CHAPTER V

Test of Quark Model and Determination of the Multipole Amplitudes above Near-threshold Energy Region*


Presented at the Symposium on 'Particles and Fields' held at the University of Madras, 30 Dec.'70-4 Jan.'71.
In Chapter III we discussed empirical determination of the multipole amplitudes $B_{0+}^q$, $B_{1+}^q$, $M_{1+}$ and $M_{1-}$ at near-threshold energy region (below 230 MeV) from the already available data for the differential cross-sections for $n^0$ meson photoproduction of spin zero nuclei like $^{12}$C, $^{16}$O etc. Whereas the multipole amplitudes at near-threshold (below 230 MeV) were all assumed to be real, the imaginary part of the multipole amplitudes are to be taken into consideration above the near-threshold region (above 230 MeV). For the empirical determination of the multipole amplitudes in the energy region we suggest to make the measurements for the elastic differential cross-sections for $n^0$ meson photoproduction on complex nuclei like $^{12}$C, $^{16}$O above 230 MeV (we note that below 200 MeV experimental data are there) along with the measurements of the inelastic differential cross-sections for $n^0$ meson photoproduction on $^9$Be leading to the second excited state at 2.43 MeV.

In this context we note that the knowledge of the photopion production multipoles allows a direct test of any symmetry scheme or quark model classification of the resonances. To date the only quantitative

---

only quantitative test of the quark model in photo-
pion production is at the first resonances: the
electric quadrupole transition \( E_{1+} \) to the final \( P_{33} \)
pion-nucleon state should vanish if the quark model
is to hold (the result at the first resonance are
also a consequence of \( SU(6) \) and \( SU(6)_{\nu} \)). The vanishing
of \( E_{1+} \) is usually predicted from the measurement
of \( I_0/C \) which is very sensitive to the ratio \( E_{1+}/M_{1+} \).
In fact, Becchi and Morpurgo (4) have shown that a value
of the ratio \( E_{1+}/M_{1+} \) much smaller than 0.17 should be
considered as a test of the quark model. But from the
measurement of \( I_0/C \) one cannot without much difficulty
infer if \( E_{1+} \) is equal to or different from zero. The
correct procedure demands one to relay on a set of
multipoles given by the theorists and then compare
the initial value of \( I_0/C \) (obtained with and without
the position \( E_{1+}^{(3)} = 0 \)) with the experimental value (5).
So far there is no direct way of measuring \( E_{1+} \) alone
at 33-resonance. In this note we also note the impor-
tance of the measurements of the elastic differential
cross-sections for \( \eta \) meson photoproduction on complex

(2) F. A. Bollits and D. G. Sutherland, Phys. Rev. 146(1966) 1190.
(3) A. Bonnachie and G. Shaw, Nucl. Phys. 87(1966) 556.
(5) G. Barbierlini et al., Frascati Preprint, INFN-69/2(1969)
nuclei along with the measurements of the inelastic
differential cross-sections for $\pi^0$ meson photoproduction
on $^9$Be leading to the second excited state at 2.43 MeV,
both the measurements being made at 350 MeV, in providing
a more direct method to determine $E_{1+}$ amplitude and thus
give a test for the quark model.

We note from eqn. (1.25) of Chapter I that the
inelastic differential cross-sections for $\pi^0$ meson photoproduction
on $^9$Be leading to the second excited state at
2.43 MeV with spin 5/2 is given by (with unpolarized
photons).

\[
(5.1) \left( \frac{d\sigma}{d\omega} \right)_{S_L} = \frac{9}{2\pi} \sum_{\ell} \left| Q \right|^2 \frac{1}{k^2}
\]

where

\[
(5.2) \left| Q \right|^2 \frac{1}{k^2} = \frac{21}{4} \left( \frac{F_{pp}}{225} \right)^2 \left( 1609 \left| k \right|^2 \left| k^* \right|^2 + 827 \left| k \cdot k^* \right|^2 \right)
\]

and

\[
(5.3) \frac{1}{2} \sum_{\ell} \left| k \cdot k^* \right| = \frac{21}{2} \left( \frac{d\sigma}{d\omega} \right) - \frac{1}{2} \sum_{\ell} LL^*
\]

From eqn. (3.25) we see

\[
(5.4) \frac{1}{2} \sum_{\ell} LL^* = \frac{21}{2} \left( \frac{d\sigma}{d\omega} \right) A(\text{form-factor term})^{-1}
\]

Thus,
In the above equations \( \frac{d\sigma}{d\Omega} \) is the \( \pi^0 \) photoproduction cross-sections with unpolarized photons on nucleon and \( \frac{d\sigma}{d\Omega}^A \) is the differential cross-sections for \( \pi^0 \) meson photoproduction on spin zero nuclei (A) with unpolarized photons.

