Chapter V

Design of Soft Computing Models for Noise Pollution
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FOR NOISE POLLUTION

5.1 Introduction

Within the last three decades, concern about the protection of the environment has grown rapidly as it has been recognized that steady rise in pollution of all kinds cannot be allowed to continue indefinitely. The acoustic environment has likewise suffered from the increase in use and power of the machines in the workplace, increasing road traffic, larger aircrafts etc. To combat this, many countries have introduced legislation making it a legal requirement to measure noise levels to reduce noise from vehicles at the source and maintain acceptable noise levels in factories to prevent hearing loss. India has emerged as a fast developing country resulting in an increase in activity of the workforce. In 1989, Central Pollution Control Board (CPCB) promulgated the Ambient Air Quality Standards for Noise, thereby establishing the noise limits for residential, commercial, industrial, and silence zone areas. For assessing the urban noise problem and suggesting the mitigation measure, a fresh look is required for environmental noise management as well it is necessary to identify levels required to protect public health and welfare [246—248].

As discussed in Chapter 2, high-level noise not only hinders communication between human beings, but, depending upon the level and exposure duration of the noise, it may also result in different type of physical, physiological and psychological effects on the human beings, especially hearing loss. Most of the studies published in the Journals, Proceedings, and Reports of WHO and other concerned agencies of the world had identified mainly six important effects of noise pollution on human beings. These are: (1) performance, (2) hearing loss, (3) annoyance, (4) health, (5) sleep, and (6) speech interference.

The studies published so far were mainly based on the data collection. After the data are collected, statistical description of the results helps understanding of the basic characteristics of the effects of noise on human beings [249]. Furthermore, statistical inference allows one to generalise the observations derived from a sample to a wider
population from which the sample was drawn. The problem with this approach is that it limits the complexity of decision-making framework since most people cannot intuitively analyse more than a few variables at one time. As a result, this approach often fails when one deals with complex system for which intuitive decision are overly simplistic. Since, the effects of noise on human beings are of complex and non-linear type, therefore, traditional quantitative techniques of systems modelling and analysis are not test worthy to deal with such systems. Moreover, in most cases, it is quite difficult to adequately describe the behaviour of a non-linear system by mathematical models, especially when the structure of the system is unknown. Even if one knows the structure, numerical model representations usually become irrelevant and computationally inefficient as the complexity grows. After all, there are a lot of uncertainties, unpredictable dynamics, mutual interactions, and other unknown phenomena in noise pollution and its effects on human beings that cannot be mathematically modelled and analysed at all [250].

As discussed in Chapter 4, more flexible and effective tools for handling and processing uncertainties in complicated and ill-defined systems are now available with the advent of soft computing techniques. With statistical techniques, the research findings of the effects of noise pollution were analysed for single input variable. In real environment, the effects of noise pollution are multi dimensional i.e. each effect is dependent on many input variables. Hence with developments in soft computing, one can consider multidimensional effects of noise pollution to make the study more appropriate and realistic. However, the scientists and engineers working in this field have not tried much in this direction.

5.2 Fuzzy Modelling of Noise Pollution

Though the term “fuzzy modelling” has not been used so often, fuzzy modelling is the most important issue in fuzzy logic or more widely in fuzzy theory. In fact we can find the seminal idea of fuzzy modelling in the early papers of Zadeh. There are many interpretations of fuzzy modelling. For instance we can consider a fuzzy set as a fuzzy model of a human concept. In this study we simply understand the fuzzy modelling to be an approach to form a system model using a description language based on fuzzy logic with fuzzy predicates. The detailed description about fuzzy modelling can be found in [227]. The most remarkable
paper related to fuzzy modelling is his paper of 1973 [225] on linguistic analysis, where he states “the principle of incompatibility,” according to which “as the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance become almost mutually exclusive characteristics. It is in this sense that precise quantitative analyses of the behaviour of humanistic systems are not likely to have much relevance to the real world societal, political, economic, and other types of problems which involve humans either as individuals or in groups.”

Based on the above considerations, Zadeh suggested linguistic analysis in place of quantitative analysis. As the main characteristics of this approach, he suggests “(1) use of so called linguistic variables in place of or in addition to numerical variables; (2) characterization of simple relations between variables by conditional fuzzy statements using IF-THEN fuzzy rules; and (3) characterisation of complex relations by fuzzy algorithms.

