CHAPTER ONE

INTRODUCTION

This work presents an analysis of the problem of optimal allocation of resources among economic activities whose locations are variable in a centralised planned economy. The context of this analysis is static. Of the models that are set up in the following chapters the first two describe the economy to be multiregional in structure; they assume that the sites of production within each region are a priori given to the central planners who allocate resources over the regions. In other words these models consider locational alternatives of production of the entire economy to be parameters and to be fixed in number. The remaining three models of this work on the other hand consider locations to be continuous variables in the two dimensional plane of geographic surface of the entire economy. In all of them, however, the discrete or continuous choice of location is considered along with the choice of product mix and the choice of technique.

The approach of the analysis that follows is basically an axiomatic one. The problem in each model is one of exploring the conditions of efficient use of economic resources in a context which takes into account the variability of locations of the productive activities — the variability being defined in a specific manner. The formulations of the different models have been so made that they help one to analyse the roles of the different locational factors like transportation and agglomeration advantages in non-pecuniary costs as determining forces behind the choice of
an efficient pattern of location and allocation under some assumptions on technical and spatial regularities. A sensitivity analysis of the efficient solution with respect to changes in the values of the parameters follows the discussions on efficiency conditions at places where such an analysis would be interesting.

All of our normative models work in reality only in an economy which is fully planned, i.e., which allows for no decentralisation in decisions on allocation and no market interventions. If the problem is alternatively posed in the background of a freely competitive economy, the efficient solutions for location and allocation as given by some of the following models would describe consistently an equilibrium of production and of its spatial organisation only in the following sense: Any such solution ensures (a) logical compatibility among the decisions of the different decision makers and (b) the capability of its being sustained in the absence of any perturbation due to external influences. This alternative way of interpreting the operational significance of an efficient solution of a model has been indicated at the relevant places. It may however be pointed out that even when the problems are visualised from this alternative standpoint, the models can highlight only the aspect of spatial equilibrium of production and distribution where a set of market prices has already been given. The analysis of consumers' equilibrium remains absent here and so will remain the full explanation of the final use prices. Thus our analysis falls short of full general equilibrium structure.
We may add that our analysis remains incomplete in another sense too. It is that the problem of dynamic stability of an equilibrium as described by some efficient solution of any model for any arbitrary initial configuration has not been dealt with anywhere in these chapters. In fact, numerous additional special problems may arise regarding the process of adjustment to an efficient equilibrium in a decentralised economy where there is no government intervention, but where there exist externalities, increasing returns and feasibility of simultaneous variation of location and allocation in a continuous manner. These factors arise very naturally in a spatial context and thus cannot be ignored in our discussions even though these would cause violation of the usual concavity-convexity conditions of well behaved models. Apart from giving rise to the problems with adjustment to equilibrium which are not discussed in the present work, the nonfulfilment of the required concavity-convexity conditions may, on the other hand, lead to a theoretical situation where one cannot assert that an equilibrium under conditions of decentralisation, as we have defined above, will necessarily achieve efficient spatial allocation of resources. This point has been, however, indicated wherever it is relevant.

The study in the following chapters has been based on the activity analysis (as given by Koopmans [11], Gale [8] and others) or on its simple nonlinear variations. Formally it is an attempt at fusing locational factors (of transportation and agglomeration) with the theory of production so that locations may be obtained as objects of choice from the very
definition of the problems. The mathematical tools of analysis used in this work are those of linear and nonlinear programming. Five models have been developed here under alternative sets of assumptions on technology and regional structure of the economy. The motivations of these models are different in the sense that they emphasise alternative aspects of different spatial factors in a theory of allocation of resources. The models have been grouped accordingly into three parts: (1) Part I: Model I and Model II which consider locational alternatives of production of the entire economy as parameters and assume a linear technology of production. (2) Part II: Model III and Model IV which consider locations of production as continuous variables and assume the technology of production to be linear for any arbitrarily given locational distribution of the productive activities. (3) Part III: Model V, which considers locations of production to be continuous variables, but assumes a nonlinear technology of production for any arbitrarily given locational distribution of the productive activities. This nonlinearity of technology accounts for the scale effect of operation and the agglomerative effect of locations on the real cost of production. A broad classification of the different models is shown in the following table for ready reference.
1.1 Concepts and Assumptions:

The discussion of this section is limited to the analysis of:

(i) the concepts which are in general applicable to the entire domain of the present work and

(ii) the alternative assumptions that would be valid for the different models.

