CHAPTER II

STUDY OF THE SYSTEM OF AUTOMATIC CONTROL WITH LIMITS IN THE OUTPUT AMPLITUDE
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2.1 Necessity of considering limits in the system under investigation.

The cutting process carried out in a vertical milling machine may be simple or complex. During simple cutting process the width and depth of the cut remains constant and the load on the instrument does not change abruptly. In the complex regime of operation, where the depth or width of cut changes abruptly, sudden change of load on the instrument takes place, and consequently, on the shaft of the driving motor. For example, when the instrument, for a certain time, moves along a relatively wider channel and then has to move along a relatively narrower channel a collision phenomenon between the metal and the instrument takes place. Collision phenomenon, may further take place under the following conditions: when each tooth of the cutting tool is cutting into the metal; due to sudden change of thickness of the casting or the semifinished products; during the movement of the stepper
motor just after the cutting process is over i.e., when the load from the instrument is suddenly withdrawn; when the table meets a rigid obstacle, during vibratory operation of the milling machine and so on. These types of processes are often met with in practice.

In all these cases there may be vibration of the table, stepper motor or the cutting tool. They are considered undesirable situations. But the presence of the undesirable situations depends on the values of such parameters as $C$, $J$, $B$. In case it is present, it may be either an elastic collision or an inelastic collision.

All this results in deterioration of quality of the finished product, specially in case of elastic collision, where presence of vibration is highly probable, consequently it necessitates verification of some of the parameters of the system under investigation. Parameters of the system must be chosen in such a way that there is a good damping to the vibration. With this object the performance of the system is investigated with limits in the output amplitude. A knowledge of the two extreme cases is very useful for predicting the behaviour of the system under actual operating conditions. The two extreme cases are: Inelastic collision and Elastic collision. Actual operating condition will always be between these two limiting cases.
Theory of operation of the system of automatic control for collision condition is not yet fully worked out. The question of obtaining the amplitude vs. phase characteristic of the kinematic system, containing elastic link, viscous friction, inertial element and limit for displacement is examined below. The block diagram for such a system is shown in figure 2.1.

Investigation of the system is carried out with viscous friction only, although in a real system both dry friction and viscous friction may be present. But it may be replaced by its equivalent viscous friction true, that in a number of cases the dry friction or damping coefficient. Here it is assumed that equivalence remains unaffected by the change in operating speed of the mechanism. However, such assumption may not always be justified. For example, with increase in frequency of input impulses to a stepper motor the motion of the mechanism becomes smoother and the sign of torque due to dry friction does not change or changes rather rarely compared to the low frequency operation. As a result, coefficient of equivalent damping depends not only on the amplitude of dry friction but also on the friction of input impulses. This dependence has been investigated in Moscow Power Institute (by Molonov) by analogue computer simulation and comparing the motions of stepper motor with dry friction to its motion with viscous friction - proportional to speed. Curves
FIG 2.1 BLOCK DIAGRAM OF A SYSTEM WITH ELASTIC LINK (EL), INERTIAL ELEMENT (IE), VISCous FRICTION (VF), AND LIMITING ELEMENTING (LE).

FIG 2.2 PLOT OF INPUT VS OUTPUT AT LOW FREQUENCY

FIG 2.3 PLOT OF FIRST HARMONIC AMPLITUDE AT LOW FREQUENCY
of equivalent viscous friction vs. frequency of input impulses for different values of dry friction have been constructed. The relative values of dry friction have been taken as 0.05; 0.1, 0.15, 0.2 \( T_{\text{max}} \). These values cover the range of dry friction which may be present on the stepper-motor shaft, acting as the drive for the vertical milling machine.

For obtaining the amplitude-phase characteristics of the combination mentioned above, the method of harmonic linearisation has been applied.

2.2 Investigation under inelastic collision.

The differential equation of the system may be written in the following form :-

\[
J \ddot{\theta}_2 + B \dot{\theta}_2 = C(\theta_2 - \theta_1)
\]

...(2.1)

and \( \theta_2 = \text{ Const.}, \quad \dot{\theta}_2 = 0 \quad \text{for } |\theta_2| = \theta_0 \)

Here, \( \theta_0 \) amplitude of the limit on the output angle,

\( J \) - moment of inertia of the inertial element,

\( B \) - coefficient of viscous damping,

\( C \) - coefficient of elasticity of the elastic link.