From eqns. (5.1), (5.2) and (5.4) we obtain

\[
\begin{align*}
(5.6) \quad \frac{1}{2} \sum |\hat{K} \cdot \hat{k}|^2 &= 11.66 \left( \frac{1}{8\pi} \right)^2 \frac{2}{9} \frac{d\sigma}{d\Omega}^\rho \frac{d\sigma}{d\Omega}^\rho \\
&- 1.94 \left( \frac{d\sigma}{d\Omega}^A - 2 \frac{d\sigma}{d\Omega} \right) \text{(form factor term)}^{-1}
\end{align*}
\]

Further, since the spin-dependent part \( \hat{K} \) of the photoproduction amplitude is given by

\[
\hat{K} = \hat{\tau} (\hat{F}_1 - \hat{F}_2 \cos \theta) + \hat{\sigma} (\hat{\tau} \cdot \hat{\tau}) (\hat{F}_1 + \hat{F}_3) + \hat{\sigma} (\hat{\tau} \cdot \hat{\tau}) \mathcal{F}_4
\]

and \( \hat{K} = (\hat{\tau} - \hat{\sigma}) \)

We find \( \frac{1}{2} \sum \left| \hat{K} \cdot \hat{k} \right|^2 = \left\{ \frac{2}{9} \right\} \frac{d\sigma}{d\Omega}^\rho \frac{d\sigma}{d\Omega}^\rho + \left\{ \frac{2}{9} \right\} \frac{d\sigma}{d\Omega}^A - \left\{ \frac{2}{9} \right\} \text{(form factor term)}^{-1}

\[
\begin{align*}
(5.7) \quad \frac{1}{2} \sum |\hat{K} \cdot \hat{k}|^2 &= \left\{ \frac{2}{9} \right\} \left[ \frac{2}{9} \left( \frac{2}{9} - \cos \theta \right) \Re \hat{F}_2 \hat{F}_3 - 2 \left( \frac{2}{9} - \cos \theta \right) \Re \hat{F}_3 \hat{F}_2 - \frac{2}{9} \sin \theta \right] \\
&- \frac{2}{9} \left( \frac{2}{9} - \cos \theta \right) \Re \hat{F}_3 \hat{F}_2 \left( \frac{2}{9} \right) \left( \frac{2}{9} - \cos \theta \right) \\
&+ \frac{2}{9} \left( \frac{2}{9} - \cos \theta \right) \Re \hat{F}_3 \hat{F}_2 \left( \frac{2}{9} \right) \left( \frac{2}{9} - \cos \theta \right)
\end{align*}
\]
Using eqns. (4.5) to (4.8) for the amplitudes $F_i$, the eqn. (5.7) and making the following expansion

$$
\left\{ \frac{e^{2i\alpha\theta}}{2u_0^2 + 2u_2^2 - 2uv\cos\theta} \right\}^{-1} \frac{\beta}{2} \Sigma |\vec{k}\cdot\vec{K}|^2 =

(5.8)

= A' + B' \cos\theta + C' \cos^2\theta + \ldots

We get

(5.9) \quad C' = 36 |E_{1+}|^2

Thus, from eqn. (5.6) we find that if the measurements for the differential cross-sections for inelastic photon production of $\pi^0$ meson on $^9$Be leading to the second excited state and for the elastic differential cross-sections for $\pi^0$ meson photo-production on spin zero nuclei (A) ($^{12}$C and $^{16}$O, for example) are both made at the 3.3-resonance energy, then using the available differential cross-sections data for $\pi^0$ meson photo-production on nucleons we can directly determine $E_{1+}$ amplitude from the expansion (5.8). Thus, the above experiments at first resonance would give a test for the quark model, which predicts vanishing of $E_{1+}$ at this energy.
Now as regards the determinations of other multipole amplitudes we note from eqns. (3.10) and (3.15) of Chapter III that

\[ |E_{0+}|^2 = A - I_0 - \left| (2M_{1+} + M_{1-}) \right|^2 \]

Using eqn. (3.3) and eqn. (3.26) and noting from eqn. (4.14) that

\[ \Sigma(\theta) = \frac{-I_0 \sin^2 \theta}{\frac{\sigma}{2} \left( \frac{d\sigma}{d\theta} \right)(\theta)} \]

and

\[ \Sigma(90^\circ) = \frac{-I_0}{\frac{\sigma}{2} \left( \frac{d\sigma}{d\theta} \right)(90^\circ)} \]

We get

\[ |E_{0+}|^2 = \frac{\sigma}{2} \left( \frac{d\sigma}{d\theta} \right)(90^\circ) - \frac{\sigma}{2} \left( \frac{d\sigma}{d\theta} \right)(90^\circ) \Sigma(90^\circ) - \frac{2\sigma}{2} \left( \frac{d\sigma}{d\theta} \right) A(90^\circ) \text{(form factor term)}^{-1} \]

where contributions due to \( M_{1+}^{(0)} \) and \( M_{1-}^{(0)} \) have been neglected.

In the righthand side of (11) all the quantities except the differential cross-sections for \( \pi^0 \) meson photoproduction on \(^{12}\)C in the energy range 250 MeV to about 400 MeV are available. Therefore, when
the experimental measurement of the differential cross-
sections for \(\vec{\pi}\) meson photoproduction on spin zero nuclei
are made in this energy range we can determine the
amplitude \(E_{0+}\) empirically using the relation (5.12).

From eqns. (3.10) and (3.12) we note the other
important relation

\[
(5.13) \left| M_{1-} \right|^2 = A + \frac{c}{3} - \left| E_{0+} \right|^2 - 6 \left| E_{1+} \right|^2 - 2 \left| M_{1+} \right|^2
\]

In the right-hand side of eqn. (5.13) \(E_{0+}\) is
known from eqn. (5.12) whereas \(E_{1+}\) is determined as
in eqn. (5.9). If we use the theoretical values of
the amplitude \(M_{1+}\), then from eqn. (5.13) we can
determine empirically the most uncertain amplitude
\(M_{1-}\) in this energy range.