The purpose of the third chapter of this dissertation is to present the generalized design methodology that is adopted for modeling and analysis of the MM-wave and Terahertz IMPATT diodes having various doping profiles. Also an in-depth analysis of the simulation technique adopted for studying photo-sensitivity of SiC, GaN and InP-based MM-wave and Terahertz IMPATT devices are presented in this Chapter.
3.1 Introduction:

In the MM-wave and sub MM-wave communication systems IMPATT devices are considered as the most powerful solid-state source. It is discussed in Chapter 2 that several investigations were carried out by various research groups, on the high-frequency properties of conventional Si and GaAs IMPATT diodes. Researchers are now-a-days focusing their attention on the development of high-power high-frequency IMPATT devices based on Wide-Bandgap (WBG) semiconductors, owing to their excellent material properties discussed in previous Chapters. Realistic analyses of the MM-wave and THz frequency performances of these new classes of IMPATT devices, incorporating the latest available experimental values of material parameters of base semiconductors, are not available in current literature. The present thesis is concerned with the modeling and analysis of high-frequency properties of MM-wave and THz IMPATT devices having various doping profiles and based on emerging semiconductors: SiC and GaN as well as InP. In addition to these, analysis of the photo-sensitivity of these devices will also be presented in the dissertation.

Numerical simulation of semiconductor devices is playing an increasingly important role as a design tool for the semiconductor industry. Device simulation has gained increasing relevance for the design and optimization of electronic semiconductor applications due to rising design complexity and the cost reduction achieved by reducing the number of experimental batch cycles. Analytical solutions of the basic device equations do not provide accurate information regarding the DC and high-frequency characteristics of the devices. Numerical simulation, on the other hand, provides proper guidelines for the fabrication of devices. An accurate knowledge of the electric field distribution is a prerequisite for understanding the high-frequency behavior of IMPATT devices. Vital information such as DC breakdown voltage, avalanche layer voltage, depletion layer width and efficiency of the devices are available from DC analysis. The DC results from the static analysis of the devices are used as the input parameters for the high-frequency analysis of IMPATT devices.

Earlier several workers have carried out computer based studies of the static field properties of the devices [3.1 -3.4] [2.5] [2.78]. A successful computer design demands that it would be generalized and will be flexible enough to model devices of various doping profiles. The previous methods, however, had certain shortcomings. Some of the methods did not consider the effects of mobile space-charge, unequal ionization rates and drift velocities of charge carriers. Some of the methods had given rise to numerical instability in matching the boundary condition for IMPATT devices having multilayered doping profile operating at high dc current density. Misswa in 1986 formulated an iterative computer method for the analysis of IMPATT devices taking into account the mobile space-charge effect and the
realistic field variation of ionization rates and drift velocities, which was subsequently used by a number of workers for the analysis of one-sided abrupt junctions [3.2]. In their computer methods the non-linear Poisson's and carrier current continuity equations are solved numerically starting the computation from one edge of the depletion layer. The location (x_i) of one edge of the depletion layer is adjusted by iteration until the boundary conditions for the electric field and carrier current are satisfied at the other edge. However, the matching of the boundary conditions is difficult because the relative changes of the electric field are extremely sharp at the two edges of the depletion layer and a slightly wrong choice of x_i may lead to numerical instability [3.2]. To overcome this problem there arise a need to develop new method. One interesting point is to note that the electric field profile of IMPATT diodes with abrupt or linearly graded junctions exhibits a field maximum near the metallurgical junction.

Sze and Gibbons [3.1] in their studies on the breakdown characteristics of linearly graded and one-sided abrupt junction avalanche diodes made use of this fact and solved the above-mentioned equations starting the computation from the position (x_0) of field maximum (E_0). In their study, the mobile space-charge effect on the electric field was neglected and hence the field maximum was assumed to be located at the metallurgical junction. The magnitude of E_0 was adjusted subject to boundary condition. This field profile calculations are thus strictly valid at the threshold of avalanche breakdown where the mobile space-charge effect can be neglected. But under proper operating conditions the dc current level is quite high and hence the mobile charge density is quite considerable in the depletion layer. Under this condition there occurs not only a change of magnitude of E_0 but also a shift in its position x_0 from the metallurgical junction. The non linear Poisson's and current continuity equations cannot be solved independently as the carrier distribution affects the field distribution and this in turn affects the carrier distribution through changes in ionization rates. So, a simultaneous solution of Poisson's and current continuity equations is needed to incorporate the effect of mobile space-charge. The field maximum would be located exactly at the metallurgical junction if the carrier ionization rates and drift velocities of the base semiconductor material are equal. The shift in x_0 depends on the inequality of ionization rates of electrons and holes of the base material. For GaAs diodes it was found that α < β and the field maximum was situated on n-side of the junction, while this was on the p-side of the junction for Si for which α > β [2.79]. Though the magnitude of the shift of field-maximum position is small, it is important for obtaining computer solution of necessary device equations. Hence the location of the field maximum is to be properly chosen at the starting point of computation of single drift (p++ n n++ and n++ p p++) and double drift (p++ p n n++) IMPATT devices.

Considering all the above-mentioned facts, Roy et al. [2.79] suggested a generalized simple and accurate DC computer simulation method incorporating the effect of mobile space-charge in the depletion
layer, inequality of carrier ionization rates and drift velocities and their realistic field and temperature dependence. This method involves simultaneous computer solution of the non-linear Poisson's and current continuity equations by starting the computation from the field maximum within the depletion layer, located near the metallurgical junction. In this way, the numerical instability encountered by starting the computation from the edge as mentioned earlier, has been avoided.

3.2 DC Simulation Scheme:

3.2.1 Basic Equation For DC Computer Analysis.

A schematic diagram of a reverse-biased p-n junction is shown in Figure 2.2. The direction of the electric field (E), electron current density (Jn) and hole current density (Jp) are shown. The electron and hole reverse saturation currents entering the edges of the depletion layer on the n and p sides are represented by Jns and Jps, respectively. The basic equation that governs the avalanche multiplication phenomenon and the charge carrier flow in the depletion layer of the reverse-biased p-n junction are the Poisson's equation and the current continuity equations. In a one dimensional case they may be written as follows:

\[ \frac{\partial E}{\partial x} = \frac{q}{\varepsilon} [N_D - N_A + p - n] \]  

where, E = electric field, q = electric charge, ND = ionized donor density, NA = ionized acceptor density, p = hole density, n = electron density, \( \varepsilon \) = permittivity of the semiconductor

The current continuity equations:

\[ \frac{\partial p}{\partial t} = - \frac{1}{q} \frac{\partial J_p}{\partial x} + G \]  

and,

\[ \frac{\partial n}{\partial t} = \frac{\partial J_n}{\partial x} + G \]  

where G = generation rate = \( \alpha v_n n + \beta v_p p \)  

and \( \alpha \) electron ionization rate, \( \beta \) hole ionization rate, \( v_n \) drift velocity of electrons and \( v_p \) drift velocity of holes. Since the effect of diffusion current is negligible compared to drift current [2.104 -2.106], this effect has not been taken into account in the present analysis and the electron and hole current densities are therefore given by:

\[ J_n = q n v_n \]  
\[ J_p = q p v_p \]
In the DC analysis, the time derivative of p and n is zero. Thus, equations (3.2 and 3.3) reduces to:

\[
\frac{1}{q} \frac{\partial j_p}{\partial x} = g \quad (3.7)
\]

and,

\[
\frac{1}{q} \frac{\partial j_n}{\partial x} = -g \quad (3.8)
\]

Substituting the value of g in equations (3.7) and (3.8), one can find:

\[
\frac{1}{q} \frac{\partial j_p}{\partial x} = \alpha v_n n + \beta v_p p \quad (3.9)
\]

i.e. \( \frac{\partial j_p}{\partial x} = \alpha J_n + \beta J_p \) \quad (3.10)

