CHAPTER TEN

CONCLUSION

This concluding chapter gives a resume of our discussions of the preceding chapters. The following review of our models of Parts I - III makes the results of the present work readily available in a classified summary form.

**Part I**: Broad assumptions: Locations are a priori fixed; technology of basic productive activities is linear.

This part focuses attention on the influence of locational factor of transportation on the efficient use of economic resources in a general linear model of production. The assumption of all locations as parameters helps us to retain the linearity of framework of the entire analysis. It thereby indicates the direction in which the linear model of production of Koopmans (11) would be modified within that framework so as to accommodate the reality of spatial flows of goods among the different fixed points of the spatial models. Model I and Model II of this part differ in the character of transport cost of the different goods.

**Fixed locations, linear technology**

**Model I**: Transport cost - zero or infinity

Assumptions and Motivation:

Model I assumes all the reproducible goods to be perfectly mobile (1.1.a) and all the non-reproducible goods to be perfectly immobile (1.2.a). Each of the reproducible goods explicitly considered in the model has got
some final consumption use (1.6), while all the non-reproducible goods have only input use (1.4). Knowledge is not locationally confined so that the technology of basic productive activities remains the same at all the locations of production (3.1.a). The possible locations of industrial production and consumption activities are represented by a priori fixed points (finite in their number) on the geographic surface of the economy (2.1, 4.1, 4.3.a). The production of any basic industry is locationally divisible only to a limited extent (4.4.a). All the exogenously given points of location are however distinct and no three of them are collinear (4.6). The technology of production which is assumed to be ‘general’ linear in character (5.1.a, 5.3) specifically rules out the possibility of joint production (5.2,b) and admits of the validity of a factor indispensability hypothesis (5.5, 5.6) and of the possibility of intermediate input use of the reproducible goods, the intermediate inputs being all currently produced (5.4, 5.7). The transport problem is absent in this model, but the perfect immobility of primary factors will play a decisive role in determining:

(i) the conditions of regional interdependence of an economy, and
(ii) the nature of allocation of resources and of values of their imputed efficiency prices at the different places.

The analysis has been made with the help of the techniques of linear programming.

**Description**:

The economy is one consisting of K number of different regions where the regional demarcation is arbitrary. In each region the local authority decides the sites of all production of the different basic industries that this perfect immobility of the primary factors constitutes the locational factor of transportation in this model.
will take place within its boundary by some unspecified mechanism of choice. The economy has got a Central Planning Authority (C.P.A.) which allocates resources regionally on the basis of the given locational decisions of local authorities of the regions. Since the present model is concerned only with the analysis of the problem of regional resource allocation of the C.P.A., the locational alternatives of production (lying before the C.P.A.) in the entire economy are considered here only as parameters. The model further supposes that the local authority in each region informs the C.P.A. of its decision of single producing all output of all the basic industries only at some common point of location. Thus in this model there exists only one location of production as exogenously given in each region. The locations of consumption, on the other hand, are also given exogenously and their number is a finite one in each region. (The number of consumption locations need not be the same for all the regions.)

The location of production of every region of our model is endowed with a vector of fixed supplies of the different non-reproducible resources which cannot move. In all the regions there prevails a common set of final use prices of all the reproducible goods. Since transport cost is here not a variable in effective sense and since technology is identical everywhere, the locational parameters of the productive activities enter this model of production in the form of regional subscripts attached to the variables representing activity levels and to the vectors of endowments of primary resources of the different production locations. Accordingly neither the transport good nor any activity producing it has been explicitly introduced in the model.
Finally, Chapter Three introduces a concept of interregional irreducibility. The model is then described as one of a multiregional economy irreducible in interregional sense.

Results and Observations:

Theorems 3.1 and 3.2 of Chapter Three give the necessary and sufficient conditions of interregional irreducibility as defined in the model. They also provide criteria for empirical tests of interregional connectedness for any given situation under the respective conditions of indecomposability or decomposability of interindustrial consumption technology matrix. The validity of the factor indispensability hypothesis has greatly facilitated the formulation of the empirically verifiable conditions of such irreducibility.

