_x(\gamma) \quad \ldots(3.28)$$

Thus the procedure for determining $t_1$ and $t_2$ is :-

1. To write down equations (3.21) and (3.24) for the co-ordinate involving larger change of speed;

2. To evaluate $T_x(\gamma) = (k_1 - k_2 - k_3 + k_4) \cdot C_d/(B \alpha_x)$ for this co-ordinate;

3. To substitute this value of $T_x(\gamma)$ into equation (3.28);

4. To put this equation into equation (3.21) or (3.24) and
determine the value of $t_1$ and hence $t_2$.

5. Given the deceleration and acceleration rate (they are of equal value) along the contour, to determine the instant when deceleration will start (i.e. point A) and the moment when acceleration will be over (i.e. point C).

Once the points A and C along with $t_1$ and $t_2$ are obtained the strategy of minimising error around a turning point in the contour is defined. But a few points of interest are to be noted in this connection. It may not always be possible to satisfy the continuity equations in the forms they are represented. For example this condition may arise when the differences of $\dot{V}(t)$ and $\theta_d$ with $\theta_C$ at the point B are of opposite signs as the point B is approached from the left hand and the right hand sides with the contour speeds reduced to zero at this point. In such a case the situation has to be dealt with in the following way: While approaching B from the left hand side point A should be so chosen that the speed reduces to zero at the point B, then to apply $\dot{V}(t)$ of sufficient magnitude but without imparting any motion to the table or sliding block so that the discrepancies of $\dot{V}(t)$ and $\theta_d$ with $\theta_C$ are equal to or less than their values which they would have attained when the point B is approached from the right hand side in such a way that $\dot{\theta}_C$ reduces to zero.
The condition under which the continuity equation may not hold good may be detected from the values of $t_1$ and $t_2$. This takes place when either $t_1$ or $t_2$ is found to be greater than their limiting values, the limiting value being the interval necessary to bring down the contour speed to zero from its nominal value.

$T_{L1}$ and $T_{L2}$ are, strictly speaking functions of time. But in the expression for $T_x(Y)$ the difference of their values rather than their actual values is of significance. Hence, if they are taken to be their nominal values (corresponding to nominal contour speed) the error introduced will be quite small.

3.3. First harmonic linearisation of the N-L system and error minimisation.

Let the portions of the contour for which the first harmonic linearisation technique is being applied be divided into two functions of time $X(t)$ and $Y(t)$. These functions may be represented by Fourier Series:

$$X(t) = a_0x/2 + a_1x \cos \omega t + a_2x \cos 2\omega t + \ldots$$
$$+ b_{1x} \sin \omega t + b_{2x} \sin 2\omega t + \ldots$$
$$Y(t) = a_0y/2 + a_1y \cos \omega t + a_2y \cos 2\omega t + \ldots$$
$$+ b_{1y} \sin \omega t + b_{2y} \sin 2\omega t + \ldots$$

...(3.29)
Now, to determine the input signal for a given output signal as given by equation (3.29) one should obtain the amplitude-phase characteristics of the system as a function of output signal. For obtaining such characteristic the method of harmonic linearisation for dry friction outlined in Popov's work has been adopted. However, the frequency response curve has to be adopted for operation under colliding conditions. For evaluation of dynamic performance the nature of dry friction has been assumed to be of the form shown in figure 3.7. For non-linearity of this type the solution is obtained by assuming the speed to be given by

\[ S_x = A_v \sin \omega t \]

where, \( A_v \) - maximum value of speed; 
\( \omega \) - frequency (rad/sec) at which speed changes.

Since the nonlinear function \( F(S_x) \) does not depend explicitly on the speed of change of input, the coefficients of harmonic linearisation will depend upon the amplitude of oscillation \( A_v \) only and will be given by:

\[
a(A_v) = \frac{1}{\kappa A_v} \int_0^{2\pi} F(A_v \sin \psi) \cdot \cos \psi \cdot d\psi
\]

\[
b(A_v) = \frac{1}{\kappa A_v} \int_0^{2\pi} F(A_v \sin \psi) \cdot \sin \psi \cdot d\psi
\]
FIG 3.7 REPRESENTATION OF DRY FRICTION FOR FIRST HARMONIC LINEARISATION