(a) Goods: their classification

The different goods of our economy may be classified into two groups: (i) reproducible goods and (ii) non-reproducible goods. The set of reproducible goods may again be divided into two subsets: (i) one consisting of those reproducible items each of which has got final consumption use, while it may have intermediate input use as well, (ii) the other consisting of purely intermediate goods which have got, by definition, no final use for consumption. All the non-reproducible goods are, on the other hand, assumed to have only input (and no final consumption) use. (The non-reproducible goods have in fact often been referred to as primary factors.)
The goods may alternatively be classified according to their respective degrees of mobility over space: (i) perfectly mobile, i.e. transportable at zero cost, (ii) perfectly immobile, i.e. non-transportable, (iii) imperfectly mobile i.e. transportable at some positive cost.

The relevant set of goods of any model considers the presence of the locational factor of transportation in a non-trivial sense includes the transport item. This specific good of transportation is actually a service that helps to effect any locational change of any other imperfectly mobile good of the economy. It is always a purely intermediate good. Since it is a service made available over space, it involves no problem of transport-ability of itself.

(b) Concepts of economic activities: basic productive activities and trading activities

Economic activities in our models fall into two groups: (i) the basic productive activities representing processes of technical transformation and (ii) the trading or shipment activities representing processes of spatial transformation involving only locational changes. The transport service may accordingly enter only some of the basic productive activities as output, while it should be directly required as input only in the shipment activities. It may be here noted that the latter activities are explicitly introduced only in those models where there exist some goods for whose shipment transport input would be required in positive finite quantity according to a functional rule.
All the basic productive activities are industrial by the very assumptions on their technical nature. They may alternatively be termed as industrial activities. The set of these activities may again be partitioned into two subsets. One of the subsets would consist of those activities which produce non-transport reproducible goods. The other subset includes only those activities that produce the transport good. The activities of the former subset will henceforth often be referred to as activities of the basic industrial sector (or of basic industries) while those of the latter subset as activities of the transport sector (or of transport industry). The possibility of such partitioning of the entire set of industrial (or basic productive) activities crucially hinges upon the validity of an assumption of no admission of joint production in the activities producing transport service. We shall see later that only the basic industries of our economy would involve locational problems, while for the transport industry such problems would be irrelevant.

(c) Concepts of locational factors: transportation and agglomeration

The present study considers only two of the locational factors — transportation and agglomeration — which were also considered by theorists of industrial location economics of the Weberian1/ tradition. Because of its assumptions on spatial regularities it does not account for the role of the locational factor of labour as it was done in the studies made by the Weberians.

1/ Alfred Weber's contribution to the theory of industrial location is a pioneering work in the field of location economics. His main work has been noted in reference no. [25].
Transport factor of location: For the locations of industries this factor induces a choice which minimises directly the use of transport input and thereby helps indirectly to economise the total use of primary resources by the transport industry. It thus helps to distribute industries regionally. The requirement of transport input by any trading activity for shipping an amount of any of the goods would be zero, infinite or positive finite depending on its being perfectly mobile, perfectly immobile or imperfectly mobile respectively.

Agglomeration factor: This factor appears only in one model in the present study. It causes greater savings in costs of the different industries when they get locationally closer, without necessarily clustering at the same point. In Losch's terminology the present study allows for the possibility of economies to arise not only from punctiform agglomeration, but also from an aerial variety of it. Consideration of this factor of agglomeration would in any case cause, as Weber has pointed out, a modification of the degree of concentration of production within the framework of its regional distribution as primarily determined by the transport factor.

(d) Price vectors: social valuation

The method of programming analysis that has been adopted in the present work which formalises the problem of planning for allocation of resources is used to find out the relevant optimum point on the consumption possibility frontier of the economy. Mathematically, it implies

\[ \text{Consumption possibility frontier of an economy is defined by the set of all attainable programmes of consumption at different consumption locations, no element of which is Pareto better than any other.} \]
a problem of solving for activity levels and locational values of productive activities which would maximise the aggregate social value of production subject to the constraints of resource availabilities. The very construction of such a programme would obviously require an assumption of values of final use prices for different locations of consumption. These prices are determined by the planners for distributing the goods to consumers at different places. The planners' dictation of these values may be supposed to have been made in accordance with their notion of welfare relations among spatially separated consumers. The discussions of all the models begin with assumption of such a social judgement and avoid the complicated question of its determinateness. At places where the models are alternatively interpreted in the setting of a purely competitive economy the assumed final use prices should be interpreted as market prices prevailing at different locations of consumption.