In the most general form, the encoded knowledge of a multi-input and multi-output (MIMO) system can be interpreted by fuzzy models consisting of IF-THEN rules with multi-antecedent and multi-consequent variables (with r antecedents, s consequents, and n rules) as expressed by equation (4.46). Conceptually, a system with multiple independent outputs can be considered as several groups of single output systems, separately. Consequently, the general rule structure of a MIMO fuzzy system can also be presented as a collection of multi-input single-output (MISO) such that for a system with s output, each multi-consequent rule is broken into s single-consequent rules. Although the number of rules in the new fuzzy system will be increased, modelling and inference would be more straightforward for MISO fuzzy systems. That is the reason why the literature concentrates on MISO rules as a generic representation of fuzzy systems. The model under consideration is of MISO type associated with fair amount of uncertainties. The architecture and fuzzy modelling of the system is described in ensuing sub-sections [250].

5.2.1 Input-Output Relationships of the Noise Pollution Model

The information systems have an important role to play in environmental remediation efforts. Problems in the environmental area are not suited to conventional data processing.
This is due to the lack of information that exists in such situations. Often it is difficult to precisely describe input parameters, which are responsible for the ill effects of noise pollution on human beings. A model for noise pollution with inputs and outputs as a system is shown in Figure 5.1. In the figure, we have selected only seven inputs that affect the outputs to a large extent. Although there are other factors, for example, gender, race and motor-sidedness etc., which may have some little contributions to affect the output parameters.

\[
\begin{align*}
\text{Noise Level} &= X_1 \\
\text{Exposure Time} &= X_2 \\
\text{Type of Task} &= X_3 \\
\text{Duration} &= X_4 \\
\text{Age} &= X_5 \\
\text{Status} &= X_6 \\
\text{Distance} &= X_7
\end{align*}
\]

\[
\begin{align*}
\rightarrow \text{Performance} &= Y_1 \\
\rightarrow \text{Hearing Loss} &= Y_2 \\
\rightarrow \text{Sleep} &= Y_3 \\
\rightarrow \text{Annoyance} &= Y_4 \\
\rightarrow \text{Health} &= Y_5 \\
\rightarrow \text{Speech Interference} &= Y_6
\end{align*}
\]

**Figure 5.1 Inputs and outputs of noise pollution model**

The hierarchical structure of noise pollution and its effect is shown in Figure 5.2. In this diagram, it has been shown that each output has only three inputs. However, there are other factors to be considered for the effects on human beings. The cause and effect relationship includes many rules and hence with all the inputs to be included, the system becomes very complex and hard to handle. Among these noise level and exposure time play the most important part because most of the studies have been conducted based on these two parameters to find the effects of noise pollution on human beings. With the simplified hierarchy shown in Figure 5.2, the logical rules and facts can be used to handle the system more easily and accurately. Because the cause effect relations may also involve some type of uncertainty, the logic to be utilized in this model must also handle the uncertainty issues. To satisfy this requirement, fuzzy logic is incorporated into this modelling approach, which is described in the next subsection.
5.2.2 System Variables

As with all modelling problems, the first step is to identify the relevant quantities whose interaction the model will specify. These quantities can be classified into input and output variables called system variables. The numbers of variables as well as the universe of discourse of each variable are dictated by the physical nature of the problem, but the number of linguistic terms for each variable needs to be chosen. Quite often the number of linguistic terms is known from experience about the system. Each linguistic term is defined by its interval. However, if this is not the case, five or seven linguistic terms would be adequate for most practical cases. For practical purposes a minimum of three linguistic terms is required, while using more than ten linguistic terms can be confusing in normal human contexts. It is also recommended that the number of terms should be odd, in order to represent the "middle" or "medium" state [251]. Therefore, a reasonable number of linguistic terms can
be 3, 5, 7, or 9. The user of the program can choose any number of linguistic terms from 3 to 9, but five is considered the most reasonable and is used by default (for example: very low, low, medium, high, very high). As discussed in the previous sections, six input and six output variables have been identified for our noise pollution model.