Assuming a harmonic signal \( M \sin \omega t \) applied at the input to the system the mutual relationship between the input ( \( \theta_1 \) ) and output ( \( \theta_2 \) ) of the system at low operating frequencies may be illustrated through figure 2.2.
Assuming that harmonic linearisation technique can be applied to the system $\theta_2$ may be expressed by the following equation:

$$\theta_2(t) = A_0 + A_1 \sin \omega t + B_1 \cos \omega t$$

...(2.2)

neglecting higher order terms of the Fourier Series.

Here,

$$A_0 = \frac{1}{2\pi} \int_{0}^{2\pi} \theta_2(t) \, d(\omega t)$$

$$A_1 = \frac{1}{\pi} \int_{0}^{2\pi} \theta_2(t) \sin \omega t \, d(\omega t)$$

$$B_1 = \frac{1}{\pi} \int_{0}^{2\pi} \theta_2(t) \cos \omega t \, d(\omega t)$$

...(2.3)

For the system under investigation

$$A_0 = 0$$

$$A_1 = \frac{2}{\pi} \left( \arcsin \frac{K}{1-K^2} \right) \quad \text{for } K < 1$$

$$= 1 \quad \text{for } K \geq 1$$

$$B_1 = 0$$

$$K = \frac{\theta_0}{M} = \sin \theta_a$$

...(2.4)
Variation of $A_1$ against $K$ is illustrated in figure 2.3.

In the dynamic regime of operation, e.g., at relatively higher frequencies the output signal may or may not attain the limiting value even though the input amplitude is higher than the limiting value, and depending on this, the portion of the system of automatic control under investigation may behave as a non-linear or linear system. The amplitude and phase of the system in absence of the limit depend not only on the amplitude of the input signal but on its frequency also. Naturally, it is expected that in presence of limit the amplitude and phase of the output signal will depend on the following three quantities: amplitude and frequency of the input signal, and the amplitude of the limit. Two modes of operation are obtained - Non-linear and linear. The first mode may be illustrated through figure 2.4. Here $\theta_2(t)/M$ attains unity at or before $\omega t = \pi$. When this condition is not satisfied the transient process becomes as shown in figure 2.5. Equation (2.1) is valid for the first mode and the solution of this equation is given in equation (2.5).

For the portion a - b - c:

$$\theta_2(t)/M = G \sin(\omega t - \theta_{0} + \psi) + e^{-\gamma \omega t} \left( P \cos \omega_2 t + Q \sin \omega_2 t \right)$$

...(2.5)

for the portion c - d: $\theta_2(t)/M = 1$. 
FIG 2.4 

FIG 2.5 

FIG-2.4 & 2.5 ILLUSTRATING DIFFERENT MODES OF NON-LINEAR OPERATION FOR INELASTIC COLLISION
Here, \( \omega_n = \sqrt{c/j} \), \( \gamma = B/2\sqrt{cJ} \)

\[
\omega_2 = \omega_n \sqrt{1 - \gamma^2}
\]

\[
\psi = \arctan \left[ \frac{2 \gamma \omega_n}{(\omega_n^2 - \omega^2)} \right]
\]

\[
\Delta = (\omega_n^2 - \omega^2)^2 + (2 \gamma \omega_n)^2
\]

\[
G = \frac{\omega_n^2}{\sqrt{\Delta}}
\]

\[
P = G \sin(\theta_a + \psi) - \theta_0/M
\]

\[
Q = \frac{\omega}{\sqrt{1 - \gamma^2}} P - \frac{\omega}{\omega_n \sqrt{1 - \gamma^2}} \cos(\theta_a + \psi)
\]

\[
\theta_a = \arcsin(\theta_0/M)
\]