Chapter Three also analyses the nature of the optimal solutions for allocation of resources and for their imputed efficiency prices in the different regions of our economy. Any optimal solution of the central problem of this model also been shown to be an equilibrium one under conditions of competition in an economy which allows for market interventions. Theorem 3.3 of this chapter gives a set of sufficient conditions for the equalisation of primary factor prices over regions. It has been constructed after McKenzie's model in his article "Equality of Factor Prices in World Trade" [17].

Finally, this theorem leads to a corollary to the following effect: if the interregional model is irreducible, the equalisation of primary factor prices may be only a possibility, and not necessarily a reality in an optimal situation.
Fixed Locations — Linear Technology

Model II: Transport Cost — Variable

Assumptions and Motivations

 Unlike Model I, Model II assumes that movement of any of the reproducible goods (excepting transport) and of some (not all) of the non-reproducible goods involves positive transport cost (1.1.b, 1.2.b). The remaining non-reproducible resources are perfectly immobile over space. All the reproducible goods (excepting transport) have got final consumption use (1.9), while the non-reproducible goods can be used only as input (1.4) in the productive activities of the model. The most important difference between the this and Model I γ-model is that in the present one transport input has to be expended for ensuring movements of imperfectly mobile goods over space according to the functional rule given in assumption (5.10). The transport good and the activities producing it obviously enter the model explicitly. This reproducible good is in fact a service which has got only input use in the trading activities (1.5). It involves no problem of transportability with reference to itself (1.3). The amount of resources that the transport industry requires as input do not have also to be transported (5.11).

Like Model I, the present model assumes the technical knowledge not to be locationally confined anywhere so that the technology of production remains the same at all the locational alternatives of production of the different industries (3.1.c). The regularity assumption on space now additionally ensures two things. First, the transport cost of trading is uniform everywhere when the trading involves shipment of any given weight over a measure of distance (3.2). Second, the cost of providing any given ton-mile of
transport service would be uniform in the different parts of geographic surface of the economy (3.3).

The locations of production of the basic industries and the locations of consumption and of deposit of imperfectly mobile resources are all spatially separated and represented by a priori fixed points in the plane of geographic surface (3.1, 4.1, 4.2, 4.3, a) of the economy. No three of these points are collinear (4.6). The production of any given industry may be locationally divisible only to a limited extent (4.4.a). The transport sector, on the other hand, has got no locational problem (2.2) and it geographically describes only a network of linear routes connecting in a relevant way the spatially separated fixed points of location (4.5).

Finally, the technology of the basic productive activities is retained here to be general linear in form (5.1.a, 5.3) without any further restriction on the possibility of joint production except in relation to the activities of the transport industry (5.2.a). This technology allows for the possibility of use of some reproducible goods as input in the production processes (5.7). All these intermediate inputs would be currently produced (5.4). Besides these, there is a further restrictive assumption that the transport industry requires none of the perfectly immobile primary resources as input (5.8.b).

The existence of the locational factor of transportation turns out to be an effective factor in our model of allocation of resources consisting of the above assumptions. The framework of the linear model of production of Chapter One gets accordingly considerably modified in order to account for the altered technical situation. The motivation of discussions of this model
is to analyse in this modified schema the problem of spatial allocation of resources and distribution of products, just as it would be done by the neoclassical writers, with the help of linear programming techniques.

Description:

The economy of this model consists of two arbitrarily demarcated regions. In each region the local authority decides the sites of all production of the different basic industries that will take place within its boundary by some unspecified mechanism of choice. The economy has got a Central Planning Authority (C.P.A), which allocates resources regionally on the basis of the given locational decisions of the regional authorities. Since the present model is concerned only with the analysis of the problem of regional resource allocation of the C.P.A, the locational alternatives of production (lying before the C.P.A) in the entire economy are considered here only as parameters. The model further supposes that the local authority in each region informs the C.P.A of its decision of producing all output of all the basic industries only at some common single point of location. Thus in this model there exists only one location of production as exogenously given in each region. The locations of consumption and of deposit of imperfectly mobile resources, on the other hand, are also exogenously given in each region. There exist in fact one point of location where all the consumption activities of a region take place and another point of location where all the imperfectly mobile primary resources of a region lie initially localised. Since the economy consists of two regions and since the a priori given locations of consumption, production, and of deposit of resources are all spatially separated, the regional structure of
the entire economy would be completely described by a set of altogether six points which are to be connected in a relevant way among themselves by the transport network.

