Chapter 11

Modeling of Polyphase System

Chapter at a Glance
The chapter develops a passive model of a polyphase system in presence of harmonics. Equivalent circuit and layer based representation of the model have been developed. After describing the main limitation of this model, activity based model has been introduced. Its equivalent circuit and layer based representation have been developed. A case study on induction machine has been done on the basis of activity based model.
11.1 Introduction
Modeling of a poly phase system in presence of harmonics has been performed. In the first part of this chapter, a passivity-based model (PBM) of a poly phase system has been developed; its equivalent circuit and layer-based representation of the passive impedances have been drawn. Discussing its limitations, in the second part of the chapter, an activity-based model (ABM) has been developed in presence harmonics. Its equivalent circuit, control system and layer based representation of active impedances are developed. On the basis of the developed models, a case study has also been made to test the suitability of the model.

11.2 Passivity Based Model
11.2.1 Mathematical Model
Any poly phase system has its own impedance matrix by which voltages and currents are related.

Let,

\[ V \] be the voltage matrix
\[ I \] be the current matrix and
\[ Z \] be the impedance matrix

They can be related as

\[ V = Z \cdot I \] (11.1)

In (11.1), voltages and current have the same frequencies. This implies that, impedance matrix when operating on current matrix, does not change frequency and relates current with voltage having same frequencies. In other words, impedance matrix does not give any information of generation of any new signal having new frequency other than existing system frequencies. Therefore, this impedance can be treated as passive impedance. The impedance matrix \[ Z \] is renamed as passive impedance \( Z_p \), where the suffix ‘p’ indicates passive behavior of impedance. Thus (11.1) is rewritten as

\[ V = Z_p \cdot I \] (11.2)

Passive impedance \( Z_p \) is defined as the impedance which does not give any information of generation of any new signal having new frequency other than existing system frequencies. It relates voltage and current having same frequency.
In (11.2), the frequency of waveforms of voltage matrix is equal to the frequency of waveforms of current matrix and magnitude will depend on the impedance matrix. If the system has only fundamental frequency, the voltage and current matrix will have only fundamental frequency. Thus, for voltage and current with fundamental frequency (11.2) becomes

\[
[V_1] = [Z_{p1}] [I_1]
\]

(11.3)

where,

\[
[V_1] = \text{matrix having fundamental frequency voltage}
\]

\[
[I_1] = \text{matrix having fundamental frequency current and}
\]

\[
[Z_{p1}] = \text{matrix consisting of elements having fundamental frequency impedance}
\]

In the same way, if the voltage and current consist of one second order frequency, (11.2) can be written as

\[
[V_2] = [Z_{p22}] [I_2]
\]

(11.4)

where, \([V_2], [I_2]\) and \([Z_{p22}]\) indicate voltage, current and impedance matrices respectively corresponding to the frequency of second order harmonics.

In the same way, voltage and current of higher order harmonics can be expressed as

\[
[V_3] = [Z_{p33}] [I_3]
\]

\[
[V_4] = [Z_{p44}] [I_4]
\]

\[
[V_n] = [Z_{pnn}] [I_n]
\]

where, \([V_n], [I_n]\) and \([Z_{pnn}]\) indicate voltage, current and impedance matrices respectively corresponding to the frequency of \(n\)th order harmonics (11.3), (11.4) and (11.5) can be expressed by a relation

\[
\sum_n [V_n] = \sum_n [Z_{pnn}] [I_n]
\]

(11.6)

(11.6) represents all components of voltage and current having same frequency and it consists of passive impedances, hence represents the passive model of a poly phase system.
11.2.2 Equivalent Circuit of Passive Model of a Polyphase System

Fig. 11.1a represents equivalent circuit of (11.6). It shows that the system has impedance matrix \[ \sum_n [Z_{p,m}] \], voltages applied to the system \[ \sum_n [V_n] \] and currents flowing through the system \[ \sum_n [I_n] \], where \( n = 1, 2, 3, \ldots \). The impedance of this circuit relates voltage and current of the same frequency. The circuit for \( n = 1, 2, 3 \) are shown in Figs. 11.1b, 11.1c and 11.1d respectively. Each circuit is independent in nature and does not have any mutual interaction among them.
11.2.3 Layer Based Representation of Passive Impedances

