CHAPTER 4

The objective of the
in Chapter 4 was to present
voltage response of the mul
shock-excited by a train of
current pulses through it.
by the response function is
output waveforms of the exp
presented in the next and t

In this chapter, variation
possible modes of operation

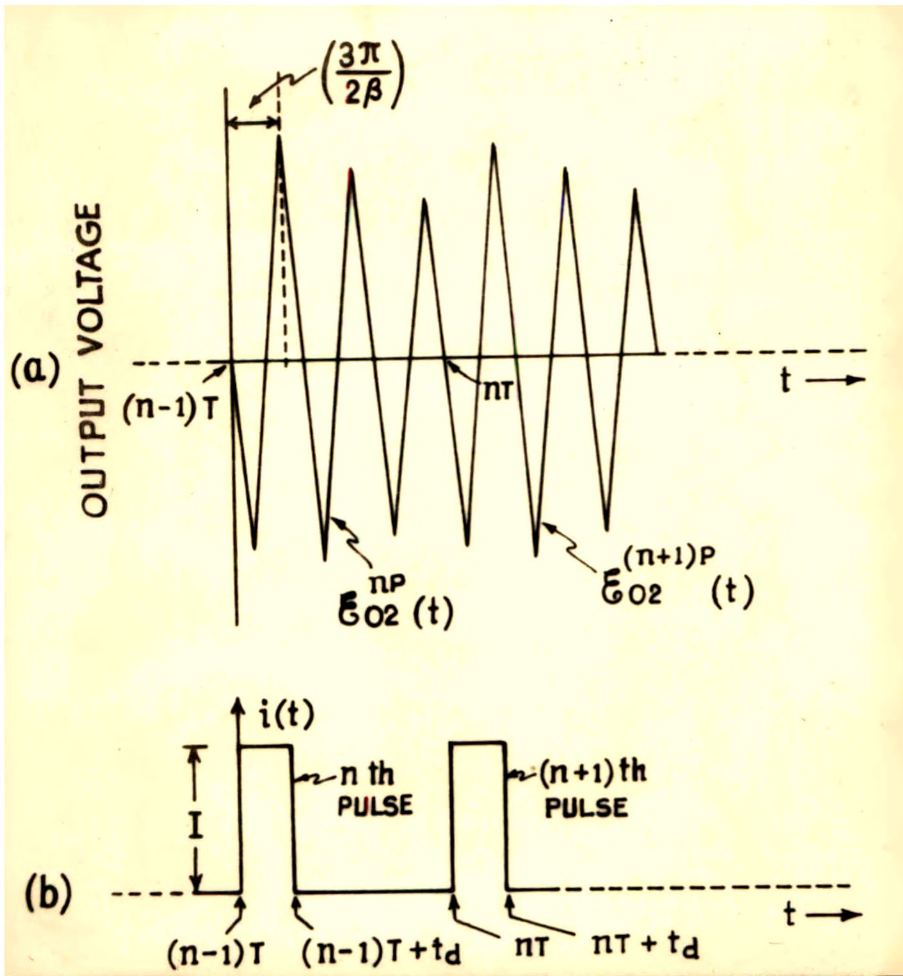

Fig.4.1 Schematic diagram of the multiplier output (a) excited by the nth and (n + 1)th current pulses (b).

4.1 Calculation of the peak amplitude of the first positive half cycle of the multiplier output

The purpose of this calculation is to provide a theoretical understanding of the variation of the output in two different modes of operation of the multiplier (see Section 4.2).

As represented by Eq. (3.43), Chapter 3, the voltage response of the multiplier output circuit to a current pulse train of 'n' rectangular pulses (cf. Fig. 3.5), is a sinusoidal oscillation of decaying amplitudes between its excitation pulses. Hence, the variation of the output has been studied in terms of the peak amplitude of the first positive half cycle of the output.

Figure 4.1a represents schematically the output voltage waveform of the multiplier, shock-excited by the n th and (n + 1) th pulses in the current pulse train of Fig. 4.1b. It is evident from Fig. 4.1 that, the peak amplitude of the first positive half cycle of the decaying sinusoid excited by the n th current pulse, occurs at $t = \left[(n-1)T + 3\pi/2\beta\right]$.

Substituting for $t = \left[(n-1)T + 3\pi/2\beta\right]$ in Eq. (3.43), the peak amplitude of the first positive half cycle becomes

$$A_1 = \left(\frac{I}{C\beta}\right)\left[\frac{e^{\alpha t_d}+1}{e^{\alpha T}-1}\right]\left(e^{n\alpha T}-1\right)e^{-\left[(n-1)T+3\pi/2\beta\right]\alpha}$$

$$= \left(\frac{I}{C\beta}\right)\left[\frac{e^{\alpha t_d}+1}{1-e^{-\alpha T}}\right]\left(1-e^{-n\alpha T}\right)e^{-(3\pi/2\beta)\alpha} \qquad \ldots \quad (4.1)$$

For large values of 'n', $\bar{e}^{n\alpha T}$ becomes very small and can be neglected.

Thus

$$A_1 = \left(\frac{I}{C\beta}\right)\left[\frac{e^{\alpha t_d}+1}{1-\bar{e}^{\alpha T}}\right] e^{-(3\pi/2\beta)\alpha}$$

... (4.2)

Furthermore, in the interesting resonance case ($Q_r \gg 5$), β becomes almost equal to ω_r (the natural resonant frequency of the multiplier output circuit).

