Chapter 2

Tobin’s q, Uncertainty, Irreversibility and Investment

In this chapter I shall, at first, focus on the neo-classical form of Tobin’s q theory of investment and then extend the analysis to find out the impact of uncertainty on firm level investment decisions.

The impact of uncertainty on investment has been an active area of research since the early 1980’s. A handful of papers have devoted their attention to various aspects of investment - uncertainty relationship. A closer look at this work reveals two different lines of development. The first line considers an investment option to be nondelayable (i.e., if one fails to take any investment opportunity at a particular point of time, it will never be available to him) and tries to find out the optimal time path of investment that maximizes the value of the firm. The second approach, on the other hand, assumes that the firm has a scope to hold an investment opportunity for some time and tries to find out the optimal time for a given amount of investment.

In real world, managers of firms face the problem of simultaneous determination of the amount of investment as well as the optimal time to invest. Therefore, an integrated approach is necessary to provide a theoretical background of joint determination of the two variables. Moreover, the partial analyses often produce mutually contradictory results. For example, while the first approach predicts increase in investment with increase in the degree of uncertainty under perfectly competitive environment, the second approach predicts a negative relationship between uncertainty and investment.
To get a way out in this chapter, I have developed a model that has taken into account, both the problems simultaneously. As suggested in the model, there is no uniform relationship between investment and uncertainty and the result depends entirely on the specification of profit and cost functions.

2.1 Survey of Literature:

In the post war period two theories of investment gained extreme popularity – the neo-classical theory of Jorgenson (1967) and the q theory of Tobin (1969). Jorgenson developed his theory in terms of explicit profit maximization objective of firms. Tobin, on the other hand, concentrated on the marginal benefit and marginal cost associated with an incremental unit of investment. Abel (1979) showed that both the theories are, in fact, similar in terms of their implications. From then onwards attempts have been made to incorporate the q theory in the neo-classical frame. The popularity of q-theory gave the investment literature a new vista. It showed how the investment decision of a firm is influenced by the movement of the stock markets. In terms of macro-economic policy perspective the q-theory highlighted that liberal policies aiming at boosting the share market are more preferred to the policies of financial repression. This policy implication has given strong support to the privatization of the state funded old age pension funds in USA. Moreover, to quote Palley (2001), the q theory also calls for “the fostering of equity markets in developing countries. If q matters for investment then developing countries should be encouraged to promote local stock markets”.
2.1.1 Neo-Classical and Tobin's q: An Integrated Approach:

The pioneering effort to integrate the neo-classical theory and q-theory was provided by Hayashi (1982). To have an insight into it I shall present a simplified version of the theory. For the purpose of simplification I shall assume that there is no tax savings due to depreciation and investment tax credit. But I shall retain the assumption of existence of corporate tax.

The net return to the firm at period t can be expressed as:

\[ R(t) = [1 - u(t)] \{ p(t) F(K(t), N(t)) - w(t)N(t) - P_i(t)I(t) \} \quad \text{(1)} \]

Where
- \( u(t) \) is the corporate tax rate
- \( p(t) \) is the price of the product
- \( P_i(t) \) is the price of the investment goods
- \( I(t) \) is the amount of investment
- \( K(t) \) and \( N(t) \) are the inputs capital and labour

The task of firm is to maximize its value, i.e.,

\[ \max V(0) = \int_0^\infty R(t) \exp \left\{ -\int_0^t r(s) ds \right\} dt \quad \text{(2)} \]

subject to

\[ \dot{K}(t) = \psi(I, K; t) - \delta K(t) \quad \text{(3)} \]

The term \( \psi(I, K; t) \) is used instead of \( I(t) \) to accommodate adjustment cost. This indicates that one unit of gross investment does not completely turn into one unit of capital. A part of it is reduced in the process of adjusting the firm to accommodate the new capital (such as the training for installation of a new
machine, required adjustment to accommodate the machine etc.). The installation
cost function \( \psi(I, K; t) \) obeys the following restrictions: \( \psi_t > 0 \), \( \psi_K > 0 \), \( \psi_{tt} < 0 \), \( \psi_{KK} < 0 \), \( \psi_{IK} < 0 \).

The current value Hamiltonian for the problem is:

\[
H = (1-u)[p F(K, N) - wN] - P_t I + \lambda \left[ \psi(I, K) - \delta K \right]
\]

---(4)

The first order conditions for this problem are:

\[
H_N = 0 \Rightarrow pF_N = w
\]

\[
H_I = 0 \Rightarrow -P_t + \lambda \psi_t = 0
\]

\[
\dot{\lambda} - r\lambda = -H_K \Rightarrow \dot{\lambda} = (r + \delta - \psi_K) \lambda - (1-u) pF_K
\]

---(7)

and the transversality condition is

\[
\lim_{t \to \infty} \lambda(t)K(t) \exp(-\int_0^t r(s) ds) = 0
\]

---(8)

From (7) we find

\[
\lambda(t) = \int_0^t [1-u(s)] p(s)F_K(s) \exp(-r + \delta - \psi_K(s-t)) ds
\]

Hence \( \lambda \) can be termed the present discounted value of additional future profits that are due to one additional unit of current investment.

Tobin's marginal \( q \) can be defined as \( q = \lambda / \lambda_t \) and average \( q \) as \( h = V /P_t K \)

Hence equation (6) and (7) can be now written as:

\[
q = \frac{1}{\psi_t}
\]

---(6')

\[
\dot{q} = [r + \delta - (P_t - \psi_K) q - (1-u) p F_K / P_t
\]

---(7')

where \( \hat{P}_t = \hat{P}_t / \hat{P}_t \)

Equation (6') helps us to obtain the optimal investment rule while (7') sorts out the optimal time path of \( q \).
From (6') we can say that
\[ I = \alpha(q, K, t) \]  
\[ \text{where } \alpha' > 0 \]  

Moreover, if the value of \( q \) is known, the optimal rate of investment can be determined from the knowledge of the installation function \( \psi(.) \) only. If the installation function is linear homogeneous in \( I \) and \( K \) then-

\[ \frac{I}{K} = \beta(q, t) \]  
\[ \text{where } \beta' > 0, \beta'' > 0 \]  

Hence the rate of investment is determined solely by the value of Tobin's \( q \).

**An example:**

If the installation function takes the form \( \psi(I, K) = I^\alpha K^{1-\alpha} \) then \( \psi_I = \alpha(K/I)^{1-\alpha} \)

Hence equation (6) transforms to
\[ q = \frac{1}{\alpha} \cdot (I/K)^{1-\alpha} \]

or,
\[ \frac{I}{K} = (\alpha q)^{1/(1-\alpha)} \]

Since \( 0 < \alpha < 1, (1/1-\alpha) > 1 \), hence \( \frac{I}{K} \) is an increasing function of \( q \) which increase at an increasing rate

The major problem in empirical verification of this theory is the fact that marginal \( q \) is not observable. What we can observe is the value of average \( q \). Hayashi introduced two propositions to provide a solution for the problem. As these propositions are highly dependent on the value of tax savings due to depreciation, it is beyond our scope to interpret these propositions form our formulation of the problem.

However, a general observation can be made this regard. When average \( q \) of a firm increases, the value of marginal \( q \) increases too. Similarly, a decrease in the value of average \( q \) is accompanied by a decrease in the value of marginal \( q \). Hence, the movement of average \( q \) gives some indication of the movement of
marginal q. The fact gave the empirical researchers the justification to use average q as proxy for marginal q.

An alternative form of this model was specified by Gould (1968), Treadway (1969) and others. Their difference lies in the specification of the adjustment cost function. Instead of setting the adjustment cost function in the capital stock adjustment equation (equation 3) they introduced the adjustment cost directly in the return function. This adjustment cost is assumed to be a function of both the levels of investment and capital stock. More specifically, these costs are increasing with the absolute size of investment or disinvestment and furthermore rise at an ever-increasing rate. They are zero only when gross investment is zero. Moreover, the adjustment cost function obeys the restrictive assumption that its partial derivative with respective to investment goes to infinity as investment goes to infinity and its partial derivative goes to negative infinity as investment goes to negative infinity. The shape of the adjustment cost function is shown in the Figure 2.1.
An increase in capital stock shifts the curve downwards. Introduction of this type of adjustment cost function modifies the return function (equation 1) which now takes the form -

\[ R(t) = (1-u(t))(p(t) F(K(t), N(t)) - w(t)N(t)) - P_{i}(t) I(t) - C(I, K; t) \]  \(1'\)

The capital accumulation equation becomes

\[ K(t) = I(t) - \delta K(t) \]  \(3'\)

The adjustment cost function now follows the properties: \( C_i \geq 0 \) for \( I \geq 0 \), \( C_i < 0 \) for \( I < 0 \), \( C_n > 0 \), \( C_n < 0 \).