(e) Assumptions

We state and discuss here all the alternative assumptions that have been used in the different models regarding the nature of the goods, the relation of the basic productive activities with the use of land, the regularity of space, the regional structure, the technology and the inter-sectoral mobility of inputs. The main assumptions in these respects are represented below by arithmetic numbering. The alternatives of a main assumption are indicated by a, b, c, ... posited immediately after such numbering. However all the main assumptions need not be present in every model.
Assumptions on goods:

(1.1.a.) All the reproducible goods (excepting transport) are perfectly mobile over space.

(1.1.b.) All the reproducible goods (excepting transport) are imperfectly mobile over space.

(1.2.a.) All the non-reproducible goods are perfectly immobile over space.

(1.2.b.) Of the different non-reproducible goods some are perfectly immobile while others are imperfectly mobile over space.

(1.2.c.) All the non-reproducible goods are imperfectly mobile over space.

(1.3.) The transport good is a service that involves no problem of transportability of itself.

(1.4.) All the non-reproducible goods have only input use in the different productive activities.

(1.5.) Transport service is a purely intermediate good having input use only in the trading activities.

(1.6.) Each of the reproducible goods (excepting transport) has got some final consumption use.

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3/ Each of these two subsets of non-reproducible goods should be nonempty.

4/ This implies that any good that has got some final consumption use is a reproducible one.

5/ This implies that the set of purely intermediate goods would be a single element one containing only the transport good.
(2) Assumptions on the relation of the basic productive activities with the use of land: locations are points, not areas:

The fundamental assumption is the following:

All the basic productive activities are industrial by their very technical nature and accordingly have got, in general, very little use of land in their respective processes. Locations of such activities would, therefore, be represented by points and not areas of the two dimensional continuous plane of geographic surface.

This assumption makes the problem of allocation of land irrelevant for our purpose. The problem of locations of industrial activities other than transport (in point sense — fixed or variable) would, however, still remain. The transport sector produces its service by techniques of industrial processing. It describes geographically a network of linear routes connecting in a relevant way all the points of spatially separated locations of production, consumption and of resource deposit. (See assumption (4.5)). Again an assumption of zero requirement of own output as input has been made on the technology of this sector. (See assumption (5.11)).

All these imply the irrelevance of all locational problems for the transport industry. In other words we make the following assumptions in this connection:

(2.1.) Any single point of location of production would be able to accommodate spatially any number of plants or establishments of any number of basic industries which may operate any arbitrary levels.

(2.2.) The transport industry has no locational problem.

Unlike the industrial sector, production in the agricultural sector is quite land intensive. Any given piece of land can produce only one agricultural product, its per hectare yield having a finite upper bound whatever the intensity of cultivation may be. There presumably exists no such bound on the magnitude of per hectare yield of industrial production.
(3) **Regularity assumptions on space:**

Space is defined to be regular (or homogeneous) if any process of technical or spatial transformation be uniformly efficient irrespective of locational choice. The following assumptions elucidate the meaning of this uniformity in efficiency.

1. **(3.1.a.)** Any given basket of inputs\(^7\) will produce the same amount of output of any given basic industry for all possible locations of it.

2. **(3.1.b.)** For any given degree of locational closeness among the different basic industries operating at given levels, the requirements of inputs by each of those industries would be independent of the actual location of the industrial complex.\(^8\)

3. **(3.2.)** The amount of transport input required by any trading activity for shipping a given amount of any good over a given measure of linear distance would be the same irrespective of geographic positions of the terminal points of shipment.

4. **(3.3.)** Transport service can be made available for facilitating shipment of goods between any two points of location along a linear route joining them. The per unit real cost of providing such a service would be independent of actual geographic positions of the terminal points of shipment.