Substituting (2.5) into (2.2)

\[
A_1 = \frac{4}{T} (g_1 + g_2 + g_3 + g_4) \quad \cdots (2.7)
\]

\[
B_1 = \frac{4}{T} (b_1 + b_2 + b_3 + b_4)
\]
Where, \( T = \frac{2\pi}{\omega} \)

\[
\begin{align*}
\theta_1 &= G \int_{0}^{T} \sin(\omega t - \theta_0 + \psi) \sin \omega t \, dt \\
&= G \left[ \frac{\cos \Phi(t_c - \frac{\sin 2\theta c}{2\omega})}{2} + \frac{\sin \Phi(\cos 2\theta c - 1)}{4\omega} \right] \\
\theta_2 &= P \int_{0}^{T} e^{-\gamma_0 t} \cos \omega t \sin \omega t \, dt \\
&= \frac{P}{2} \frac{1}{\omega_3^2 + (\gamma_0^2 \omega_1^2)^2} \left[ (\omega_3 \cos \omega_3 t_c - \gamma_0 \omega_1 \sin \omega_3 t_c)e^{-\gamma_0 t_c} - \omega_3 \right] \\
&+ \frac{P}{2} \frac{1}{\omega_4^2 + (\gamma_0^2 \omega_1^2)^2} \left[ (\omega_4 \cos \omega_4 t_c - \gamma_0 \omega_1 \sin \omega_4 t_c)e^{-\gamma_0 t_c} - \omega_4 \right] \\
\theta_3 &= Q \int_{0}^{T} e^{-\gamma_0 t} \sin \omega t \sin \omega t \, dt \\
&= -\frac{Q}{2} \frac{1}{\omega_5^2 + (\gamma_0^2 \omega_1^2)^2} \left[ (\omega_5 \sin \omega_5 t_c - \gamma_0 \omega_1 \cos \omega_5 t_c)e^{-\gamma_0 t_c} + \gamma_0 \right] \\
&+ \frac{Q}{2} \frac{1}{\omega_6^2 + (\gamma_0^2 \omega_1^2)^2} \left[ (\omega_6 \sin \omega_6 t_c - \gamma_0 \omega_1 \cos \omega_6 t_c)e^{-\gamma_0 t_c} + \gamma_0 \right] \\
\theta_4 &= \sin \theta_0 \int_{t_c}^{T/2} \sin \omega t \, dt \\
&= \sin \theta_0 (1 + \cos \theta_c) / \omega
\end{align*}
\]
\[
\begin{align*}
  b_1 &= G \int_0^t \sin(\omega t - \phi) \cdot \cos \omega t \cdot dt \\
        &= G \left[ \frac{\sin \phi}{2} \left( t - \frac{\sin 2\theta_e}{2\omega} \right) + \frac{\cos \phi}{4\omega} (\cos 2\theta_e - 1) \right] \\
  b_2 &= P \int_0^t e^{-\gamma \omega t} \cdot \cos \omega t \cdot \cos \omega t \cdot dt \\
        &= \frac{P}{2} \frac{1}{\omega^2 + (\gamma \omega)^2} \left[ (\omega_3 \sin \omega_3 t e^{-\gamma \omega_3 t} + \gamma \omega_3 \cos \omega_3 t e^{-\gamma \omega_3 t}) \right] \\
        &\quad + \frac{P}{2} \frac{1}{\omega_4^2 + (\gamma \omega_4)^2} \left[ (\omega_4 \sin \omega_4 t e^{-\gamma \omega_4 t} + \gamma \omega_4 \cos \omega_4 t e^{-\gamma \omega_4 t}) \right] \\
  b_3 &= Q \int_0^t e^{-\gamma \omega t} \cdot \sin \omega t \cdot \cos \omega t \cdot dt \\
        &= \frac{Q}{2} \frac{1}{\omega^2 + (\gamma \omega)^2} \left[ (\omega_3 \cos \omega_3 t e^{-\gamma \omega_3 t} - \gamma \omega_3 \sin \omega_3 t e^{-\gamma \omega_3 t}) \right] \\
        &\quad - \frac{Q}{2} \frac{1}{\omega_4^2 + (\gamma \omega_4)^2} \left[ (\omega_4 \cos \omega_4 t e^{-\gamma \omega_4 t} - \gamma \omega_4 \sin \omega_4 t e^{-\gamma \omega_4 t}) \right] \\
  b_4 &= \sin \theta_a \int_{t_0}^{t/2} \cos \omega t \cdot dt \\
        &= -\sin \theta_a \sin \theta_a / \omega.
\end{align*}
\]
where, \[ \Phi = \theta_a + \psi \]
\[ \omega_3 = \omega_n \sqrt{1 - q^2} + \omega = \omega_2 + \omega \]
\[ \omega_4 = \omega_2 - \omega \]

Hence, the amplitude and phase of the first harmonic are:
\[ A = M \sqrt{A_1^2 + B_1^2} \]
\[ \varphi = \arctan \frac{A_1}{B_1} - \theta_a \]

...(2.11)

When the system operates in the other non-linear region e.g., when (2.5) does not attain unity before and the condition \( G > 1 \) is satisfied, the nature of variation of output may be portrayed as in figure 2.5.