Impedances have been represented in voltage-current-frequency plane. Passive impedances relate voltages and currents of same frequency as shown in Figs. 11.1b, 11.1c and 11.1d. Thus passive impedances lie in the voltage and current plane of same frequency. Fig.11.2 shows the layer based representation of passive impedance.
Horizontal layers of Fig. 11.2 show that each harmonic component has a distinct layer of impedance and there is no interaction between two horizontal layers. Each horizontal layer relates voltage and current matrix of same frequency. For example elements or slopes representing elements of \([Z_{p11}]\) will lie on its layer which is the lower horizontal layer in the figure. Elements or slopes representing elements of \([z_{ij}]\) will lie on its layer, which is the upper horizontal layer in the figure. In general, \([Z_{pmn}]\) relates voltage and current matrix of \(n^{th}\) order of harmonics. This describes the situation where all the harmonic components are supplied from external circuit and no new harmonics are generated inside the system.

11.2.4 Limitation of Passive Model
Passive model represented by (11.6) does not give any information about generation of new harmonics inside the system due to interaction of different harmonics present. Since the
layers in passive model are parallel to each other, passive model of the system is not capable of giving any information about the harmonics, which may be generated inside the system due to its non-linear behavior.

11.3 Activity Based Model
11.3.1 Mathematical Model
The limitation of passive model has been overcome by introducing activity based model. In this model, following factors have been included.

1. Harmonic may be produced inside the system
2. Highest order of harmonic in voltage may not be equal to the highest order of harmonic in current.
3. Any order of harmonics can produce any other order of harmonics.

Let voltage matrix consists of harmonics up to order ‘m’ and current matrix consists of harmonics up to order ‘n’. Then (11.6) can be rewritten as

$$\sum_{m} \left[ V_m \right] = \sum_{m,n} \left[ Z_{p mn} \right] \left[ I_n \right] = \sum_{m,n} \left[ Z_{p mn} \right] \left[ I_n \right] + \sum_{m=n} \left[ Z_{p mn} \right] \left[ I_n \right]$$

(11.7)

In (11.7), $\sum_{m=n} \left[ Z_{p mn} \right]$ represents the impedance due to active sources contributed by interaction among harmonics inside the machine by all causes and it relates mth order voltage matrix with nth order current matrix. $\left[ Z_{p mn} \right]$ represents passive impedance part of the system which does not generate new harmonics and relates nth order voltage and current matrices Considering all frequencies, (11.7) can be rewritten as

$$\left[ V_n \right] = \left[ Z_{p m1} \right] \left[ I_1 \right] + \left[ Z_{p m2} \right] \left[ I_2 \right] + \left[ Z_{p m3} \right] \left[ I_3 \right] + \ldots \ldots \left[ Z_{p mn} \right] \left[ I_n \right]$$

(11.8)

$$\left[ V_2 \right] = \left[ Z_{p 21} \right] \left[ I_1 \right] + \left[ Z_{p 22} \right] \left[ I_2 \right] + \left[ Z_{p 23} \right] \left[ I_3 \right] + \ldots \ldots \left[ Z_{p 2n} \right] \left[ I_n \right]$$

(11.9)

$$\left[ V_3 \right] = \left[ Z_{p 31} \right] \left[ I_1 \right] + \left[ Z_{p 32} \right] \left[ I_2 \right] + \left[ Z_{p 33} \right] \left[ I_3 \right] + \ldots \ldots \left[ Z_{p 3n} \right] \left[ I_n \right]$$

(11.10)

$$\ldots \ldots$$

$$\left[ V_n \right] = \left[ Z_{p m1} \right] \left[ I_1 \right] + \left[ Z_{p m2} \right] \left[ I_2 \right] + \left[ Z_{p m3} \right] \left[ I_3 \right] + \ldots \ldots \left[ Z_{p mn} \right] \left[ I_n \right]$$