At each location of production of the economy there is available a given supply of perfectly immobile primary resources. At each location of consumption, on the other hand, there prevails either a perfectly inelastic vector of demand or a given vector of final use prices of the reproducible goods. Trading activities appear now explicitly in the model and ensure the possibility of flows of goods among the different given points of locations of it. The formal presence of a nontrivial transport factor makes some of the technical parameters of the trading activities to be dependent on the structure of fixed locations of the economy.

In connection with the description of the economy, Chapter Four finally restates the definition of interregional irreducibility and examines its meaning in this context just as it was done in Model I.

Results and Observations:

In Chapter Four the model, first of all, discusses a consistency problem of planning. The problem is whether it is possible to ensure that the economic system, as described above, can satisfy, after obeying all the resource constraints, any arbitrary pattern of spatial distribution of consumption demand \( d_k \), \( k = 1, 2 \) where \( \mathbf{0} \leq d_k \leq \mathbf{c}^k \).

(i) \( d_k \) denoting the vector of consumption demand of the consumption location of the \( k \)th region.
(ii) \( \mathbf{c}^k \) \( k = 1, 2 \) describing a spatial consumption distribution for which there exists a consistent solution in the model.

The results of analysis of this model would be generalizable to the case considering any finite number of points of consumption (or of production or of resource deposit).
The answer is 'no'. Theorem 4.1 establishes this result. The discussion of this chapter in this context also notes that the answer to this question will be in affirmation when the structure of mobility of different goods is defined to be exactly similar to that of Model I.

The other problem that Chapter Four takes up for discussion is one of analysing the nature of optimal allocation of resources and of optimal patterns of trade flows among the different points of location of the model when a final social valuation of consumption of the different goods at the different places of consumption is given. To be more specific, theorems of this chapter (Theorems 4.2 - 4.5) derive and prove, under some conditions, some results on:

(i) the choice of production location from which a consumption location should import any given product,

(ii) the choice of production location from which the transport industry should draw its requirement of any particular intermediate input,

(iii) the choice of resource deposit from which a particular location of production should draw its requirements,

(iv) the necessary conditions for ensuring the possibility of all consumption locations of the economy being simultaneously served with a particular commodity by a single location of production,

(v) the general impossibility of requirements of a particular product (or of a resource) being supplied in more than one consumption location (or production location) by each of the two production locations (or resource deposits) of the economy at the same time,
(vi) the general impossibility of a location of production exporting more than one final product to all the locations of consumption of the economy at the same time.

All these results have, however, been derived for an optimal situation where imputed efficiency price of the transport input is obtained positive, i.e., where the transport input is scarce and imposes a binding constraint. In conclusion the chapter notes the validity of all the above results in an equilibrium situation when the central problem of the model is posed in an alternative setting of a competitive economy.

Part II: Broad assumptions: Locations are continuous variables, technology of basic productive activities is linear and specifically Leontief in nature.

This part discusses the problem of transport orientation of production and location. Here the transport factor of location has been introduced typically in Weberian manner into the Leontief production schema. Since both the activity levels of the industries and their locations appear as matters of continuous choice, the departure of the overall structure of analysis from linearity cannot but be helped in spite of linear assumptions on the basic production technology. Model III and Model IV of this part differ mainly on two points:

(i) they try to solve two somewhat different problems on production,
(ii) they differ in respect of intersectoral mobility of inputs — the sectors being the basic industrial and the transport ones.
Variable Locations, Linear Technology

Model III : Fixed Consumption Targets

Assumptions and Motivation:

The central problem of this model is to find out, for an arbitrarily given set of prices of the non-reproducible goods for their use in the different sectors (the sectors being ones of basic industries and of transport), the least cost solution for locations and for techniques of production which is to satisfy some given vectors of consumption demand at the different consumption locations. It is presumed here that these given targets of consumption at the different locations describe an attainable programme. The model presupposes the solution of the problem of allocation of resources between the transport and the basic industrial sectors. Accordingly it begins with an assumption of perfect intersectoral immobility of inputs (6.1).