Similarly,

\[
\frac{1}{q} \frac{\partial j_n}{\partial x} = -\alpha v_n n - \beta v_p p \quad (3.11)
\]

i.e. \( \frac{\partial j_n}{\partial x} = -\alpha J_n - \beta J_p \) \quad (3.12)

The total DC current density \( J_0 \) is given by:

\[
J_0 \text{ or } J_0 = J_n + J_p \quad (3.13)
\]

This is constant throughout the depletion region. Combining equations (3.10) and (3.12) one may get:

\[
\frac{\partial j_p}{\partial x} - \frac{\partial j_n}{\partial x} = 2\alpha J_n + 2\beta J_p \quad (3.14)
\]

Dividing throughout by \( J \) and simplifying, one obtain:

\[
\frac{\partial}{\partial x} \left( \frac{(j_p - j_n)}{J} \right) = (\alpha + \beta) - (\alpha - \beta) \frac{P(x)}{J} \quad (3.15)
\]

where, \( P(x) = \frac{(j_p - j_n)}{J} \), and is defined as the normalized difference between hole and electron currents. Hole and electron currents can be separately obtained from the equations given by:

\[
J_p (x) = 0.5 J [1 + P(x)] \text{ and } J_n (x) = 0.5 J [-P(x)].
\]

Equation (3.10) can be rewritten in the following way:

\[
q \frac{\partial}{\partial x} (T \frac{v_p}{V_p}) = \alpha J_n + \beta J_p
\]

\[
\text{or, } q \frac{\partial p}{\partial x} = \frac{\alpha J_n + \beta J_p}{v_p} \frac{v_p}{\partial x} \quad (3.16)
\]

Similarly, equation (3.12) can be rewritten as,

\[
q \frac{\partial n}{\partial x} = \frac{-\alpha J_n - \beta J_p}{v_n} \frac{v_n}{\partial x} \quad (3.17)
\]

From equations (3.16) and (3.17), one gets,

\[
q \frac{\partial (p-n)}{\partial x} = (\alpha J_n + \beta J_p) \left( \frac{1}{v_p} + \frac{1}{v_n} \right) + \frac{J_n}{v_n} \frac{\partial v_n}{\partial x} - \frac{J_p}{v_p} \frac{\partial v_p}{\partial x}
\]
or, \[ q \frac{\partial(p-n)}{\partial x} = J_p \frac{\partial}{\partial x} \left( \frac{1}{v_p} \right) - J_n \frac{\partial}{\partial x} \left( \frac{1}{v_n} \right) - (\alpha - \beta)q(p-n) + J \left( \frac{\alpha}{v_p} + \frac{\beta}{v_n} \right) \] (3.18a)

Equation (3.18a) can be rearranged as:

\[ q \frac{\partial(p-n)}{\partial x} = \left[ J_p \frac{\partial}{\partial E} \left( \frac{1}{v_p} \right) - J_n \frac{\partial}{\partial E} \left( \frac{1}{v_n} \right) \right] \frac{\partial E}{\partial x} - (\alpha - \beta)q(p-n) + J \left( \frac{\alpha}{v_p} + \frac{\beta}{v_n} \right) \] (3.18b)

3.2.2 BOUNDARY CONDITIONS FOR DC ANALYSIS.

In this Sub-section the author will discuss the boundary conditions which are to be satisfied by the E(x) and P(x) profiles at the two edges of the depletion layer. In Figures 2.2(a-b), \( x_1 \) and \( x_2 \) represents the two edges of the depletion layer. At \( x = x_1 \), the hole current consists entirely of the reverse saturation current \( J_{ps} \).

Thus at \( x = x_1 \),

\[ J_p - J_n = J_{ps} - (1 - J_{ps}) = 2(J_{ps} - J) \]

i.e. \( (J_p - J_n)/J = (\frac{2J_{ps}}{J} - 1) \)

or, \( (J_p - J_n)/J |_{x = x_1} = (\frac{2J_{ps}}{J} - 1) \)

(3.19a)

where, \( M_p = \frac{J}{J_{ps}} \) is the hole current multiplication factor.

Similarly, at \( x = x_2 \), the electron current consists entirely of the reverse saturation current \( J_{ns} \). Thus at \( x = x_2 \),

\[ J_p - J_n = (1 - J_{ns}) - J_{ns} \]

i.e. \( (J_p - J_n)/J = (1 - \frac{2J_{ns}}{J}) \)

or, \( (J_p - J_n)/J |_{x = x_2} = (1 - \frac{2J_{ns}}{J}) \)

(3.19b)

where, \( M_n = \frac{J}{J_{ns}} \) is the electron current multiplication factor.

The field boundary conditions are given by:

\[ E(x_1) = 0, \text{ and } E(x_2) = 0 \]

(3.20)

3.2.3 DESCRIPTION OF THE COMPUTER METHODOLOGY.

Let the electric field be a maximum (\( E_0 \)) at \( x = x_0 \) near the metallurgical junction. Then,

\[ \frac{\partial E}{\partial x} \bigg|_{x = x_0} = 0 \]

It can be shown that the value of \( P(x) \) at \( x = x_0 \) is determined by \( x_0 \) and \( E_0 \). Thus,
CHAPTER 3: SIMULATION METHODS FOR DESIGNING MM-WAVE AND TERAHERTZ IMPATTS

\[ P(x_0) = \frac{\left[ J_p(x_0) - J_n(x_0) \right]}{J} \]

\[ = \frac{\frac{\partial}{\partial x} \left[ q \left( p(x_0) v_p(E_0) - n(x_0) v_n(E_0) \right) \right]}{q p(x_0) v_p(E_0) - q n(x_0) v_n(E_0)} \]  

(3.21)

Since, at \( x = x_0, \frac{\partial E}{\partial x} = 0 \), one can obtain from equation (3.1)

\[ q p(x_0) - q n(x_0) = -q N(x_0) \]

(3.22)

where, \( N(x_0) = N_D - N_A \mid_{x = x_0} \)

Total current density = \( J \) (or \( J_0 \)) = \( J_n + J_p \) = constant,

Hence, \( J = q p(x_0) v_p(E_0) + q n(x_0) v_n(E_0) \)

(3.23)

Combining equations (3.22) and (3.23)

\[ q p(x_0) = \frac{\left[ J - q N(x_0) v_n(E_0) \right]}{v_p(E_0) + v_n(E_0)} \]

Similarly, \( q n(x_0) = \frac{\left[ J + q N(x_0) v_p(E_0) \right]}{v_p(E_0) + v_n(E_0)} \)

Substituting this in equation (3.21), one can obtain:

\[ P(x_0) = \frac{v_p(E_0) - v_n(E_0) - \frac{N(x_0)}{v_p(E_0) + v_n(E_0)} v_p(E_0) v_n(E_0)}{v_p(E_0) + v_n(E_0)} \]

(3.24)

The numerical computation, for the simultaneous solution of equations (3.1) and (3.15), starts from the field maximum with some arbitrarily chosen values of \( E_0 \) and \( x_0 \), \( P(x_0) \) is determined by \( E_0, x_0 \) and the doping profile as can be seen from equation (3.22). The space step for the simulation is chosen as \( \sim 10^{-9} \) m, for MM-wave devices and \( 10^{-10} \) m for Terahertz devices. Choosing an appropriate space step, equations (3.1) and (3.15) are solved at every space step starting from the field-maxima towards the right hand side of the junction, i.e. the p-region until the field becomes zero or changes sign. Let \( P(x) \mid_R \) be the value of \( P(x) \) at this point. In the numerical solution of (3.1), the space variation of mobile charge density \( q(p-n) \) is obtained from equation (3.18b). Now the computation again starts from field maximum point and the equations are solved at every space-step towards the left side of the junction (i.e. in the n-region) until the field becomes zero or changes sign. The value of \( P(x) \) at this point is expressed as \( P(x) \mid_L \). If the boundary conditions for \( P(x) \) and \( E(x) \) are not satisfied simultaneously, then the values of \( E_0 \) and \( x_0 \) are adjusted suitably. This requires a double iteration over \( E_0 \) and \( x_0 \). The double-iteration is carried out until boundary conditions of \( P(x) \) and \( E(x) \), as shown in equations (3.19 a-b) and (3.20), are satisfied simultaneously at the two edges of the depletion region. Once the values of \( E_0 \) and \( x_0 \) are fixed for a particular value of DC current density, one can obtain the electric field and carrier current profiles through simultaneous solution of (3.1) and (3.15) by starting the computation from the correct location of the field maximum and the corresponding value of electric field. The algorithm of the DC computation scheme is shown in Appendix 1.
3.3 High-Frequency Simulation Scheme:

If the ac field developed across the diode is much smaller than the breakdown field, the variation of \( \alpha \) and \( \beta \) with electric field can be assumed to be linear and the small-signal solution can be carried out by linearizing the diode equations. Accurate information regarding the effect of mobile space-charge at high values of DC current density on the high-frequency properties of the diodes can be obtained through small-signal calculations. In general, small-signal results predict the range of frequency in which the IMPATT diodes will oscillate if placed in a proper resonant circuit and also the operating current range. Small-signal analysis also provides the magnitude of maximum negative conductance that can be generated by the diode and also the optimum frequency of oscillation at which device exhibit maximum negative conductance.