Here, for the portion \( a-b-c \):
\[ \frac{\theta_2(t)}{M} = G \sin(\omega t - \theta_a - \psi) + \]
\[ + (p' \cos \omega_2 t + q' \sin \omega_2 t) e^{-\gamma \omega_2 t} \]

...(2.12)

This equation must satisfy the following equality:
\[ \frac{\theta_2(t)}{\dot{M}} = 1 \text{ at } \omega t = \pi \]

...(2.13)

As distinct from the previous case, now \( \theta_a' \) changes with frequency. This quantity may be positive or negative so as to satisfy equation (2.13).

Replacing \( \theta_a \) by \( \theta_a' \) in equation (2.6),
\[ p' = G \sin(\theta_a' + \psi) - \theta_a / M \]
\[ q' = \frac{1}{\sqrt{1 - q^2}} p' - G \frac{\omega}{\omega_2} \cos(\theta_a + \psi) \]

...(2.14)
From equations (2.3) and (2.12):

\[ A'_1 = \frac{4}{T} (q'_1 + q'_2 + q'_3) \]

\[ B'_4 = \frac{4}{T} (b'_1 + b'_2 + b'_3) \]

\[ \ldots (2.15) \]

\[ g'_1 = G \int_0^{T/2} \sin(\omega t - \theta'_a - \psi) \sin \omega t \, dt \]

\[ = G \cos(\theta'_a + \psi) \cdot T/4 \]

\[ g'_2 = \rho' \int_0^{T/2} e^{-q \omega t} \cdot \cos \omega_2 t \cdot \sin \omega t \, dt \]

\[ = \frac{\rho'}{2} \frac{1}{\omega_2^2 + (q \omega_2)^2} \left[ (\omega_3 \cos \omega_3 T/2 - q \omega_2 \sin \omega_3 T/2) e^{-q \omega_3 T/2} - \omega_3 \right] \]

\[ + \frac{\rho'}{2} \frac{1}{\omega_4^2 + (q \omega_4)^2} \left[ (\omega_4 \cos \omega_4 T/2 - q \omega_2 \sin \omega_4 T/2) e^{-q \omega_4 T/2} - \omega_4 \right] \]

\[ \ldots (2.17) \]

\[ g'_3 = q' \int_0^{T/2} e^{-q \omega t} \cdot \sin \omega_2 t \cdot \sin \omega t \, dt \]

\[ = -\frac{q'}{2} \frac{1}{\omega_2^2 + (q \omega_2)^2} \left[ (\omega_5 \sin \omega_5 T/2 - q \omega_2 \cos \omega_5 T/2) e^{-q \omega_5 T/2} + \omega_5 \right] \]

\[ + \frac{q'}{2} \frac{1}{\omega_4^2 + (q \omega_4)^2} \left[ (\omega_4 \sin \omega_4 T/2 - q \omega_2 \cos \omega_4 T/2) e^{-q \omega_4 T/2} + \omega_4 \right] \]
\[ b_1' = G \int_0^{T/2} \sin(\omega t - \theta_0 - \psi) \cdot \cos \omega t \, dt \]
\[ = - G \sin(\theta_0 + \psi) \cdot T/4 \]

\[ b_2' = p' \int_0^{T/2} e^{-j\omega_n t} \cdot \cos \omega_2 t \cdot \cos \omega t \, dt \]
\[ = \frac{p'}{2} \frac{1}{\omega_2^2 + (j\omega_n)^2} \left[ (\omega_3 \sin \omega_3 T/2 - j\omega_n \cos \omega_3 T/2) e^{-j\omega_n T/2} + j\omega_n \right] \]
\[ + \frac{p'}{2} \frac{1}{\omega_4^2 + (j\omega_n)^2} \left[ (\omega_3 \sin \omega_4 T/2 - j\omega_n \cos \omega_4 T/2) e^{-j\omega_n T/2} + j\omega_n \right] \ldots (2.17) \]

