III. LOW ENERGY K-N SCATTERING

On the basis of the current algebra method, Balachandran, Gundzik and Nicodemi obtained (6), as outlined in Appendix II, the \( K^+p \) and \( K^0p \) s-wave scattering lengths in terms of the matrix elements, between proton states, of the third components of the \( V \) and \( V \)-spin vector currents. These same scattering lengths may also be obtained from the vector theory of strong interactions by considering the contributions of the \( \phi \), \( \phi \), and \( \omega \) exchanges in the t-channel. The results of these calculations roughly agree with each other and with the experimentally determined scattering lengths. These observations suggest that these two different approaches may be related by assuming the vector-meson dominance of vector-current matrix elements (this is what is expected on the basis of the field current identity). This immediately suggests the following ansatz: in the absence of t-channel singularities (i.e., \( \rho \), \( \phi \), \( \omega \)) low energy s-wave kaon-nucleon scattering is absent.

Such an ansatz for pion-nucleon scattering has led to sum rules for the pion-nucleon coupling constant and the subtraction constant in the dispersion relation for the crossing even pion-nucleon scattering amplitude, which have been found (8) to be in very good agreement with experiment. In the case of kaon-nucleon scattering, however, the testing of our ansatz is more complicated because of the occurrence of many unknown
coupling constants and the fact that the crossed channel involves KN scattering. We shall, therefore, adopt a more heuristic attitude in testing the hypothesis by considering the contributions of the nearby singularities in the narrow-width resonance (or polelog) approximation rather than the formulation in terms of a dispersion relation framework.

As discussed in Section II, the transition amplitude for KN scattering may be expressed in terms of the invariant amplitudes through

\[ T^I_{f,i} = \mathcal{C}(\psi') \left[ - A^I(z, t, w) + i \frac{K + K'}{2} B^I(z, t, w) \right] \psi(p). \]  

(1)

Now, at threshold \((s = s_0 = (m + \mu)^2)\) and \(t = 0\) the invariant amplitudes may be expressed \(9\) in terms of the scattering lengths thus:

\[ \frac{A^I(z_0, 0)}{4\pi} = \left( 1 + \frac{1}{2m} \right) a^I_{s_{1/2}} - 2m \mu \left( \alpha^I_{p_{3/2}} - \alpha^I_{p_{1/2}} \right) \]  

(2)

\[ \frac{B^I(z_0, 0)}{4\pi} = - \frac{1}{2m} a^I_{s_{1/2}} + 2m \left( \alpha^I_{p_{3/2}} - \alpha^I_{p_{1/2}} \right) \]  

(3)

Denoting by \(\tilde{A}^I\) and \(\tilde{B}^I\) the invariant amplitudes in the absence of \(t\)-channel exchanges; by virtue of our ansatz, we have

\[ \frac{\tilde{A}^I(z_0, 0)}{4\pi} = - 2m \mu \left( \alpha^I_{p_{3/2}} - \alpha^I_{p_{1/2}} \right) \]  

(4)

\[ \frac{\tilde{B}^I(z_0, 0)}{4\pi} = 2m \left( \alpha^I_{p_{3/2}} - \alpha^I_{p_{1/2}} \right) \]  

(5)
We next exploit the fact that up to moderate energies the scattering of kaons from nucleons, unlike what occurs for pion-nucleon scattering, appears to be isotropic and thus p-waves seem to be conspicuously absent. This enables us to write

\[ \tilde{A} I (s_0, 0) \sim 0 \quad (6) \]
\[ \tilde{B} I (s_0, 0) \sim 0 \quad (7) \]

In the framework of Regge asymptotics, if a Regge trajectory with trajectory function \( \alpha(t) \) contributes, then for large \( s \) at fixed \( t \) we have

\[ A (s, t) \sim \alpha(t) \]
\[ B (s, t) \sim \alpha(t) \]