In this present section, a description of an accurate small-signal analysis of IMPATT diode following Gummel-Blue approach [2.63] will be presented. An iterative and generalized computer method is framed to tackle the small-signal analysis of various problems included in the thesis. This method is free from any simplifying assumption regarding material parameters and also it is applicable to any diode structure. The DC data for a particular diode structure under particular operating conditions, as obtained from the DC simulation scheme described in Section 3.2, are used as input for small-signal analysis. The boundaries of the depletion region are fixed by DC analysis and are taken as starting and end points for small-signal analysis. A description of the analysis and the associated computer methodologies will be presented now.

3.3.1 Description of The Small-Signal Analysis.

To keep the computation effort within the feasible limits, the following assumptions are made in this present method:

1. One dimensional model of p-n junction is considered.
2. The electron and hole velocities are taken to be saturated and independent of the electric field throughout the space-charge region.

One dimensional model of the p-n junction, shown in Figures 2.2 (a-b), is considered for the understanding of the small-signal analysis. In the figures, \( x = -x_1 \) and \( x = x_2 \) represents the boundaries of the depletion region on n and p sides, respectively. The region close to the metallurgical junction represents the avalanche region. The conduction current density due to the flow of generated carriers in the depletion region is given by:

\[
J_{\text{con}} = q (v_{sn} n + v_{sp} p)
\]

(3.25)
where, $J_{con} =$ conduction current density, $q =$ charge of electron, $v_{sn} =$ saturated drift velocity of electron, $v_{sp} =$ saturated drift velocity of holes, $n =$ electron density, $p =$ hole density.

The total electric current density is the sum of the conduction current density and the displacement current density. Thus,

$$J_{total} = J_{con} + J_{dis} = q (v_{sn} n + v_{sp} p) + \frac{\partial (\epsilon E)}{\partial t}$$

(3.26)

Where, $\epsilon =$ permittivity of the semiconductor, $E =$ electric field. The spatial variation of the electric field in the space-charge region is given by the Poisson's equation as shown in equation (3.1):

$$\frac{d\epsilon E}{dx} = \frac{q}{\epsilon} [N_D - N_A + p - n]$$

Let $E_m$ represents the portion of the electric field caused by the space-charge of the mobile carriers in the depletion layer.

$$E_m = E - \frac{q}{\epsilon} \int (N_D - N_A) dx$$

(3.27)

The 2nd term in the above equation represents the portion of the field caused by the impurity space-charge. The equation (3.26) and the Poisson's equation can be re-written in terms of the field due to the mobile space-charge in the following form:

$$J_{total} = J_{con} + J_{dis} = q (v_{sn} n + v_{sp} p) + \frac{\partial E_m}{\partial t}$$

(3.28)

$$\frac{\partial E_m}{\partial x} = \frac{q}{\epsilon} [p - n]$$

(3.29)

From equation (3.29) it can be easily seen that

$$n = p - \frac{\epsilon}{q} \frac{\partial E_m}{\partial x}$$

(3.30)

$$p = n + \frac{\epsilon}{q} \frac{\partial E_m}{\partial x}$$

(3.31)

Substituting equation (3.30) in equation (3.28) one obtains:

$$J_{total} = J_{con} + J_{dis} = q \left[ v_{sp} p + v_{sn} \left\{ p - \frac{\epsilon}{q} \frac{\partial E_m}{\partial x} \right\} \right]$$

(3.32)

or, $p = \frac{J_{total} - \left( \frac{\partial E}{\partial t} + \epsilon \frac{\partial v_{sn}}{\partial x} \right) E_m}{q (v_{sn} + v_{sp})}$

(3.33)

In the similar way the following equation can also be derived:

$$n = \frac{J_{total} - \left( \frac{\partial E}{\partial t} + \epsilon \frac{\partial v_{sp}}{\partial x} \right) E_m}{q (v_{sn} + v_{sp})}$$

(3.34)

Now, the carrier generation rate at any point in the depletion region is given by:

$$g = \alpha v_m n + \beta v_{sp} p$$

Combining carrier continuity equations, (3.2) and (3.3), one can obtain:
Putting equations (3.3), (3.33) and (3.34) in equation (3.35) one can obtain the fundamental time and space varying dynamical differential equation involving the electric field and the total current:

\[
\left[ \frac{d^2}{dx^2} - k^2 + (\alpha - \beta + 2 \beta k) \frac{d}{dx} + 2 \bar{\alpha} k \right] E_m = \frac{J_{\text{total}}}{\bar{\nu} \varepsilon} (2 \bar{\alpha} - k)
\]  

(3.36)

where the following notations have been introduced:

\[
\dot{\bar{\nu}} = \sqrt{\left( v_{mp} - v_{sp} \right)}
\]

\[
\bar{\alpha} = \frac{\beta v_{sp} + \alpha v_{mp}}{2 \bar{\nu}}
\]

\[
\bar{\beta} = \frac{v_{mp} - v_{sp}}{2 \bar{\nu}}
\]

\[
k = \frac{1}{\bar{\nu}} \frac{\delta}{\delta t}
\]

Now, the small-signal operation will be discussed. Let the alternating electric field superimposed on the DC electric field, \( E_m \) be small and denoted by \( \tilde{E} \) and also let \( \tilde{J} \) be the corresponding small-signal ac total current density. It is to be noted that equation (3.36) is non-linear in \( E \) since the ionization coefficients, \( \alpha \) and \( \beta \) depend on electric field. In order to obtain a linear equation for the small-signal quantities from equation (3.36), one has to include an operator \( H \), in the left hand side of the equation, which contain the derivatives of \( \alpha \) and \( \beta \) with respect to the electric field. This is shown below in the following way:

\[
H = \frac{1}{\bar{\nu} \varepsilon} \left[ 2 \frac{\partial \bar{\alpha}}{\partial E} + \bar{\nu} \frac{d}{dx} (\alpha - \beta) \right]
\]

(3.37)

\[
y = \frac{\bar{\nu} \varepsilon}{\bar{J}} \frac{d}{dx} E_m
\]

(3.38)

Using equations (3.37) and (3.38) in equation (3.36) and separating the DC and ac parts one can obtain the small-signal equation given by

\[
\left[ \frac{d^2}{dx^2} - k^2 + (\alpha - \beta + 2 \beta k) \frac{d}{dx} + 2 \bar{\alpha} k - H \right] \tilde{E} = \frac{\bar{J}}{\bar{\nu} \varepsilon} (2 \bar{\alpha} - k)
\]

(3.39)