So that

$$A_1 = \left(\frac{I}{C\omega_r}\right)\left[\frac{e^{\alpha t_d}+1}{1-\bar{e}^{\alpha T}}\right] e^{-(3\pi/2\omega_r)\alpha}$$

... (4.3)

Also

$$\begin{aligned}(3\pi/2\omega_r)\alpha &= (3\pi/4\pi f_r)(\pi f_r/Q_r) \\ &= (3/4)\delta\end{aligned}$$

... (4.4)

where $\delta\left(=\pi/Q_r\right)$ is the logarithmic decrement of the oscillation.

Thus, the expression (4.3) finally becomes

$$A_1 = \left(\frac{I}{C\omega_r}\right)\left[\frac{e^{\alpha t_d}+1}{1-\bar{e}^{\alpha T}}\right] e^{-(3/4)\delta}$$

... (4.5)

4.2 Study of the modes of operation of the multiplier

The multiplier output as represented by Eq. (4.5), is evidently a function of the two independent variables ω_k and T. Hence, two independent modes of operation of the multiplier are possible. The two operations are described below with all possible details.

(a) First Mode (I, ω_k , α , δ and t_d remaining constant)

Here, the multiplier output circuit is kept resonant at frequency ω_k . The desired harmonics are then obtained by suitable adjustments of the excitation current pulse frequency 1/T or the input pulse frequency. In this typical operation of the multiplier, input frequency is thus a submultiple of the output frequency ω_k . Because of constant ω_k , the outputs are developed at constant impedance level. This makes possible the study of the multiplier output voltages as a function of the harmonic number.

Now as shown in Chapter 3, the multiplier outputs will be a continuous oscillation of frequency ω_k provided that phase relationships (3.21) and (3.29) are satisfied simultaneously. In this operational mode of the multiplier, the output frequency is kept constant at ω_k . Hence, according to Eq. (3.21), the value of 't_d' requires to be adjusted to $t_d = \pi/\omega_k$. Furthermore, for all integral values of ratio (f_r/f_{in}), the phase angle

βT or $2\pi \times (f_r/f_{in})$ remains an even integer of π. Thus for $t_d = \pi/\omega_k$ and for the integral values of ratio (f_r/f_{in}), both the conditions remain satisfied in this mode of operation of the multiplier. It is then clear from Eq. (4.5) that the peak amplitude 'A_1' of the harmonic voltages decreases in magnitude by a factor $1/[1-e^{-\alpha T}]$ or $1/[1-e^{-\delta(f_k/f_{in})}]$ with the increase in the value of the ratio (f_r/f_{in}) — the harmonic order.

The practical design and operation of the multipliers, operated in this mode, is simple and straightforward in as much as the multiplier operation is possible with all variables, except the input frequency, held intact.

(b) <u>Second Mode</u> (I and T remaining constant)

This is the conventional method of generating the harmonics. Here, the excitation current pulse frequency $1/T$ or the input pulse frequency is kept constant. The harmonic outputs are then obtained by tuning the multiplier output circuit to the harmonics of the input pulse frequency. In this operation, it should be noticed from Eq. (4.5) that $1/[1-e^{-\delta(f_k/f_{in})}]$ is again the major factor (another factor sometimes is $e^{-(3/4)\delta}$) responsible for lowering the multiplier output at harmonic frequencies. This is because 'αT' which is equal to ($\pi f_r/Q_r f_{in}$) increases with the order of the harmonic. Nevertheless, this effect is likely to be masked by the tendency of the factor $1/\omega_k C$ to increase with the order of the harmonic.

As said above, in this operational mode, the harmonic outputs are obtained by tuning the multiplier output circuit to the harmonics of the input pulse frequency. Thus to satisfy the phase condition (3.21), each time the circuit is tuned to a new harmonic, 't_d' should be changed to $t_d = \pi/\omega_{rn}$ where ω_{rn} is the frequency of that harmonic. Condition (3.29) is satisfied for all integral values of the ratio (f_{rn}/f_{in}).

The actual variation of the multiplier harmonic output is thus a complex function of the variables α, δ, ϵ and t_d (all vary with ω_r). Hence, it depends on the compromise between the effects due to factors $1/\omega_r C$, $1/[1-e^{-\alpha T}]$ and to a smaller extent on $[e^{\alpha t_d}+1]$ and $e^{-(3/4)\delta}$.

Now since

$$\alpha = \frac{\pi f_r}{Q_r} = \frac{\pi f_r}{2\pi f_r C Z_L} = \frac{1}{2 C Z_L}$$

$$\left(Z_L = \frac{L}{RC} = \frac{Q_r}{\omega_r C}\right) \qquad \ldots \quad (4.6)$$

the variations of $[e^{\alpha t_d}+1]$ and $1/[1-e^{-\alpha T}]$ will also depend on the variation of 'C' and 'Z_L' (multiplier output impedance). Because of the dependence of 'Z_L' on frequency, a study of the variation of the harmonic current rather than voltage with harmonic frequency has been pursued.

The practical design of the multiplier for this mode of operation, is not simple. This is because 't_d' requires adjustment each time a new harmonic is attempted.