\[ b_3' = q' \int_0^{T/2} e^{-j\omega_n t} \cdot \sin \omega_2 t \cdot \cos \omega t \, dt \]
\[ = \frac{q'}{2} \frac{1}{\omega_2^2 + (j\omega_n)^2} \left[ (\omega_3 \cos \omega_3 T/2 - j\omega_n \sin \omega_3 T/2) e^{-j\omega_n T/2} - \omega_3 \right] \]
\[ - \frac{q'}{2} \frac{1}{\omega_4^2 + (j\omega_n)^2} \left[ (\omega_4 \cos \omega_4 T/2 - j\omega_n \sin \omega_4 T/2) e^{-j\omega_n T/2} - \omega_4 \right] \]
In the non-linear region of operation, knowing to vide figure 2.4 , the values of terms on the left hand side of equations (2.8) and (2.9), and similarly, knowing \( \theta_\alpha \) (vide figure 2.5), - values of terms on the left hand side of equations (2.16) and (2.17) may be evaluated. Consequently, with the help of (2.7) and (2.11) or (2.15) and (2.11), as the case may be, the amplitude and phase of the output signal may be determined.

However, \( t_c \) or \( \theta_\alpha \) are not known beforehand. They may be determined by trial and error method. This gives rise to difficulty in the theoretical determination of amplitude-phase characteristics of the system. Qualitative results and amplitude-phase characteristics of the given system were obtained by analogue computer simulation. More accurate results were obtained from digital computer MINSK-22. These results tally (within the permissible range of variation) with those obtained from the analogue computer MN-7. The flow diagram for the digital computer programme is given in figure 2.6.

When the system operates in the linear region, e.g., when the output quantity does not attain the limiting value, then the amplitude and phase of the output signal is easily obtained from the solution of the linear differential equation (2.1).
FIG 2.6 SEQUENCE OF CALCULATION BY THE COMPUTER TO GET THE FIRST HARMONIC IN CASE OF INELASTIC COLLISION.
FIG 27 AMPLITUDE-PHASE CHARACTERISTIC OF ELASTIC LINK WITH VISCOS FRICTION AND INERTIA WITH LIMIT ON THE OUTPUT AMPLITUDE, FOR INELASTIC COLLISION (\( \delta = 1.0 \))
FIG 2.8 AMPLITUDE-PHASE CHARACTERISTIC OF ELASTIC LINK WITH VISCOSOUS FRICTION AND INERTIA WITH LIMIT ON THE OUTPUT AMPLITUDE, FOR INELASTIC COLLISION ($\xi = 0.8$)
FIG 2.9 AMPLITUDE-PHASE CHARACTERISTIC OF ELASTIC LINK WITH VISCOUS FRICTION AND INERTIA WITH LIMIT ON THE OUTPUT AMPLITUDE, FOR INELASTIC COLLISION (\( \xi = 0.6 \))
FIG.- 2.10 AMPLITUDE-PHASE CHARACTERISTIC OF ELASTIC LINK WITH VISCOUS FRICTION AND INERTIA WITH LIMIT ON THE OUTPUT AMPLITUDE, FOR INELASTIC COLLISION ($\xi=0.4$)
From the results obtained from digital computer, the amplitude vs. phase plots are made and illustrated in figures 2.7 to 2.12.

2.3. Interpretation of computer results for the case of inelastic collision.

Amplitude-phase characteristics, obtained from computer results for the combination "elastic link plus viscous friction plus inertia plus element with limit" under inelastic collision are shown in figures 2.7 - 2.12. Here, inphase and quadrature components of first harmonic of the output signal are plotted along X and Y axes respectively. Each family of characteristics corresponds to a given value of relative damping, and the individual characteristic of a particular family corresponds to a particular value of the limiting amplitude. The amplitude-phase characteristics are represented by continuous lines, whereas, the dotted lines are the curves for constant frequency.

The above mentioned characteristics reveal the following features.
(1) Presence of inelastic stop, which reduces the output velocity to zero immediately after collision modifies the amplitude and phase of the first harmonic of the output signal if the amplitude of the output signal in absence of the inelastic stop had exceeded the limiting value, that is, the presence of the stop is perceived under such condition only.

(2) The system may operate in the non-linear region even though the amplitude of the input signal is less than the limiting value, provided the first condition is satisfied.