\( \tilde{J} \) is small ac total current density. To convert the small-signal operation from time domain to frequency domain, \( \frac{\delta}{\delta t} \) will be replaced by \( j \omega \), i.e. \( k = \frac{j \omega}{\bar{\nu}} \). Thus equation (3.39) can be re-written as:

\[
\left[ \frac{d^2}{dx^2} + \frac{\omega^2}{\bar{\nu}^2} + (\alpha - \beta + 2j \bar{\beta} \frac{\omega}{\bar{\nu}}) \frac{d}{dx} + j2 \bar{\alpha} \frac{\omega}{\bar{\nu}} - H \right] \tilde{E} = \frac{\bar{J}}{\bar{\nu} \varepsilon} (2 \bar{\alpha} - \frac{j \omega}{\bar{\nu}})
\]

(3.40)

or,

\[
\left[ \frac{d^2}{dx^2} + \frac{\omega^2}{\bar{\nu}^2} + (\alpha - \beta + 2j \bar{\beta} \frac{\omega}{\bar{\nu}}) \frac{d}{dx} + j2 \bar{\alpha} \frac{\omega}{\bar{\nu}} - H \right] Z (x, \omega) = \frac{1}{\bar{\nu} \varepsilon} (2 \bar{\alpha} - \frac{j \omega}{\bar{\nu}})
\]

(3.41)

where, \( Z (x, \omega) = \frac{\tilde{E}}{\tilde{J}} \) = the ratio of the ac electric field to the ac total current density.
Writing the space dependent quantities explicitly in equation (3.41), one can obtain
\[
\left[ \frac{d^2}{dx^2} + \frac{\alpha^2}{\psi^2} + (\alpha(x) - \beta(x) + j2 \psi \frac{\omega}{\psi} \frac{d}{dx} + j2 \bar{\alpha}(x) \frac{\omega}{\psi} - H(x) \right] Z(x, \omega) = \frac{1}{e} (2 \bar{\alpha}(x) - \frac{j \omega}{\psi}) \tag{3.42}
\]
Denoting the operator on the L.H.S. of equation (3.42) by \( L \), one can write the following equation:
\[
L Z(x, \omega) = \frac{1}{e} (2 \bar{\alpha}(x) - \frac{j \omega}{\psi}) \tag{3.43}
\]
This is a second order linear differential equation for the device impedance \( Z \).

### 3.3.2 BOUNDARY CONDITIONS FOR SMALL-SIGNAL ANALYSIS.

From Figures 2.2(a-b) it can be easily seen that all the particle current at the edge of the left drift region is only due to electron \( \bar{n} \), the hole current \( \bar{p} \) is being negligible there. Thus under this condition, equation (3.26) can be written as
\[
\bar{J} = q (v_{sn} \bar{n}) + \frac{\partial \bar{n}}{\partial t} \tag{3.44}
\]
where, \( \bar{n} \) is the small-signal electron concentration. \( \bar{n} \) may be expressed in terms of \( \bar{E} \) using Poisson's equation, when the hole concentration (\( \bar{p} \)) is negligible. Thus one may write:
\[
q \bar{n} = -\frac{\partial}{\partial x} \varepsilon \bar{E} \tag{3.45}
\]
From (3.44) and (3.45) one can write the left boundary condition as:
\[
\left( \frac{\bar{E}(-x_1)}{v_{sn} \varepsilon} \right) = -\frac{\bar{J}}{v_{sn} \varepsilon}, \text{ at } x = -x_1
\]
i.e. \( \left( \frac{\bar{E}}{v_{sn} \varepsilon} + \frac{\partial}{\partial x} \right) Z(x, \omega) = -\frac{1}{v_{sn} \varepsilon}, \text{ at } x = -x_1 \tag{3.46} \)
Similarly the right boundary condition in the p-side is derived as,
\[
\left( \frac{\bar{J}}{v_{sp} \varepsilon} + \frac{\partial}{\partial x} \right) Z(x, \omega) = -\bar{J}, \text{ at } x = x_2 \tag{3.47}
\]

### 3.3.3 DESCRIPTION OF THE SMALL-SIGNAL COMPUTATION TECHNIQUES.

The small-signal analysis of a diode provides information regarding its high-frequency performance. (i) Diode impedance (\( Z \)), (ii) device quality factor (-Q), (iii) range of frequency exhibiting negative resistance, (iv) device negative resistance (-ZR) and (v) diode admittance can easily be computed from the numerical solution of equation (3.42). It is to be noted that the spatially dependent quantities such as \( \alpha(x) \), \( \beta(x) \), \( \bar{\alpha}(x) \) and \( H(x) \) are evaluated and fed as input data for the small-signal analysis. The edges of the diode depletion region are fixed by the DC analysis and they are used as starting and end point for the small-signal analysis. In addition to the above-mentioned five integrated diode parameters, small-signal analysis also provides information regarding the spatial variation of negative resistance.
(R(x)) and reactance (X(x)) in the depletion layer. The spatial distribution of R(x) and X(x) in the depletion region is vital for getting clear knowledge regarding the microscopic properties of the devices. This gives an insight into the region of depletion layer that contributes much to the output power.

In order to obtain R(x) and X(x) profiles for a diode under particular operating condition, the easiest approach is to split the complex quantity Z(x, ω) into its real part R(x) and imaginary part X(x) which in turn will split equation (3.42) into two second order differential equation in R and X. These two equations should be solved simultaneously to obtain both R(x) and X(x) profiles and also the diode integrated parameters like conductance (G), susceptance (B), impedance (Z) and quality factor (Q).

On splitting equation (3.42) into R(x) and X(x), the following equations are obtained [3.5]:

\[
\frac{d^2 R}{dx^2} + (\alpha - \beta) \frac{dR}{dx} - 2R \frac{\omega}{v} \frac{dx}{dx} + \left(\frac{\omega^2}{v^2} - H\right) R - 2\alpha \frac{\omega}{v} X - \frac{2\beta}{v^2} = 0
\]

and,

\[
\frac{d^2 X}{dx^2} + (\alpha - \beta) \frac{dX}{dx} + 2R \frac{\omega}{v} \frac{dx}{dx} + \left(\frac{\omega^2}{v^2} - H\right) X + 2\alpha \frac{\omega}{v} R + \frac{\omega}{v^2} = 0
\]

where, R, X, H, α, β are functions of x.

Correspondingly, the boundary conditions as shown in equations (3.46) and (3.47) are modified respectively as:

at x = -X_1

\[
\frac{dR}{dx} + \frac{\omega X}{v_{sn}} = -\frac{1}{v_{sn} \varepsilon}
\]

\[
\frac{dX}{dx} - \frac{\omega R}{v_{sn}} = 0
\]

(3.50)

and, at x = X_2

\[
\frac{dR}{dx} - \frac{\omega X}{v_{sp}} = \frac{1}{v_{sp} \varepsilon}
\]

\[
\frac{dX}{dx} + \frac{\omega R}{v_{sp}} = 0
\]

(3.51)

Now, the problem reduces to the solution of equations (3.48) and (3.49) subject to the boundary conditions (3.50) and (3.51). The 'modified Runge Kutta method [3.6] is adopted for solving these two second order differential equations simultaneously.