**Input Variables**

The input variables are basically the causes or stimuli, which produce the effects or response in a system. In our case the input data have been identified on the basis of literature survey. These input variables have been taken as the linguistic variables. The numbers of selected inputs for the present study are as follows:

1. Noise level \((X_1)\)
2. Exposure Time \((X_2)\)
3. Type of Task \((X_3)\)
4. Age \((X_4)\)
5. Status \((X_5)\)
6. Distance \((X_6)\)
7. Duration \((X_7)\)

**Output Variables**

The output variables are actual effects on human beings due to the input parameters. They have also identified on the basis of literature survey. In the present study, the output variables considered are:

1. Human Performance \((Y_1)\)
2. Hearing Loss \((Y_2)\)
3. Sleep \((Y_3)\)
4. Annoyance \((Y_4)\)
5. Health \((Y_5)\)
6. Speech Interference \((Y_6)\)

**5.2.3 Membership Functions of the Variables**

The concept of linguistic variable plays an important role in the applications of fuzzy logic. A linguistic variable is a variable whose values are words or sentences in a natural or synthetic language. Linguistic values are expressed in the form of fuzzy sets. A fuzzy set is usually defined by its membership functions. For most control applications, the sets that
have to be defined are easily identifiable. However, for other applications they have to be determined by knowledge acquisition from an expert or group of experts. Once the fuzzy sets have been established, one must consider their associated membership functions. The approach adopted for acquiring the shape of any particular membership function often depends on the application [252]. The membership values of a fuzzy set do not convey any absolute significance, but they only reflect an ordering of the elements in the domain with respect to a vague predicate or not well-defined concept, and this ordering is more important than the membership values by themselves [253]. For this reason, the normalized triangular fuzzy sets (NTFS) have been considered as the basic component in the proposed architecture. In addition, this choice has been determined by the simplicity of the definition of a NTFS and by the efficiency from the computational point of view [254].

5.3 IF-THEN Rules

Fuzzy rule based systems (FRBSs) have been successfully applied to a wide range of real world problems. In order to design an intelligent system of this kind, several task have to be performed. One of the most important and difficult ones is the derivation of the fuzzy rule base, which will contain the information needed to solve the problem in the form of fuzzy rules. There exist two different kinds of FRBSs in the literature, Mamdani and TKS models. In the first category, both antecedent and consequent parts of IF-THEN rules usually consist of vague predicates. In these models, fuzzy quantities are associated with linguistic labels and fuzzy model is essentially a qualitative expression of the system that uses natural language expressions. The second category of fuzzy models is formed by logical rules that have a fuzzy antecedent part and a functional consequent part; essentially they are a combination of fuzzy and nonfuzzy models. TSK fuzzy models have effective potential for expressing quantitative information and are computationally efficient. In fuzzy modelling, the fuzzy arithmetic is used for computation and “if-then rules” are used for inference together with fuzzy reasoning based on fuzzy logic. Fuzzy reasoning is interpreted as approximate reasoning based on vague or incomplete knowledge. In many applications of fuzzy rule-based systems, fuzzy if-then rules have been obtained from human experts. A number of techniques have been proposed in the literature for automatic generation of rules.
5.3.1 Mamdani Model

The ordinary fuzzy model based on Mamdani’s approach represents the dynamics of a system using a set of fuzzy linguistic propositions. In this model fuzzy IF-THEN rules can be easily understood by human beings and often an initial rule-base can be provided by an expert, there is no problem of readability. Hence, fuzzy rule-based models are to a certain degree transparent to interpretation and analysis. This FRBS has been widely used and it has obtained very good results in many different applications. Because of its simplicity Mamdani model is still most widely used for solving many real world problems. A typical rule is of the form:

IF antecedent THEN consequent

Depending on the form of the antecedent and consequent, this rule takes on forms ranging from very simple to complex. A fuzzy rule relates $m$ antecedent variables $X_1, X_2, \ldots, X_m$ to $n$ consequent variables, $Y_1, Y_2, \ldots, Y_n$ and has the form:

$$\text{IF } X_1 \text{ is } A_1 \text{ AND } X_2 \text{ is } A_2 \text{ AND } \ldots \text{ AND } X_m \text{ is } A_m$$

$$\text{THEN } Y_1 \text{ is } B_1 \text{ AND } Y_2 \text{ is } B_2 \text{ AND } \ldots \text{ AND } Y_n \text{ is } B_n$$

where $X = (X_1, X_2, \ldots, X_m)$ and $Y = (Y_1, Y_2, \ldots, Y_n)$ are linguistic variables and $(A_1, A_2, \ldots, A_m)$ and $(B_1, B_2, \ldots, B_n)$ their linguistic values. For example

1. IF noise level is “high” AND type of task is “complex” AND exposure time is “long”
   THEN degradation in human performance is “very high”.