Thus the \( B \) amplitude may be expected to satisfy an unsubtracted dispersion relation whereas the \( A \) amplitude would not. In the present context what this implies is that the saturation of the dispersion relation for \( B \) by nearby singularities corresponds to the present approach of considering the contributions of the low-lying states, whereas the subtraction constants in the case of \( A \) amplitude would necessitate the introduction in some way of the contributions of distant singularities. This situation is to be contrasted with that in pion-nucleon scattering where the p-wave scattering is by no means small. To test the compatibility of the eq. (5) in pion-nucleon case, we calculated the s- and u-channel ('poleology') contributions to \( B^I (s_0, 0) \) of \( N \) and \( N^*_{33} (1236) \). Eq. (5) for pion-nucleon scattering follows from the general equation (3) on making a similar ansatz that low-energy s-wave \( \pi N \) scattering
is absent in the absence of t-channel singularities. This ansatz has been satisfactorily tested in the \( \pi N \) case by H. Bauerjee and B. Dutta-Roy. Taking the values 14.6 for \( \frac{g}{\sqrt{2}} \) and 0.072 for \( (\alpha_{\pi}^{1/2}_{3\pi} - \alpha_{\pi}^{3/2}_{1\pi}) \) given by Hamilton and Woolcock, we calculated, by solving the two equations analogous to eq. (5) in the pion-nucleon case for \( I = 1/2 \) and \( I = 3/2 \), \( (\alpha_{\pi}^{3/2}_{3\pi} - \alpha_{\pi}^{3/2}_{1\pi}) \), and \( \gamma_{33} \) for the \( N^{*}_{33} \) coupling to \( \pi N \). They are:

\[
(\alpha_{\pi}^{3/2}_{3\pi} - \alpha_{\pi}^{3/2}_{1\pi}) = 0.260 \\
\gamma_{33} = 0.061
\]

Hamilton and Woolcock give for the same 0.253 and 0.066 respectively. Thus we see that the Eq. (6) is quite compatible with experiment. Also, this agreement may be taken as an aposteriori justification in this case, of our procedure of taking the polynomial contributions. We would therefore demand that the B amplitudes both in isospin \( I = 0 \) and 1 states, when saturated by nearby singularities, in channels other than the t-channel should in this approximation vanish at threshold. There are no well established s-channel resonances in the KN channel indeed such resonances would have hypercharge \( Y = 2 \) and are called exotic. Thus the contribution we must consider come from the u-channel. In this connection it is useful to consider the s \( \leftrightarrow u \) crossing matrix

\[
C_{s,u} = \begin{pmatrix}
\frac{1}{2} & -\frac{3}{2} \\
-\frac{2}{3} & \frac{7}{3}
\end{pmatrix}
\]
This relates the contribution to isospin $I = 0, 1$ amplitudes in the $s$-channel arising from an exchange in the $u$-channel.

To calculate the contributions of the closest singularities to the invariant amplitudes we consider the following exchanges in the $u$-channel:

$$\gamma (\Lambda \alpha \Lambda \Sigma): \frac{1}{2}^+; \quad \gamma_0^* (1405): \frac{1}{2}^-;$$

$$\gamma_1^* (1385): \frac{3}{2}^+; \quad \gamma_0^* (1520): \frac{3}{2}^-.$$

The contribution of $\gamma_1^* (1660)$ is found to be negligible. The contributions of these resonances will be expressed in terms of the coupling constants defined through the Lagrangian:

$$\mathcal{L} = i \frac{\gamma_{NK}}{\sqrt{2}} \overline{\psi}_N \gamma_5 \psi_N \phi_K + \frac{1}{2} \frac{\gamma_{NK}}{m} \left( \overline{\psi}_N \gamma_5 \phi_K - \overline{\psi}_N \phi_K \right) \psi_N$$

where $\psi_N$ is the Rarita-Schwinger field, and $\gamma_0^* (1520)$. As discussed in Appendix IV, the contributions of the various singularities to the amplitude may be calculated to yield:

$$A^1 = \frac{m - m}{m^2 - \mu} \frac{g^2}{\sqrt{2} \phi_K},$$

$$B^1 = \frac{1}{m^2 - \mu} \frac{g^2}{\sqrt{2} \phi_K}.$$
\[
\Sigma = \frac{\mu_{+}^{2}}{2} \\
A' = \frac{M-m}{M^2-u} g^2 \Sigma K \\
B' = \frac{1}{M^2-u} g^2 \Sigma K
\]  

(10)

\[\gamma_0^* (1405) : \frac{1}{2}\]

\[
A' = -\frac{M+m}{M^2-u} g^2 \gamma_0^* K \\
B' = \frac{1}{M^2-u} g^2 \gamma_0^* K
\]  

