CHAPTER 1

Computation of the Fourier Series Coefficients of some Waveforms used in the study of Harmonic Generators

In this chapter, the Fourier Series Coefficients A_n, B_n, C_n and (A_{av}) of the Fractional sine wave, Isosceles Triangular and Rectangular pulse waveforms have been computed by the Fourier Analysis method. Such a computation is essential in studying the harmonic response of the Harmonic Generators.

The computation finds useful application in the next chapter. It is also made use of in the subsequent chapters in the interpretation of some experimental results.

1.1 Computation of the Fourier Coefficient pulse waveform - (General)

The method of calculating the Fourie A_n , B_n and (A_{av}) for a repeating fu such as a series of pulses with the pulse r $f_p = \frac{1}{T}$, consists in representing the fu Fourier series

$$f(t) = (A_{av}) + \sum_{n=1}^{\infty} \left(A_n \cos n\omega_p t \; + \right.$$

where the coefficients A_n , B_n and by

$$(A_{av}) = \frac{1}{T}\int_0^T f(t)\, dt$$

$$\left.\begin{matrix} A_n \\ B_n \end{matrix}\right\} = \frac{2}{T}\int_0^T f(t) \left\{\begin{matrix} \cos n\omega_p t \\ \sin n\omega_p t \end{matrix}\right\} ($$

Eq. (1.1) may also be expressed as

$$f(t) = (A_{av}) + \sum_{n=1}^{\infty} C_n \sin\left(n\omega_p t + \right.$$

where

$$C_n = \sqrt{A_n^2 + B_n^2}$$

$$\phi_n = \tan^{-1}\left(\frac{A_n}{B_n}\right)$$

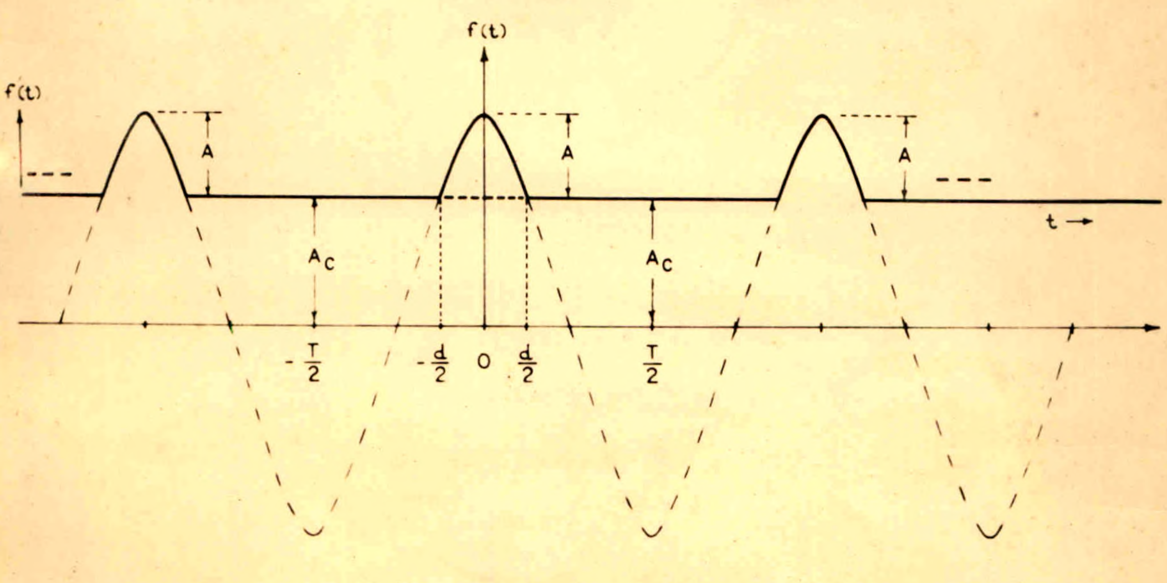

FIG. 1.1 FRACTIONAL SINE WAVE PULSE WAVEFORM OF AMPLITUDE A, PERIOD T AND DURATION d

The coefficients (A_{av}) and C_n in Eqs. (1.2) and (1.5) denote respectively the average or the d-c value of the pulse and the amplitude of the n th harmonic. Their values depend on the shape, amplitude, duration and period of the pulse form.