2. IF noise level is “medium” AND type of task is “simple” AND exposure time is “medium”
   THEN degradation in human performance is “negligible”.

The above general fuzzy rule structure corresponds to the multi input and multi output. For simplicity the system models considered for practical purposes are multi input and single output. In this situation the $i^{th}$ fuzzy rule can be represented with the following structure:

$$R_i : \text{IF } X_1 \text{ is } A_{i1} \text{ AND } X_2 \text{ is } A_{i2} \text{ AND } \ldots \text{ AND } X_m \text{ is } A_{im}, \text{ THEN } Y \text{ is } B_i$$
In each rule $i$ ($i = 1, 2, \ldots, m$), the input membership functions $A_{ij}$ ($j = 1, 2, \ldots, r$) are aggregated via AND connection, which is expressed by a triangular norm operator. In the present noise pollution model we have taken $m = 3$ and $n = 1$. That is for each output only three inputs will be considered at one time.

The various effects of noise pollution on human beings produced by corresponding input parameters are described below in IF-THEN rule forms with their associated schematic diagrams.

i) **Human Performance**

\[ R_i : \text{IF } X_1 \text{ is } A_{1i} \text{ AND } X_2 \text{ is } A_{12} \text{ AND } X_3 \text{ is } A_{13} \text{ THEN } Y_1 \text{ is } B_i \]

![Figure 5.3(a) Input-output diagram for human performance](image)

ii) **Hearing Loss**

\[ R_i : \text{IF } X_1 \text{ is } A_{11} \text{ AND } X_2 \text{ is } A_{12} \text{ AND } X_5 \text{ is } A_{15} \text{ THEN } Y_2 \text{ is } B_i \]

![Figure 5.3(b) Input-output diagram for hearing loss](image)
iii) Sleep

\[ R_i : \text{IF } X_1 \text{ is } A_{11} \text{ AND } X_4 \text{ is } A_{14} \text{ AND } X_3 \text{ is } A_{13} \text{ THEN } Y_3 \text{ is } B_i \]

![Schematic diagram for the effects on sleep](image)

Figure 5.3(c) Schematic diagram for the effects on sleep

iv) Annoyance

\[ R_i : \text{IF } X_1 \text{ is } A_{11} \text{ AND } X_4 \text{ is } A_{14} \text{ AND } X_6 \text{ is } A_{16} \text{ THEN } Y_4 \text{ is } B_i \]

![Schematic diagram for the effects on annoyance](image)

Figure 5.3(d) Schematic diagram for the effects on annoyance

v) Health

\[ R_i : \text{IF } X_1 \text{ is } A_{11} \text{ AND } X_2 \text{ is } A_{12} \text{ AND } X_3 \text{ is } A_{13} \text{ THEN } Y_5 \text{ is } B_i \]

![Schematic diagram for the effects on health](image)

Figure 5.3(e) Schematic diagram for the effects on health
vi) Speech Interference

\[ R_i : \text{IF } X_1 \text{ is } A_{i1} \text{ AND } X_2 \text{ is } A_{i2} \text{ AND } X_7 \text{ is } A_{i7} \text{ THEN } Y_6 \text{ is } B_i \]

Figure 5.3(e) Schematic diagram for the effects on speech interference

5.3.2 TSK Model

The direct approach fuzzy modelling has some inherent limitations. The main drawbacks of this approach are subjectivity and lack of generalization and dependence on expert’s knowledge that sometimes could be faulty. In search for more objectivity in constructing fuzzy models, scientists and engineers tried to develop more formal techniques that could use available data to augment human knowledge or even generate new knowledge. Therefore, the second direction in the development of fuzzy models is based on the use of input-output data. Sugeno and Yasukawa [227] proposed an approach for deriving linguistic rules from fuzzy IF-THEN rules with fuzzy sets in consequent parts. A fuzzy model, proposed by Takagi and Sugeno [232], is described by fuzzy IF-THEN rules which represent linear input-output relations of a system. This fuzzy model has the following form [142]:

Rule \( i \): IF \( X_1 \) is \( A_{i1} \) AND \( X_2 \) is \( A_{i2} \) AND ...... AND \( X_m \) is \( A_{im} \)