FIG 3.8 ILLUSTRATING CO-ORDINATE TYPE OF CONTOUR
The function $F(x)$ does not have any hysteresis and hence,

$$a(A_N) = 0$$

$$b(A_N) = \frac{4B}{\pi A_N}$$

Therefore, the formula for harmonic linearisation of dry friction becomes:

$$F(Sx) = b(A_N)Sx$$

Putting

$$x = A \sin \omega t$$

$$Sx = A \omega \cos \omega t$$

Therefore,

$$A_N = A \omega$$

Denoting

$$a(A) = \frac{4B}{\pi A}$$

The formula for harmonic linearisation can be written as

$$F(x) = \frac{a(A)}{\omega}Sx \quad \cdots (3.30)$$

Therefore, with the linearisation of dry friction, the input signal can be obtained for a given output, which has been represented by a Fourier Series. But this has to be carried out in two steps: First to obtain the stepper motor displacement and then the input signal to the stepper motor. Of course, the input signal will be the sum of the inputs for all the harmonic components of the output. The order of harmonic up to which calculation will have to be carried out will be dependent on the desired degree of accuracy. The input for both the axes will be obtained
in the following form:

\[ \begin{align*}
\dot{v}_x(t) &= v_{0x} + v_{1x} \cos(\omega t + \theta_{1x}) + v_{2x} \cos(2\omega t + \theta_{2x}) + \cdots \\
\dot{v}_y(t) &= v_{0y} + v_{1y} \cos(\omega t + \theta_{1y}) + v_{2y} \cos(2\omega t + \theta_{2y}) + \cdots \\
\end{align*} \]  \quad \ldots(3.31)

It is worth mentioning here that the harmonic analysis method is to be applied not only at turning points of the contour, but at the beginning and end of the contour also. Hence, generally speaking this is to be applied wherever there is a change of speed along the co-ordinate axes.

One difficulty may arise to realise the input signal when the system works through an interpolator where the programme is given through the starting and end point of a particular segment, and the correction is to be fed automatically. For such an application the input command has to be given with such constant acceleration/deceleration that the deviation of the actual input signal from the desired one is minimum, and hence the error also will be minimum (within the scope of harmonic linearisation).

3.4. Co-ordinate type of control and error minimisation.

It is well known that in the co-ordinate type of control the two axes are given displacements alternately. A control of this type may be intentional or circumstances may dictate such an operation. One essential
feature is that the time of working out a contour by this method is larger. However, due to technological reasons an engineer may be compelled to choose a slower speed of operation and then the control automatically becomes converted to a co-ordinate type. The worked-out contour for obvious reason becomes stepped and not smooth. The working out of such a contour is illustrated in figure 3.8.

As distinct from the contour type of control here the frequency of the input impulses may be kept constant. In the contour type of control the motion of the two axes are corelated and for a particular contour speed the frequencies of the input impulses for both the axes are fixed. With the type of motion being discussed under this article the motion of one axis is not co-related to that of the other and hence, so long the motion is confined along one axis the control also is needed only along that axis. The main problem of minimising error (optimisation) now reduces to that the initial and final points in the phase-space of the system are given and to determine the control strategy so that at the final point the error is minimum. But before defining the error it will be helpful to describe the operation of the system by means of figure 3.8 as below: From $t_1$ to $t_2$ the input command is applied to the x-axis-drive, which brings the system from state $A_1$ to state $A_2$. 
This is followed by motion of y-axis drive from $t_2$ to $t_3$ at the end of which the system moves to state $A_3$. In this way the system-states are changed during the intervals $t_3$ to $t_4$, $t_4$ to $t_5$ and so on, and the contour $K$ is worked out. The drive chosen for the installation under consideration is a stepper motor, the rotor of which moves through discrete steps when impulses are applied at the input. Hence, the programmed distances moved through for each horizontal or vertical segment must be some integral multiplies of the elementary step. Consequently, it will not be difficult to approximate the given contour by a set of horizontal and vertical segments. This approximate contour comprising horizontal and vertical segments has to be formed in such a way that the mean square error becomes minimum. After this is obtained, attention is focussed on the problem of realisation of this configuration, by considering the motion along the two axes independently. The problem is now converted to obtaining the algorithm of control (e.g., the no. of forward impulses, the no. of backward impulses and the sequence of their application), which will move the system successively from state $A_1$ to $A_2$, then from $A_2$ to $A_3$ and so on. Here it is assumed that the points $A_1$, $A_2$, etc., fix up not only the position of the table relative to the centre of the tool but also spatial position of the mmf and shaft of the stepper motor. The derivatives of the
quantities at the beginning and end of any segment are assumed to be zero. Such conditions assure that at the end of each interval the system comes to a state of equilibrium and elastic deformation of the elastic link is reduced to a minimum. When such conditions are fulfilled each segment can be analysed independent of the other and thereby simplifying the problem.