Regarding the nature of goods the model assumes all its non-reproducible goods to be imperfectly mobile over space (1.2,c). The number of such resources directly used in the different industries as input is taken to be one. All the reproducible goods (excepting transport), two in number (2), are also imperfectly mobile (1,1,b) and are produced only by a Leontief technology (5.1,b, 5.3). The basic industries producing them are connected among themselves through interindustrial consumption relations (5.4, 5.7, 5.9). The transport good, on the other hand, is also produced by a technology of Leontief character which would, however, admit of no possibility of consumption of any intermediate input (5.8,a). Moreover, the resources required in its production do not have to be transported (5.11). The functional nature

2 The results of this model are perfectly generalisable to the case where the number of reproducible goods (excepting transport) is n, (n is any positive integer.)
of requirement of the transport good as input in the trading activities follows, on the other hand, exactly the Weberian rule (3.10).

The problem of choice of locations of the basic industries has been here considered in a continuous plane of geographic surface assumed to be homogeneous from the point of view of efficiency of production (3.1.a). This implies that the model takes locations as continuous variables (4.3.b).

Among the different constraints of the optimisation problem of location of this model, there is one of perfect locational indivisibility of production (4.4.b). Accordingly our model considers only one variable point of location for each basic industry. The locations of consumption and of deposit of the only imperfectly mobile primary resource are on the other hand represented by a finite number of a priori fixed points in the plane of geographic surface of the economy (4.1, 4.2). These points of fixed locations are all distinct and no three of them are collinear (4.6). The transport sector connecting geographically describes a network the spatially separated locations (fixed or variable) of consumption, production and of deposit of resources among themselves in a relevant way (4.5).

The other assumptions regarding the nature of use of the different types of goods, the specific nature of the transport good, the relation of the basic productive activities with the use of land and the regularity of space with its reference to trading and transport activities are exactly the same as those of Model II. 4/ 

4/ The remaining assumptions of Model III that are all common with those of Model II are the following:

(1.3), (1.4), (1.5), (1.6), (2.1), (2.2), (2.2), (3.3).
The best manner of fulfilling the given consumption programme of the problem of our present model would lead to a technical and locational organization of production for which the total cost due to expenditure on the primary factor is kept at a minimum. The above mentioned assumption of indecomposability of the structure of interindustrial consumption of the basic industries would further imply interdependence of locations of those industries in a strong sense.

Description:

Model III assumes a centralised structure of all decision makings on production. It considers a problem of regional allocation of resources with locations as continuous variables in space and as such, objects of simultaneous choice before the Central Planning Authority (C.P.A.) of the economy. Since the local authorities of any part of the economy are allowed no significant autonomy in the planning of production, the explicit description of the economy as consisting of a number of demarcated regions will no longer be interesting for the present model. The regional structure of the economy has therefore been here completely described by a number of points of locations, some of which are fixed and some variable. It has been assumed that the entire supply of the only primary input of the model lies initially localised at a single point of source, given to the model. The consumption locations of the economy are also taken as parametrically given and their number is assumed to be two. Although the location of production of any basic industrial activity is, on the other hand, variable, there exists a constraint of locational indivisibility of production of any basic industry in this model. It is therefore impossible.
to split up locationally the production of an industry although any such splitting may help attainment of greater efficiency in meeting the requirements of the different consumption locations. Because of such a description of regional structure of the economy and of the assumptions on technology, the gross production levels of the different basic industries, the total requirement of the non-reproducible input in those industries, the number of spatial flows and the magnitude of weight of each such flow between any two relevant points of location are all immediately determined at the very moment when the demand requirements of the different consumption locations are stipulated. It is only the requirement of transport input, the programme of production of the transport industry and the locations of the basic industries that would remain variable in effective sense thereafter. Because of in Chapter One, the points noted in the comment on assumption (6.1) we find the part of analysis of production of the transport industry to be a trivial one in the context, and therefore consider often its explicit introduction to be unnecessary. In fact any solution of the model minimising requirement of the transport input should also minimise the total cost on the primary resource for any arbitrarily given prices for its use in the different sectors. As a result the optimisation problem of the model ultimately reduces to one of finding the minimum transport cost locations of the two basic industries when the industries have to produce a given configuration of outputs. This derived problem would mathematically be equivalent to one of determining the points of location that minimise the weighted sum of all the relevant distances among

5/ The feasibility of such determination of values of some of the variables is ensured by the hypothesis of attainability of the stipulated consumption programme of the central problem of this model.
the points of locations over which shipments have to take place. It thus looks like a typically Weberian problem of transport-oriented location.