(11)

\[\gamma_1^* (1385) : \frac{3}{2}\]

\[
A' = \frac{g^2 \gamma_1^* K}{m^2 (M^2-u)} \left[ -\frac{1}{2} (M+m)(2m^2+2m^2-2-M^2) \\
- g^2 \left\{ (M+m) + \frac{M-m}{3} \frac{E^*+m}{E^*-m} \right\} \right]
\]  

(12)

\[
B' = \frac{g^2 \gamma_1^* K}{m^2 (M^2-u)} \left[ \frac{2m^2+2m^2-3-M^2}{2} \\
+ g^2 \left\{ 1 - \frac{E^*+m}{3(E^*-m)} \right\} \right]
\]
In the above \( M \) stands for the mass of the exchanged "particle" under consideration; \( E^* \) and \( q^* \) refer to the nucleon energy and momentum in the centre-of-mass system at the resonance energy.

The eq. (7) for the B-amplitude, namely \( \mathcal{B}^I (s_0, 0) = 0 \), provides us with two extremely useful sum rules for the coupling constants, one in each isospin channel. In kaon mass units, these sum rules read:

\[
\mathcal{B}^I (s_0, 0) = 0.233 g^2_{\Lambda K^-} + 0.199 g^2_{\Sigma^+ K^-} + 0.137 g^2_{Y^0 K^-} - 0.329 g^2_{\Sigma^- K^-} - 0.131 g^2_{Y^0 K^-} = 0 \quad (14)
\]

\[
\mathcal{B}^O (s_0, 0) = 0.233 g^2_{\Lambda K^-} - 0.597 g^2_{\Sigma^+ K^-} + 0.137 g^2_{Y^0 K^-} + 0.987 g^2_{\Sigma^- K^-} - 0.121 g^2_{Y^0 K^-} = 0 \quad (15)
\]

Rewriting the above sum rules so that the interdependence of the

various coupling constants becomes more apparent, we obtain:

\[
0.796 \frac{g^2}{\pi} \eta^* K^- = 1.216 \frac{g^2}{\pi} \eta^* K^- \tag{16}
\]

\[
0.932 \frac{g^2}{\pi} \eta^* K^- = 0.524 \frac{g^2}{\pi} \eta^* K^- - 0.548 \frac{g^2}{\pi} \eta^* K^- \tag{17}
\]

These sum rules may be tested in the following way.

We shall substitute the values of the coupling constants on the right hand side - obtained as delineated below - and calculate the \( \Lambda_{\pi K} \) and \( \Sigma_{\pi K} \) coupling constants which can be compared with their values from other estimates. Of the \( \eta^* \) resonances, \( \eta^* (1520) \) gives directly the coupling constant \( \frac{g^2}{\pi} \eta^* K^- \) because it has well established \( \pi N \) decay mode, with a known branching ratio. The decay

\[ \eta^* (1520) \rightarrow \pi + N \]

is a \( d \)-wave decay and its partial width \( \Gamma_{\pi K} \) can be derived using the projection operator \( \Lambda_{\mu\nu} \) for the Rarita-Schwinger field \( \Psi_\mu \) (cf. Appendix IV, where \( \Lambda_{\mu\nu} \) has been derived) to yield:

\[
\Gamma_{\eta^* \rightarrow K^-} = \frac{g^2}{4\pi} \frac{2 \cdot \varphi^5}{3 m^2 \left[ (m+m)^2 - \mu^2 \right]} \tag{18}
\]

where \( \varphi \), the centre-of-mass three-momentum is given by

\[
\varphi^2 = \frac{\left[ m^2 - (m+m)^2 \right] \left[ m^2 - (m-\mu)^2 \right]}{4 m^2} \]
Taking (12) the masses as \( M = 1618.8, m = 938.256, \mu = 493.2 \)
and the width to be (with \( \lambda \) and \( \Gamma_{\text{total}} \) as the branching into \( \bar{K}N \) mode and the total width respectively)
\[
\Gamma_{\gamma^* \to \bar{K}N} = \frac{1}{2} \Gamma_{\gamma^* \to \pi N} = \frac{1}{2} \lambda \Gamma_{\text{total}} = 0.39 \times 16 \text{ MeV} = 3.12 \text{ MeV}
\]
we get from eq. (18) for the coupling constant:
\[
\frac{q^2}{\pi \gamma^* p \bar{K}^-} = 2.9 \tag{19}
\]