Let it be assumed that the system has to move from state \( A_1 \) (defined as \( \nu_i = \theta_d = \theta_c = \eta_1 \alpha \); \( \dot{\theta}_d = \dot{\theta}_c = 0 \)) to state \( A_2 \) (defined as \( \nu_2 = \theta_d = \theta_c = \eta_2 \alpha \); \( \dot{\theta}_d = \dot{\theta}_c = 0 \)).

Here, \((\eta_1 \sim \eta_2) \alpha = n \alpha\) distance between two successive states \( A_1 \) and \( A_2 \).

It is to be noted here that the input signal may be applied only at discrete moments and hence the exact transition from one state to the next one may not be possible. But it is possible to go to a value very near to it and to minimise the error. The criterion for error is chosen to be,

\[
\Delta = \left[ n \alpha - (\theta_{cf} - \theta_{ci}) \right]^2 + \left[ n \alpha - (\theta_{df} - \theta_{di}) \right]^2 + (\dot{\theta}_{cf})^2 \leq \epsilon
\]
where, $\theta_{cf}, \theta_{ci}$ - are the final and initial values for the position of the table;

$\theta_{df}, \theta_{di}$ - are the final and initial values for the position of S.M. shaft,

$\epsilon$ - permissible error.

Each control or input impulse moves the mmf axis of the stepper motor shaft through a definite angle, $\alpha$. If $+\alpha$ denotes a 'forward' control signal and $-\alpha$, a backward control signal, then the control engineer knows only the number of effective impulses but not the total number of forward impulses or the backward impulses. The system equation being of the 4th order with two nonlinearities, theoretical analysis for obtaining the total number of forward impulses and backward impulses for a given number of effective impulses to minimise the error as defined earlier becomes a highly difficult job. The practical method adopted for arriving at the desired result by using a digital computer is a trial and error method. For a given number of effective impulses a particular order for the forward and backward impulses is chosen and the corresponding trajectory for the motion of the system is obtained. Then a different order is chosen and the corresponding trajectory is plotted. In this way all the possible types of trajectories are obtained. Out of all these the best possible trajectory will be picked up, the
corresponding control algorithm being already known. For making the trial and error procedure systematic, the following procedure is followed:-

(a) The first command to the system may be a +α or -α. +α will bring the system to one state and -α to another. The computer stores these values and the corresponding states of the system at the end of the first interval.

(b) The second impulse, now, may be again a +α or -α. But the final state of the system will be determined not only by the input command during this interval but the command during the previous interval also. Thus four possible states can be obtained at the end of the second interval. These states along with the commands for which they are due are again stored in the computer memory.

(c) Application of both 'forward' and 'backward' signal during the third interval will thus produce 8 (= 2^3) states at the end of this interval and at the end of 4th interval, 16 (= 2^4) possible states. Knowing all the possible states and the input signals to which they are due it is possible to construct all the possible trajectories and separate out the best from the aggregate. The corresponding input signal will be the required control algorithm.
For obtaining the points on the trajectories by means of a digital computer, the following relationships were utilised (34):

\[ J_d \ddot{\theta}_d + B \dot{\theta}_d + (c_d + c_{dc}) \dot{\theta}_d + \text{Sign}(\dot{\theta}_d) T_d = c_d v(t) + c_{dc} \theta_c \]

\[ J_c \ddot{\theta}_c + c_{dc} \theta_c + T_K = c_{dc} \theta_d \]

where, \( T_K = T_L + \text{Sign}(\dot{\theta}_c) T_c \)

\[ T_L = T_{\text{cut},x} \text{ or } T_{\text{cut},y} \quad \text{depending on the axis which is being considered.} \]

\[ T_{\text{cut},x} = P_{\text{cut},x} \cdot 6 \times 10^{-3} / (2\pi) \]

\[ T_{\text{cut},y} = P_{\text{cut},y} \cdot 6 \times 10^{-3} / (2\pi) \]

\[ P_{\text{cut},x} = P_z \sin \phi + P_r \cos \phi \]

\[ P_{\text{cut},y} = P_z \cos \phi - P_r \sin \phi \]

or,

\[ P_{\text{cut},x} = P_z \sin (90^\circ - \xi - \psi_i) + P_r \cos (90^\circ - \xi - \psi_i) \]

\[ P_{\text{cut},y} = P_z \cos (90^\circ - \xi - \psi_i) - P_r \sin (90^\circ - \xi - \psi_i) \]

\[ P_z \] - Tangential cutting force,

\[ P_r \] - Radial cutting force,
\( \xi \) - angle made by the direction of cutting with \( X \)-axis.

\( \psi_i \) - instantaneous angle of contact of the tool with the material being cut. Conventionally, this angle is taken to be half of the angle subtended at the centre by the arc of contact between the drill bit and the material being cut.