(11.11)
(11.8) to (11.11) represent voltages of first, second, third....mth order harmonic, which are related to currents of first, second, third, nth order harmonic. These equations can be written in matrix form as

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
\vdots \\
V_m
\end{bmatrix} =
\begin{bmatrix}
Z_{p11} & Z_{p12} & \cdots & Z_{p1m} \\
Z_{p21} & Z_{p22} & \cdots & Z_{p2m} \\
Z_{p31} & Z_{p32} & \cdots & Z_{p3m} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{pm1} & Z_{pm2} & \cdots & Z_{pmn}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
\vdots \\
I_n
\end{bmatrix}
\]

(11.12)

As \(Z_{pmn}\) gives information of interaction among of voltage and current harmonics of different frequencies, it may be resymbolized as \(Z_{anm}\) and renamed as active impedance, where suffix 'a' indicates the active nature of the impedance.

Active impedance is defined as the impedance which gives information of generation of new harmonics inside the system and relates voltage and current of different frequencies. Using active impedances, (11.11) can be written as

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
\vdots \\
V_m
\end{bmatrix} =
\begin{bmatrix}
Z_{p11} & Z_{a12} & \cdots & Z_{a1m} \\
Z_{a21} & Z_{a22} & \cdots & Z_{a2m} \\
Z_{a31} & Z_{a32} & \cdots & Z_{a3m} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{an1} & Z_{an2} & \cdots & Z_{anm}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
\vdots \\
I_n
\end{bmatrix}
\]

(11.13)

Where, \(Z_{p11}, Z_{p22}, Z_{p33}, \ldots\) are passive impedances and \(Z_{a12}, Z_{a13}, \ldots\) are active impedances. (11.13) represents an active model of a system consisting of fundamental as well as other harmonic components which may be generated inside the system or supplied from external circuits.

11.3.2 Equivalent Circuit of Active Model

Equivalent circuit of (11.13) is drawn in Fig. 11.3. It shows that system has impedance matrix \(\sum_n Z_{pmn}\). Voltages in the system are \(\sum_m V_m\). Currents flowing through the system
are $\sum_n [I_n]$. All the frequencies supplied by from external circuits are present in passive impedance matrix $\sum_n [Z_{pm}]$. $\sum_n [Z_{pm}] [I_n]$ represents the source or generation of harmonics inside the system, where $|Z_{am}|$ is the active impedance generated by mutual interaction of $m^{th}$ order voltage and $n^{th}$ order current.

Fig. 11.3 Active model of a polyphase machine

11.3.3 Layer Based Representation of Active Model
Impedance has been represented in voltage-current-frequency plane. Passive impedance relates voltage and currents of same frequency and lies in the voltage and current plane of same frequency. Active impedance relates voltage and currents of different frequency and hence lies in the inclined plane connecting two different frequencies. Fig. 11.4 shows the layer based representation of active model of a system having first and second order frequencies.
In Fig. 11.4, there are two horizontal layers and two inclined layers. Each horizontal layer relates voltage and current of same frequency and carries passive impedances. Inclined layers show that each harmonic component is generated from other harmonic component and there is distinct interaction between two layers. Thus inclined planes represent active layers of the system, which describes the situation where the harmonic components are generated inside the system due to mutual interaction of the layers.
In Fig. 11.5, \([Z_{f1}]\) is the only passive impedance, which is responsible for producing fundamental current waveform from fundamental supply voltage. \([Z_{a1}], [Z_{a2}], [Z_{a3}], [Z_{a4}], [Z_{a5}], [Z_{a6}]\) are active impedances responsible for generation of voltage harmonics of \([V_1(\omega_1)], [V_3(\omega_3)], [V_5(\omega_5)], [V_7(\omega_7)]\) from fundamental current \([I_1(\omega_1)]\) or vice versa.