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\(^7\) This basket would obviously exclude the transport input since the latter may be directly required only in a spatial transformation process and not in any process of technical transformation.

\(^8\) We note it later that our assumption (3.1.a.) would be valid only in a model that considers presence of no agglomeration factor. Assumption (3.1.b.) replaces it whenever such factors are present in a model.
In assumption (3.3) by real cost we mean the consumption of different goods as input by the transport activities when operated at unit levels. This real cost would include the expenditure on inputs for constructing all the necessary routes, rolling stocks, etc., for providing a given ton-mile of transport service. The infrastructure of our economy which includes these constructions is, by itself, supposed to be variable. We have incorporated all problems in connection with the construction or laying of this type of infrastructure in the transport activities. Besides, we also make a strong hypothesis that our long run is sufficiently long so that all investments on infrastructure would be consumed up in the processes of production of the economy during the period concerned, and that we face no problem of indivisibility with respect to such investments.

It may be noted that assumptions (3.1.a.) - (3.3) would imply the following:

(i) The technological parameters of any basic productive activity would be independent of all locational connotations.

(ii) The transport input coefficient of any trading activity would depend on locations only to the extent that they determine the distance over which the trading or shipment is to take place.

(4) Assumptions on the regional structure of the economy:

(4.1.) The consumption of outputs of the basic industries can take place at point formed centres of location which are exogenously given to a model and are finite in number.
(4.2.) The imperfectly mobile non-reproducible resources are initially localised at point-formed centres of deposit which are exogenously given to a model and are finite in number.

(4.3.a.) For any basic industry of a model the locational alternatives of production are exogenously given and are finite in number.

(4.3.b.) The location of any basic industrial activity is a continuous variable in the two-dimensional plane of geographic surface of the economy.

(4.4.a.) Production of any basic industry is locationally divisible only to a limited extent.

(4.4.b.) Production of any basic industry is locationally indivisible.

Assumption (4.4.a.) means that the productive activity of any basic industry can take place at the same time at a finite (possibly more than one) number of locations. Assumption (4.4.b.), on the other hand, implies that the entire production of an industry has to take place at a single location in a single establishment.

We shall note it later that assumption (4.4.a.) is valid only in a model which considers locations of production as parameters while the alternative assumption (4.4.b.) accompanies the consideration of locations of production as variables. These combinations of assumptions may be interpreted from the viewpoint of planning as follows.
Assumption (4.3.a.) is valid in Model I and Model II. These models assume the economy to be multiregional in structure. The local authority enjoys the autonomy of choosing the sites of all production that would take place within its boundary. The decisions on locations that should be made by any regional authority are determined by some unspecified mechanism of choice and are supplied as given values to the central planners who allocate resources over regions on their basis. At any such location of any basic industry it is presumed that the local authority can produce any amount of the good concerned according to some linear technical rule provided that the resources are available in required amounts. While framing its allocation problem the central planning authority finds that it can locationally distribute resources at most among a finite number (possibly more than one) of given points in the entire economy where production of any good can possibly take place. Thus the combining assumption (4.3.a.) with assumption (4.4.a.) becomes imperative in the models of Part I of this thesis.

The models of Part II and Part III, on the other hand, do not extend the autonomy of choosing locations of production to the regional authorities; they consider the problems of regional allocation of resources with locations as continuous variables and as such, objects of simultaneous choice before the central planners. These models impose a constraint that production of no basic industry can be locationally split up. They also assume among other things the validity of assumption (4.6) and assumption (5.9). The following assumptions are in fact underlying the constraint of perfect locational indivisibility of production of these models: 
(i) The marginal factor of production is indispensably required in all industrial activities (including transport) and it has got no problem of transportability of itself. Managerial staff cannot organise production at a particular place from a different location.

(ii) The period of analysis of a model is not so long that the overall supply of the managerial factor of production of the economy can be augmented.

(iii) The managerial input is (a) indivisible and (b) limited in supply so that our model permits of only one production unit per industry (including transport). 9/

Because of the validity of assumption (4,6) and assumption (5,9), any nontrivial feasible solution of the location-allocation programme of any model of Part II or Part III would require that all the industries (including transport) are operated at positive levels. Again the assumptions of perfect inelasticity of supply, indivisibility and overall scarcity of the managerial factor of production imply that all output of any industry would be produced in a single production unit whenever all the industrial

9/ At this point we assume that the running of any single establishment of production of any given industry would technically require an indivisible minimum of managerial capacity. These minimum values as required by the different industries may be quite significant when compared to the total supply of the managerial resource in the entire economy.

