CHAPTER 4.

XIII. Jet issuing in all directions from a thin slit in a circular cylinder of small radius containing liquid under pressure.

Two infinite circular cylinders of equal radii are placed near each other with their axes along the axis of and contains liquid under pressure. The liquid issues through the opening in all directions and mixes with the surrounding liquid of the same kind at rest. We take the origin on the axis at the level of the slit and use cylindrical coordinates \( r, \theta, z \). The motion in the jet is symmetrical about the \( z \)-axis.

Let \( u, v, w \) be the components of velocity in the directions \( r, \theta, z \) and \( w \) are independent of \( \theta \). Assuming the motion to be steady, Stokes-Navier equations in cylindrical coordinates reduce to two, viz.,

\[
\begin{align*}
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} &= 0 \\
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{\partial^2 v}{\partial z^2} &= 0
\end{align*}
\]

The equation of continuity is

\[
\frac{\partial u}{\partial r} + \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0
\]

Assuming \( w \) to be of \( \frac{1}{r} \) in the jet, we see from the equation of continuity that

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0
\]

Therefore \( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \) where \( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \) is the thickness of the jet.

Since \( w \) is within the jet.

The left hand side of the equation (13.1) is therefore of
On the right hand side the terms \( \frac{\partial^2 u}{\partial t^2} \) are of \( \gamma \) while \( \gamma \frac{\partial u}{\partial t} \) is of \( \gamma \). Therefore the coefficient of \( t \) on the right hand side of equation (13.1) is of order \( \gamma \). If the motion in the jet is to be influenced by viscosity, it must be of \( \gamma \). Hence the first equation of motion reduces to

If we consider now the second equation of motion, its left hand side is seen to be of order \( \gamma \). On the right hand side \( \gamma \frac{\partial u}{\partial t} \) are of \( \gamma \), while \( \gamma \frac{\partial v}{\partial t} \) is of \( \gamma \); so that

is of \( \gamma \). Hence we get from the second equation of motion

If we neglect terms of \( \gamma \) the pressure in the jet can be considered to be a function of \( \gamma \) alone, and for a given value of \( \gamma \) it is given by the pressure of the liquid just outside the jet. Since we consider the liquid outside the jet to be at rest, we have \( \gamma \). Therefore the equation of motion in the jet becomes

\[
(13.4)
\]

and the equation of continuity is

\[
(13.5)
\]

If \( \psi \) be the stream function of the flow in the jet.
To obtain a similarity solution of this equation, we assume
\[ \psi = \alpha z^r \]  
where \( \alpha, r \) are constants. Then
\[ r_i L_i = \ldots \]  
(13.7)

Substituting in the equation (13.4) and simplifying we get
\[ \alpha, r \]  
(13.8)

In order that this equation may reduce to an ordinary
differential equation in \( \alpha \) and only, we must have
\[ \ldots \]  
(13.9)

The momentum which crosses a section of the jet perpendi-
cular to \( x \) between two axial planes inclined at a small angle \( \omega \) is
\[ \ldots \]  

Since the motion is steady and the pressure is constant,
\( \omega \) must be independent of \( \alpha \). Therefore
\[ \ldots \]  
(13.10)

From (13.9) and (13.10), we have
\[ \ldots \]  
(13.11)
Substituting in (13.8) we get

\[ \frac{\partial}{\partial y} \left( \frac{\partial G}{\partial y} \right) = \frac{\partial^2 \phi}{\partial y^2} \]

Taking \( \psi \) we see that \( f \) satisfies the equation

\[ f = \text{constant} \quad \text{when} \quad x = 0, \quad \text{i.e.,} \]

Integrating (13.11) with these conditions, we get

This equation is identical with the corresponding equation given by Bickley in his solution of the problem of the two-dimensional jet. Integrating again we get

\[ \phi = \text{constant} \]

or

On substituting

this equation reduces to

which gives

Hence

Since \( \phi = \text{constant} \) we have \( \psi = \text{constant} \) when \( x = 0 \), i.e., when \( x = 0 \) is satisfied. We have, therefore
Replacing \( i \), \( \beta \) by \( a \), \( \alpha \) respectively, we do not alter the value of the ratio \( \frac{A}{i} \). This means that we can take \( i = A \). We then have

\[
- \end{align*}
\]

Therefore, if \( y \) is prescribed

\[
Hence
\]

The total flux of mass across the surface of the cylinder constant, is