One more layer formation of passive and active impedance in an active model responsible for generation of fundamental component as well as other harmonics are shown in Fig. 11.6. It includes some more passive and active impedances of the system. The current matrix consists of harmonics up to 6th order which are partly injected by the power supply and partly produced inside the system. Passive layers up to 6th order and have been shown. Also interaction between two consecutive layers and interaction between first layers with all other layers have also been shown.
In all the cases above, harmonics up to 6th order are considered. Fig. 11.6 shows the passive impedances up to 6th harmonics, active impedances due to interaction between 1st and 2nd, 2nd and 3rd harmonics. Figures show the passive impedances and active impedances due to interaction among different harmonics.

### 11.4 Mutual Interaction of Voltage and Current of Different Frequencies in Park Plane

Fig. 11.7 shows activity based model of a multi-harmonic three-phase system in Park Plane. Supply consists of n\textsuperscript{th} order voltage and current and they are related by passive impedances \([Z_{rnm}]\). Due to the presence of active impedances, voltage of m\textsuperscript{th} order are produced from n\textsuperscript{th} order current by \([Z_{amn}]\). Voltage of m\textsuperscript{th} order are produced from m\textsuperscript{th} order current by \([Z_{ram}]^{-1}\). Voltage of n\textsuperscript{th} order will be produced from m\textsuperscript{th} order current by \([Z_{arn}]\). Current of n\textsuperscript{th} order may flow due to m\textsuperscript{th} order voltage by \([Z_{aam}]^{-1}\).
11.5 Active Model having Harmonics up to Third order: A Case Study

For simplicity consider the possibility of harmonics up to third order and in that situation the active model (11.13) can be written as

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3
\end{bmatrix} =
\begin{bmatrix}
Z_{p11} & Z_{a12} & Z_{a13} \\
Z_{a21} & Z_{p22} & Z_{a23} \\
Z_{a31} & Z_{a32} & Z_{p33}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\]  \hspace{1cm} (11.14)

In (11.14), \([Z_{p11}] [I_1]\), \([Z_{p22}] [I_2]\) and \([Z_{p33}] [I_3]\) are the part of voltage matrices which consist of the harmonics supplied by the source. Similarly, \([Z_{a12}] [I_2]\), \([Z_{a13}] [I_3]\), \([Z_{a23}] [I_3]\) and \([Z_{a32}] [I_2]\) are the part of voltage expressions which consist of harmonics were not supplied by the source but have been generated inside the system.
Now consider there was no other frequency except the fundamental at initial stage. By passive impedance, voltage matrix will contain only fundamental frequency given by

$$[V_1] = [Z_{p_{11}}]I_1$$

(11.15)

But by the active impedances, harmonics will be created in voltage represented by

$$\begin{bmatrix} [V_1'] \\ [V_2'] \\ [V_3'] \end{bmatrix} = \begin{bmatrix} [Z_{p_{11}}] & [Z_{a_{12}}] & [Z_{a_{13}}] \\ [Z_{a_{21}}] & [Z_{p_{22}}] & [Z_{a_{23}}] \\ [Z_{a_{31}}] & [Z_{a_{32}}] & [Z_{p_{33}}] \end{bmatrix} \begin{bmatrix} [I_1] \\ [0] \\ [0] \end{bmatrix}$$

(11.16)

where $[V_2']$ and $[V_3']$ have been created by active components $[Z_{a_{21}}]$ and $[Z_{a_{31}}]$. Now $[V_2']$ and $[V_3']$ will produce current $[I_2]$ and $[I_3]$ given by the inverse of $[Z_{p_{22}}]$ and $[Z_{p_{33}}]$. Then $[I_2]$ and $[I_3]$ will produce other harmonic in the voltage expressions controlled by active impedance $[Z_{a_{12}}]$, $[Z_{a_{13}}]$, $[Z_{a_{23}}]$, $[Z_{a_{32}}]$ and two passive impedance $[Z_{p_{22}}]$ and $[Z_{p_{33}}]$. Then (11.16) becomes,

$$\begin{bmatrix} [V_1'] \\ [V_2'] \\ [V_3'] \end{bmatrix} = \begin{bmatrix} [Z_{p_{11}}] & [Z_{a_{12}}] & [Z_{a_{13}}] \\ [Z_{a_{21}}] & [Z_{p_{22}}] & [Z_{a_{23}}] \\ [Z_{a_{31}}] & [Z_{a_{32}}] & [Z_{p_{33}}] \end{bmatrix} \begin{bmatrix} [I_1] \\ [I_2] \\ [I_3] \end{bmatrix}$$