To calculate the coupling constant to \( \bar{K}N \) of \( \gamma^*_0 (1405) \)
whose width almost entirely is due to its decay to \( \pi \pi \), one can calculate the \( \gamma^*_0 \pi \pi \) coupling from its decay rate and use \( SU(3) \) to get its coupling to \( \bar{K}N \). This leads to the value 0.045 for \( q^2/\pi \gamma^*_0 p \bar{K}^- \). In obtaining this, however, one has to assume that \( \gamma^*_0 (1405), J = 1/2^- \) is an \( SU(3) \) singlet. However, the coupling constant can also be obtained if one assumes that \( \gamma^*_0 (1405) \) is a virtual s-wave \( \bar{K}N \) bound state a proposition first made by Dalitz and Tuan (5). This follows from the fact that \( \bar{K}N \) system has the same quantum numbers as \( \gamma^*_0 (1405) \), whose mass is less by just about 32 MeV than the \( \bar{K}N \) threshold. The recent multichannel effective range parametrization of Kim (13)
also gives when used to study the coupled channel reactions \( \eta \pi \rightarrow \pi \pi \) and \( \pi N \rightarrow \pi N \) support to this idea. Indeed, this analysis reproduces the width and the position of the \( \gamma_c^*(1405) \) resonance quite well.

So, following Dalitz and Tuan \(^{(5)}\) we take the imaginary part of the \( \bar{K}N \) scattering amplitude to be

\[
\text{Im} f = -\frac{m \left| q^* \right|}{E^* \omega^*} \pi \delta \left( \omega'' - \omega' \right)
\]

(20)

where \( E^* \) and \( \omega^* \) are the nucleon and \( K^- \)-meson energies in the centre-of-mass system of the resonance, at the resonance energy \( \omega' \), the mass of \( \gamma_c^*(1405) \). \( \left| q^* \right| \) is the magnitude of the centre-of-mass momentum \( i \left| q^* \right| \) (which is due to the fact that \( \gamma_c^*(1405) \) is below the \( \bar{K}N \) threshold). We obtain, now, the real part of the amplitude from the dispersion relation:

\[
\text{Re} f = \frac{1}{\pi} \int \frac{\text{Im} f(\omega)}{\omega'' - \omega} \, d\omega'
\]

This gives on using eq. \((20)\):

\[
\text{Re} f = -\frac{m \left| q^* \right|}{E^* \omega^*} \frac{1}{\omega'' - \omega}
\]

(21)

Now, \( \text{Re} f \) can also be obtained by calculating the Born amplitude for the exchange of \( \gamma_c^*(1405) \) in the \( u \)-channel. The invariant amplitudes \( A \) and \( B \) for this exchange are given in eq. \((11)\) and these lead to:
Comparing the residues of the eqs. (21) and (22) we obtain:

$$\frac{g^2 y^*_8 \pi^-}{4 \pi} = \frac{m^2 |y^*_8|}{\epsilon^*_\omega^* (\epsilon^*_\omega^* + m)}$$

which leads to:

$$\frac{g^2 y^*_9 \pi^-}{4 \pi} = 0.31$$  \hspace{0.5cm} (2.3)

Also, Kim and von Hippel\(^{(14)}\) calculated the $y^*_8 N\pi$ coupling constant by using Kim's multichannel effective range parametrization. They obtain

$$\frac{g^2 y^*_8 \pi^-}{4 \pi} = 0.32 \pm 0.04$$

$\gamma^*_8(1385)$ is the $I = 1$ and $\gamma = 0$ member of the $SU(3)$ decuplet. So one can calculate the coupling constant