This type of specification of the problem changes equations \(6'\) and \(7'\) to

\[ q = (1+C_1/P_1) \]  \(6''\)

and \[ q = (r+\delta) - [(1-u)p F(K)/P_{i}] \]  \(7''\)

However, equation \(8\) and \(9\) remains unaltered. Since both the approaches reach the same conclusion, we would use these two approaches interchangeably.

**2.1.2 Introduction of Irreversibility:**

So far I have taken investment to be perfectly reversible, i.e., a firm can buy and sell their capital with equal ease. But such a frictionless capital market does not exist in real world. The investment decision of a firm is, in fact, an irreversible decision. As Kenneth J. Arrow (1968) argues, "there will be many situation in which the sale of capital goods can not be accomplished at the same price as their purchase.......For simplicity, we will make the extreme assumption that resale of capital goods is impossible, so that gross investment is constrained to be non-
negative”. Hence, his assumption adds an additional constraint to the maximization problem, viz., the gross investment is non-negative (I ≥ 0). The papers of Robert E. Lucas Jr. (1981) and Lucas and Prescott (1971) are built on this type of formulation.

Caballero (1991) released the assumption of strict irreversibility and made investment partially reversible by assuming different costs adjustment for positive and negative investments. His cost function was specified by the form:

\[ C(I) = I + [I>0] \gamma_1 I^\beta + [I<0] \gamma_2 |I|^{\beta} \quad \text{where } \beta \geq 1 \]

This cost function has the scope to incorporate to cases of symmetric adjustment costs (\(\gamma_1 = \gamma_2 > 0\)) as well as the case of perfectly irreversible investment (\(\gamma_1 = 0, \gamma_2 = \alpha \)).

A more detailed analysis of the cost function was done by Abel and Eberly (1994). According to them, when a firm undertakes the decision to invest or disinvest it incurs three types of costs- (1) Purchase/sale cost, (2) Adjustment cost and (3) Fixed cost.

(1) **Purchase/sale cost:**

The purchase/sale cost is the cost of buying or selling capital. If \(P_k^+\) is the purchase price of an unit of capital and \(P_k^-\) be the selling price, then the purchase and sell costs are \(P_k^+ I\) (for \(I>0\)) and \(P_k^- I\) (for \(I<0\)) respectively. Due to the possibility of adverse selection in the market for capital goods or for the firm specificity of capital goods \(P_k^+ \geq P_k^-\). This cost function is (weakly) convex and non-decreasing which takes the value zero when investment is zero. This cost function is assumed to be twice differentiable everywhere except at \(I=0\).
(2) Adjustment cost:

Adjustment costs are non-negative costs, which attain their minimum value zero when gross investment is zero. This cost depends on both the level of investment as well as on capital stock. As is typical in the adjustment cost literature we assume that the adjustment costs are continuous, strictly convex in $I$ and decreasing in $K$. The adjustment cost function is twice differentiable except at $I=0$.

(3) Fixed cost:

Fixed cost of investment are non-negative costs that are independent of the level of investment. This cost emerges each time when the investment is non-zero. The firm can avoid this cost be setting the level of investment at zero.

The total cost associated with investment is the sum of purchase/sale cost, adjustment cost and fixed cost and is represented by the function $C(I,K)$. Abel and Eberly assumed that

$$\lim_{I \to 0^-} C(I,K) = \lim_{I \to 0^+} C(I,K) = C(0,K)$$

and defined it as the fixed cost of investment.

This cost function is continuous, strictly convex and twice differentiable everywhere except at $I = 0$. Let $C_i(0,K)^-$ and $C_i(0,K)^+$ denote the left hand and right hand partial derivatives of $C(I,K)$ with respect to $I$ evaluated at $I=0$. Hence, $C_i(0,K)^+ \geq C_i(0,K)^-$.

\footnote{For a detail analysis see Abel and Eberly (1994)}
With this type of specification of the cost function the problem of the firm becomes:

\[ V(0) = \max_{I,N} \int_0^\infty \left\{ [1 - u(t)] \left[ p(t) F(K(t),N(t)) - w(t) N(t) \right] - C(I(t),K(t)) \right\} \exp(\int_0^t r(s) ds) dt \quad (10) \]

subject to

\[ \dot{K}(t) = I(t) - \delta K(t) \quad (11) \]

The current value Hamiltonian of this problem is:

\[ H = \left\{ (1 - u) p F(K,N) - wN \right\} - C(I,K) + \lambda(I - \delta K) \quad (12) \]

The first order conditions for this problem are:

\[ H_N = 0 \Rightarrow p F_N = w \quad (13) \]

\[ H_I = 0 \Rightarrow \lambda = C_I(I,K) \text{ for } \lambda < C_i(0,K) \text{ or } \lambda > C_i(0,K) \quad (14a) \]

\[ I = 0 \text{ for } C_i(0,K) \leq \lambda \leq C_i(0,K) \quad (14b) \]

\[ \dot{\lambda} - r \lambda = -H_K \Rightarrow \dot{\lambda} = (r + \delta) \lambda - p F_K - C_K(I,K) \quad (15) \]

Now, defining \( q = \lambda / P_t \), the equations (14a), (14b) and (15) transform to:

\[ q = C_i(I,K) / P_t \text{ for } q < C_i(0,K) / P_t \quad \text{or } q > C_i(0,K) / P_t \quad (14'a) \]
\( I = 0 \) for \( C_l(0,K)^- \leq q \leq C_l(0,K)^+ \) \hspace{1cm} (14'b)

\[ q = (r + \delta - P_l) q - \frac{p}{P_l} F_K - C_\frac{K(I,K)}{P_l} \] \hspace{1cm} (15')

Hence, if the augmented adjustment cost function is linear homogeneous in \( I \) and \( K \) then the optimal investment rule becomes-

\[
I/K = \begin{cases} 
< 0 & \text{for } q < C_l(0,K)^- / P_l \\
0 & \text{for } q = C_l(0,K)^- / P_l \leq q \leq C_l(0,K)^+ / P_l \\
> 0 & \text{for } q > C_l(0,K)^+ / P_l 
\end{cases}
\] \hspace{1cm} (16)

Thus the investment path of the firm is not continuous as there is a range of inaction. This range of inaction depends only on the specification of the adjustment cost function.

### 2.2 Investment and Uncertainty:

After a brief introduction to the neo-classical and \( q \) theory of the investment we now extend our analysis to find out the impact of the uncertainty on the investment decision of firms. To be more specific, we would focus on impact uncertainty on investment for a risk neutral firm. A vast literature has developed in this particular line. But unfortunately their results are often mutually conflicting. This made Caballero (1991) state that "the relationship between
changes in price uncertainty and capital investment under risk neutrality is not robust. ............... it is very likely that it will be necessary to turn back to risk aversion, incomplete markets and lack of diversification to obtain a sturdier negative relationship between investment and uncertainty."

The relationship between uncertainty and investment is very important for suggesting appropriate macroeconomic policy. For example, if the existence of uncertainty made the firms reluctant to take new investment (disinvestment) decision in an otherwise favourable (unfavourable) environment then the aggregate level of output and employment will not vary very much in accordance to fluctuations in the current demand conditions of the economy. Hence for an economy with high unemployment rate, very large wage fluctuation will be needed to attain full employment. Moreover, maintenance of macroeconomic stability would be more important than a cut in investment rate or tax to stimulate investment. Hence a major cost of political and economic instability is its depressing effect on investment.

In the remaining part of this section I shall discuss the existing theories of investment under uncertainty. For the purpose of discussion we shall follow an approach-oriented sequence. That is, we will cluster together those models which follow similar approach even if their outcomes may differ. To be more concrete, we shall, at first concentrate on the papers of Abel (1983) and Caballero (1991) and then focus on the papers of Dixit (1991) and Pindyck (1991).

Abel (1983) developed his model as an attempt to solve the controversy that arose from the conflicting results of Hartman (1972) and Pindyck (1982) regarding the impact of uncertainty of investment. Hartman showed that for a perfectly competitive firm with a linear homogeneous production function increased uncertainty of output prices lead to an increase in the level of
investment. Pindyck, on the other hand, showed that the result entirely depends on the specification of the adjustment cost function. When the adjustment cost function is convex, increase in price uncertainty leads to increase in investment. But when the adjustment cost function is concave increased uncertainty leads to a decrease in the level of investment. According to Pindyck the difference in the two results occurs specifically due to the difference in the stochastic specification of the price of output.

Abel showed that the result of Hartman holds true even with the stochastic specification of Pindyck. According to him, for a competitive firm with linearly homogeneous production function, increased price uncertainty leads to an increase in the level of output as long as the marginal revenue product of capital is convex function of price of output.