(11.17)

Thus the generated harmonics depend on the values of active impedances of the circuit. The inter-relations between the harmonic components can be well understood by a flow diagram as shown in Fig. 11 8.
11.6 Nature of Active Impedance

(11.12) is the most general case where harmonics are generated inside the system due to the presence of \([Z_{\text{amn}}]\). Active impedance \([Z_{\text{arna}}]\) performs two jobs: cancels the term containing \(n\)th order of harmonic and generates \(m\)th order of harmonic. Adding with this, \([Z_{\text{amn}}]\) controls the amplitude of voltage waveform of frequency of order \(m\). This indicates that \([Z_{\text{amn}}]\) should have time dependent function \(f(t)\) represented as (11.17) multiplied by time independent function assumed to be equal in form like passive impedance \([Z_{\text{pmm}}]\).

\[
\text{f}(t) = k_{\text{am}} \frac{\sin m\omega t}{\sin n\omega t} \tag{11.18}
\]

\(k_{\text{am}}\) is constant for a particular system. This constant depends on the design parameter which may vary during a fault. Thus,

\[
[Z_{\text{amn}}] = k_{\text{am}} \frac{\sin m\omega t}{\sin n\omega t} [Z_{\text{pmm}}] \tag{11.19}
\]

Thus it seems that active impedance is both time and frequency dependent.

11.7 Case Study of Active Model on Poly-phase Induction Machine

A case study has been carried out by developing activity based model of a poly-phase induction machine. First a general activity based model has been considered for a poly-phase induction machine. Then an active model has been developed considering the machine an ideal one, which does not produce harmonics inside the machine. Then a real
induction machine has been considered, in which new harmonics are produced and corresponding active model has been developed.

Voltage and current matrix of a rotating machine consist of stator and rotor components which may again be subdivided into d axis and q axis components. Writing these components in voltage and transformed matrix, \(11.13\) becomes

\[
\begin{bmatrix}
V_{q1} \\
V_{d1} \\
V_{q3} \\
V_{d3} \\
V_{q2} \\
V_{d2} \\
V_{q4} \\
V_{d4} \\
\vdots \\
V_{q_m} \\
V_{d_m}
\end{bmatrix}
=
\begin{bmatrix}
Z_{q1} & Z_{d1} & Z_{q1} & \cdots & \cdots & Z_{q1(n-1)} & Z_{d1(n-1)} \\
Z_{q2} & Z_{d2} & Z_{q2} & \cdots & \cdots & Z_{q2(n-1)} & Z_{d2(n-1)} \\
Z_{q3} & Z_{d3} & Z_{q3} & \cdots & \cdots & Z_{q3(n-1)} & Z_{d3(n-1)} \\
\vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
Z_{q_m} & Z_{d_m} & Z_{q_m} & \cdots & \cdots & Z_{q_m(n-1)} & Z_{d_m(n-1)}
\end{bmatrix}
\begin{bmatrix}
I_{q1} \\
I_{d1} \\
I_{q3} \\
I_{d3} \\
I_{q2} \\
I_{d2} \\
I_{q4} \\
I_{d4} \\
\vdots \\
I_{q_m} \\
I_{d_m}
\end{bmatrix}
\]

where, stator voltage and current can be written as

\[
\begin{bmatrix}
V_{d1} \\
V_{q1} \\
V_{d3} \\
V_{q3} \\
V_{d4} \\
V_{q4} \\
\vdots \\
V_{d_m} \\
V_{q_m}
\end{bmatrix} = [Park\ Matrix] \times \begin{bmatrix}
V_{am} \\
V_{bm}
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_{d1} \\
I_{q1} \\
I_{d3} \\
I_{q3} \\
I_{d4} \\
I_{q4} \\
\vdots \\
I_{d_m} \\
I_{q_m}
\end{bmatrix} = [Park\ Matrix] \times \begin{bmatrix}
I_{an} \\
I_{bn}
\end{bmatrix}
\]