**Results and Observations:**

The discussion on results of Model III, as given in Chapter Five, derives and analyses the necessary and sufficient conditions that a locational scatter of the basic industries should satisfy in order to describe an efficient locational organisation of production which is to meet a given consumption programme. It also attempts to derive, in particular, the conditions for occurrence of any locational coincidence between a fixed and a variable point of location and/or between two variable points of location in an efficient situation. It then interprets the mathematical conditions so derived in economic terms and also in terms of conditions of equilibrium of locational forces with the help of a mechanical analogy.

A few words on sensitivity analysis have also been added in the chapter. This sensitivity analysis consists of formulation of the relevant inequalities which would define a range of variability of any parameter over which the original solution for locations would preserve optimality. The concluding section of Chapter Five makes a note of the direction and the extent of modification of the mathematical structure and the results of the model in case of a generalisation of the regional structure of the economy in each of the following senses:

(i) any number of consumption locations,

(ii) any number of deposits of the given primary resource,

(iii) any number of production locations for any single basic industry (i.e., perfect locational divisibility of production of the basic industries).
Chapter Six continues the discussion on results of Model III. The central problem of the model, as it has been posed in Chapter Five, has been shown to be mathematically reducible to one of an unconstrained problem of minimising the weight sum of the distances of the two variable points of location from a set of fixed points and of the distance between those two variable points of location. Chapter Six attempts, first of all, to pose a mathematical dual to this unconstrained minimisation problem. This dual programme is nonlinear in character; its statement does not require any solution of the primal problem beforehand, and yet it possesses almost all the useful properties of duality as defined in linear programming. The construction of this dual has been made after H. W. Kuhn who gave it as a Fasbender duality generalised in a context where the primal is the General Fermat problem of asking for a point which minimises the weighted sum of distances (weights being a priori fixed) from a given points in a Euclidean plane. The mathematical arguments of duality as given by Kuhn have been restated and extended in Chapter Six so as to cover the central problem of Model III. Moreover, the chapter attempts to interpret this dual problem along with the primal one in an economically meaningful way. Kuhn hinted at a possible economic meaning of the General Fermat problem in the context of industrial location theory; but he gave no meaning of the dual in real terms. Chapter Six tries to find, though in a bit roundabout way, the meaning of mathematics of both the primal and the dual problems in a consistent way. Besides, it has been noted how an algorithm for a transport oriented efficient solution for locations of Model III may incidentally be derived from the mathematical arguments given for establishing the duality.
Variable Locations, Linear Technology
Model IV: Fixed Social Valuation of Final Goods

Assumptions and Motivations:

The central problem of Model IV is one of finding an efficient point on the consumption possibility frontier of the economy for a given social valuation of final goods at the different consumption locations. The problem of the present model is then different from that of Model III.

The assumptions of the two models (Model III and Model IV) differ only in the following two respects:

(i) First, Model IV imposes no restriction on intersectoral mobility of inputs — the sectors being the basic industrial and the transport ones. Accordingly it considers the problem of allocation of resources among all the industries including transport simultaneously. (Absence of assumption (6.1) that was considered in Model III.)

(ii) Second, in Model IV, the technology of the transport industry is of Leontief nature and it admits of the possibility of intermediate input consumption. (Absence of assumption (5.8.a) that was considered in Model III.)

In all other respects the structure of the present model as defined by the different other assumptions remains exactly the same as that of Model III.2/

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2/ The assumptions of Model IV that are all common with those of Model III are the following:

- (1.1.b), (1.2.a), (1.3), (1.4), (1.5), (1.6), (2.1), (2.2), (2.1.a), (3.2), (3.3), (4.1), (4.2), (4.3.b), (4.4.b), (4.5), (4.6), (5.1.b), (5.3), (5.4), (5.7), (5.9), (5.10), (5.11).
Description:

The regional description of the economy of Model IV is exactly the same as that of Model III.