We should also note that the task of gathering various inputs at any location of production and of sending the product to the locations of consumption is managed by the managers of the respective industries. The problem of management of any industry then incorporates all problems with managing the relevant trading activities.
activities are operated at positive levels. The validity of assumption (4.4.b) is thus obtained as a consequence of the assumptions on the nature and the availability of the managerial input and of other accompanying assumptions of the models concerned. For all feasible situations allocation of the managerial input would therefore be the same and the problem of determining the optimal allocation of this input would reduce to be a trivial one. The problem of allocation of the managerial input has not, accordingly been considered in any of the formal models explicitly; the constraint of perfect every locational indivisibility of production of \( \mathcal{B} \) basic industry has instead been explicitly taken into account through assumption (4.4.b). The imposition of this constraint ensures that the optimality of the location-allocation solutions of our models remains consistent with the making of the assumptions (i) – (iii) of the preceding paragraph about the managerial factor of production which would exist in reality, but would not be considered in the formal models.

Finally, we suppose that the models assuming (4.3.a) and (4.4.a) have no such scarcity of the managerial resource so that the production of any basic industry may be locationally divisible.

(4.5) The transport sector geographically links up the points of location (fixed or variable) of our economy by a network of linear routes as follows:

(i) Each of the locations of the basic industries is linked with the locations of consumption which import goods of final use from it.
(ii) Each of the locations of the basic industries is linked with the locations of resource deposit which export imperfectly mobile non-reproducible resources to it.

(iii) All the locations of the interindustrially related basic industries are linked among themselves.

It may, however, be argued here that the possibility of various locations of consumption (or of resource deposit) being connected among themselves should also be considered in our models so that movements of goods among those locations may take place and our economy may achieve a greater return in value. The point to be noted here is that for an optimal solution of location and allocation no such trading of final (or of non-reproducible) goods would be profitable if the transport cost of any given amount of any good varies proportionately with distance. This is because the movement of any imperfectly mobile good between any two points along a linear route would be more efficient than that along a broken linear one. As we are interested only in the properties of the efficient solutions, we do not introduce the existence of those routes in the statement of our assumption (4.5), nor consider explicitly the tradings along such routes as possibilities in our models.

(4.6.) The fixed locations (of consumption, imperfectly mobile resource deposit and possibly of production) of our economy are all distinct and no three of them are collinear.
Assumptions on technology:

The following assumptions describe the relation of inputs (other than the managerial one) with outputs of the different productive activities. We assume that for any given production unit, its managerial input is not a scarce resource over the domain of all feasible solutions, permitting us to retain the assumption of constant returns to scale.

(5.1.a.) For any given locational distribution the technology of the basic productive (or industrial) activities is general linear in character.

(5.1.b.) For any given locational distribution the technology of the basic productive (or industrial) activities is Leontief in character.

(5.1.c.) For any given locational distribution the technology of the basic productive (or industrial) activities is nonlinear in character, so that it admits of the presence of agglomerative factors through externalities and scale effects of production.

(5.2.a.) The basic productive (or industrial) activities producing transport service do not admit of joint production.

(5.2.b.) None of the basic productive (or industrial) activities admits of joint production.

(5.3.) Each productive activity requires at least one input.

(5.4.) All the intermediate inputs are currently produced.
(5.5.) Each of the non-reproducible immobile resources is indispensably required as input in the activities of at least one basic industry, although the ratio of its requirement with respect to other inputs may be different in the different activities of the same basic industry.

(5.6.) If some of the activities of a basic industry require a particular non-reproducible immobile good as input in positive amount, all the activities of that industry would require that good indispensably as input.¹

(5.7.) The technology allows for the possibility of intermediate input use of the reproducible goods in any basic productive (or industrial) activity.

(5.8.a.) The transport industry does not require any reproducible good as input.

(5.8.b.) The transport industry does not require any perfect immobile non-reproducible good as input.

(5.9.) The basic industrial activities are connected¹¹ among themselves through relations of inter-industrial consumption.