Rotor voltage and current can be written as
In case of poly phase induction machine, rotor circuit is shorted and hence rotor voltages are zero, (11.20) can be modified as

\[
\begin{bmatrix}
V_{q_1} \\
V_{d_1} \\
V_{q_2} \\
V_{d_2} \\
V_{q_3} \\
V_{d_3} \\
V_{q_m} \\
V_{d_m} \\
\end{bmatrix}
= \begin{bmatrix}
Z_{p11} & Z_{p12} & Z_{p13} & \cdots & \cdots & \cdots & Z_{p1(n-1)} & Z_{p1n} \\
Z_{p21} & Z_{p22} & Z_{p23} & \cdots & \cdots & \cdots & Z_{p2(n-1)} & Z_{p2n} \\
Z_{p31} & Z_{p32} & Z_{p33} & \cdots & \cdots & \cdots & Z_{p3(n-1)} & Z_{p3n} \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\
Z_{pm1} & Z_{pm2} & Z_{pm3} & \cdots & \cdots & \cdots & Z_{pm(n-1)} & Z_{pmn} \\
\end{bmatrix}
\begin{bmatrix}
I_{q_1} \\
I_{d_1} \\
I_{q_2} \\
I_{d_2} \\
I_{q_3} \\
I_{d_3} \\
I_{q_m} \\
I_{d_m} \\
\end{bmatrix}
\] (11.25)

Or, \( \begin{bmatrix}
V_{\text{Induction Machine}} \\
\end{bmatrix}
= \begin{bmatrix}
Z_{\text{Induction Machine}} \\
\end{bmatrix}
\times
\begin{bmatrix}
I_{\text{Induction Machine}} \\
\end{bmatrix}
\] (11.26)
\[ \begin{bmatrix}
V_{q1} \\
V_{d1} \\
0 \\
0 \\
V_{q2} \\
V_{d2} \\
0 \\
0 \\
\vdots \\
V_{qN} \\
V_{dN} \\
0 \\
0 \\
\end{bmatrix} \]

\[ I^{a} = \begin{bmatrix}
I_{q1} \\
I_{d1} \\
I_{q2} \\
I_{d2} \\
\vdots \\
I_{qN} \\
I_{dN} \\
\end{bmatrix} \]

\[ I^{\text{Induction Motor}} = \begin{bmatrix}
I_{q1}^{a} \\
I_{d1}^{a} \\
I_{q2}^{a} \\
I_{d2}^{a} \\
\vdots \\
I_{qN}^{a} \\
I_{dN}^{a} \\
\end{bmatrix} \]

\[ (11.27) \]

\[ (11.28) \]
**a. Activity Based Model of an Ideal Poly phase Induction Motor**

If an ideal poly phase induction motor does not produce harmonics, then all active impedance matrices will become null. Thus the stator components will carry the harmonics which have been supplied by the source which can be represented only by passive impedance. Then (11.26) becomes

\[
\begin{bmatrix}
V_{\phi_1} \\
V_{\phi_2} \\
V_{\phi_3} \\
V_{\phi_m}
\end{bmatrix} =
\begin{bmatrix}
Z_{p_{11}} & [0] & [0] & \ldots & [0] \\
[0] & Z_{p_{22}} & [0] & \ldots & [0] \\
[0] & [0] & Z_{p_{33}} & \ldots & [0] \\
[0] & [0] & [0] & \ldots & [0]
\end{bmatrix}
\begin{bmatrix}
I_{\phi_1} \\
I_{\phi_2} \\
I_{\phi_3} \\
I_{\phi_m}
\end{bmatrix}
\]