The equations (3.48) and (3.49) can be expressed in the simple form as:

\[
\frac{d^2 R}{dx^2} = f_1 (x, R, X, \frac{dR}{dx}, \frac{dX}{dx})
\]

\[
\frac{d^2 X}{dx^2} = f_2 (x, R, X, \frac{dR}{dx}, \frac{dX}{dx})
\]

(3.52)

Let \( \frac{dR}{dx} = Y \), and \( \frac{dX}{dx} = U \), then equation (3.52) becomes
\[ \frac{dy}{dx} = f_1(x, R, X, Y, U) \]
\[ \frac{du}{dx} = f_2(x, R, X, Y, U) \quad (3.53) \]

For an increment of \( \delta x \) in \( x \), the quantities \( R, X, Y, U \) will undergo changes by increments \( a, b, c \) and \( d \), respectively. Separate expressions for these increments are as follows:

The increment ‘\( a \)’ for \( R \) will be:
\[ a = \frac{1}{6} (a_1 + 2a_2 + 2a_3 + a_4) \quad (3.54) \]
\[ a_1 = Y \delta x, \quad a_2 = (Y + \frac{a_1}{2}) \delta x, \quad a_3 = (Y + \frac{a_2}{2}) \delta x, \quad a_4 = (Y + c_3) \delta x \]

The increment ‘\( b \)’ for \( X \) will similarly be:
\[ b = \frac{1}{6} (b_1 + 2b_2 + 2b_3 + b_4) \quad (3.55) \]
\[ b_1 = U \delta x, \quad b_2 = (U + \frac{d_1}{2}) \delta x, \quad b_3 = (U + \frac{d_2}{2}) \delta x, \quad b_4 = (U + d_3) \delta x \]

The increment ‘\( c \)’ for \( Y \) will similarly be:
\[ c = \frac{1}{6} (c_1 + 2c_2 + 2c_3 + c_4) \quad (3.56) \]
\[ c_1 = f_1(x, R, X, Y, U) \delta x, \quad c_2 = f_1(x + \delta x/2, R + a_1/2, X + b_1/2, Y + c_1/2, U + d_1/2) \delta x, \]
\[ c_3 = f_1(x + \delta x/2, R + a_2/2, X + b_2/2, Y + c_2/2, U + d_2/2) \delta x, \]
\[ c_4 = f_1(x + \delta x/2, R + a_3, X + b_3, Y + c_3, U + d_3) \delta x. \]

and, increment ‘\( d \)’ for \( U \) is taken as:
\[ d = \frac{1}{6} (d_1 + 2d_2 + 2d_3 + d_4) \quad (3.57) \]
\[ d_1 = f_2(x, R, X, Y, U) \delta x, \quad d_2 = f_2(x + \delta x/2, R + a_1/2, X + b_1/2, Y + c_1/2, U + d_1/2) \delta x, \]
\[ d_3 = f_2(x + \delta x/2, R + a_2/2, X + b_2/2, Y + c_2/2, U + d_2/2) \delta x, \]
\[ d_4 = f_2(x + \delta x, R + a_3, X + b_3, Y + c_3, U + d_3) \delta x. \]

In order to solve equation (3.48) and (3.49) the values of \( R \) and \( X \) are chosen arbitrarily (these are \( R_0 \) and \( X_0 \)) at \( x = -x_1 \), i.e. at the left edge of the depletion region. With these values of \( R \) and \( X \), the values of ‘\( Y \) (dR/dx)’ and ‘\( U \) (dX/dx)’ are computed from the left boundary conditions, as shown in equation (3.50). With the knowledge of \( R, X, Y, U \) at \( x = -x_1 \), one can compute \( f_1 = \frac{dy}{dx}, \quad f_2 = \frac{du}{dx}, \quad c_1 = f_1 \delta x, \) and \( d_1 = f_2 \delta x \). From the values of \( f_1, f_2, c_1 \) and \( d_1 \), the increments ‘\( a \)’, ‘\( b \)’, ‘\( c \)’, and ‘\( d \)’ respectively for \( R, X, Y, \) and \( U \) are computed through several steps. The values of \( R, X, Y, \) and \( U \) for the next step are obtained as:
\[ R|_{x = -x_1 + \delta x} = R|_{x = -x_1} + a \]
\[ X|_{x = -x_1 + \delta x} = X|_{x = -x_1} + b \]
Chapter 3: Simulation Methods for Designing MM-Wave and Terahertz IMPATTs

\[ Y|_{x = -x_1 + s \delta x} = Y|_{x = -x_1} + c \]
\[ U|_{x = -x_1 + s \delta x} = U|_{x = -x_1} + d \] (3.58)

This process continues till the right edge \((x = x_2)\) of the active layer is reached. Now the boundary values of \(Y\) and \(U\) at \(x = x_2\) are computed. If these do not satisfy the boundary conditions stated in equations (3.51), further iteration is carried out by suitably adjusting the initial values of \(R\) and \(X\), i.e. \(R_0\) and \(X_0\) at \(x = -x_1\). This process is repeated till the boundary conditions are satisfied at the right edge of the depletion region. A very fast logic is framed to ensure speedy convergence to the boundary values.

When the small-signal numerical solution program is in progress, there are 4 ways to iterate over initial values of \(R_0\) and \(X_0\): both are increased or both are decreased or one is increased and other is decreased. One of these four logics will lead to the desired convergence. It is important to note that the convergence logic is not unique and is found to be different for different structures, frequencies and different base semiconductor materials. The program is framed in such a way that it shifts automatically from one logic to another when any divergence is sensed. This switching over from a particular logic to another continues till convergence track is achieved by the simulation program.

Once the boundary conditions are satisfied, the \(R(x)\) and \(X(x)\) profiles can be easily obtained. The numerical integration of \(R(x)\) and \(X(x)\) profiles over the depletion layer give the total negative resistance \((Z_R)\) and \((Z_X)\) of the diode:

\[ Z_R = \int_{-x_1}^{x_2} R \, dx \quad \text{and} \quad Z_X = \int_{-x_1}^{x_2} X \, dx \] (3.59)

The diode impedance \(Z_D\) at a particular frequency, \(\omega\), is given by:

\[ Z_D(\omega) = \int_{-x_1}^{x_2} Z(x, \omega) \, dx = Z_R + j \, Z_X \] (3.60)

The admittance of the diode is given by:

\[ Y_D = Z^{-1} = G_D + j \, B_D = \frac{1}{(Z_R + j \, Z_X)} \] (3.61)

or, \(G_D = \left| \frac{Z_R}{(Z_R + j \, Z_X)} \right| \quad \text{and} \quad B_D = \left| -\frac{Z_X}{(Z_R + j \, Z_X)} \right| \) (3.62)

The values of \(Z_R, Z_X, Y_D, G_D\) and \(B_D\) for the diode at a particular frequency are thus obtained.

The computer algorithm for small-signal analysis is shown in Appendix 2.

Following are the important parameters which provide insight about the performance of the device and can easily be obtained from the above discussed small-signal analysis.

(a) Avalanche Resonance Frequency \((f_a)\):

The avalanche frequency \((f_a)\) is the frequency at which the imaginary part \((B)\) of the admittance, i.e. device susceptance, changes its nature from inductive to capacitive. Again it is the minimum frequency at which the real part \((G)\) of admittance becomes negative. At the avalanche frequency
oscillation starts to build up in the circuit. For an ideal Read diode, according to Gilden and Hines [2.59], [2.63], \( f_s \) is given by:

\[
f_s = \frac{1}{2 \pi \sqrt{\frac{2 \alpha' v_s}{e}}}
\]

(3.63)

where, \( \alpha' = \frac{d\alpha}{dE} \), \( \alpha = \alpha \) (for electron) \( = \beta \) (for holes) and \( v_s = v_{m} = v_{tp} \)

It is observed that the IMPATT operation is strongly current dependent and the resonance frequency is a function of the operating bias current density. As the current density increases, the optimum frequency increases and so does the diode output power. The current density can be extended until space-charge effects cause power saturation and efficiency reduction. For CW diodes the maximum current density is limited thermally, but for pulsed diodes this limit is extended many times depending on the pulse-width and duty factor.

It is important to mention here that the computer program adopted in the present thesis, for modeling and analyzing the small-signal behavior of high-frequency IMPATT devices, is free from simplifying assumptions of Gilden and Hines. In the present generalized simulation technique, the value of \( f_s \) at any DC current density is obtained from the admittance plot of the diode corresponds to the frequency at which \( B \) changes its sign from negative to positive.

(b) Quality Factor (Q):

The most useful small-signal parameter for predicting the oscillator performance is the 'Q-factor' of the device. The Q-factor is defined as the ratio of time average of the ac field energy \( <E_d> \) stored in the device to the ac power dissipation \( <\frac{dE_d}{dt}> \), times the angular frequency.

\[
Q = \frac{\omega <E_d>}{- <\frac{dE_d}{dt}>} = - \frac{B_D}{G_D}
\]

(3.64)

The 'Q-factor' of the diode is negative wherever it posses negative resistance. For a stable oscillation, 'Q-factor' of the circuit should be equal to the magnitude of diode Q. When the magnitude of the diode 'Q-factor' is smaller than the circuit Q, oscillation starts to build up and diode Q starts decreasing as the oscillation amplitude increases, until the steady state condition is reached. As soon as the device negative conductance reaches its peak, the device negative Q reaches its minimum value.