Abel assumes a risk neutral firm with the object of maximizing its value, that is,

$$\max V (K_t, p_t) = \int_t^\infty \left[ p_s L^{\alpha_s} K^{1-\alpha_s} - w N_s - \gamma I_s \right] \exp(-r(s-t))ds$$

$$I_s, N_s$$------------------(17)

Subject to

$$dK_t = (I_t - \delta K_t) dt$$------------------(18)

$$dp_t/p_t = \sigma dz.$$------------------(19)

Where dz is a Wiener process with mean zero and unit variance.

Thus the uncertainty arises in the economy due to uncertainty in the price of output. Increase in uncertainty is denoted by an increase in the value of \(\sigma\). Solving the problem, Abel reached the following equations:
\[ V(K_t, p_t) = q_t K_t + \left[ (\beta - 1) \gamma (q_t / \gamma) \alpha^{\beta-1} \right] / \left[ \beta \left( 1 - \alpha \gamma \right) \omega / 2 \left( 1 - \alpha \right)^2 \right] \]

\[ \text{(20a)} \]

where \( q_t = \left[ h p_t \right] / \left[ r + \left( a - \alpha w \right) / 2 \left( 1 - \alpha \right) \right] \) \[ \text{(20b)} \]

Here, \( q_t \) represents the expected present value of marginal revenue product accruing to the undepreciated proportion of capital flow from the time \( t \) onwards with \( h = (1-\alpha)(\alpha/w)^{\alpha/1-\alpha} \).

Implicit solution of equation (20) gives

\[ I_t = (q_t / \beta \gamma)^{1/\beta-1} \]

\[ \text{(21)} \]

Hence, the optimal rate of investment is an increasing function of \( q_t \).

To trace out the impact of uncertainty Abel pointed out that uncertainty influences the value of \( q_t \). From (20b) it can be said that for a given level of current price \( p_t \), any increase in uncertainty would increase the value of \( q_t \), thereby increasing the level of investment.

Caballero (1991) introduced a two period model to show that the investment-uncertainty relationship is not so robust for all the cases. He showed that the relationship between investment and uncertainty depends entirely on the market structure. According to him

(a) When the market structure is perfectly competitive, the findings of Abel hold true and hence there is a positive relationship between uncertainty and investment irrespective of the nature of adjustment cost.
(b) When there is imperfection in market, the result of increase in uncertainty is not robust. When the adjustment cost is symmetric there may be a positive relationship between uncertainty and investment. However, market imperfection has a dampening effect on the amount of investment. Caballero highlighted two reasons for such dampening effect:

(1) When the market becomes imperfect, the convexity of the marginal profitability of capital with respect to price uncertainty is reduced. Under perfect competition any increase in price uncertainty raises total revenue both directly ($Q\Delta p$) and indirectly through the corresponding increase in total output. But under imperfect competition, the demand curve is downward sloping. Hence any increase in output lowers the price level, this dampens the direct effect partially.

(2) When the market becomes imperfect the marginal profitability of capital decreases more with a given increase in capital compared to the perfectly competitive situation. This also has a dampening effect on the level of investment.

However, when the adjustment cost becomes asymmetric the relationship becomes more complicated. Any change in the price level may bring either a ‘good’ or a ‘bad’ shock to firm. The more the degree of asymmetry in the market the greater is the impact of the ‘bad’ shock relative to the ‘good’. Hence, it becomes optional for the firm to restrict its investment decision as the degree of uncertainty increases.

Dixit (1991) and Pindyck (1991) introduced a different line of thinking on investment—uncertainty relationship. Dixit pointed out that, in practice, U.S firms
do not invest until price rises three or four times above the long run average cost. Similarly, on the downside, firms stay in business for lengthy periods absorbing operating losses till price falls substantially below average variable cost. For example, the appreciation of US dollars from 1980 increased the competitive position of the importers to USA. But the import volume started to increase only from 1983. On the other hand, the value of dollar started to fall from 1985. But imports did not decrease for another two years. According to Dixit, this particular feature of investment cannot be explained by the existing theories of investment.

The Dixit –Pindyck framework assumes that most investment decisions exhibit three common features:

**First**: The investment expenditures undertaken by the firms are mostly irreversible in the sense that all of them entail some sunk cost, which cannot be recovered if the action is reversed on a later date. This sunk cost arises because capital is, in general, firm or industry specific. A different firm or a different industry cannot use the capital as productively as the firm which owns the particular capital.

**Second**: The economic environment is full of uncertainty and information emerges gradually.

**Third**: The investment opportunity can be delayed for some time, giving the firm the opportunity to wait for new information to arrive about prices, costs and other markets conditions.

Giving the firm the opportunity to delay its investment decision for some time gave the literature a major breakthrough. When such an opportunity is
present, the standard neo-classical results often become invalidate. Note that, the
decision to postpone the investment decision is not the same as setting I=0 in the
neo-classical models. A neo-classical firm does not invest when the
additional investment does not add any value to the firm. On the other hand when,
the firm gets the option to postpone its investment for a particular time period
waiting gets some positive value. Hence, a firm may refrain from a positive NPV
project when its value of waiting exceeds the value of additional
investment.

Hence, while the managers of the neo-classical firms determine
how much to invest, the managers of the new line of literature have two
responsibilities—(a) to determine how much to invest and (b) to
determine when to invest.

Thus, a firm capable to postpone an investment decision faced a
trade-off between waiting and investing. As argued by Dixit "..........
the value of waiting must be set against the sacrifice of current profit. If
current conditions become sufficiently favourable, one should
eventually take the action that is optimal according to current
calculation, and not wait any longer. ............... firms that refuse to
invest even when the currently available rates of returns are far in
excess of the cost of capital may be optimally waiting to be surer that
this state of affairs is not transitory. Likewise, farmers who carry
operational losses may be rationally keeping their operation alive on
chance that the future can be brighter."

To show how, under an uncertain environment, the scope to
delay an investment project changes the investment plan of a firm
Pindyck (1991) provided a simple two period example. He considered a
firm which wants to invest $ 800 in a widget factory. The price of
widgets fluctuates, depending on the market conditions. In the current period the price is $100. But in the next year it can rise to $150 with probability 0.5 or fall to $50 with probability 0.5. The price will remain there forever. If the firm has no option to postpone his investment decision then he has to take decision instantly at the current period. This decision is taken by calculating the NPV of the project. The NPV at the current period assuming a discount rate of 10% is

\[
\text{NPV} = -800 + \sum_{t=0}^{\infty} \frac{100}{1.1^t} = $300
\]

Since, NPV is positive, it seems that the firm should go ahead with investment.

On the other hand, if the firm can wait for a year then it can consider a second alternative: wait for a year, invest if price rises, reject if price falls. This option yields an NPV of

\[
(0.5)[-800/1.1 + \sum_{t=1}^{\infty} \frac{150}{1.1^t}] = $386.
\]

As this NPV is greater than the previous case, a firm with the option to wait for a period would always take the second option. This changes the time path of investment.

To consider the investment-uncertainty relationship, taking into account the option of waiting, Dixit and Pindyck presented two separate papers. Both of them have shown that introduction of irreversibility creates three zones: one with positive, one with negative and one with zero investment. An increase in uncertainty widens the zone of inaction and hence there is an inverse relationship between uncertainty and investment. Since, the implication of both the papers are the same, for the rest of this section I shall concentrate on Pindyck type specification of the problem.
To present his view on investment-uncertainty relationship Pindyck used the framework of Mc. Donald and Siegel (1986). Suppose a firm decides, at any particular point of time, what would be the optimal time to invest in a project of value ‘V’ by incurring a sunk cost ‘I’. The value of the project is uncertain and evolves around a geometric Brownian motion:

\[ dV = \alpha dt + \sigma V dz \] \hspace{1cm} (22)

where \( dz \) is the increment of a Wiener process.

Hence, at any particular point of time \( t \), the firm tries to find out the optimal timing of investment which maximizes the value of the investment opportunity.

\[ F(V) = \max.E_t[(V_T - I)e^{-\delta T}] \] \hspace{1cm} (23)

Where \( E_t \) denotes the expectation at time \( t \), \( T \) is the (unknown) future time when investment is to be made and \( \mu \) is the discount rate. This maximization is done subject to equation (22). To make waiting worthwhile Pindyck assumed \( \mu > \alpha \) and defined \( \delta = \mu - \alpha \).