In what follows, the Fourier Coefficients $\left[(A_{av}),\ C_n\right]$ of the Fractional sine wave, Isosceles Triangular and Rectangular pulse wave forms will be found.

1.2 Computation of A_n, B_n, C_n and (A_{av}) of some periodic pulse wave forms used in the study of harmonic generators

(a) Fractional sine wave pulse wave form

Consider the pulse wave form of Fig. 1.1. We can represent the function $f(t)$ in the period from $-T/2$ to $+T/2$ by the relations

$$
\begin{aligned}
f(t) &= A_c\,, && \text{for } -T/2 \leqslant t \leqslant -d/2 \\
f(t) &= (A+A_c)\cos\left(\frac{2\pi t}{T}\right), && \text{for } -d/2 \leqslant t \leqslant d/2 \\
f(t) &= A_c\,, && \text{for } d/2 \leqslant t \leqslant T/2
\end{aligned}
\qquad \ldots \quad (1.7)
$$

Substituting from Eqs. (1.7), we have

$$
(A_{av}) = \frac{1}{T}\left[\int_{-T/2}^{-d/2} A_c\,dt + \int_{-d/2}^{d/2} (A+A_c)\cos\left(\frac{2\pi t}{T}\right)dt + \int_{d/2}^{T/2} A_c\,dt\right] \qquad \ldots \quad (1.8)
$$

$$= \frac{1}{T}\left\{ \left| A_c \cdot t \right|_{-T/2}^{-d/2} + \left| (A + A_c)\cdot\frac{T}{2\pi}\cdot \sin\left(\frac{2\pi t}{T}\right)\right|_{-d/2}^{d/2} + \left| A_c \cdot t \right|_{d/2}^{T/2} \right\}$$

$$= \frac{1}{T}\left[A_c\left(T/2 - d/2\right) + \frac{T}{\pi}(A + A_c)\sin\left(\frac{\pi d}{T}\right) + A_c\left(T/2 - d/2\right)\right]$$

$$= \left[\frac{1}{\pi}(A + A_c)\sin\left(\frac{\pi d}{T}\right) + A_c\left(1 - \frac{d}{T}\right)\right]$$

or

$$= \frac{1}{\pi}\left[(A + A_c)\sin\left(\frac{\pi d}{T}\right) + A_c\left(\pi - \frac{\pi d}{T}\right)\right] \qquad \ldots \qquad (1.9)$$

Now reference to Fig. 1.1 shows that

$$A_c = (A + A_c)\cos\left(\frac{\pi d}{T}\right)$$

Therefore,

$$A_c = \left[\frac{A\cos\left(\frac{\pi d}{T}\right)}{1 - \cos\left(\frac{\pi d}{T}\right)}\right] \qquad \ldots \qquad (1.10)$$

Substituting Eq. (1.10) in Eq. (1.9) gives

$$A_{av} = \frac{1}{\pi}\left\{ A\sin\left(\frac{\pi d}{T}\right) + \left[\frac{A\cos\left(\frac{\pi d}{T}\right)}{1 - \cos\left(\frac{\pi d}{T}\right)}\right]\sin\left(\frac{\pi d}{T}\right) + \left(\pi - \frac{\pi d}{T}\right)\left[\frac{A\cos\left(\frac{\pi d}{T}\right)}{1 - \cos\left(\frac{\pi d}{T}\right)}\right]\right\}$$

$$= \frac{A}{\pi\left[1 - \cos\left(\frac{\pi d}{T}\right)\right]}\left\{ \left[\sin\left(\frac{\pi d}{T}\right)\right]\left[1 - \cos\left(\frac{\pi d}{T}\right)\right] + \sin\left(\frac{\pi d}{T}\right)\cos\left(\frac{\pi d}{T}\right) + \pi\cos\left(\frac{\pi d}{T}\right) - \left(\frac{\pi d}{T}\right)\cos\left(\frac{\pi d}{T}\right)\right\}$$

$$= \frac{A}{\pi\left(1 - \cos\frac{\pi d}{T}\right)}\left[\sin\left(\frac{\pi d}{T}\right) - \left(\frac{\pi d}{T}\right)\cos\left(\frac{\pi d}{T}\right) + \pi\cos\left(\frac{\pi d}{T}\right)\right] \qquad \ldots \quad (1.11)$$