$$\frac{g^2 y^*_8 \pi^-}{4 \pi} \text{ using } SU(3). \quad N^*_33(1236), \text{ which is the } I = 3/2, \gamma = 1 \text{ member of the same decuplet, is a resonance in } \pi^- N \text{ system and if one uses its decay rate to calculate the } N^*_33 N\pi \text{ coupling and then uses } SU(3) \text{ to obtain the } y^*_8 \pi^- \text{ coupling; the value obtained is: } \frac{g^2 y^*_8 \pi^-}{4 \pi} \approx 2.4. \quad \text{However, one can also use the decay data of } \gamma^*_8(1385) \to \Lambda \pi \text{ and then use } SU(3) \text{ to obtain the } y^*_8 \pi^- \text{ coupling from the } y^*_8 \Lambda \pi$
coupling constant. This procedure gives:

$$\frac{q^2 \gamma_i \gamma^* \gamma^*}{4\pi} = 1.9$$

The coupling constant has also been calculated by Wall and Warnock using a specific model for the broken decuplet. Their model assumes that the forces responsible for the decuplet are mainly due to baryon exchanges in meson-baryon scattering. For a given isospin and hypercharge, they retain only the coupled two-particle channels and perform an N/D calculation for the partial-wave amplitude

$$\gamma_{ij} \equiv \frac{T_{ij}}{q_i \cdot q_f}$$

where $T_{ij}$ is the invariant scattering amplitude for $J = 3/2$ partial wave and $q_i$ and $q_f$ are initial and final momenta. Their results show a definite improvement in agreement with experiment, particularly as regards the branching ratios of $\gamma^*(1385)$. They obtain for the coupling constant a value, which also satisfies the sum rules for the decuplet-octet-octet coupling constants derived by Gupta and Singh, taking into account the $SU(3)$ symmetry breaking interaction to first order, as:

$$\frac{q^2 \gamma_i \gamma^* \gamma^*}{4\pi} = 1.9$$

This is in very good agreement with the estimate presented in eq. (24).
So, if we substitute the values:

\[
\frac{g^2_{\gamma^* K^-}}{4\pi} = 2.9 \quad \frac{g^2_{\gamma^* K^-}}{4\pi} = 0.31 \quad \frac{g^2_{\gamma^* K^-}}{4\pi} = 1.9
\]

in the sum rules (16) and (17) we obtain:

\[
\frac{g^2_{\gamma^+ K^-}}{4\pi} = 1.16 \quad \frac{g^2_{\gamma^+ K^-}}{4\pi} = 3.14 \quad (2.6)
\]

Other estimates for these couplings can be divided, to start with, into two categories: those which are compatible with SU(3) invariance and those which are not. \( ^\wedge \text{NK} \) and \( \Sigma \text{NK} \) coupling constants have been calculated by using both current algebras and dispersion relations.

Several authors (17) have employed forward KN dispersion relations to calculate the \( ^\wedge \text{NK} \) coupling constant. The typical form of the dispersion relations used are (these were originally derived by Mathews and Salam (18))

\[
D_-(\omega) - D_+(\omega) = \frac{\omega L}{2\pi^2} \int \frac{d\omega' \left[ \sigma_-(\omega') - \sigma_+(\omega') \right]}{\omega' \omega' - \omega^2} \\
- \frac{2\omega L}{\pi} \int_{\omega L}^{\infty} \frac{A_-(\omega')}{\omega' \omega' - \omega^2} d\omega' \\
+ 2\omega L \left[ \frac{\chi(\wedge)}{\omega \omega' - \omega^2} + \frac{\chi(\Sigma)}{\omega \omega' - \omega^2} \right] \quad (2.7)
\]
in which the subscripts $+$ and $-$ refer to $\Lambda K^p$ and $\Lambda K^-$ scatterings, and the $\sigma$'s refer to the total cross sections. $\omega_\Lambda$ is the incident $\Lambda$-meson energy in the laboratory system and the lower limit of the second dispersion integral refers to the $\Lambda\pi$ threshold. $D$ and $A$ are the real and imaginary parts of the forward scattering amplitude and

$$
\times (\gamma) = \frac{g^2}{8\gamma_{\Lambda K^-}} \left[ \frac{(m^2 - m)^2 - s^2}{4m^2} \right]
$$

$$
c_\Lambda^\gamma = \frac{n^2 - m^2 - k^2}{2m} 
$$

The values obtained by various authors\(^{(17)}\) have not been in agreement with each other, primarily because of different treatments of the low-energy and unphysical regions for $\bar{K}N$ scattering. For example, Rood\(^{(17)}\) has found that the use of a $\Lambda$-matrix parametrization of the low-energy $\bar{K}N$ interaction in the dispersion relations of the type (27) leads to $\varphi^\gamma_{\Lambda K^-}/4\pi = 7.4 \pm 1.2$ compared with $4.8 \pm 1.0$ obtained by Lusignoli et al.\(^{(17)}\) using essentially the same input data, but assuming constant scattering lengths instead, to calculate the contributions due to the unphysical cuts below threshold. The values thus obtained are in the region 4 to 8 and are less than the $SU(3)$ estimate by a factor of 3 to 2.