The descriptions of mathematical structure of the two models (Model III and Model IV) presenting their respective central problems, as stated above, differ, however, considerably because of the differences in respect of some of the assumptions as mentioned above. The mathematical situation described by the nonlinear programme of location and allocation of Model IV deviates from that of the programme of Model III. As a result the procedure of solving the problem of Model IV is no longer immediately reducible into a two stage analysis as it was in the case of the problem of Model III. The values of allocation variables can no longer be solved prior to the finding of locational solution since all the variables of the system are now interdependent.

Results and Observations:

While analysing the central problem of the model, Chapter Seven defines, first of all, the set of all possible locational choices for the basic industries of the economy for each of which there would exist at least one nontrivial solution for allocation of resources and distribution of final outputs. Some of the lemmas and theorems of the chapter state and prove the following different properties and characteristics of this set:

(i) First, the conditions (necessary and sufficient) that any locational choice for basic industries should satisfy in order to be an element of this set are derived and proved.
(ii) Second, the property of openness of the set and the conditions necessary and sufficient for its boundedness have been stated and proved.

(iii) Third, it has been pointed out that for any given locational distribution of the basic industries which corresponds to an element of the set, the derived model of allocation of resources (conditional upon that locational choice) would turn out to be linear and also solvable under usual assumptions of productivity and of impossibility of cockaigne.

(iv) Fourth, it has been established how the optimum value of the conditional allocation programme of (iii) would be a continuous function of locational choice defined over the set concerned having a maximum over it.

From the lessons of all the above results the chapter finally proves a theorem on solvability of the programme of location-allocation of the model. The existence of a supporting system of efficiency prices of inputs (i.e., a set of values of Lagrange multipliers) associated with each optimal solution of the programme has also been established. It has been pointed out in this context, how the non-fulfilment of the required concavity-convexity conditions of well behaved neoclassical models in the programme of our location-allocation may raise problems with the following:

(i) non-convexity of the set of optimal solutions when multiplicity of solutions is possible.

(ii) the possibility of the existence of an efficiency price system for inputs which supports some optimal solutions of the location-allocation programme, but not all such solutions.
Chapter Eight continues discussions on the analysis of the nature of optimal solutions of the programme of location-allocation of Model IV. It defines the concept of an extreme locational solution for basic industries as a configuration of locations for which each variable point of location of production coincides with one of the fixed points of the model. The theorem of the chapter establishes that set of optimal solutions of the programme of location-allocation/chapter seven contains at least one solution whose locational component is an extreme one. The discussion in this context indicates incidentally a method of computation of the optimal solutions of the programme.

Chapter Eight also contains a discussion on sensitivity analysis of the optimal location-allocation solutions with respect to variations in the parameters of the model. The method of analysis at this point consists of determining the inequalities which variations in any parameter should satisfy for ensuring the stability of the initial extreme locational solution and of the associated initial basic allocation pattern of resources.

In the concluding section remarks have been made on the direction of modification of the framework of analysis and of the results of our model when generalisation of the regional structure of the economy is made in ray of the following senses:

(i) any number of consumption locations,
(ii) any number of deposits of the primary resource,
(iii) any number of production locations for any single basic industry (i.e., perfect locational divisibility of production of the basic industries).
Finally, the concluding section adds a note on how to look at the problem of equilibrium allocation of resources in locational context like the one described in the model, in a setting of competitive market conditions. It has been here incidentally pointed out that the violation of the neoclassical concavity-convexity conditions as mentioned above may stand in the way of two-way correspondence between an efficient solution and an equilibrium one. The problem of adjustment difficulties to the efficient point equilibrium from any initial condition may at this point arise due to the existence of multiple local equilibria some of which may be sub-optimal for the central problem of Model IV in global though not in local sense. This point has only been hinted at and has not been discussed in any details.

Part III: Broad Assumptions: Locations are continuous variables; technology of basic productive activities is nonlinear accounting for externalities and returns to scale.