Again

$$f(t) = A_c + f'(t) \qquad \ldots \quad (1.12)$$

Therefore,

$$f'(t) = -A_c + f(t) \qquad \ldots \quad (1.13)$$

Now since the periodic function as shown in Fig. 1.1 possesses symmetry with respect to vertical axis at $t = 0$, all coefficients B_n must vanish. Its Fourier expansion can thus contain only the d-c term and cosine terms.

Therefore,

$$f'(t) = \left[-A_c + A_{av}\right] + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2n\pi t}{T}\right) \qquad \ldots \quad (1.14)$$

Thus, from Eq. (1.14) we have

$$(A_{av})' = (A_{av}) - A_c \qquad \ldots \quad (1.15)$$

$$= \frac{A}{\pi\left[1 - \cos\left(\frac{\pi d}{T}\right)\right]}\left[\sin\frac{\pi d}{T} - \frac{\pi d}{T}\cos\frac{\pi d}{T} + \pi\cos\left(\frac{\pi d}{T}\right)\right] - \left[\frac{A\cos\left(\frac{\pi d}{T}\right)}{1 - \cos\left(\frac{\pi d}{T}\right)}\right]$$

or

$$= \frac{A}{\pi\left(1 - \cos\frac{\pi d}{T}\right)}\left[\sin\frac{\pi d}{T} - \frac{\pi d}{T}\cos\frac{\pi d}{T}\right] \qquad \ldots \quad (1.16)$$

Again, substituting from Eqs. (1.7), we have

$$A_n = \frac{2}{T}\left[\int_{-T/2}^{-d/2} A_c \cos\left(\frac{2\pi nt}{T}\right) dt + \int_{-d/2}^{d/2} (A + A_c) \cos\left(\frac{2\pi nt}{T}\right) \cos\left(\frac{2\pi t}{T}\right) dt + \int_{d/2}^{T/2} A_c \cos\left(\frac{2\pi nt}{T}\right) dt\right] \qquad \ldots \quad (1.17)$$

Now

$$\cos\left(\frac{2\pi t}{T}\right) \cos\left(\frac{2\pi nt}{T}\right) = \frac{1}{2}\left[\cos (n-1)\frac{2\pi t}{T} + \cos (n+1)\frac{2\pi t}{T}\right] \qquad \ldots \quad (1.18)$$

Substituting Eq. (1.18) in Eq. (1.17) gives

$$(T/2)A_n = \left[\int_{-T/2}^{-d/2} A_c \cos\left(\frac{2\pi nt}{T}\right) dt + \int_{-d/2}^{d/2} \left(\frac{A+A_c}{2}\right)\left\{\cos (n-1)\frac{2\pi t}{T} + \cos (n+1)\frac{2\pi t}{T}\right\} dt + \int_{d/2}^{T/2} A_c \cos\left(\frac{2\pi nt}{T}\right) dt\right]$$

$$= A_c \left| \frac{T}{2\pi n} \sin\left(\frac{2\pi nt}{T}\right)\right|_{-T/2}^{-d/2} + \left(\frac{A+A_c}{2}\right)\left| \frac{T}{2\pi(n-1)} \sin (n-1)\frac{2\pi t}{T} + \frac{T}{2\pi(n+1)} \sin (n+1)\frac{2\pi t}{T}\right|_{-d/2}^{d/2} + A_c \left| \frac{T}{2\pi n} \sin\left(\frac{2\pi nt}{T}\right)\right|_{d/2}^{T/2}$$

$$\ldots \quad (1.19)$$

$$= \left(\frac{TA_c}{2\pi n}\right)\left[\sin(n\pi) - \sin\left(\frac{n\pi d}{T}\right)\right] + \left(\frac{A+A_c}{2}\right)\left[\frac{T}{2\pi(n-1)}\sin(n-1)\frac{\pi d}{T}\right.$$