Raman\(^{(19)}\), on the other hand, obtains from current algebra calculations $\varphi^\gamma_{\Lambda K^-}/4\pi = 16.8$, but his derivation is dependent on various parameters (like the $d/f$ ratio for axial vector current
octet and the off-mass shell meson-baryon form factors) and the variation in the values of these parameters can be as much as to make 

\[ q^2 / m^2 \pi^2 \approx 5 \]

The multichannel effective range parametrization of Kim\(^{(13a)}\) has provided a better framework for extrapolating the \(\overline{K}N\) amplitudes to the unphysical region. He considered the coupled two-body channels \(\overline{K}N\), \(\pi\pi\) and \(\pi\pi\) in the \(\mathcal{M}\)-matrix formalism defined through the relation:

\[ \frac{k^2}{m - i(k_{L+1}^2)} \]

The \(\mathcal{M}\)-matrix can be expanded in the following effective range form:

\[ \mathcal{M}(s) = \mathcal{M}(E_0) + \frac{1}{l} \sum_{l} \mathcal{C}_l (k^2 - k_0^2) \]

where \(k\) is the diagonal matrix of the centre-of-mass channel momenta, \(r\) is the diagonal effective range matrix and \(\mathcal{C}_0 = 1\) for s-wave and \(\mathcal{C}_1 = -3\) for p-wave, and \(\mathcal{M}(E_0)\) is referred to as zero-energy parameter. He determined the values of the parameters by making a phase-shift analysis of \(\overline{K}N\) interaction and making a fit with experiment in the region 0 to 550 MeV/c of incident meson momentum in the laboratory system. The values of the relevant parameters used in this thesis (particularly in Section IV) are given in Table 2. Kim's parameters reproduce the mass and width of the \(\gamma^*(1405)\) and \(\gamma^*(1385)\) resonances.
Kim's multichannel effective range parameters (13a). $M_{AB}$ is the $M$-matrix element, where $I$ indicates isospin, $A$ and $B$ indicate the initial and the final channel. $M$ is in units of $F = (2I + 1)$ and $r$ is in $F$.

<table>
<thead>
<tr>
<th></th>
<th>$z_{1/2}$</th>
<th>$p_{1/2}$</th>
<th>$p_{3/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M^0_{KK}$</td>
<td>$-0.00 \pm 0.02$</td>
<td>$17.5 \pm 16.4$</td>
<td>$10.9 \pm 0.9$</td>
</tr>
<tr>
<td>$M^0_{K\Sigma}$</td>
<td>$-1.11 \pm 0.04$</td>
<td>$-10.6 \pm 4.5$</td>
<td>$-1.4 \pm 0.2$</td>
</tr>
<tr>
<td>$M^0_{\Sigma\Lambda}$</td>
<td>$2.04 \pm 0.10$</td>
<td>$-11.4 \pm 4.8$</td>
<td>$5.6 \pm 0.5$</td>
</tr>
<tr>
<td>$M^I_{KK}$</td>
<td>$-3.60 \pm 0.02$</td>
<td>$-18.1 \pm 0.7$</td>
<td>$5.2 \pm 1.0$</td>
</tr>
<tr>
<td>$M^I_{K\Sigma}$</td>
<td>$-2.86 \pm 0.03$</td>
<td>$7.2 \pm 1.1$</td>
<td>$-13.8 \pm 1.6$</td>
</tr>
<tr>
<td>$M^I_{\Sigma\Lambda}$</td>
<td>$2.08 \pm 0.07$</td>
<td>$4.7 \pm 0.4$</td>
<td>$-11.9 \pm 1.6$</td>
</tr>
<tr>
<td>$M^I_{\Sigma\Xi}$</td>
<td>$-1.40 \pm 0.06$</td>
<td>$0.3 \pm 1.1$</td>
<td>$-15.8 \pm 0.9$</td>
</tr>
<tr>
<td>$M^I_{\Xi\Lambda}$</td>
<td>$1.81 \pm 0.04$</td>
<td>$-6.8 \pm 1.0$</td>
<td>$-13.6 \pm 0.2$</td>
</tr>
<tr>
<td>$M^I_{\Xi\Xi}$</td>
<td>$-2.31 \pm 0.11$</td>
<td>$3.4 \pm 1.3$</td>
<td>$-16.3 \pm 2.0$</td>
</tr>
</tbody>
</table>