(c) Growth Rate Factor (g):

The rate of growth of oscillation is determined by the 'Q-factor' and is given by:

\[
g = \left| \frac{1}{2Q} \right|
\]
It is large when the magnitude of diode ‘Q-factor’ is small. Thus a small value of diode Q-factor produces a rapid build-up of the oscillation. If the oscillation growth rate at the small-signal level is found to be large, it is to be expected that the oscillation will build up to larger amplitude and that in turn will increase the oscillator efficiency. Thus a small magnitude of the ‘Q-factor’ is an indication of a high-efficiency oscillator. As the ‘Q-factor’ is defined under steady-state conditions, it may not accurately predict the transient behavior of the device. However, in practice it’ll be a useful guide in predicting the high-frequency characteristics and operating conditions for different diode design.

(d) **Peak Operating Frequency ($f_p$)**

At a given bias current density, the peak frequency ($f_p$) is the frequency at which the magnitude of negative conductance of the diode is a maximum and the ‘Q-factor’ is a minimum. Thus the value of $f_p$ is obtained from the admittance characteristics of the device at a particular current density when the magnitude of diode negative conductance is a maximum.

(e) **Output Power Density:**

The maximum output power density is estimated from the following equation:

$$P_{max} = \frac{(V_{RF}^2 |G_p| )}{2}$$

where, $G_p$ is the diode negative conductance at peak operating frequency as obtained from the admittance characteristics. $V_{RF}$ (amplitude of the RF swing) is taken as $V_B/2$, assuming 50% modulation of the breakdown voltage $V_B$. However, a more precise estimate of the power capability of IMPATT diode requires full consideration of the voltage swing, the thermal limitations and the device-circuit interaction.

(f) **Positive Series Resistance:**

Series resistance ($R_S$) is a crucial parameter that limits power dissipation and causes burn-out problem in IMPATTs. From the device side, the parasitic series resistance consists of the sum of the substrate resistance and metallic contact resistance. The exact value of $R_S$ is difficult to estimate as it varies with doping profile, the width of the depletion layer, current density, contact technology used and the chip mounting conditions i.e. the load conductance of the circuit [2.51]. The measurement of $R_S$ by a network analyzer is difficult, owing to circuit modeling difficulties and network analyzer error [3.7 -3.8]. The value of $R_S$ also depends on the load conductance of the circuit.

The estimation of $R_S$ is thus extremely important particularly for high-frequency (Terahertz) IMPATTs, since the negative resistance become very small at Terahertz frequencies. Adlerstein et al.
[3.9] proposed a simple technique of determining $R_s$ for DDR IMPATTs from the measurement of the threshold oscillation frequency assuming equal ionization rates and drift velocities for electrons and holes. In 1983, Mitra et al. [3.10] calculated $R_s$ for SDR IMPATT diode in the X-band with an assumption on unequal ionization rates and drift velocities of charge carriers. In this dissertation the author has estimated the values of $R_s$ (baring the contribution of contact resistances) from the high-frequency admittance characteristics using the realistic analysis of Gummel-Blue [2.63] and Adlerstein et al [3.9] without any drastic assumption. The diode negative conductance $-G_D$ and susceptance $B_D$ can be calculated from equation (3.62). $G_D$ and $B_D$ are function of RF voltage, $V_{RF}$ and frequency such that the steady state condition for oscillation is given by [3.9]:

$$G_L(\omega) = -G_D(\omega) - (B_D(\omega))^2 R_s(\omega)$$

Under the small-signal condition, $V_{RF}$ is small and $R_s$ can be calculated by considering the value of load conductance $G_L$, as nearly equal to the diode conductance $G_D$, at resonance. The relation provides minimum uncertainty in $G_L(\omega)$ at low power oscillation threshold. The equivalent circuit for an IMPATT oscillator at resonance is shown in Figure 3.1. At resonance, the reactance of the resonant cavity is mainly the capacitive reactance of the diode and power is absorbed in the load conductance $G_L$ when $|G_D| = G_L$. Once the value of $R_s$ is estimated in this way, the p- and n-type ohmic contact resistances are added to the value of $R_s$. This total value of series resistance is called $R_{s,\text{total}}$. The effect of $R_{s,\text{total}}$ is considered in determining the values of $G_D$ and $P_{\text{max}}$ for the THz IMPATT diodes.

It is to be noted that the small-signal analysis describes quite well the characteristics of the diode when operated as a small-signal amplifier. However, because of the approximation that the a.c. signal level is small, this analysis specifically excludes the large-signal oscillator case. However, it should be mentioned that the small-signal analysis is quite a useful guide in predicting the properties of the oscillator.
3.4 Simulation Scheme for Studying Optical Illumination Effects on IMPATT Diodes:

Several experimental and theoretical investigations have shown that the leakage current or reverse saturation current flowing in a reversed biased p-n junction below breakdown has a very important role in determining the power output and frequency of oscillations in the IMPATT mode of operation. The saturation current mainly depends on thermally generated carriers and the stored minority carriers near the substrate. The basic process involved in the experiment of optical illumination on an IMPATT diode is that, when optical radiation with a photon energy \((E = hv, \nu = \text{frequency of incident radiation})\) greater than the bandgap of the semiconductor, is absorbed at the edges of the reversed biased p-n junction of an IMPATT diode, photo-excitation of electron-hole pairs occurs within the active region of the diode. These photo-generated carriers give rise to photocurrent and thereby enhance the existing thermal leakage current in the IMPATT diode. The time required for the building up of avalanche current depends on the magnitude of the reverse saturation current. Increase of reverse saturation current due to optical illumination leads to a decrease of the build-up time of avalanche current, i.e. alters the avalanche phase.
delay which, in turn, modifies the phase and magnitude of the terminal current in the device oscillator circuit. A modified simulation scheme has been used to study the effects of optical illumination on the IMPATT diodes.

In an un-illuminated IMPATT, the current multiplication factors due to thermally-generated electrons and holes are given by:

\[ M_n = \frac{J}{(J_{ns})_{th}} \quad \text{and} \quad M_p = \frac{J}{(J_{ps})_{th}} \]  

(3.67)

where, \( J \) is the bias current density and \( (J_{ns})_{th} \) and \( (J_{ps})_{th} \) are leakage current density due to thermally generated electrons and holes respectively. The values of \( M_n \) and \( M_p \) are very large for the un-illuminated IMPATT, since, \( (J_{ns})_{th} \) and \( (J_{ps})_{th} \) are very small compared to \( J \). The accepted value of \( 10^6 \) is used for both \( M_n \) and \( M_p \) in the simulation [3.11].

The current multiplication factors under optical illumination are given by:

\[ M_n' = \frac{J}{(J_{ns})_{th} + (J_{ns})_{opt}} \quad \text{and} \quad M_p' = \frac{J}{(J_{ps})_{th} + (J_{ps})_{opt}} \]  

(3.68)

where, \( (J_{ns})_{opt} \) and \( (J_{ps})_{opt} \) are saturation current densities due to photo-generated electrons and holes, respectively, which depend on the incident optical power according to the following relation [2.64]:

\[ (J_{ns} \text{ or } J_{ps})_{opt} = q \eta \frac{P_{opt}}{A h \nu} \]  

(3.69)

where, \( \eta \) is the quantum efficiency and \( A \) is the surface area over which absorption of incident optical power, \( P_{opt} \) takes place corresponding to photon energy \( h \nu \). If recombination is neglected, a linear response of avalanche breakdown can be assumed, where, \( (J_{ns})_{opt} \) or \( (J_{ps})_{opt} \) would increase linearly with \( P_{opt} \) over a particular range of wavelengths in which appreciable absorption takes place.