The Bellman’s equation for this problem is

\[ \frac{\mu F}{\delta} = \frac{1}{dt} E_t dF \] \hspace{1cm} (24)

Using Ito’s Lemma and the relationship \( dz^2 = dt \) we get:

\[ dF = \alpha VF \ dt + \sigma VF \ dz + 1/2 \sigma^2 V^2 \ F_{VV} \ dt \] \hspace{1cm} (25)

Substituting the value of (25) in (24) and the relationship \( E_t (dz) = 0 \) and \( \alpha = \mu - \delta \) we get:
\[
\frac{1}{2}\sigma^2 V^2 F_{vv} + (\mu - \delta) V F_v - \mu F = 0
\]

Apart from satisfying the differential equation, \( F(V) \) must satisfy three boundary conditions:

\[
F(0) = 0 \quad (27a)
\]

\[
F(V^*) = V^* - 1 \quad (27b)
\]

\[
F_v(V^*) = 1 \quad (27c)
\]

Where \( V^* \) is the optimal value of the project. Solving equations (26), (27a), (27b) and (27c) we get-

\[
F(V) = aV^\beta \quad (28)
\]

and \( V^* = \beta \ln(\beta - 1) \quad (29) \)

where \( \beta = \frac{1}{2} - (\mu - \delta) \sigma^2 + \left\{ \left[ (\mu - \delta) \sigma^2 - \frac{1}{2} \right]^2 + 2\sigma^2 \right\}^{1/2} \)

The value \( V^* \) is the critical value for which the firm is indifferent between waiting and investing. When the value of the firm is greater than \( V^* \), investment is preferred to waiting. Similarly, when the value of the firm is smaller than \( V^* \), it is optimal for the firm to defer the option to invest.

An increase in uncertainty increases the value of \( V^* \). Thus the firm becomes more restrictive in taking any investment decision. Hence contrary to the results of Abel and Caballero, Dixit and Pindyck found a negative relationship
between uncertainty and investment. This relationship neither appears for the concavity of the adjustment cost nor for imperfection in the market structure. Increase in uncertainty raises the value of waiting, thereby restricting the firm's option to invest.

Ingessell and Ross (1988) pointed out that when the firms have the option to withhold investment for some time then for long lived project, investment can not be boosted by a decrease in the interest rate. A decrease in the rate of interest not only raise current profit but also reduces the cost of waiting. Hence, the effect on investment is not ambiguous.

2.3 Optimum Investment Under Uncertainty :

After a brief survey of the literature on investment I now enter into the main part of our analysis. In this part I shall extend the analysis of determining the optimal time path of investment to incorporate the option to delay the investment decision for some time. Under these circumstance the firm faces two problems - (a) determine when to invest and (b) determine how to invest. While the existing literature with the option to delay deals with the first problem in detail, by fixing the level of investment (I) exogenously, it has bypassed the second problem. Thus our frame is wider in the sense that it is capable of addressing both the questions.

As in Caballero (1991) I also failed to find any robust relationship between uncertainty and investment. However the model does point out certain cases, which were beyond the scope of the earlier papers. For example, under perfect competition, the papers of Abel (1983) and Caballero (1991) obtained an increase in the level of investment following an increase in uncertainty. This was
contradicted by Dixit (1991) and Pindyck (1991) who postulated a decrease in the level of investment. My findings point out that in making any decision, the firm considers the relative merits of the two options: the option to invest and the option to wait. When the value of the first option exceeds that of the second, the firm invests and there is a positive relationship. In the reverse case there is a negative relationship. Thus my formulation appears to be more general than the earlier papers. Similar considerations appear in the case of imperfect competition. As the result suggests, small firms often adhere to more aggressive investment strategies than their bigger counterparts.

The Model:

The basic frame of the model is somewhat similar to that of Abel and Eberly (1994). Consider a firm that uses capital and labor to produce a non-storable output. Labor can be adjusted costlessly and is adjusted to maximize the value of revenue net of expenditure to labor. Hence, the only task the firm faces is to determine the optimal capital stock and adjust the investment decision accordingly.

Let \( \pi(K_t, \varepsilon_t) \) denotes the maximized value of the instantaneous operating profit of the firm at time \( t \), where \( K_t \) represents the amount of capital at time \( t \) and \( \varepsilon_t \) a random variable that influence the profit function. This random variable can be interpreted as the price or the level or the demand faced by the firm. The profit function is assumed to satisfy the following conditions:

\[
\pi_K(K_t, \varepsilon_t) > 0, \quad \pi_{\varepsilon}(K_t, \varepsilon_t) > 0 \quad \text{and} \quad \pi_{KK}(K_t, \varepsilon_t) < 0.
\]

The movement of \( K_t \) and \( \varepsilon_t \) are governed by the following equations -
where $z$ is a standard Wiener process, $I$ represents the amount of investment and $\delta$ the depreciation rate.

The cost function consists of three components—purchase/sell cost, adjustment cost and fixed cost. We represent the total cost function as $C(I,K)^3$. The cost function is continuous, strictly convex and twice differentiable except at $I=0$. We assume that

$$\lim_{I \to 0^-} C(I, K) = \lim_{I \to 0^+} C(I, K) = C(0, K)$$

and refer it as fixed cost. To make investment partially irreversible, we assume $C(I, K)^+ \geq C(I, K)^-$ where the terms denote the left hand and right hand partial derivatives of $C(I, K)$ with respect to $I$ evaluated at $I = 0$.

Abel and Eberly assumed investment to be a now or never phenomenon. This means, if the firm fails to undertake an investment opportunity at any point of time, it can never attain it in future. We depart from this specification and assume that the firm has the scope to hold its investment opportunity for some time. With the introduction of this assumption, at any particular point of time, the firm faces two options:

**Option 1:** Invest/Disinvest

**Option 2:** Hold the investment opportunity for some time

---

2 the specification of the components of the cost function is given in detail in section 2.1
Assume the firm to be risks neutral, its choice will depend on the relative values of the two options. This means if $V_0(K,e)$ denotes the value of waiting and $V_1(K,e)$ the value of investing then the value of the firm $V(K,e) = \max\{V_0(K,e), V_1(K,e)\}$.

The firm selects its option in two steps:

**Step 1:**
Assuming the only option is to invest/disinvest, the firm finds out the optimal investment rule and the consequent maximized value.

**Step 2:**
Choice between waiting and investing/disinvesting.

**The Optimal Investment Rule:**

When the firm has to choose between investment and disinvestment, the task of the firm is to find out the investment level that maximizes the present value of its expected profit. In other words, the task of the firm is to maximize its value:

$$\max V(K, e) = \int_0^\infty E_t\{\pi(K_{t+s}, e_{t+s}) - C(L_{t+s}, K_{t+s})\} e^{-rs} ds \quad (32)$$

Where the maximization is subject to (30) and (31).

Bellman's equation is (suppressing the time subscripts):

$$rV(K, e) = \max \{ \pi(K, e) - C(I,K) + 1/dt E(dV) \} \quad (33)$$
Using Ito's Lemma in equation (30) and (31) and the relation \((dK)^2 = (dK)(dt) = (dt)^2 = (dz)(dt) = 0 = E(dz)\) and \(dz^2 = dt\) we get-

\[ E(dV) = [V_K(I-\delta K) + \mu \sigma \epsilon V_e + \frac{1}{2} \sigma^2 \epsilon^2 V_{\infty}] dt \]  \hspace{1cm} (34)

Substituting (34) in (33) we get-

\[ rV(K,\epsilon) = \max \{ r(K,\epsilon) - C(I,K) + V_K(I-\delta K) + \mu \sigma \epsilon V_e + \frac{1}{2} \sigma^2 \epsilon^2 V_{\infty} \} \]  \hspace{1cm} (35)

To solve the maximization problem of the right hand side, note that the only term involving the decision variable \(I\) are \(q_I\) and \(-C(I,K)\). Hence the optimal value is obtained by solving-

\[ \max_I [q_I - C(I,K)] \hspace{1cm} \]  \hspace{1cm} (36)

Let \(\Psi(q,K) = \max_I [q_I - C(I,K)]\) \hspace{1cm} (37)

Since \(I\) is differentiable everywhere except at \(I=0\), if \(I^*(q,K)\) denote the value of \(I\) that maximizes the maximand in (37), the F.O.C. determining \(I^*(q,K)\) are-

\[ C_I [I^*(q,K),K] = q \text{ for } q < C_I(0,K) \text{ and } q > C_I(0,K)^+ \]  \hspace{1cm} (38a)

\[ I^*(q,K) = 0 \text{ for } C_I(0,K)^- \leq q \leq C_I(0,K)^+ \]  \hspace{1cm} (38b)

As \(C_I > 0\), investment is a strictly increasing function of \(q\) in the relevant region. Thus the optimal investment rule becomes:
When investment is optimally made, the value of the maximand-

\[ \Psi(q, K) = q I^*(q, K) - C[I^*(q, K), K] \]  \hspace{1cm} (40)

The firm undertakes new investment only when the value of this maximand is greater than zero. Now, we know that in the range of inaction \( I^*(q, K) = 0 \).
Hence, \[ \Psi(q, K) = -C(0, K) \]  \hspace{1cm} (41)