$$+ \frac{T}{2\pi(n+1)}\sin(n+1)\frac{\pi d}{T} + \frac{T}{2\pi(n-1)}\sin(n-1)\frac{\pi d}{T}$$

$$\left. + \frac{T}{2\pi(n+1)}\sin(n+1)\frac{\pi d}{T}\right] + \left(\frac{TA_c}{2\pi n}\right)\left[\sin(n\pi) - \sin\left(\frac{n\pi d}{T}\right)\right]$$

$$= \frac{T(A+A_c)}{2\pi}\left[\frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)} + \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)}\right] - \left(\frac{TA_c}{2\pi n}\right) 2\sin\left(\frac{n\pi d}{T}\right)$$

or

$$A_n = \frac{1}{\pi}\left[(A+A_c)\left\{\frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)} + \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)}\right\} - \frac{2A_c}{n}\sin\left(\frac{n\pi d}{T}\right)\right]$$

$$= \frac{1}{\pi}\left[A\left\{\frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)} + \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)}\right\}\right.$$

$$\left. + A_c\left\{\frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)} + \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)} - \frac{2}{n}\sin\left(\frac{n\pi d}{T}\right)\right\}\right]$$

... (1.20)

Substituting Eq. (1.10) in Eq. (1.20) gives

$$A_n = \frac{1}{\pi}\left[A\left\{\frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)} + \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)}\right\}\right.$$

$$+ \left(\frac{A\cos\left(\frac{\pi d}{T}\right)}{1-\cos\left(\frac{\pi d}{T}\right)}\right)\left\{\frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)} + \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)}\right.$$

$$\left.\left. - \frac{2}{n}\sin\left(\frac{n\pi d}{T}\right)\right\}\right]$$

.... (1.21)

$$= \frac{A}{\pi\left(1-\cos\frac{\pi d}{T}\right)}\left[\frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)} + \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)} - \frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)}\cos\left(\frac{\pi d}{T}\right)\right.$$

$$- \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)}\cos\left(\frac{\pi d}{T}\right) + \frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)}\cos\left(\frac{\pi d}{T}\right)$$

$$\left. + \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)}\cos\left(\frac{\pi d}{T}\right) - \frac{2}{n}\sin\left(\frac{n\pi d}{T}\right)\cos\left(\frac{\pi d}{T}\right)\right]$$

$$= \frac{A}{\pi\left(1-\cos\frac{\pi d}{T}\right)}\left[\frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)} + \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)}\right.$$

$$\left. - \frac{2}{n}\sin\left(\frac{n\pi d}{T}\right)\cos\left(\frac{\pi d}{T}\right)\right] \quad \ldots \quad (1.22)$$

Now

$$\sin\left(\frac{n\pi d}{T}\right)\cos\left(\frac{\pi d}{T}\right) = \frac{1}{2}\left[\sin(n+1)\frac{\pi d}{T} + \sin(n-1)\frac{\pi d}{T}\right]$$

Therefore,

$$A_n = \frac{A}{\pi\left(1-\cos\frac{\pi d}{T}\right)}\left[\frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)} + \frac{\sin(n+1)\frac{\pi d}{T}}{(n+1)}\right.$$

$$\left. - \frac{\sin(n+1)\frac{\pi d}{T}}{n} - \frac{\sin(n-1)\frac{\pi d}{T}}{n}\right]$$

$$= \frac{A}{\pi\left(1-\cos\frac{\pi d}{T}\right)}\left[\left\{\sin(n-1)\frac{\pi d}{T}\right\}\left\{\frac{1}{(n-1)} - \frac{1}{n}\right\}\right.$$

$$\left. + \left\{\sin(n+1)\frac{\pi d}{T}\right\}\left\{\frac{1}{(n+1)} - \frac{1}{n}\right\}\right]$$

$$= \frac{A}{\pi\left(1-\cos\frac{\pi d}{T}\right)}\left[\frac{\sin(n-1)\frac{\pi}{T}}{n(n-1)}\right.$$

or

$$A_n = \frac{A\left(\frac{\pi d}{T}\right)}{n\pi\left(1-\cos\frac{\pi d}{T}\right)}\left[\frac{\sin(n-1}{(n-1)}\right.$$