THEN \( Y = b_i \quad i = 1, 2, ..., m \)

where \( b_i \) are constants (singleton consequents). If the numerical values of \( X_1, X_2, \ldots, X_m \) are \( u_{i1}, u_{i2}, \ldots, u_{im} \), respectively, and the grades of membership of \( u_{i1}, u_{i2}, \ldots, u_{im} \), in \( A_{i1}, A_{i2}, \ldots, A_{im} \) are \( \mu_{i1}(u_{i1}), \mu_{i2}(u_{i2}), \ldots, \mu_{im}(u_{im}) \), then the combined degree to which the input \( n \)-tuple \( X(u_{i1}, u_{i2}, \ldots, u_{im}) \) matches the antecedents is taken to be the product
Then, defining the normalized weight $w_i$ as

$$w_i = m_i / (m_1 + m_2 + \ldots + m_m)$$

The output is expressed as

$$Y = \sum w_i p_i$$

The significant point in this architecture is that there is no defuzzifier because the inputs $X_1, X_2, \ldots, X_m$ are assumed to be singletons. In the application of gradient programming to this architecture, the membership functions of $A_{i1}, A_{i2}, \ldots, A_{im}$ are assumed to be triangular, trapezoidal, or Gaussian in form. Then, using backward iteration, the values of membership function parameters are computed from right to left [255, 256]. In this way, from the knowledge of I/O pairs we can compute the values of parameters and thereby induce the rules from observations.

### 5.4 Fuzzy Reasoning Algorithms

Fuzzy logic has been developed to provide decision-making capabilities in the presence of uncertainty [257–261]. Its structure is rule-based. The uncertainty in statements and conditions is modelled by possibility distributions. The antecedent clause, the consequent clause or both, may be represented as possibility distributions. A system of inference, called approximate reasoning, has been developed to make deductions from statements expressed in terms of possibility distributions. It is shown in [259] how the concept of a possibility distribution provides a basis for the representation of the meaning of propositions expressed in a natural language. Thus, fuzzy premises are translated into expressions in a language PRUF (“possibilistic relational universal fuzzy”) to which the rules of inference associated with this language can be applied. The entire approach to reasoning in fuzzy logic is linguistic. As claimed by Zadeh [258], “Informally by approximate, or equivalently, fuzzy reasoning we mean the process or processes by which a possibly imprecise conclusion is deduced from a collection of imprecise premises. Such reasoning is, for the most part, qualitative rather than quantitative in nature…” Schematically, the process of reasoning here is shown in Figure 5.4 [262].
The fuzzy reasoning method is the kernel of a fuzzy rule-based system, which employs fuzzy If-THEN rules from the rule base to infer the output by a fuzzy reasoning method. In short, the main issues of fuzzy modelling are about the reasoning mechanisms, i.e.: 1) the way information is implied in each rule; 2) the way it is aggregated among a set of rules; 3) the way new information is inferred from the set of rules; and 4) the way fuzzy inferred values are translated into their crisp correspondences.

For multi input and single output, the $i^{th}$ fuzzy rule is represented as:

$$P_i: \text{IF } X_1 \text{ is } A_{i1} \text{ AND } X_2 \text{ is } A_{i2} \text{ AND } \ldots \text{ AND } X_m \text{ is } A_{im} \text{ THEN } Y \text{ is } B_i$$

Each rule $i$ ($i = 1, 2, \ldots, m$), is associated with fuzzy relation $R_i$, treated as a fuzzy intersection of the antecedent and consequent fuzzy sets:

$$R_i = A_{i1} \cap A_{i2} \cap \ldots \cap A_{im} \cap B_i$$

Fuzzy relation $R_i$ is defined on the Cartesian product space $X_1 \times X_2 \times \ldots \times X_m \times Y$ and its joint possibility distribution is:

$$\Pi(x | y) = R_i$$

The possibility distribution of fuzzy relation $R_i$ is generally represented in terms of its membership function as

$$R_i(x_1, x_2, \ldots, x_r, y) = A_{i1}(x_1) \land A_{i2}(x_2) \land \ldots \land A_{ir}(x_r) \land Y_i(y)$$

where $A_{i1}(x_1), A_{i2}(x_2), \ldots, A_{ir}(x_r),$ and $B_i(y)$ are the membership functions of $X_1, X_2, \ldots, X_m,$ and $Y$ respectively.
Fuzzy relations $R_i$, $i=(1, m)$, associated with the individual rules are aggregated using fuzzy union, resulting in the fuzzy relation $R$ associated with the multi input and single output:

$$R = \bigcup_i R_i = \bigcup_i (A_{i1} \cap A_{i2} \cap \ldots \cap A_{ir} \cap B_i)$$