Outside this interval \( \psi(q, K) \geq -C(0, K) \). Thereby the minimum value of \( q \) is obtained in the interval \([C_l(0, K)^-, C_l(0, K)^+]\). Outside this interval \( \psi(q, K) \) is twice differentiable with respect to \( q \). Differentiating (40) with respect to \( q \) and using equation (38a), (38b) and (39), we get

\[ \psi_q(q, K) = I^*(q, K) \begin{cases} 
< 0 \text{ if } q < C_l(0, K)^- \\
= 0 \text{ if } C_l(0, K)^- \leq q \leq C_l(0, K)^+ \\
> 0 \text{ if } q > C_l(0, K)^+ 
\end{cases} \]  \hspace{1cm} (42)

Again \[ \psi_{qq}(q, K) = I^*_q(q, K) > 0 \text{ if } q < C_l(0, K)^- \text{ or } q > C_l(0, K)^+ \]  \hspace{1cm} (43)
Thus the function $\psi(q, K)$ is a convex function that attains its minimum value: $-C(0,K)$ when $q$ is in the interval $[C_1(0,K)^-, C_1(0,K)^+]$. The relationship between $q$ and $\psi(q, K)$ is shown in Figure 2.2. Let $q_1$ and $q_2$ represent the smallest and largest root of $\psi(q, K) = 0$. Then the optimum behaviour of investment is characterized by:

$$I(q, K) = \begin{cases} 
= I^*(q, K) < 0 \text{ if } q < q_1 \\
= 0 \text{ if } q_1 \leq q \leq q_2 \\
= I^*(q, K) > 0 \text{ if } q > q_2 
\end{cases}$$

(44)
Value of Waiting:

After finding the optimal investment path, the next task of the firm is to find out the value corresponding to the alternative investment rules. Since the value of investment depends on the nature of the market structure, we, at first, skip the problem and concentrate on finding out the value of waiting.

Since waiting provides no return, the only gain from waiting is the change in the expected value of the project. Following Bellman's equation (equation 33) we find that if the firm invests, it would invest up to the point where the maximized expected return from investing is equal to the normal rate of return. The value of waiting, in this case, is less than the return from investment. Similarly, when the firm prefers waiting, optimal waiting requires the value of waiting to equal its normal return. The value of waiting, in this case, exceeds that of investment.

Thus we find that, optimal waiting requires

\[ rV = E(\frac{dV}{dt}) \] \hspace{1cm} (45)

Using Ito’s Lemma and the conditions stated in equation (34), we get

\[ dV = V_\kappa d\kappa + (\mu \varepsilon V_\varepsilon + \frac{1}{2} \sigma^2 \varepsilon^2 V_{ee} ) \, dt. \]

When the firm waits, \(d\kappa = 0\). Therefore

\[ \frac{dV}{dt} = \mu \varepsilon V_\varepsilon + \frac{1}{2} \sigma^2 \varepsilon^2 V_{ee} \] \hspace{1cm} (46)

Substituting (46) in (45) we get
\[
\frac{1}{2} \sigma^2 e^V \kappa + \mu e V e^{-rV} = 0
\]

This is second order homogeneous differential equation. The general solution of the equation is

\[
V(K, \varepsilon) = A_1 e^{\beta_1} + A_2 e^{\beta_2}
\]

where

\[
\beta_1 = -[(\mu/\sigma^2) - \frac{1}{2}] + \left[ (\mu/\sigma^2 - \frac{1}{2})^2 + 2r/\sigma^2 \right]^{1/2}
\]

\[
\beta_2 = -[(\mu/\sigma^2) - \frac{1}{2}] - \left[ (\mu/\sigma^2 - \frac{1}{2})^2 + 2r/\sigma^2 \right]^{1/2}
\]

As shown in appendix 1, \( \beta_1 > 1 \) and \( \beta_2 < 0 \).

Since waiting gets a positive value only in the presence of uncertainty \( V(K, \varepsilon) \rightarrow 0 \) when \( \sigma \rightarrow 0 \). Using this boundary condition we get \( A_1 = 0 \).

Hence the value of waiting is \( V_0(K, \varepsilon) = A_2 e^{\beta_2} \).

Note that this value is constructed with the assumption of no depreciation. Incorporating depreciation we get the value of waiting as

\[
V_0(K, \varepsilon) = A_2 e^{\beta_2} - \delta K
\]

\[\text{Value of Investment / Disinvestment:}\]

The value of investment is derived from equation (35). Putting \( I = I^* \), optimal investment requires

\[
\frac{1}{2} \sigma^2 e^V \kappa + \mu e V e^{-rV} + (I^* - \delta K) V_K - rV = C(I^*, K) - \pi(K, \varepsilon)
\]

The method of solution is shown in Appendix 1.
Clearly, the solution depends on the specification of the cost function and the profit function. Caballero (1991) showed that for a firm facing an isoelastic demand function

$$P_t = Q_t \left(1 - \theta \phi \right) \epsilon_t$$

(50a)

and constant returns to scale production function

$$Q_t = AL_t^a K_t^{1-a}$$

(50b)

the profit function takes the form

$$\pi \left(K_t, \epsilon_t \right) = h K_t \epsilon_t^\phi$$

(51)

where

$$h = \left(1 - \alpha /\theta \right) A^{\theta (1-a/\theta)} \left(\alpha /\theta \right) (a/\theta) (1 -a/\theta)$$

$$\phi = \left[ \frac{1}{(1-a/\theta)} \right] > 1$$

$$\gamma = \left[ (1-a/\theta) /\theta \right] / [1-a/\theta] \leq 1$$

Following the properties of the cost function, for $I \neq 0$ we define $C(I,K) = C(0,K) + C_1 I^\alpha K^{-1}$ where $C(0,K)$ represents the fixed cost, where as $C_1 I^\alpha K^{-1}$ the variable cost. For notational simplicity, we define $C(0,K)$ as $\bar{C}$. The term $C_j$ is used to denote partial irreversibility of investment. We assume that $C_j$ can take two values $C_1$ and $C_2$. $C_j$ takes the value $C_1$ when the firm invests, it becomes $C_2$ when the firm disinvests. To ensure $C_2(0,K)^\gamma \geq C_1(0,K)^\gamma$ we assume $C_1 \geq C_2$. Hence the final form of the cost function becomes

$$C(I,K) = \bar{C} + C_2 I^\alpha K^{-1} \quad \text{when } I < 0$$

$$= \bar{C} + C_1 I^\alpha K^{-1} \quad \text{when } I > 0$$

(52)

where $\alpha$ is an even no ($\geq 2$)

Hence, equation (49) transforms to

$$\frac{1}{2} \sigma^2 \epsilon^2 V_{\epsilon x} + \mu \epsilon V_{\epsilon x} + (I^* - \delta K) V_K - rV = \{ \bar{C} + C_1 I^\alpha K^{-1} \} - h K^\phi$$

(53)
The nature of solution depends on the market structure. Here we consider two cases —

**Case 1: Perfect Competition**

Under perfect competition \( \theta = 1 \). Hence—

\[
\begin{align*}
    h &= (1-a) A^{1+\theta} (a/w)^{1+\theta} \\
    \phi &= 1/(1-a) > 1 \\
    \gamma &= 1
\end{align*}
\]

Hence the profit function becomes: \( \pi(K, \varepsilon) = h K \varepsilon^\theta \). Substituting the value in (53) we get the value on investment\(^4\)

\[
V_1(K, \varepsilon) = qK + h \varepsilon^\theta I^* / \left[ \frac{1}{2} \sigma^2 \phi (\phi - 1) + \mu \phi - \delta - r \right]
\]

---------(54)

The firm prefers to invest if \( V_1(K, \varepsilon) > V_0(K, \varepsilon) \). Waiting becomes preferable if \( V_0(K, \varepsilon) > V_1(K, \varepsilon) \). The firm is indifferent between waiting and investing if \( V_0(K, \varepsilon) = V_1(K, \varepsilon) \). If we define the corresponding value of \( q \) as \( q^* \) then —

\[
q^* K + h \varepsilon^\theta I^* / \left[ \frac{1}{2} \sigma^2 \phi (\phi - 1) + \mu \phi - \delta - r \right] = A_2 \varepsilon^{\delta^2} K
\]

\[
\Rightarrow q^* = A_2 \varepsilon^{\delta^2} / K - h \varepsilon^\theta I^* / K \left[ \frac{1}{2} \sigma^2 \phi (\phi - 1) + \mu \phi - \delta - r \right]
\]

---------(55)

When the alternative to wait is to invest \( I^* > 0 \). We define the corresponding value of \( q^* \) as \( q_2^* \). Hence the firm would invest only if \( q^* > q_2^* \). Similarly, when

---

\(^4\) the method of solution is shown in Appendix 2
the alternative to wait is to disinvest, \( I^* < 0 \). If we define the corresponding value of \( q^* \) as \( q_{i1}^* \), then the firm disinvests if \( q^* < q_{i1}^* \).