Since B_n is equal to zero, C_n is eq
C_n in terms of $(A_{av})'$, we obtain

$$C_n = A_n = \frac{(A_{av})'\left(\frac{\pi d}{T}\right)}{n\left[\sin\left(\frac{\pi d}{T}\right)-\left(\frac{\pi d}{T}\right)C\right.}$$

$$\times\left[\frac{\sin(n-1)\frac{\pi d}{T}}{(n-1)\frac{\pi d}{T}}\right.$$

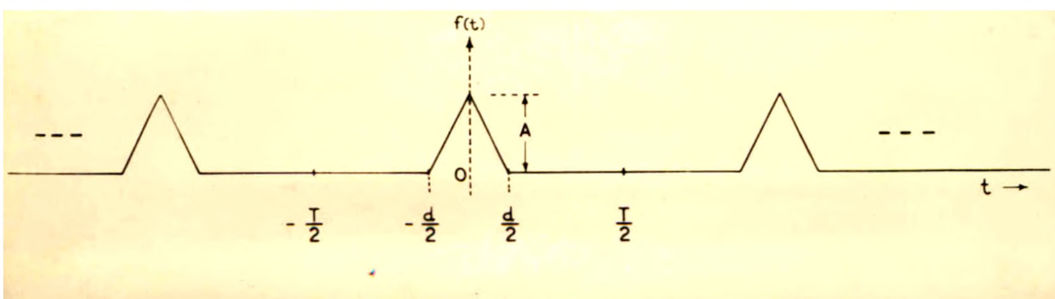

Fig.1.2 Triangular pulse waveform of amplitude A, duration d and period T.

(b) Isosceles Triangular pulse wave form

The periodic function of Fig. 1.2 can be represented in the period from $-T/2$ to $T/2$ by the relations

$$f(t) = 0, \qquad \text{for} \quad -T/2 \leqslant t \leqslant -d/2$$

$$f(t) = \left[1 + \frac{2t}{d}\right]A, \qquad \text{for} \quad -d/2 \leqslant t \leqslant 0$$

$$f(t) = \left[2 - \left(1 + \frac{2t}{d}\right)\right]A, \qquad \text{for} \quad 0 \leqslant t \leqslant d/2$$

$$f(t) = 0, \qquad \text{for} \quad d/2 \leqslant t \leqslant T/2$$

... (1.25)

Substituting from Eqs. (1.25), we have

$$A_{av} = \frac{1}{T}\left[\int_{-d/2}^{0}\left(1+\frac{2t}{d}\right)A\,dt + \int_{0}^{d/2}\left[2-\left(1+\frac{2t}{d}\right)\right]A\,dt\right]$$

$$= \frac{1}{T}\left| A.t + \left(\frac{2A}{d}\right)\left(\frac{t^2}{2}\right)\right|_{-d/2}^{0} + \frac{1}{T}\left| 2.A.t - \left\{A.t + \left(\frac{2A}{d}\right)\left(\frac{t^2}{2}\right)\right\}\right|_{0}^{d/2}$$

$$= \frac{A}{T}\left[\frac{d}{2} - \left(\frac{2}{d}\right)\left(\frac{d^2}{8}\right)\right] + \frac{A}{T}\left[2\left(\frac{d}{2}\right) - \left(\frac{d}{2}\right) - \frac{2}{d}\left(\frac{d^2}{8}\right)\right]$$

$$= A\left(\frac{d}{2T}\right)$$

... (1.26)

Again substituting from Eqs. (1.25), we have

$$A_n = \frac{2}{T}\left[\int_{-d/2}^{0}\left(1+\frac{2t}{d}\right)A\,\mathrm{Cos}\left(\frac{2\pi nt}{T}\right)dt + \int_{0}^{d/2}\left[2-\left(1+\frac{2t}{d}\right)\right]A\,\mathrm{Cos}\left(\frac{2\pi nt}{T}\right)dt\right]$$

or

$$(T/2)A_n = A\int_{-d/2}^{0} \cos\left(\frac{2\pi nt}{T}\right) dt + \frac{2A}{d}\int_{-d/2}^{0} t \cos\left(\frac{2\pi nt}{T}\right) dt$$
$$+ A\int_{0}^{d/2} \cos\left(\frac{2\pi nt}{T}\right) dt - \frac{2A}{d}\int_{0}^{d/2} t \cos\left(\frac{2\pi nt}{T}\right) dt$$