(11.30)
being reflected in the stator voltage. If the supply voltage consists of only fundamental frequency then the equation will be \[ V_s = \left[ Z_p \right] \left[ I_1 \right], \] which is obviously not a real case.

b. Activity Based Model for a Real Poly Phase Induction Motor

From (11.26) active model for real induction machine can be written as

\[
\left[ V_{Real} \right] = \left[ Z_{Real} \right] \times \left[ I_{Real} \right]
\]

(11.31)

where,

\[
\left[ I_{Real} \right] = \left[ I_{Induction\machine} \right]
\]

(11.32)

\[
\left[ V_{Real} \right] = \left[ V_{Induction\machine} \right]
\]

(11.33)

\[
\left[ Z_{Real} \right] = \left[ Z_{Induction\machine} \right]
\]

(11.34)

Like ideal machine, in a real induction machine rotor voltages are zero. Also, mesh connection of a balanced three phase system cancels all possibilities of third harmonics and hence. Thus, from (11.29) and (11.34), \( Z_{Real} \) can be written as

\[
\begin{bmatrix}
Z_{p11} & Z_{p12} & [0] & Z_{p14} & [0] & \cdots & Z_{p1(n-1)} & Z_{p1n} \\
Z_{p21} & Z_{p22} & [0] & Z_{p24} & [0] & \cdots & Z_{p2(n-1)} & Z_{p2n} \\
[0] & [0] & [0] & [0] & [0] & \cdots & [0] & [0] \\
Z_{p41} & Z_{p42} & [0] & Z_{p44} & [0] & \cdots & Z_{p4(n-1)} & Z_{p4n} \\
Z_{p51} & Z_{p52} & [0] & Z_{p54} & [0] & \cdots & Z_{p5(n-1)} & Z_{p5n} \\
Z_{p61} & Z_{p62} & [0] & Z_{p64} & [0] & \cdots & Z_{p6(n-1)} & Z_{p6n} \\
Z_{p71} & Z_{p72} & [0] & Z_{p74} & [0] & \cdots & Z_{p7(n-1)} & Z_{p7n} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
Z_{p81} & Z_{p82} & [0] & Z_{p84} & [0] & \cdots & Z_{p8(n-1)} & Z_{p8n} \\
\end{bmatrix}
\]

(11.35)

In most of the cases even harmonics are not generated inside an induction motor and they may be present if they are supplied by the source. Thus, (11.35) can be modified as
If the possibility of supply of even harmonics by source is neglected, \( [Z_{p22}] \) becomes a null matrix and then (11.36) can be modified as

\[
[Z_{rev}] = \begin{bmatrix}
Z_{p11} & [0] & [0] & [0] & [0] & [Z_{a15}] & [0] & [Z_{a17}] & \cdots & \cdots & [Z_{a(n-4)}] & [Z_{a(n-3)}] \\
[0] & [Z_{p23}] & [0] & [0] & [0] & [0] & [0] & [0] & \cdots & \cdots & [0] & [0] \\
[0] & [0] & [Z_{a15}] & [0] & [0] & [0] & [0] & [0] & \cdots & \cdots & [0] & [0] \\
[0] & [0] & [0] & [Z_{a17}] & [0] & [0] & [0] & [0] & \cdots & \cdots & [0] & [0] \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \cdots & \cdots \\
[0] & [0] & [0] & [Z_{a(n-4)}] & [0] & [0] & [0] & [0] & \cdots & \cdots & [0] & [0] \\
\end{bmatrix}
\]

(11.37)

11.8 Discussion

A first, a passivity-based model (PBM) has been developed; its equivalent circuit and layer-based representation of the passive impedances have been drawn. Discussing its limitations, an activity-based model (ABM) has been introduced in presence harmonics. Its equivalent circuit, model and layer based representation of active impedances have been developed. On the basis of the developed models, a case study on activity-based model has been made for poly-phase induction machine. ABM has been developed for an ideal and a real induction machine.
Publications