$r^0_K$ | $0.54 \pm 0.08$ |
$r^0_\Sigma$ | $0.89 \pm 0.31$ |
$r^I_K$ | $0.13 \pm 0.07$ | $0.66 \pm 0.20$ |
$r^I_\Xi$ | $6.78 \pm 0.23$ | $0.27 \pm 0.05$ |
$r^I_\Lambda$ | $1.22 \pm 0.45$ | $0.31 \pm 0.13$ |
quite well. He also ascertains that $\gamma^+(1385)$ is mainly a $\pi^-$ resonance and that its coupling to $\bar{K}N$ is very weak.

Kim uses \cite{13b} his parametrization to extrapolate the $\bar{K}N$ amplitudes to the unphysical region, and uses the forward dispersion relations (at threshold) of the type (27) and obtains:

\[ \frac{g^2 \Lambda_{\Lambda K^-}}{4\pi} = 13.5 \pm 2.1 \]

and

\[ \frac{g^2 \sum_{\Lambda K^-}}{4\pi} = 0.2 \pm 0.4 \]

These differ markedly with the other estimates \cite{17} particularly with those of Lusignoli et al. \cite{17} who use constant scattering length approximation for the extrapolation procedure. Kim's values are also consistent with SU(3) invariance. Thus we note that the effective range parametrization, which undoubtedly is to be favoured on theoretical grounds, gives values quite different from other estimates.

Chan and Meiere \cite{20} provide an independent method of determining the $\Lambda N K$ and $\Sigma N K$ coupling constants. They employ Kim's effective range parametrization but use modified dispersion relations. They write dispersion relations for the modified amplitude

\[ F_\rho (\gamma) = \frac{\tau (\gamma)}{(\gamma - \mu)^{1/2} (\gamma - \nu)^{1/2}} \]
where $T(v)$ is the invariant forward $\bar{K}N$ scattering amplitude. $\xi_c$ is the lowest $\pi\gamma$ threshold and $\beta$ is an arbitrary parameter, $0 < \beta < 1$. The factor $(\omega - \mu)^{-1 - \beta} (\omega - \xi_c)^\beta$ is chosen so that it introduces a short cut joining the branch points $\xi_c$ and $\mu$ (mass of the $K$-meson), and is taken to be positive for real $\omega > \mu$ and negative for real $\omega < \xi_c$. In the physical region, $\Im F_\beta(v) \propto \Im T(v)$, but in the unphysical region it is proportional to $\cos \pi \beta \Im T(v) + \sin \pi \beta \Re T(v)$. Thus, as $\beta$ varies, contributions from the unphysical region are determined by combinations that range from $\Re T$ to essentially $\pm \Im T$; while the contributions from the physical region are determined always by the total cross-sections. Thus, this modification has the distinct advantage that, in the physical region only $\Im T(v)$ occurs, which is given by experiment and $\Re T$ (for which little direct information is available over most of the physical region) does not appear in the physical region; and once the parameters used to extrapolate from the physical to the unphysical region are determined, they give both real and imaginary part of the amplitude in the unphysical region with the same accuracy. Thus the input to the modified dispersion relations is as well known as for the conventional dispersion relations. Also the high energy behaviour of the modified amplitude being

$$F_\beta(v) \sim \frac{T(v)}{v}$$

which is independent of $\beta$, it does not restrict the allowed values of $\beta$. 
Starting from the KN amplitude corresponding to isospin $I$, their basic equations are obtained from the dispersion relation for $F_\rho(v)$, evaluated at KN threshold $v = -\mu$.