The multiplication factors, \( M_n \) and \( M_p \) can be related to \( P_{opt} \) and thereby \( (J_{ns})_{opt} \) and \( (J_{ps})_{opt} \) by the following simple relations:

\[ M_n' = \frac{J}{(J_{ns})_{opt}} \quad \text{and} \quad M_p' = \frac{J}{(J_{ps})_{opt}} \]  

(3.70)

Thermally-generated leakage currents are neglected in the above relations due to their relative insignificance compared with the optically generated leakage current. The values of \( M_n \) and \( M_p \) decrease from \( 10^6 \) (un-illuminated diode) to \( \ll 10^6 \) (illuminated diode).

The effect of shining light from the p++ side in TM (Top Mounted) SDR and DDR IMPATT diodes (Figure 2.10 (b) and Figure 3.2 (a)) are to generate an electron-dominated photocurrent that changes the expression for the electron current multiplication factor \( M_n \) to much smaller values, while the
value of $M_P$ remains unchanged at $10^6$. Similarly, the effect of shining light from the substrate side ($n^{++}$ edge) in a FC (Flip Chip) SDR and DDR IMPATT structures (Figure 2.10 (a) and Figure 3.2 (b)) are to generate a hole dominated photo current that reduces the expression for hole current multiplication factor to much smaller values, keeping the value of $M_n$ unchanged at $10^6$.

Computer simulation of the DC and high-frequency properties of IMPATT diodes of various doping profiles are carried out for the following three cases:

1. Un-illuminated diode for which $M_n$ and $M_P$ are both large $\sim 10^6$.
2. Illuminated TM diode, whose junction is at the top and substrate is at the bottom so that $M_n$ can have finite values $\ll 10^6$ but $M_P = 10^6$.
3. Illuminated FC diode whose substrate is at the top and junction is at the bottom so that $M_P$ can have finite values $\ll 10^6$ but $M_n = 10^6$.

Using a double-iterative field-maximum computer method, as described in previous sections of the dissertation, DC and high-frequency properties of the diodes corresponding to above three configurations are obtained. The boundary condition for the electric field for case (1) was shown in equations (3.20). The boundary conditions for the normalized current density, $P(x)$, for case (1) are shown in 3.19(a), 3.19 (b) and re-written below:

$$P(x) = \frac{(J_p - J_n)}{J} \left|_{x=x_1} \right. = (\frac{2}{M_p} - 1) = -1 \quad \text{and} \quad P(x) = \frac{(J_p - J_n)}{J} \left|_{x=x_2} \right. = (1 - \frac{2}{M_n}) = 1.$$

This is due to the fact that under un-illuminated condition, $M_n$ and $M_P \to \infty$.

The modified boundary conditions for $E(x)$ and $P(x)$ at the edges of the depletion layer for case (2) and (3) are shown below:

$$E(-x_1) = 0, \quad \text{and} \quad E(x_2) = 0,$$

where $-x_1$ and $x_2$ are the two edges of the depletion region as shown in Figures 2.2(a-b).

For case (2),

$$P(-x_1) = \frac{(J_p - J_n)}{J} \left|_{x=x_1} \right. = \left(\frac{2}{M_p} - 1\right) = -1 \quad \text{and} \quad P(x_2) = \frac{(J_p - J_n)}{J} \left|_{x=x_2} \right. = \left(1 - \frac{2}{M_n}\right) = 1 \quad \text{(3.71)}$$

and for case (3),

$$P(-x_1) = \frac{(J_p - J_n)}{J} \left|_{x=x_1} \right. = \left(\frac{2}{M_p} - 1\right) = -1 \quad \text{and} \quad P(x_2) = \frac{(J_p - J_n)}{J} \left|_{x=x_2} \right. = \left(1 - \frac{2}{M_n}\right) = 1 \quad \text{(3.72)}$$

Thus the increase of leakage current due to photo-generated electrons and holes is manifested as the lowering of $M_n$ and $M_P$ from a high value ($10^6$), corresponding to TM and FC illumination configurations, respectively. Simulation experiments are carried out on the effect of $M_n$ (keeping $M_P$ very high $\sim 10^6$) and $M_P$ (keeping $M_n$ very high $\sim 10^6$) on (i) the small-signal admittance characteristics, (ii) the
negative resistivity profiles (iii) negative resistance of the diodes, (iv) power output and (v) ‘Q-factor’ of the designed IMPATT devices.

3.5 Validity of the DC and Small-Signal Simulation Methods:

In the simulation scheme for modeling and analysis of SiC, GaN and InP based IMPATTs, one dimensional p-n junction diode equations (Poisson and current continuity equations), considering the mobile space-charge effect, have been solved by a double iterative computer method satisfying appropriate boundary conditions as described earlier [2.79][3.5]. To check the validity of the adopted modeling scheme, the author has first designed DDR (p++ p n n++) IMPATT diode based on conventional Si for the operation at 94 GHz and 140 GHz MM-wave window frequencies and compared their performances with fabricated IMPATTs at the same operating frequencies [2.8][3.12]-[3.13]. The design parameters, such as, background doping concentration, epilayer widths and bias current densities, of these diodes are taken almost similar to that of the fabricated diodes. The DC and the small-signal results of the simulated CW IMPATT diodes at 94.0 GHz and 140 GHz are shown in Table 3.1 and Table 3.2, respectively. In order to make a comparison, the experimentally observed data are also shown in the same tables. From Table 3.1, it is observed that the simulated diode breakdown at 18V and the fabricated diode [2.8], under similar operating condition, breakdowns at 16.5V. Conversion efficiency is found to be 8.5% in case of simulated diode; while that is 8% [3.12] and 6.7% [2.8] in case of fabricated devices. Table 3.2 also reveals that the breakdown voltage in case of simulated diode at 140 GHz is 16.0V, while, it is 14.0V in case of the fabricated diode. Similarly, the conversion efficiency is 7% and 6.5%, in case of the simulated and the fabricated diodes, respectively. Thus, close agreement between the simulated results and the experimental observations are found, as far as the breakdown voltages and the efficiencies are concerned. The CW power from the designed diode is further compared with the output power level from the fabricated diodes. It is found that the small-signal analysis predicts a maximum output power of 1.05W from the designed diode at 96.0 GHz (W-band), while the actual (experimental) power output from the designed diodes are within 500 mW - 600mW [2.8][3.12]. The maximum output power that can be obtained from the simulated diode at 140 GHz is ~ 730 mW, while the fabricated device [3.13] yields an output power of 225 mW. Thus the simulated power levels are higher than those are found experimentally. However, it should be mentioned here that the reported power output from the designed diodes are the maximum power that the device can generate in an ideal case, i.e. excluding the power limitation effects due to the presence of parasitics in the realistic cases. Moreover, the discrepancy between the theoretical and the experimental results may be due to the adopted small-signal...
approach, thermal limitations, impedance matching problem, and improper heat-sink arrangement. The admittance plots of the designed diodes at 94 GHz and 140 GHz are shown in Figure 3.3.