The joint possibility distribution of the fuzzy relation $R$ is:

$$R(x_1, \ldots, x_r, y) = \bigvee_{i=1}^m R_i(x_1, \ldots, x_r, y) = \bigvee_{i=1}^m A_{i1}(x_1) \land A_{i2}(x_2) \land \ldots \land A_{ir}(x_r) \land B_i(y)$$

The fuzzy output $F$, inferred for a given input fuzzy sets $X_1 = C_1 , X_2 = C_2 , \ldots , X_r = C_r$ is obtained by extending the max-min rule of inference to the case of $r$ inputs:

$$F = (C_1, C_2, \ldots, C_r) \circ R$$

The membership function of the inferred fuzzy set $F$:

$$F(y) = \bigvee_{x_1, \ldots, x_r} \left[ C_1(x_1) \land C_2(x_2) \land \ldots \land C_r(x_r) \land R(x_1, x_2, \ldots, x_r, y) \right]$$

$$= \bigvee_{x_1, \ldots, x_r} \left[ \bigvee_{i=1}^m C_1(x_1) \land C_2(x_2) \land \ldots \land C_r(x_r) \land R_i(x_1, x_2, \ldots, x_r, y) \right]$$

$$= \bigvee_{i=1}^m \left[ \bigvee_{x_1, \ldots, x_r} C_1(x_1) \land C_2(x_2) \ldots \land C_r(x_r) \land A_{i1}(x_1) \land \ldots \land A_{ir}(x_r) \land B_i(y) \right]$$

$$= \bigvee_{i=1}^m \left( \bigvee_{x_1} [A_{i1}(x_1) \land C_1(x_1)] \land \ldots \land \left( \bigvee_{x_r} [A_{ir}(x_r) \land C_r(x_r)] \right) \land B_i(y) \right)$$

$$= \bigvee_{i=1}^m \left[ \text{Poss}[A_{i1} | C_1] \land \ldots \land \text{Poss}[A_{ir} | C_r] \land B_i(y) \right]$$

$$= \bigvee_{i=1}^m \tau_i \land B_i(y)$$

where $\tau_i$, $i=(1, m)$ denotes the DOF of the $i$th rule:

$$\tau_i = \text{Poss}[A_{i1} | C_1] \land \ldots \land \text{Poss}[A_{ir} | C_r]$$

$$= \bigvee_{x_1} [A_{i1}(x_1) \land C_1(x_1)] \land \ldots \land \left( \bigvee_{x_r} [A_{ir}(x_r) \land C_r(x_r)] \right) \land B_i(y)$$

Algorithm:

1. For each rule:

   Calculate the DOF, the $\tau_i$'s, of the rule:

   $$\tau_i = \left( \bigvee_{x_1} [A_{i1}(x_1) \land C_1(x_1)] \land \ldots \land \left( \bigvee_{x_r} [A_{ir}(x_r) \land C_r(x_r)] \right) \right)$$

   find the fuzzy set $F_i$ inferred by the $i$th rule:

   $$F_i(y) = \tau_i \land B_i(y)$$
2. Aggregate the inferred fuzzy sets \( F_i \) by using the \( \max \) operation:

\[
F(y) = \bigvee_{i=1}^{m} F_i(y)
\]

Figure 5.5 shows a block-diagram of the internal structure of multi input and single output based on the use of Mamdani reasoning method.

![Block-diagram of multi input and single output](image)

In the present study, three input variables are considered to give single output. Therefore, this architecture can be modified for different outputs (\( Y_1 \)—\( Y_6 \)) with three out of six inputs (\( X_1 \)—\( X_6 \)) according to the structures of input-output relations as shown in Figures 5.5(a)—5.5(e) respectively. For example, in order to observe the effects of noise level, exposure time, and type of tasks on human performance, the inputs will be \( X_1, X_2, \) and \( X_3 \) and \( F = Y_1 \). Similarly we can obtain different outputs depending on the selection of inputs.