... (1.27)

Now

$$\int t \cos\left(\frac{2\pi nt}{T}\right) dt = \frac{T^2}{4\pi^2 n^2} \cos\left(\frac{2\pi nt}{T}\right) + \frac{T}{2\pi n}\cdot t\cdot \sin\left(\frac{2\pi nt}{T}\right)$$

Therefore

$$(T/2)A_n = A\left|\frac{\sin\left(\frac{2\pi nt}{T}\right)}{\left(\frac{2\pi n}{T}\right)}\right|_{-d/2}^{0} + \frac{2A}{d}\left|\frac{T^2}{4\pi^2 n^2}\cos\left(\frac{2\pi nt}{T}\right)\right.$$
$$\left. + \frac{T}{2\pi n}\cdot t\cdot \sin\left(\frac{2\pi nt}{T}\right)\right|_{-d/2}^{0} + A\left|\frac{\sin\left(\frac{2\pi nt}{T}\right)}{\left(\frac{2\pi n}{T}\right)}\right|_{0}^{d/2}$$
$$- \frac{2A}{d}\left|\frac{T^2}{4\pi^2 n^2}\cos\left(\frac{2\pi nt}{T}\right) + \frac{T}{2\pi n}\cdot t\cdot \sin\left(\frac{2\pi nt}{T}\right)\right|_{0}^{d}$$
$$= \left(\frac{AT^2}{\pi^2 n^2 d}\right)\left[1 - \cos\left(\frac{n\pi d}{T}\right)\right]$$

... (1.28)

or

$$A_n = 2\left(\frac{Ad}{2T}\right)\left[\frac{\sin(n\pi d/2T)}{(n\pi d/2T)}\right]^2 \qquad \ldots \quad (1.29)$$

Similarly, the coefficients B_n are

$$B_n = \frac{2}{T}\left[\int_{-d/2}^{0}\left(1 + \frac{2t}{d}\right) A \sin (\right.$$

$$+ \int_{0}^{d/2}\left[2 - \left(1 + \frac{2t}{d}\right)\right]$$

or

$$(T/2)B_n = A\int_{-d/2}^{0} \sin\left(\frac{2\pi nt}{T}\right) dt + $$

$$+ A\int_{0}^{d/2} \sin\left(\frac{2\pi nt}{T}\right) dt -$$

Now

$$\int t \sin\left(\frac{2\pi nt}{T}\right) dt = \left(\frac{T^2}{4\pi^2 n^2}\right) \sin$$

Therefore,

$$(T/2)B_n = \left(\frac{AT}{2\pi n}\right)\left| -\cos \frac{2\pi nt}{T}\right|_{-d/2}^{0} + ($$

$$- \left(\frac{2A}{d}\right)\left(\frac{T}{2\pi n}\right)\left| t \cos \frac{2\pi nt}{T}\right.$$

$$+ \left(\frac{2A}{d}\right)\left(\frac{T}{2\pi n}\right)\left| t \cos \frac{2\pi n}{T}\right.$$

$$= \left(\frac{AT}{2\pi n}\right)\left[\cos\left(\frac{n\pi d}{T}\right) - 1\right] +$$

$$- \left(\frac{2A}{d}\right)\left(\frac{T}{2\pi n}\right)\left[\frac{d}{2} \cos\left(\frac{n\pi}{T}\right.\right.$$

$$- \left(\frac{2A}{d}\right)\left(\frac{T^2}{4\pi^2 n^2}\right) \sin\left(\frac{n\pi}{T}\right.$$