For example, for $I = 0$, they obtain:

$$
-\frac{2q^2_{\Lambda N K} \times (\Lambda)}{(\mu - \nu)^{1-\rho} (\nu_c - \nu)^{\rho}} = \frac{1}{4\pi^2} \int_0^\infty \frac{dk}{k} \left( \frac{\rho + \mu}{\rho + \nu} \right)^{\rho} \left[ 2 \sigma^{-\rho}_{-\Lambda K^+}(\nu) - \sigma^{-\rho}_{-\Lambda K^-}(\nu) \right],
$$

$$
-\frac{1}{4\pi^2} \int_0^\infty \frac{dk}{k} \left( \frac{\nu - \mu}{\nu - \nu_c} \right)^{\rho} \left[ 2 \sigma^{-\rho}_{-\Lambda K^0}(\nu) - \sigma^{-\rho}_{-\Lambda K^0}(\nu) \right],
$$

$$
+ \frac{1}{\pi} \int \frac{d\nu}{\nu + \mu} \frac{\cos \pi \rho \cos \pi (\nu + \mu) + 3 \sin \pi \rho \cos \pi (\nu - \mu)}{(\mu - \nu)^{1-\rho} (\nu - \nu_c)^{\rho}} \frac{T(-\mu)}{(2\mu)^{1-\rho} (\nu_c + \mu)^{\rho}}
$$

where $\times (\Lambda)$ is same as defined earlier and $\nu_\Lambda$ is the $\Lambda$ pole position. The analogous equation for $I = 1$ gives $q^2_{\Sigma N K}$.

Their results can be summarized as follows. The representative values for the integrals over the unphysical and physical regions which appear in the above equation for the $\Lambda N K$ case vary individually by a factor of 4 to 5 as $\rho$ is varied from 0.1 to 0.9, but the resultant coupling constant values are consistent with each other within the specified errors for them in each case. Similarly,
for the $\Sigma NK$ case. This stability of the coupling constants against variations in $\beta$ can be taken as a measure of the self-consistency of such a determination, or more specifically, it may be considered as providing a criterion for the compatibility of the extrapolated amplitude in the unphysical region with the known total cross-sections in the physical region. They repeat whole calculation with the constant scattering length approximation in which the different values obtained for each of the coupling constants for different $\beta$ disagree quite badly with each other. This indicates that the coupling constants obtained by the use of effective range parametrization are definitely to be favoured to those obtained from the constant scattering length approximation. The values obtained by Chan and Meiere are:

$$\frac{g_{\Sigma^{-}pK^{-}}}{4\pi} = 13 \pm 3$$

$$\frac{g_{\Sigma^{-}pK^{-}}}{4\pi} = 0 \pm 1$$

which are compatible with SU(3) invariance and also with the values obtained by Kim (eqs. (28)).

From the above discussion, we see that the value 16.11 for $g_{\Sigma^{-}pK^{-}}/4\pi$ obtained from our sum rule (eq. (17)) agrees quite well with the values of Kim and Chan and Meiere and is compatible with SU(3) invariance. The coupling constant $g_{\Sigma^{-}pK^{-}}/4\pi = 3.14$ obtained from our sum rule (eq. (16)) is higher than the estimates of Kim and Chan and Meiere. However, from eq. (16), we see that
\( q^2 \sum \frac{\gamma_k}{4\pi} \) is given entirely in terms of the \( \frac{\gamma_k}{\gamma'_k} \), the

\( \gamma_k \) coupling to \( \vec{K}N \). As mentioned earlier Kim's multichannel effective range analysis positively ascertains that

\( \gamma_k \) is mainly a \( \Lambda\pi \) resonance and that its coupling to \( \vec{K}N \) is very weak; also the analysis reproduces the position and the width of the resonance quite well. Our sum rule (16) is very much in agreement with this observation since in the limit of \( \gamma_k \) being completely decoupled to \( \vec{K}N \), our sum rule gives:

\[
\frac{q^2 \sum \frac{b_k}{4\pi}}{4\pi} = 0
\]

in agreement with the estimates of Kim and Chan and Meiere.

Thus, we see that our sum rules are quite in agreement with experiment and this must be taken as an posteriori justification of our hypothesis that low-energy s-wave kaon-nucleon scattering is solely due to the force provided by t-channel singularities.