(3) Even at frequencies higher than the resonance frequency, the phase of the first harmonic of output signal may remain less than 90°, if the input amplitude is higher than the limiting amplitude. This is explained by the fact, that if the limit had not existed, then the output would have reached its maximum value as determined from the linear relationship. However, in presence of the limit, upto a particular frequency, the output amplitude is limited, retains the limiting value for a certain period and then starts decreasing only when the input amplitude becomes less than the limiting value. This takes place at such a time when the input amplitude has not yet attained its zero value. Consequently, the peak of the input signal and peak of the first harmonic of output signal can not be separated by more than 90°. Therefore, under such
conditions the phase lag of the first harmonic of the output signal cannot be more than 90°. Here, equation (2.5) describes the operation of the system.

(4) For $M > \theta_0$ as soon as the phase of first harmonic of the output signal exceeds 90° it changes abruptly. This phenomenon can be explained by the fact that if the disagreement of the output amplitude from the limiting amplitude starts after the point $d$ (vide figure 2.4), then the maximum amplitude of the output signal will not afterwards reach the limiting amplitude and hence violates the initial assumption. For frequencies at which the output amplitude (considering the limit stop to be non-existent) is greater than the limiting amplitude, this, however, does not apply. Therefore, the interval between the points of disagreement of the input and output amplitude from the limiting amplitude starts becoming wider and wider as a result of which, the abrupt change in the phase of first harmonic of output signal becomes a probability.

(5) In presence of limit stop, if $M > \theta_0$ the amplitude of first harmonic of the output signal does not change appreciably so long its phase does not reach 90°. However, at lower frequencies the amplitude of first harmonic is somewhat greater than the limiting amplitude, because under such conditions the output remains constant for the portion $c - d$ (figure 2.4). The presence of this
portion explains the higher value of the first harmonic of the output signal.

(6) For the reasons mentioned in paragraphs (3) and (4) the curves of constant frequency (shown by dotted lines) show a stiff curvature to the left. In other words, the amplitude-phase characteristics approach the characteristics obtained in absence of limit stop as the effectiveness of the limit stop becomes less and less.

On the basis of the above mentioned characteristics the describing functions of the system are obtained. They are helpful in determining the equivalent scheme (linearised) for the whole of the given system of automatic control. The method of applying these characteristics is discussed in chapter III for the purpose of obtaining control algorithms.

2.4. Investigation with elastic limit stop.

As mentioned earlier, a practical limit stop will be somewhat inbetween the perfectly elastic and perfectly inelastic stop. The nature of collision with the metal to be cut, as well as the subsequent motion of the system may depend on several factors, such as speed of the table before the tool meets the stop, hardness of the metal, elasticity of the cutting tool and the elastic link, and so on. Investigation of operation with elastic stop is useful from the point of view of understanding and explaining the effect
FIG 2.13 ILLUSTRATING DIFFERENT MODES OF NONLINEAR OPERATION FOR ELASTIC COLLISION
of a practical stop and the operation is examined with this object.

As distinct from inelastic stop, here the energy of the system is not dissipated immediately after collision. Assuming that the tool holder is immovable and both the objects taking part in the collision are elastic, the table will start moving backwards after the collision. With such assumptions the equations of motion may be written down in the following way:

\[ J \frac{d^2 \theta}{dt^2} + B \frac{d\theta}{dt} = C(\theta_1 - \theta_2) \quad \text{for} \quad |\theta_2| < \theta_0 \quad \ldots (2.19) \]

Immediately after the collision, the initial conditions for the motion following the collision are,

\[ \theta_{20} = \pm \theta_0 \]
\[ \dot{\theta}_{20} = -\dot{\theta}_{20} \]

These initial conditions will be valid for collision at any instant. The nature of variation of \( \theta_2 \) for a sinusoidal input signal is shown in figures 2.13a and 2.13b. The difficulty of obtaining a theoretical solution to this problem lies in the fact that the value of \( \dot{\theta}_{20} \) is not known beforehand and that during any particular half cycle collisions may take place several times as illustrated in figure 2.13a, especially at lower frequencies.