On the other hand, putting the specific form of cost function in (40) and assuming 
\[
\Psi(q, K) = 0
\]
we get

\[
q^* = \frac{\bar{C}}{I^*} + C_i[I^* a_i/K]^{-1}
\]

\[
\Rightarrow q = \frac{\bar{C}}{I^*} + C_i[I^* a_i/K]
\]

Hence, \( q_{i1} = \frac{\bar{C}}{I^*} + C_i[I^* a_i/K] \) and \( q_2 = \frac{\bar{C}}{I^*} + C_2[I^* a_i/K] \).

The scope of temporary postponement of investment decision can influence investment if \( q_{i1}^* < q_{i1} \) and \( q_2^* > q_2 \). This requires the attainment of the following conditions-

\[
\left| \frac{\bar{C}}{I^*} + C_i[I^* a_i/K] \right| < \left| (A_2 e^{0.02}/K) - \delta - h e^I I^*/K \left[ \frac{1}{2} \sigma^2 (\phi^{-1}) + \mu \phi - r \right] \right|
\]

\[
\left| \frac{1/2 \sigma^2 (\phi^{-1}) + \mu \phi - r - \delta}{I^* < 0} \right| \quad \text{----------(a)}
\]

and

\[
\left| \frac{\bar{C}}{I^*} + C_i[I^* a_i/K] \right| > \left| (A_2 e^{0.02}/K) - \delta - h e^I I^*/K \left[ \frac{1}{2} \sigma^2 (\phi^{-1}) + \mu \phi - r \right] \right|
\]

\[
\left| \frac{1/2 \sigma^2 (\phi^{-1}) + \mu \phi - r - \delta}{I^* > 0} \right| \quad \text{----------(b)}
\]

Simultaneous attainment of these two conditions require-

\[
C_i < 0, \quad \left| C_i[I^* a_i/K] \right| < \left| \bar{C} / I^* \right|
\]

\[
0 < \frac{1}{2} \sigma^2 (\phi^{-1}) + \mu \phi - r < \delta
\]

and

\[
\left| h e^I I^*/K \left[ \frac{1}{2} \sigma^2 (\phi^{-1}) + \mu \phi - r \right] \right| > (A_2 e^{0.02}/K) - \delta
\]
When these conditions hold value of waiting does influence the investment behaviour of firms. The optimal investment rule, under these conditions, becomes:

\[
I(q, K) = \begin{cases} 
I^*(q, K) < 0 & \text{if } q < q_1^* \\
0 & \text{if } q_1^* < q < q_2^* \\
I^*(q, K) > 0 & \text{if } q > q_2^* 
\end{cases}
\]

Since \( q_1^* < q_1 \) and \( q_2^* > q_2 \), the firm now becomes more restrictive in talking any investment / disinvestment decision. In other words, getting the scope to hold investment opportunity for some time, firm does not invest until price rises substantially above long run average cost. By similar reasoning, firms stay in business for longer periods while absorbing losses. The hurdle rate of return that influences the investment / disinvestment decisions depends on the nature of uncertainty prevailing in the economy as well as on its impact on the value of waiting and investing.

**Effect of Increase in Uncertainty:**

The result of increase on uncertainty is not robust. Increase in the degree of uncertainty is captured by the increase in the value of \( \sigma \). As \( \sigma \) increases, the value of waiting \( V_0(K, \varepsilon) = A_2 \varepsilon^{3/2} \) increases (as \( \beta_2 = -\left(\frac{\mu / \sigma^2}{2} - \frac{1}{2}\right) - \left(\frac{\mu / \sigma^2}{2} - \frac{1}{2}\right)^2 + 2\varepsilon / \sigma^2 \right) < 0 \)). Again increase in \( \sigma \) raises the value of \( V_1(K, \varepsilon) \) too. Hence, given \( q = q^* \) the net result depends on the increase in relative values of \( V_0(K, \varepsilon) \) and \( V_1(K, \varepsilon) \). If the increase in the value of waiting is higher than the increase in the value of investing the firm prefers waiting to invest/ disinvest. On the other hand, if the increase in the value of investing / disinvesting is higher than increase in value of waiting, rise in uncertainty increases investment activity in the economy.
Economically speaking, increase in uncertainty raises the risk of misjudgment. This raises the chance of loss in undertaking a new investment project. As a result, the value of waiting increases. On the other hand, as under perfect competition the profit function is convex in prices, increase in uncertainty raises the value on investment too. Therefore the net result depends on the change in relative value of the two options.

**Case 2: Imperfect Competition**

Under imperfect competition $\theta > 1$ and hence $\gamma = [(1 - a) / \theta] / [1 - a/\theta] < 1$ and $\phi = \theta / (1 - a)$. Therefore the value of the firm is

$$V_1(K, \varepsilon) = \frac{qK - hI^* e^t}{\delta^2} \left\{ \frac{1}{2} \frac{\sigma^2}{\phi(\phi - 1) + \phi \mu - r} - \gamma \right\} \left\{ \frac{1}{2} \frac{\sigma^2}{\phi(\phi - 1) + \phi \mu - \delta - r} \right\}$$

$$+ \gamma \left( \frac{\gamma - 1}{\phi(\phi - 1) + \phi \mu - 2\delta - r} \right)$$

$$+ \gamma \left( \frac{(\gamma - 1) hI^* e^t e^t K^2}{2 \delta^2} \right) \left[ \frac{\sigma^2}{\phi(\phi - 1) + \phi \mu - 2\delta - r} \right]$$

$$= \frac{qK - Z_1 + Z_2}{\delta^2}$$

where

$$Z_1 = hI^* e^t \left\{ \frac{1}{2} \frac{\sigma^2}{\phi(\phi - 1) + \phi \mu - r} - \gamma \right\} \left\{ \frac{1}{2} \frac{\sigma^2}{\phi(\phi - 1) + \phi \mu - \delta - r} \right\}$$

$$+ \gamma \left( \frac{\gamma - 1}{\phi(\phi - 1) + \phi \mu - 2\delta - r} \right) \delta^2$$

$$Z_2 = \gamma \left( \frac{(\gamma - 1) hI^* e^t e^t K^2}{2 \delta^2} \right) \left[ \frac{\sigma^2}{\phi(\phi - 1) + \phi \mu - 2\delta - r} \right]$$

5 The method of solution is shown in Appendix 3
Note that both $Z_1$ and $Z_2$ are positive for $I^* > 0$ and negative for $I^* < 0$.

Now, if $q^*$ denotes the value of $q$ for which $V_0(K, \varepsilon) = V_1(K, \varepsilon)$ then:

$$q^* = A_2 \delta^{02} / K - \delta + Z_1 / K - Z_2 / K \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (57)$$

Comparing (54) and (56) we find that, under imperfect competition the firm becomes more restrictive in taking any investment / disinvestment decision. The basic cause of the dampening effect of market imperfection on investment is that, under perfect competition the demand curve of the firms are perfectly elastic. Hence, any change in output has only an output effect which alters the level of profit of the firm. When the market structure is imperfect, the demand curve faced by the firms are negatively sloped. Under this situation, any increase in the amount of output not only engenders an output effect but also reduces the price level of the commodity. This creates a dampening effect on the level of investment, thereby restricting the firms in undertaking new projects. On the other hand, when the firm reduces its output, the price level rises. This has a favourable impact on the level of profit which prevents further disinvestment. $^6$

**Effect of Increase in Uncertainty:**

An increase in uncertainty in the environment is captured by an increase in the value of $\sigma$. For a positive investment opportunity (i.e., when $I^* > 0$) as the value of $\sigma$ increases, the value of waiting increases. But the impact on the value of investment cannot be told with certainty. Increase in $\sigma$ reduces the value of $Z_1$ in equation (56). This raises the value of investment. But it also reduces $V_1(K, \varepsilon)$ by reducing the value of $Z_2$. Thus there are two opposite forces and the net result depends on their relative strengths.

$^6$ for a detail discussion in this particular aspect, see Caballero (1991)
Though the net result is uncertain, equation (56) helps us to make some general comments. When the value of $I^*$ is very low but the value of capital stock is very large, increase in uncertainty is likely to reduce the value of investment. On the other hand, for firms with low capital stock and high $I^*$, increase in uncertainty is more likely to raise the value of the option to invest. This suggests small firms have higher chance of taking aggressive investment strategies compared to the larger firms under uncertain environment.

The economic intuition behind this type of behaviour can be explained as follows. For the large firms, the loss with the emergence of an adverse economic situation is significantly large. Therefore, when the environment becomes uncertain, the large firms prefer to take a wait and watch policy rather than intensifying the loss by taking a wrong investment decision. On the other hand, for the small firms the probable loss is not so high. Therefore, under uncertain environment, the firms always attempt to undertake projects that would help them to increase their market power if the outcome becomes favourable.