Variable Locations, Non-linear Technology

Model V: Fixed Social Valuation of Final Goods

Assumptions and Motivation:

This model introduces the locational factor of agglomeration into the analytic scheme of production of Part II which considered only the locational factor of transportation. Like Part II, both locations of production and production levels of the different industries are continuous decision variables of the social planners in this model (4.3.b). But the technology of industrial production which is nonlinear has been here so defined in order to account for the agglomerative factor that it has to accommodate scale effects of production along with externalities (5.1.e). In any case, the technology
however rules out the possibility of joint production in any of the basic productive (or industrial) activities (5.2.b). The functional rule according to which agglomeration economies arise in the model states that for given levels of operation of the different basic industrial activities, input requirements would decline in any of them when its distance from the location of at least one other got shorter, while its distance from any of the remaining basic industries remains unchanged. These input requirements would, however, decrease in all the basic industries in the case of shortening of all the mutual distances between the locations of the basic industries (5.12). In all other respects the assumptions of Model IV and Model V are exactly similar.  

The present model poses for a given social valuation of final goods at the different places the problem of optimal choice for locations and production levels when the non-pecuniary economics arising out of agglomeration and returns to scale are operative within a framework of transport orientation. It also tries to develop a rough or approximate method of disentangling the influence of force of agglomeration from that of others on this optimal choice.

Description:

The regional structure of the economy of Model V is exactly similar to that of Model IV of Part II.

7/ The meaning of regularity of space will however be slightly different in the context of Model V. Since Model V considers the presence of locational factor of agglomeration in addition to that of transportation it assumes (3.1.b) while Model IV assumed (3.1.a). The remaining assumptions of Model V that are all common with those of Model IV are the following:

- (1.1.b), (1.2.a), (1.3), (1.4), (1.5), (1.6), (2.1), (2.2), (2.3), (2.4), (2.5), (2.6), (3.1), (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9), (3.10), (3.11),
The mathematical descriptions of the structures of two models (Model I and Model V) differ mainly in the definition of the functional forms of average input requirement coefficients of the different industries. Since the basic approach of the present analysis is normative and since there is no role of market in the economy, all economies that arise because of scale effect or of externalities would be non-pecuniary in character and can be well represented by a suitable mathematical assumption on the functional forms of production functions of the different industries. The specific functional forms of average input coefficients that have been assumed in this model admit of:

(i) fixity of the coefficients when production levels and locations of the different basic industries are given, and

(ii) the possibility of realisation of both internal and external economies of scale in any of the basic industries, while the transport sector experiencing only internal economies.

Advantages due to externalities, have again been so represented in these mathematical expressions of input coefficients that their intensity of basic effect in each industry varies directly with the extent of locational closeness of the different basic industries among themselves. The real factors of specialisation, division of labour and of reorganisation of production that have been cited in the model for explaining the existence of external economies of scale and the greater extent of its realisation in the event of agglomerations, are in fact also common for causing economies that arise out of internal expansion of scale of any industry. The technology of production of the model thus implies that there are three distinct ways of exploiting the advantages of
the above mentioned real factors — (a) expansion of own scale of an industry,
(b) expansion of scale of operation of other industries and (c) locational
concentration of production of all the different basic industries towards some
corner point of location. There may however happen a mixing up of the effects
of agglomerative advantages with those of independent scale effects so far as
their bearing upon an optimum choice for location and allocation are concerned.
The decomposition of such effects poses a real problem indeed.

Results and Observations:

Chapter Nine describes Model V and poses the relevant nonlinear
programme of production and location. The optimal solution that is searched
for, describes a point on the consumption possibility frontier of the economy.
The model assumes that for any given locational distribution of the basic
industries for which each location of production of the basic industries lies
within the locational polygon obtained by joining the fixed locations of the
model, the derived programme of allocation would be feasible in nontrivial
sense. Lemmas 9.1 - 9.7 and Theorem 9.1 of Chapter Nine establish solvability
of the nonlinear programme of location-allocation under the validity of this
restrictive assumption. Finally, the chapter tries to indicate an approximate
analytic method of decomposing the effects of agglomeration and of other real
factors on the optimal locational choice.