For mild steel, the empirical formula for \( P_z \) is

\[
P_z = 670.1^{0.86} \cdot W \cdot Z \cdot S_z \cdot D \cdot 10^{-6} \cdot 10^{-8}
\]

Newtons.

Here, \( t \) = depth of cut (in mm.).

\( W \) = width of cut (mm.)

\( S_z \) = feed per tooth (mm/tooth)

\( Z \) = no. of teeth in the drill bit.

\( D \) = diameter of the drill bit (in mm.)

The term tooth-feed may be replaced by rate of feed, referred to the motor shaft by means of the following expression:

\[
S_{sec} = \frac{Z \cdot Z_r \cdot N}{60} \cdot \frac{2\pi}{\alpha_{LS}} \cdot S_z
\]

where, \( Z_r \) = no. of teeth in the rotor of the stepper motor,

\( N \) = r.p.m. of the spindle turning the drill bit,

\( \alpha_{LS} \) = step of the lead screw.
Near-optimum control strategies are given in this table for the transverse axis with an input pulse-repetition frequency of 100/sec., dry friction in the motor shaft of 2.0 Nw-m, and dry friction in the sliding block of 2.0 Nw-m.

<table>
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<tr>
<th>(Deg.)</th>
<th>(Deg.)</th>
<th>(Deg.)</th>
<th>(Deg./Sec.)</th>
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The details of the computer programming for MINSK-22 in ALGAMS (a variational form of ALGOL-60) is given in Appendix-i.

Resume

1. The block diagram of figure 1.8, which is obtained by the method of identification contains element of different types which makes the problem of optimal control quite difficult. However, all the elements do not have significant influence on the system performance, and only those elements which are considered primary importance have significant influence. Strictly speaking the equation of motion of the stepper motor is non-linear, containing implicit and explicit types of non-linearities. Similarly the current flowing through the windings are non-linear functions of time, as shown in figure 3.1. But with sufficiently high degree of forcing it may be considered rectangular in nature. Then the current flowing through the windings assumes either the steady state value or reduces to zero. It is found from experience that such simplifying assumptions give sufficiently accurate results for engineering calculations.

2. The synchronising torque of a stepper motor is a non-linear periodic function of the angular
displacement of the stator mmf axis with respect to the rotor displacement \( \Delta \gamma(t) - \gamma_d \) as shown in figure 3.2 under steady state conditions. In the working range of the characteristic the applied torque on the shaft must be larger than the maximum value of the synchronising torque and hence \( \Delta \gamma(t) - \gamma_d \) remains within \( \pm \pi/2 \). In this region the static characteristic is more or less linear. Linearisation of this region gives quite accurate results from an engineer's point of view as well as for the purpose of analysing this system, since the application of a SM as a drive is limited to system of medium accuracy and relatively smaller milling machines. Considering these aspects the SM has been represented for the purpose of analysis by an elastic link.

3. The experimentally obtained static characteristics (figures 1.3 to 1.6) demonstrate the existence of some kind of non-linear friction, which has been explained as "jamming". The magnitude of this being not very great, it has been neglected.

4. Observation of Molonov\(^{34}\) shows that the total frictional torque does not vary with speed. The inference drawn from this is that the viscous friction does not play any vital role.

5. The co-related coupling between the co-ordinate
axes is almost imperceptible and the coupling through the load is of deterministic type. Hence it may be stated that each coordinate may be examined separately and to write down equations (3.5) and (3.6).

6(a). These equations of motion of the kinematic chain at once give the representative block scheme as shown in figure 3.3. Non-linearity is contained in the two blocks 1 and 2.

6(b). For the contour type of control the mathematical description of the system of automatic control has been rendered easier by linearisation of the non-linearity through the first harmonic of the Fourier Series. Frequency response curves were obtained by means of digital computer MINSK-22. How these curves are to be utilised for obtaining the control algorithm around a turning point in the contour is also explained. For straight line segments, a method is given by which the error at the turning points can be minimised and the input signal may be synthesised.

6(c). In obtaining the algorithm due consideration has been paid to simultaneous and co-ordinated control of both the coordinate axes. However, one algorithm is valid only around one turning point in the contour.

7(a) The method of obtaining the control algorithm by trial runs on the digital computer is examined in
art. 3.3. The results obtained by this method can be used for any type of contour after the approximated contour consisting of horizontal and vertical lines has been obtained.

7(b). Here each co-ordinate axes is examined separately, it being assumed that the motion is confined along one axis only at a time. The input signals are obtained for parts constituting the contour.

7(c). For this analysis only the effect of "jamming" has not been taken into consideration. Except this, all other factors are taken care of properly.