2.4 Comparison With Earlier Results:

In this model we have incorporated the option of waiting in the value-maximizing framework of a firm to find out a more accurate relationship between investment and uncertainty. Unfortunately, our finding does not lead to any definite conclusion. But the results do point out some particular aspects of investment behaviour which were beyond the scope of earlier papers.

(a) Hartman (1972), Abel (1983) and Caballero (1991) pointed out that, in a perfectly competitive market, increase in uncertainty promotes more investment by raising the value of investment. But our findings suggests that, increase in uncertainty not only raises the value of
investment but also increase the value of waiting. When the value of waiting increases at a higher rate than the value of investment, increase in uncertainty may reduce rather than increase the level of investment.

(b) Dixit and Pindyck (1991) concluded that increase in uncertainty, by raising the value of waiting, reduces investment. But, in a perfectly competitive market, as profit function is convex in prices; increase in uncertainty raises the value of investment, too. When the later exceeds the former, increase in uncertainty leads to an increase in the level of investment. Even, under certain circumstances, increase in uncertainty leads to an increase in the level of investment in an imperfectly competitively market.

(c) Caballero (1991) found that under asymmetric adjustment costs increase in uncertainty reduces the level of investment in an imperfectly competitive market. Our result suggests that, under certain circumstances, the small firms may takes increased uncertainty as the opportunity of reaping out some unpredictable gains try to increase their market share by raising the level of investment.
Appendix

Appendix 1: The Value of Waiting

From the optimal condition of waiting, we get -

\[(1/2) \sigma^2 \varepsilon^2 \left( \frac{\partial^2 V}{\partial \varepsilon^2} \right) + \mu \varepsilon \left( \frac{\partial V}{\partial \varepsilon} \right) - rV = 0\]

This is second order non-homogeneous differential equation with variable coefficients. To transform it into fixed coefficient we put \( \varepsilon = e^t \). Hence,

\[
\left( \frac{\partial}{\partial \varepsilon} \right) = \left( \frac{\partial}{\partial t} \right) \left( \frac{\partial t}{\partial \varepsilon} \right) = \left( \frac{\partial}{\partial t} \right) \left( \frac{\partial \varepsilon}{\partial \varepsilon} \right) = \left( \frac{\partial}{\partial t} \right) e^t = \left( \frac{\partial}{\partial t} \right) \varepsilon
\]

Therefore -

\[\varepsilon \left( \frac{\partial}{\partial \varepsilon} \right) = \left( \frac{\partial}{\partial t} \right) \varepsilon\]

Let \( \left( \frac{\partial}{\partial \varepsilon} \right) = D \) and \( \left( \frac{\partial}{\partial t} \right) = D' \). Then the relationship transforms to —

\[\varepsilon D = D' \] ————(i)

Again,

\[\left( \frac{\partial^2}{\partial \varepsilon^2} \right) = \left( \frac{\partial}{\partial \varepsilon} \right) \left( \frac{\partial}{\partial \varepsilon} \right) = \left( \frac{\partial}{\partial t} \right) \left( \frac{\partial t}{\partial \varepsilon} \right) = \left( \frac{\partial}{\partial t} \right) \left( \frac{\partial \varepsilon}{\partial \varepsilon} \right) - \left( 1/ \varepsilon^2 \right) \left( \frac{\partial}{\partial \varepsilon} \right) \varepsilon
\]

\[= \left( 1/ \varepsilon^2 \right) \left( \frac{\partial}{\partial \varepsilon} \right) \left( \frac{\partial \varepsilon}{\partial \varepsilon} \right) - \left( 1/ \varepsilon^2 \right) \left( \frac{\partial}{\partial \varepsilon} \right) \varepsilon
\]

\[= \left( 1/ \varepsilon^2 \right) \left( \frac{\partial}{\partial \varepsilon} \right) \left( \frac{\partial}{\partial \varepsilon} \right) \varepsilon - \left( 1/ \varepsilon^2 \right) \left( \frac{\partial}{\partial \varepsilon} \right) \varepsilon
\]

\[= \left( 1/ \varepsilon^2 \right) \left( \frac{\partial}{\partial \varepsilon} \right) \left[ \left( \frac{\partial}{\partial \varepsilon} \right) - 1 \right]
\]

Therefore

\[\varepsilon^2 \left( \frac{\partial^2}{\partial \varepsilon^2} \right) = \left( \frac{\partial}{\partial t} \right) \left[ \left( \frac{\partial}{\partial t} \right) - 1 \right] \]

or,

\[\varepsilon^2 D^2 = D' (D' - 1) \] ————(ii)

Thus the equation transforms to

\[(1/2) \sigma^2 \varepsilon^2 D^2 V + \mu \varepsilon DV - rV = 0\]

Using the relationship (i) and (ii) we get -

\[(1/2) \sigma^2 D' \left[ \frac{D'}{\partial v} - 1 \right] V + \mu \varepsilon DV - rV = 0\]

\[(1/2) \sigma^2 D'^2 V + \left[ \mu - \left( 1/2 \right) \sigma^2 \right] D' V - rV = 0 \] ————(iii)

To solve this equation we try a trial solution \( V = A e^{\mu t} \)

\[D' V = m A e^{\mu t} = mV\]

\[D'^2 V = m^2 A e^{\mu t} = m^2 V\]

Hence equation (iii) transforms to

\[(1/2) \sigma^2 m^2 V + \left[ \mu - \left( 1/2 \right) \sigma^2 \right] mV - rV = 0\]
or, \((1/2) \sigma^2 m^2 + [\mu - (1/2) \sigma^2] m - r = 0\) \-----------------(iv)

If \(P_1\) and \(P_2\) denotes the roots of the equation then -

\[\begin{align*}
\beta_1 &= -\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right) + \left[\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + 2 \frac{r}{\sigma^2}\right]^{1/2} \\
\beta_2 &= -\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right) - \left[\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + 2 \frac{r}{\sigma^2}\right]^{1/2}
\end{align*}\]

Applying the rule of signs we get \(\beta_1 > 1\) and \(\beta_2 < 0\)

Therefore, the solution becomes—

\[V(K, e) = A_1 e^{\beta_1 t} + A_2 e^{\beta_2 t} = A_1 e^{\beta_1 t} + A_2 e^{\beta_2 t} \ -------------(v)\]

Applying the boundary condition in (v) we get \(V_0(K, e) = A_2 e^{\beta_2 t}\).
Appendix 2: Value of Investing under Perfect Competition

Under perfect competition equation (53) transforms to –

\[
\frac{1}{2} \sigma^2 e^2 \dot{V}_{xx} + \mu \dot{V} + \left[ I^* - \delta K \right] V_K - rV = -\bar{C} + C_j I^* e^K - hK e^x \tag{i}
\]

We apply the following transformation –

\[
x = \log e \Rightarrow e^x = e
\]

and

\[
y = \log (I^* - \delta K) \Rightarrow K = (I^* - e^y/\delta)
\]

Hence, \(e(\delta/\delta x) = \delta/\delta x\) and \(e^2 (\delta/\delta e^2) = \delta/\delta y (\delta/\delta y - 1)\)

This transforms (i) to a second order partial differential equation with constant coefficient, viz.,

\[
F(D, D') v(x, y) = f(x, y) \tag{ii}
\]

where \(f(x, y) = \bar{C} + C_j I^* e^{x} (I^* - e^y/\delta)^{-1} - (hI^*/\delta)(1 - e^y/I^*) e^x = \bar{C} + C_j (I^* e^{-1/\delta} (1 - e^y/I^*)^{-1} - (hI^*/\delta) (1 - e^y/I^*) e^x
\]

\[
F(D, D') = \frac{1}{2} \sigma^2 D^2 + (\mu - \sigma^2/2) D - \delta D' - r
\]

\[D = \delta/\delta x \ ; \ D' = \delta/\delta y\]

Consider the homogeneous part of equation (ii), i.e.,

\[
F(D, D') v(x, y) = 0 \tag{iii}
\]

In general \(F(D, D')\) is irreducible. We try the following method-
\[ D^j D'^i e^{ax+by} = a^i b^j e^{ax+by} \quad \text{where} \quad i, j, \in \mathbb{Z}^+ \]

\[ \Rightarrow F(D, D') e^{ax+by} = F(a, b) e^{ax+by} \]

Hence \( e^{ax+by} \) will satisfy (iii) provided \( F(a, b) = 0 \)

Now, \( F(a, b) = \frac{1}{2} a^2 a^2 + (\mu - \delta^2/2) \) \( D - \delta D^2 = 0 \) \( \text{(iv)} \)

Equation (iv) represents the auxiliary equation. Let \( (a_i, b_i), i = 1, 2, 3, \ldots \ldots n \) be the \( n \) pair of the vectors satisfying (iv) where \( n \in \mathbb{Z}^+ \).