¹ The combination of assumption (5.5) and assumption (5.6) will at some places be referred to as factor indispensability hypothesis. It has been explicitly made only in one of our models.

¹¹ By connectedness of the basic industries we mean that the matrix of interindustrial input consumption of those industries is indecomposable.
For the movement of any imperfectly mobile good, the demand for transport input follows the functional rule of exact proportionality with respect to the weight carried and the distance covered. For a model the constant of proportionality is the same for all such goods and is assumed to be equal to unity.

The Weberian functional rule has been retained here for defining the mode of influence of the transport factor in a model of production. The requirement of transport input in any programme of production would thus depend on the locational happenings of the basic industries of a model.\textsuperscript{12}

The amounts of inputs required by the transport industry do not have to be transported.\textsuperscript{13}

\textsuperscript{12} The Weberian function of demand for transport input that has been assumed here is explained in Weber's work cited in reference \textsuperscript{25}.

\textsuperscript{13} This assumption was made by Professor Lefeber in his models on spatial equilibrium of production and distribution \textsuperscript{15, p.11}. The framework of Lefeber's analysis may be modified to account for transportation problem of inputs used in the transport industry and also for locational problem of this industry. This has been avoided here in order to keep the mathematical complexity of the problem at a minimum level without any substantial loss of theoretical points.
The functional rule according to which agglomeration economies arise assumes that for given levels of operation of the different basic industrial activities, input requirements would decline in any of them when its distance from the location of at least one other gets shorter, while its distance from any of the remaining basic industries remains unchanged. These input requirements would, however, decrease in all the basic industries in the case of shortening of all the mutual distances between the locations of the basic industries.\textsuperscript{14}

(6) \textbf{Assumption on intersectoral factor mobility:

(6.1) The inputs are perfectly immobile between the sector of basic industries and that of the transport.}

This assumption has been made only in Model III of Part II. The model presupposes the solution of the problem of allocation of non-reproducible resources between those two sectors and leaves to the planners only the problem of choice of allocation among the alternative activities of the basic industries along with the choice of their locations. We shall note that the model which imposes such a restriction on intersectoral mobility of non-reproducible resources also assumes the transport industry to have a fixed coefficient technology of production and to have no requirement of any reproducible good as input. All these would imply the following:

\textsuperscript{14} The spatial distances between industries only matter here, the agglomeration effect being indifferent to the actual location of an industrial complex.
(i) First, the transport sector should be independent of the basic industrial sector from the viewpoint of inter-industrial consumption relations.

(ii) Second, we should get the maximum producible amount of the transport good to be uniquely determined by the a priori fixed endowment of non-reproducible resources specific for the use of the transport industry.

(iii) Finally, for any arbitrarily given set of prices of the non-reproducible resources, the average cost of production of the transport industry would be constant up to the capacity output.

Under such conditions the analysis of location and allocation of the model concerned may be decomposed into two stages. In the first stage we should carry out the analysis of production and location of the basic industries without explicitly introducing the activities of the transport industry. At this stage the supply side of the balance constraint of the transport good would represent the maximum producible amount of it which is predetermined as noted above. From the solution of location-allocation of the basic industries we shall readily be able to obtain the amount of the transport service that would be required for production and distribution of the final goods in the optimal situation. In the second stage of analysis we would then solve how to produce this requirement of the transport good. The problem of the second stage reduces to a trivial one because of the fixity of technological coefficients of the transport industry. Its solution would in fact influence in no way the decisions of the planners about the programme.
of location and production of the basic industries. The model concerned actually poses the following problem:

For an arbitrarily given valuation of the non-reproducible goods, find out the least cost solution for locations of the basic industries which have to support a given feasible consumption programme.

Because of the Leontief nature of technology of all the industries (including transport) and of the other accompanying assumptions on technology and on intersectoral mobility of primary inputs, this problem would reduce to a simple one of choice of locations for which the requirement of transport input would be minimum. Accordingly, we need not always carry out the analysis of this second stage separately in the model concerned.

1.2. Conclusion:

After having discussed here the basic concepts and the assumptions of the present work, we are now going to take up in the subsequent chapters a detailed analysis of the different models. A résumé of the results of the models has been given in the concluding chapter (Chapter Ten).

15/ Chapters Three to Nine.