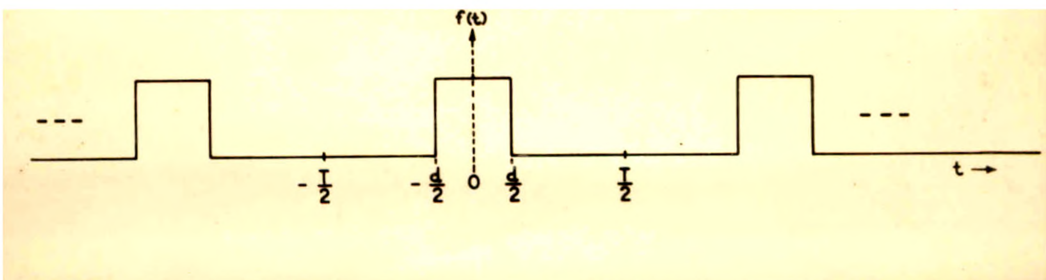

Fig.1.3 Rectangular pulse waveform of amplitude A, duration d and period T.

or

$$B_n = 0$$

... (1.31)

Since B_n is equal to zero, C_n is equal to A_n. Expressing C_n in terms of (A_{av}) , we have

$$C_n = A_n = 2(A_{av})\left[\frac{\sin(n\pi d/2T)}{(n\pi d/2T)}\right]^2$$

... (1.32)

(c) <u>Rectangular pulse wave form</u>

Consider the pulse of Fig. 1.3. Here

$$f(t) = 0 \,, \quad \text{for} \quad -T/2 \leqslant t \leqslant -d/2$$
$$f(t) = A \,, \quad \text{for} \quad -d/2 \leqslant t \leqslant d/2$$
$$f(t) = 0 \,, \quad \text{for} \quad d/2 \leqslant t \leqslant T/2$$

... (1.33)

Substituting from Eqs. (1.33), we have

$$(A_{av}) = \frac{1}{T}\int_{-d/2}^{d/2} A \, dt$$
$$= \left(\frac{Ad}{T}\right)$$

... (1.34)

and

$$A_n = \frac{2}{T}\int_{-d/2}^{d/2} A \cos\left(\frac{2\pi n t}{T}\right) dt$$

$$= \left(\frac{2A}{n\pi}\right) \sin\left(\frac{n\pi d}{T}\right)$$

... (1.35)

and

$$B_n = \frac{2}{T}\int_{-d/2}^{d/2} A \sin\left(\frac{2\pi n t}{T}\right) dt$$
$$= 0 \qquad \ldots \quad (1.36)$$

Therefore

$$C_n = A_n = 2A\left(\frac{d}{T}\right)\left[\frac{\sin(n\pi d/T)}{(n\pi d/T)}\right]$$
$$= 2\,A_{av}\left[\frac{\sin(n\pi d/T)}{(n\pi d/T)}\right] \qquad \ldots \quad (1.37)$$

For convenience, the results of the illustrations (a), (b) and (c) are summarised in Table 1.1.

TABLE 1.1

Fourier coefficients[1] (A_{av}, C_n) of the pulse wave forms of Figs. 1.1, 1.2 and 1.3

Pulse wave form	A_{av}	C_n
Fractional sine wave	$\dfrac{A\left(\sin\frac{\pi d}{T} - \frac{\pi d}{T}\cos\frac{\pi d}{T}\right)}{\pi\left(1-\cos\frac{\pi d}{T}\right)}$ * $+ A_c$	$\dfrac{(A_{av})'\left(\frac{\pi d}{T}\right)}{n\left[\sin\frac{\pi d}{T} - \frac{\pi d}{T}\cos\frac{\pi d}{T}\right]}\left[\dfrac{\sin(n-1)\frac{\pi d}{T}}{(n-1)\frac{\pi d}{T}} - \dfrac{\sin(n+1)\frac{\pi d}{T}}{(n+1)\frac{\pi d}{T}}\right]$
Isosceles Triangular	$A\left(\frac{d}{2T}\right)$	$2\,A_{av}\left[\dfrac{\sin(n\pi d/2T)}{(n\pi d/2T)}\right]^2$
Rectangular	$A\left(\frac{d}{T}\right)$	$2\,A_{av}\left[\dfrac{\sin(n\pi d/T)}{(n\pi d/T)}\right]$

* see page 7, Eq.(1.15).

REFERENCES

1. Moskowitz, S., and Racker, J., Pulse Techniques, Prentice-Hall, Inc., New York (1951), 9-13.