Then \( \nu = \sum_{i=1}^{n} B_i e^{w_i} \) \( \text{(v)} \)

where \( w = a_i x + (1/\delta) \left( r - 1/2 \sigma^2 a_i - (\mu - \sigma^2/2) a_i \right) y \)

Substituting the values of \( x \) and \( y \) we get

\[ \nu = \sum_{i=1}^{n} B_i e^{w_i^i} \left( I^* - \delta K \right)^{1/2} \text{[} r - 1/2 \sigma^2 a_i - (\mu - \sigma^2/2) a_i \text{]} y \]

Now the particular integral is--

\[ PI = \left[ 1/ \{ \frac{1}{2} \sigma^2 D^2 + (\mu - \delta^2/2) D - \delta D^2 = r \} \right] \left\{ \bar{C} + C_j (I^{* \text{-} \delta^{-1}}) (1 - e^{y/\delta^*}) \right\} \]

\[ = \left[ 1/ \{ \frac{1}{2} \sigma^2 D^2 + (\mu - \delta^2/2) D - \delta D^2 = r \} \right] \left\{ \bar{C} + C_j(I^{* \text{-} \delta^{-1}})(1 + e^{y/\delta^*} + e^{2y/\delta^*}) \right\} \]

\[ - (hI^{* \delta}) (1 - e^{y/\delta^*}) e^{y/\delta^*} \]

\[ \text{[ as } e^{y/\delta^*} < 1 \text{ we ignore higher order terms ]} \]
Where the function becomes—

\[ V(K, e) = \sum_{i=1}^{n} B_i e^{\alpha_i (I^* - \delta K)} \frac{1}{1/2 (\sigma^2 \phi^2 + (\mu - \sigma^2/2) \phi - r)} \]

We now use two boundary conditions \( V(K, 0) = 0 \) and \( V(K, \infty) = g(K) \) where \( g(K) \) is a continuous function having a maximum point (inverted U shaped).

These conditions imply:

\[ \frac{\partial V}{\partial r} + C_j I^{\alpha_j} \delta / r + C_j I^{\alpha_j} (I^* - \delta K) \delta / (r + \delta) + C_j I^{\alpha_j} (I^* - \delta K)^2 \delta / (2(r + \delta)) = 0 \]

and \( B_j = 0 \quad \forall \quad j \)

Hence the value function becomes—

\[ V_1(K, e) = -hI^* e^{\phi} \delta / [1/2 \sigma^2 \phi^2 + (\mu - \sigma^2/2) \phi - r] \]

\[ + h(I^* - \delta K)e^{\phi} \delta / [1/2 \sigma^2 \phi^2 + (\mu - \sigma^2/2) \phi - \delta - r] \]

\[ = q K + hI^* e^{\phi} / [1/2 \sigma^2 \phi (\phi - 1) + \mu \phi - r] \]

\[ = q \frac{K}{1/2 \sigma^2 \phi (\phi - 1) + \mu \phi - \delta - r} \]

where \( q = V_{1K} \)
Appendix 3 : Value of Investing under Imperfect Competition

Under imperfect competition, equation (53) takes form:

\[
\frac{1}{2} \sigma^2 e^2 V_{ee} + \mu e V_{e+} (I^* - \delta K) V_K - rV = \bar{C} + C_j I^{*a} K^{-1} - hK\gamma e^g
\]

(viii)

Using the transformation we get:

\[
F(D, D') v(x, y) = f'(x, y) \\
\text{where } f'(x, y) = \bar{C} + C_j (I^* r/\delta) (1 - e^y/I)^0
\]

\[
- (hI^*/\delta) (1 - e^y/I^*) e^{\delta x}
\]

\[
= F_1 - F_2
\]

where

\[
F_1 = \bar{C} + C_j (I^* a^*/\delta^*) (1 - e^y/I)^0
\]

\[
F_2 = (hI^*/\delta^*) (1 - e^y/I^*) e^{\delta x}
\]

The complementary function remains the same as in appendix 2. Now the particular integral -

\[
PI = \{1/ [ \frac{1}{2} \sigma^2 D^2 + (\mu - \sigma^2/2)D - \delta D' - r ] \} (F_1 - F_2)
\]

As shown in Appendix 2,

\[
\{1/ [ \frac{1}{2} \sigma^2 D^2 + (\mu - \sigma^2/2)D - \delta D' - r ] \} F_1
\]

\[
= - \bar{C}/r - C_j I^{*a^1}/\delta^1 r - C_j I^{*a^2} (I^* - \delta K)/\delta^2 (\delta + r)
\]

\[
- C_j I^* a^{*3} (I^* - \delta K)^2 /\delta^3 (2\delta + r)
\]
Now,

\[
\frac{1}{\sqrt{\frac{1}{2} \sigma^2 D^2 + (\sigma^2/2) D - 8D - r}} \left\{ (hI^* e^{\gamma \theta}) \left( 1 - e^{\gamma \theta} \right) e^{\delta x} \right\}
\]

\[
= (hI^* e^{\gamma \theta}) \left\{ \frac{1}{\sqrt{\frac{1}{2} \sigma^2 D^2 + (\sigma^2/2) D - 8D - r}} \right\} \left\{ 1 - (\gamma I^*) e^{\gamma \theta} \right\} e^{\delta x} + \left[ (\gamma - 1) / 2I^* \right] e^{\delta x}
\]

[as \( e^{\gamma \theta} / I^* > 1 \) we ignore higher order terms]

\[
= (hI^* e^{\gamma \theta}) \left\{ \frac{1}{\sqrt{\frac{1}{2} \sigma^2 D^2 + (\sigma^2/2) D - 8D - r}} \right\} \left\{ e^{\delta x} - (\gamma I^*) e^{\delta x + \gamma \theta} \right\} + \left[ (\gamma - 1) / 2I^* \right] e^{\delta x + \gamma \theta}
\]

\[
= hI^* e^{\gamma \theta} / \delta^2 \left[ \frac{1}{2} \sigma^2\phi(\phi - 1) + \mu \phi - r \right] + \gamma hI^* e^{\delta x + \gamma \theta} / \delta^2 \left[ \frac{1}{2} \sigma^2\phi(\phi - 1) + \mu \phi - r \right]
\]

\[
+ \left[ (\gamma - 1) / 2I^* \right] e^{\delta x + \gamma \theta} / \delta^2 \left[ \frac{1}{2} \sigma^2\phi(\phi - 1) + \mu \phi - r \right]
\]

Therefore, the value of the firm:

\[
V(K,\epsilon) = \sum_{i=1}^{n} Bi e^{\alpha_i (I^* - \delta K) / \delta^2} - C_i I^* e^{\alpha_i / \delta^2} - C_i I^* e^{\alpha_i - 2(I^* - \delta K) / \delta^2 (\delta + r)} - C_i I^* e^{\alpha_i - 3(I^* - \delta K)^2 / \delta^3 (2\delta + r) - hI^* e^{\gamma \theta} / \delta^2 \left[ \frac{1}{2} \sigma^2\phi(\phi - 1) + \mu \phi - r \right] - \gamma hI^* e^{\gamma \theta} / \delta^2 \left[ \frac{1}{2} \sigma^2\phi(\phi - 1) + \mu \phi - r \right] - \gamma (\gamma - 1) hI^* e^{\gamma \theta} / \delta^2 \left[ \frac{1}{2} \sigma^2\phi(\phi - 1) + \mu \phi - r \right]
\]

\[\text{...(x)}\]
Putting the boundary conditions, we get

\[ V_1(K, \varepsilon) = - h_1^{*} e^\delta / \delta' \left[ \frac{1}{2} \sigma^2 \phi (\phi - 1) + \mu \phi - r \right] \]

\[ + h_1^{*} e^\delta (1 - \delta K) / \delta' \left[ \frac{1}{2} \sigma^2 \phi (\phi - 1) + \mu \phi - \delta - r \right] \]

\[ - \gamma(\gamma - 1) h_1^{*} e^\delta (1 - \delta K)^2 / 2 \delta' \left[ \frac{1}{2} \sigma^2 \phi (\phi - 1) + \mu \phi - 2\delta - r \right] \]

\[ = q K - h_1^{*} e^\delta / \delta' \left[ \frac{1}{2} \sigma^2 \phi (\phi - 1) + \mu \phi - r \right] \]

\[ + h_1^{*} e^\delta / \delta' \left[ \frac{1}{2} \sigma^2 \phi (\phi - 1) + \mu \phi - \delta - r \right] \]

\[ - \gamma(\gamma - 1) h_1^{*} e^\delta / \delta' \left[ \frac{1}{2} \sigma^2 \phi (\phi - 1) + \mu \phi - 2\delta - r \right] \]

\[ + \gamma(\gamma - 1) h_1^{*} e^\delta (1 - \delta K)^2 / 2 \delta' \left[ \frac{1}{2} \sigma^2 \phi (\phi - 1) + \mu \phi - 2\delta - r \right] \]

where \( q = V_{1K} \).