For the above reason qualitative investigation was
FIG 2.14 SEQUENCE OF CALCULATION BY THE COMPUTER I
TO GET THE FIRST HARMONIC IN CASE OF ELASTIC COLLISION

START

IF \( \frac{\theta_0}{M} < 1 \)

YES \( S_1 \)

IF \( G \leq \theta_0/M \)

YES \( S_2 \)

USE LINEAR FORMULA

NO \( S_3 \)

GO TO \( S_2 \)

IF \( G \leq \theta_0/M \)

YES \( S_2 \)

NO \( S_3 \)

GO TO \( S_3 \)

SET INITIAL VALUES

SOLVE BY RUNGE KUTTA METHOD FOR 10 CYCLES

CALCULATE THE FIRST HARMONIC

CHANGE \( \omega \), GO TO \( S_0 \)

STOP
FIG-2.15 AMPLITUDE - PHASE CHARACTERISTIC OF ELASTIC LINK, VISCOUS FRICTION AND INERTIA WITH LIMIT ON OUTPUT AMPLITUDE, FOR ELASTIC COLLISION ($\xi=0.9$)
FIG. 216: AMPLITUDE-PHASE CHARACTERISTIC OF ELASTIC LINK, VISCOUS FRICTION AND INERTIA WITH LIMIT ON OUTPUT AMPLITUDE FOR ELASTIC COLLISION ($\gamma = 0.8$)
FIG-2.17 AMPLITUDE-PHASE CHARACTERISTIC OF ELASTIC LINK, VISCOUS FRICTION AND INERTIA WITH LIMIT ON OUTPUT AMPLITUDE, FOR ELASTIC COLLISION ($\xi=0.6$)
FIG 2.18 AMPLITUDE-PHASE CHARACTERISTIC OF ELASTIC LINK, VISCOUS FRICTION AND INERTIA WITH LIMIT ON OUTPUT AMPLITUDE, FOR ELASTIC COLLISION ($\xi=0.4$)
carried out in analogue computer. More accurate results were obtained through digital computer MINSK-22. Flow diagram of calculation is shown in figure 2.14.

Amplitude-phase characteristics are drawn from the digital computer results, which are shown in figures 2.15 to 2.19.

Under static condition ($\omega \approx 0$) equation (2.4) may be used. Further, formula (2.18) may be used if $g < 1$.

2.5. Interpretation of computer results for the case of elastic collision.

As already mentioned above, amplitude-phase characteristics, obtained with the help of digital computer, for the combination "elastic link + viscous friction + initial element + limiting element" for an elastic limit stop are shown in figures 2.15 to 2.19. Here also, as in case of inelastic collision, real and quadrature parts of the first harmonic of the output are plotted along X and Y-axes respectively. Each family of characteristics is related to only one value of relative damping, each characteristic corresponding to a particular value of limiting amplitude. Amplitude-phase characteristics are drawn in continuous lines, the
frequencies being indicated on the curves.

The characteristics for an elastic limit stop reveal the following peculiarities.

1) Presence of elastic limit stop modifies the amplitude and phase of first harmonic of the output only when the maximum amplitude of the output signal is more than the limiting amplitude if the limit did not exist i.e., when \( G > \theta_0 \). Then the system operates in the non-linear region.

2) If the input peak has an amplitude greater than the limiting amplitude, then the phase of the first harmonic of the output may remain less than \( 90^\circ \) even at frequencies higher than the resonance frequency. This can be explained from the fact that as soon as the output signal reaches the limit its derivative immediately changes sign and this phenomenon takes place at an instant before the input signal reaches its peak value. Consequently, the phase lag becomes limited to a value less than \( 90^\circ \).

3) Because of collision the output in this case differs from the linear case. In a certain range of frequencies, the first harmonic of the output signal may be less than the limiting amplitude, even though the presence of the limit is felt by the system. This is explained by the presence of sharp peaks in the output waveform for this range of frequencies. This thing can
take place only in case of elastic collision since the first harmonic amplitude does not change appreciably unless the phase of the output signal reaches $90^\circ$.

4) As soon as $A$ (as determined by equation 2.18) falls below $\theta_0$, the amplitude-phase characteristics suddenly turns leftwards and follows the linear nature since the presence of the limiting amplitude is no longer to be felt by the system, in whose dynamics we are interested in.

5) Due to the reasons mentioned above in paragraphs (2) and (4) the constant frequency lines abruptly turns leftwards. In other words, the polar plot approaches the linear characteristic as the effectiveness of the limit goes down.

As mentioned in case of inelastic collision, here also the output oscillations and the equivalent block diagram for the whole system for a given value of input amplitude may be obtained by means of these characteristics. They are useful in analysing the dynamic operation of the system under consideration.