Table 3.1:
Si based DDR IMPATT for CW operation at 94.0 GHz window frequency: Comparison of simulation results with experimental published data.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Simulation conditions &amp; Results</th>
<th>Experimental conditions &amp; Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating frequency band (CW mode)</td>
<td>W-band (Experiments by Dalle et. al. (Ref: [3.12]))</td>
<td>W-band (Experiments by Luy et. al. (Ref: [2.8]))</td>
</tr>
<tr>
<td>$N_n (10^{23} m^{-3})$</td>
<td>1.4</td>
<td>1.5</td>
</tr>
<tr>
<td>$N_d (10^{23} m^{-3})$</td>
<td>1.4</td>
<td>1.5</td>
</tr>
<tr>
<td>$J_0 (A m^{-2})$</td>
<td>$4 \times 10^8$</td>
<td>$4.5 \times 10^8$</td>
</tr>
<tr>
<td>$W_n (\mu m)$</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>$W_p (\mu m)$</td>
<td>0.28</td>
<td>0.25</td>
</tr>
<tr>
<td>$A (m^2)$</td>
<td>$10^9$</td>
<td>$10^9$ (35$\mu$m diameter)</td>
</tr>
<tr>
<td>$\eta (%)$</td>
<td>8.5%</td>
<td>8%</td>
</tr>
<tr>
<td>$P_{max}$</td>
<td>1.09 W</td>
<td>&gt; 500 mW</td>
</tr>
</tbody>
</table>

Table 3.2
Si based DDR IMPATT for CW operation at 140.0 GHz window frequency: Comparison of simulation results with experimental published data.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Simulation conditions &amp; Results</th>
<th>Experimental conditions &amp; Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating frequency band (CW mode)</td>
<td>D-band ($f_p = 140$ GHz) (Experiments by Wollitzer et. al. (Ref: [3.13]))</td>
<td>D-band ($f_p = 140$ GHz)</td>
</tr>
<tr>
<td>$N_n (10^{23} m^{-3})$</td>
<td>2.5</td>
<td>3.0</td>
</tr>
<tr>
<td>$N_d (10^{23} m^{-3})$</td>
<td>3.0</td>
<td>4.0</td>
</tr>
<tr>
<td>$J_0 (A m^{-2})$</td>
<td>$7 \times 10^8$</td>
<td>$6 \times 10^8$</td>
</tr>
<tr>
<td>$W_n (\mu m)$</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>$W_p (\mu m)$</td>
<td>0.20</td>
<td>0.17</td>
</tr>
<tr>
<td>$A (m^2)$</td>
<td>$3.8 \times 10^{-10} (22 \mu m$ diameter)</td>
<td>$3.8 \times 10^{-10} (22 \mu m$ diameter)</td>
</tr>
<tr>
<td>$\eta (%)$</td>
<td>7.0%</td>
<td>6.5%</td>
</tr>
<tr>
<td>$P_{max}$</td>
<td>730 mW</td>
<td>225 mW</td>
</tr>
</tbody>
</table>
Chapter 3: Simulation methods for designing mm-wave and terahertz IMPATTs

External Radiation from Laser

Optical window

Figure 3.2 (a): Schematic diagram of Top Mounted DDR IMPATT diode under optical-illumination.

External Radiation from Laser

Optical window

Figure 3.2 (b): Schematic diagram of Flip Chip DDR IMPATT diode under optical-illumination.
CHAPTER 3: SIMULATION METHODS FOR DESIGNING MM-WAVE AND TERAHERTZ IMPATTs

Figure 3.3: Admittance plots of Si DDR IMPATTs at (a) W-band and (b) D-band.

3.6 Summary:

A double iterative computer simulation method for obtaining electric field and normalized current density profiles of various IMPATT structures in the MM-wave and Terahertz region are presented in this Chapter. The computation for static analysis (DC analysis) starts from the field maximum position near the metallurgical junction and iterates over the values of field maximum and its location in the depletion region. This method takes into account the effect of mobile space-charge in the depletion region and incorporates the realistic doping profiles in the form of exponential and complementary error function, realistic field and temperature dependence of carrier ionization rates, saturated drift velocities and mobilities of charge carriers. This computer method is also used for studying the optical-illumination effects on the IMPATT diodes. The modified boundary conditions for studying optical-illumination effects are described in this Chapter. The method is simple, generalized, accurate and free from any numerical instability as the boundary conditions at the edges of the depletion layer are easily satisfied by initiating the computation from a pre-determined field maximum at the metallurgical junction. The DC data are used as input data for the small-signal analysis following the Gummel-Blue approach. The small-
signal approach involves no simplifying assumptions and is much generalized. The analysis is very much sensitive to the initial choice of R and X at the left edge of the depletion region and the logic of iterations varies widely with diode structures and frequencies. However, a very fast converging logic is framed and incorporated in the computer program. The small-signal diode admittance characteristics, as well as the spatial distribution of negative resistivity in the depletion region of the diodes are obtained through this analysis. The admittance characteristics provide knowledge of the total MM-wave and Terahertz frequency performance of the device. Spatial variation of negative resistivity provides considerable physical insight into the regions in the avalanche and drift layers of the device which contribute maximum to the output power generation.

The estimation of parasitic series resistance is also presented in this chapter. The effect of parasitic resistance is incorporated in the estimation of power output from the devices. It should be mentioned in the conclusion that the large-signal simulation of IMPATTs, incorporating the device-circuit interaction, may provide an improved quantitative idea regarding the influence of optical illumination on the frequency chirping and power output, but qualitatively, there will be no change in the results as obtained from generalized small-signal simulation model. Moreover, the small-signal simulation program provides useful and interesting information regarding the intrinsic properties of the designed IMPATT diodes, which can be further used for the practical realization of the devices.
CHAPTER 3: Reply to 2nd Examiner's comment

Examiner's 2nd comment:

a. Author claims that shining the bandgap light on IMPATT reduces the avalanche multiplication factor \( M_n \) or \( M_p \) depending on the configuration. However she does not provide any physical description on how this reduction occurs. \( M_n \) or \( M_p \) are defined under dark and lighted condition on page 19 of Chapter 3. In this definition value of \( J_{\text{total}} \) is kept fixed for both dark and light condition. No explanation is provided for this. A discussion of this issue is important. Author may refer to literature on this topic.

Reply:

The physical implication of optical-illumination on IMPATT devices is given first in Chapter 2, page 48. There it is clearly mentioned that above bandgap illumination results in creation of excess electron-hole pairs in the device, which increases the reverse saturation current and decreases the multiplication factors, since \( M_n, p \propto \frac{1}{U_{ns, p/\text{opt}}} \). Thus increase of leakage current is manifested as decrease of \( M_n, p \). It is mentioned in the thesis (in Chapter 4, page 31, Chapter 6, page 49) that by varying the incident optical power, the leakage current has been increased from 1 nA to 500 µA, i.e. by a factor of \( 5 \times 10^5 \) (reference in the thesis: [4.29]). In this thesis, \( M_n \) or \( M_p \) has been reduced from (i) \( 10^6 \) to \( 10^2 \), i.e. by a factor of \( 10^4 \), (ii) \( 10^6 \) to 50, i.e. by a factor of \( 2 \times 10^4 \), (iii) \( 10^6 \) to 25, i.e. by a factor of \( 4 \times 10^4 \), assuming an increase of leakage current by the same factors, as has been experimentally demonstrated in real devices (reference in the thesis: [4.29]).

- **Reason for keeping \( J \) fixed for both dark and light conditions**

It is mentioned in the thesis (Chapter 3, page 19) that \( J \) (or \( J_n, J_p \)) is the applied dc bias current density which is kept constant at the operating frequency for direct comparison between un-illuminated and illuminated devices. The output power and frequency of oscillation of IMPATT diodes can be modulated by changing the bias current density, applied through the electrical terminal. Besides electrical control, the device properties can also be modulated optically by controlling the intensity of incident optical radiation. As discussed in the thesis, the change of the illumination level modulates device performance by controlling the reverse saturation current in the devices. Since the modulation of IMPATT characteristics by optical means is reported in the thesis, thus the electrical bias conditions have been kept fixed by keeping \( J \), bias current density, constant.

A simple mathematical calculation proves that under steady-state condition the total current \( I \), throughout the p-n junction remains constant and is related to \( M_n \) and \( M_p \) by the relation, \( I = M_n I_n \) and \( I = M_p I_p \), where \( I_n \) and \( I_p \) are reverse saturation current ("Physics of Semiconductor Devices" by Sze and K K Ng, 3rd Edition, page 105). The following diagrams (reference in the thesis: [1.35]) show that optical
illumination only influences the reverse saturation current but not the bias current which is constant under both illuminated and un-illuminated conditions.

Figure (a): Current density profiles in active layer of a reverse biased FC IMPATT diode with light (-----) and without light (------).

Figure (b): Current density profiles in active layer of a reverse biased TM IMPATT diode with light (-----) and without light (------).
3.7 Bibliography:


