Chapter 5

Results
5.1 Introduction

In the present chapter, results pertaining to simulation study of the closed loop performance of the EHAS have been presented. In addition experimental results with the real-time system have also been discussed. The sliding mode controllers – both 1-SMC and 2-SMC – used in this investigation, have been augmented with bias voltages to compensate for the hard nonlinearities along with PI control within a boundary layer near the regulation demand to remove control chatter.

For the simulation analysis, the plant has been represented by the nonlinear mathematical model of the electro-hydraulic actuation system – as formulated in Chapter 3 has been used. This model has been encoded in MATLAB-SIMULINK. The plant model consists of modules which represent the sub-system characterization of the actual experimental hardware. All the oil properties and physical sizes of the components are consistent with the actual system used during experimentation.

Detail parametric studies of each of the controllers have been carried out to arrive at the suitable parameter choice in the simulation study for a regulation demand. Effects of uncertainties associated with the friction model have been studied. Both symmetric and asymmetric cylinders with different friction hysteresis characteristics have been explored. Simulation studies have been carried out for sinusoidal tracking demands. Robustness of the present control schemes in the face of load disturbances has been studied. Comparisons of the performances with 1-SMC and 2-SMC have been presented. Experimental results using a 1-SMC with bias voltage and PI-control and a 2-SMC with bias voltage and I-control have also been presented.

5.2 Simulation

The numerical simulations have been carried out in Matlab-Simulink using the Dormand-Prince ode45 variable time step solver with the maximum time step size of $10^{-4}$ s. The simulation block diagram has been shown in Fig.5.1 which consists of the following modules – interconnected to execute the closed loop performance simulation of the EHAS:
The Demand Block which generates the demand signal. For Figs. 5.2 – 5.17, the position demand is constant at 0.12m with the initial position of the cylinder at 0.1m. The velocity and acceleration demands are both zero for this case of regulatory control. For Fig. 5.18, a tracking position demand comprising of a rise phase with a velocity of +0.02m/s for 1s followed by a hold phase at a position of 0.12m for 2s and subsequently a falling phase with a velocity of -0.02m/s for the next 1s is used.

The Error Calculator which calculates the state errors from the demands and the state feedbacks using Eqs. 4.1a to c.

The Controller which generates the control signal \( u \) based on the state errors using Eqs. 4.5, 4.6 and 4.9 for 1-SMC and Eqs. 4.10, 4.12a to c for 2-SMC.

The Voltage Extraction Block which calculates the control voltage \( e \) from the control signal \( u \) based on the Eqs. 3.7a, 3.7b, 3.14a to c and 4.25a to 4.29b.

Corresponding to a given set of demands \( \{ y_d(t), v_d(t), a_d(t) \} \), the output of the simulation model are \( \{ y(t), v(t), a(t) \} \) with control voltage \( e(t) \). All the individual blocks in Fig. 5.1 have been illustrated in detail in Appendix I. The values of the different geometric sizes, oil properties, leakage coefficient, dynamic parameters of the proportional valve, pressure conditions, moving mass of the cylinders, oil volumes and spring stiffness have been chosen according to the hardware specifications of the actual experimental system. These values are listed in Table 5.1. The friction parameter values, both for plant model and controller model have already been presented in Chapter 3 in Tables 3.1 – 3.2 and Eqs. 3.14d to 3.14f. The controller parameters have been discussed along with corresponding results.
Figs. 5.2 to 5.14 pertain to 1-SMC simulation for identifying the appropriate parameter settings, many of which are retained also for the 2-SMC study. Fig. 5.15 presents a simple 2-SMC analysis. While bulk of the results corresponds to the double-rod cylinder \( D \), Figs. 5.12 and 5.13 also involve the single-rod cylinder \( S \). In the controller, the parameter values of the simple friction model given by Eq. 3.14e have been replaced by Eq. 3.14f for studying the controller robustness in tackling uncertainties. This combination of high static friction and negative viscosity coefficient is a good representation for only very low piston speed regime. The SMC parameter values of 1200m/s\(^2\) and 3s\(^{-1}\) for \( \alpha_c \) and \( s_y \) respectively have been used in Figs. 5.2 to 5.7 and 5.9. Biases of ±2V have been used in all the cases, unless explicitly mentioned to be otherwise.

<table>
<thead>
<tr>
<th>Table 5.1: Simulation Data</th>
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<tr>
<td><strong>Description</strong></td>
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<tr>
<td>Tank pressure ( (P_T) )</td>
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<tr>
<td>Threshold voltage ( (e_0) )</td>
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<tr>
<td>Proportional valve natural frequency ( (\omega) )</td>
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<td>Proportional valve damping ratio ( (\zeta) )</td>
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<tr>
<td>Proportional valve leakage coefficient ( (C_l) )</td>
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<tr>
<td>Oil volumes on either side of actuator disc when piston is in null position ( (V_{10} \text{ and } V_{20}) )</td>
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<tr>
<td>Piston pressure bearing areas of symmetric actuator ( (A_{a1} \text{ and } A_{a2}) )</td>
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<tr>
<td>Piston pressure bearing area on the left side of asymmetric actuator ( (A_{a1}) )</td>
</tr>
<tr>
<td>Piston pressure bearing area on the right side of asymmetric actuator ( (A_{a2}) )</td>
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<tr>
<td>Oil compressibility ( (\beta) )</td>
</tr>
<tr>
<td>Spring stiffness</td>
</tr>
</tbody>
</table>

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5.3 Parametric Study of 1-SMC

Fig. 5.2 demonstrates the effectiveness of the PI switching within the boundary layer in mitigating the stick-slip problem, which is a sequence of a sudden extension or retraction, a hold up followed by another sudden extension or retraction. Over the duration of the study, shown in Fig. 5.2(a), the PI switching predicts only two such occurrences. One takes place at $t = 0.57s$ and another at $t = 8.5s$. The withdrawal of this switching exhibits dramatically different response. Two such cases without PI switching have been investigated. In one, a negative bias of $-2.0V$ has been employed and the other has a non-default setting of $-2.5V$.

The closed loop performance without PI and $-2.0V$ bias is presented next in Figs. 5.3 to 5.6, each in the form of a set (a) to (d). In order to appreciate the severe nonlinear effects of the cylinder friction and the valve deadband, different phases of the dynamics have been presented in these figures in a sequential manner. Set (a) shows the piston position dynamics, Set (b) the variation of the control voltage to the proportional valve, Set (c) the dynamics of velocity and acceleration of the piston and Set (d) the variation of the forces acting on the piston along with the sliding mode control variable $u_v$. Different forces presented in Set (d) include the pressure force $F_p$, the force $F_k$ of the compression spring, the net external force $F_n = F_p - F_k$ and the resistive friction $F_f$. Certain insets have been included in Fig. 5.3 in order to pay attention to some critical issues.

As long as the piston motion is not initiated, the voltage remains constant in each case shown in Fig. 5.2(b), one of which is included also in Fig. 5.3(b). This constant value comprises of the perturbation voltage and the same force compensating term corresponding to friction through Eqs. 4.26c and 4.27, respectively. The net force remains equal to friction up to the instant corresponding to $A_1$ in the first inset of Fig. 5.3(d). At $A_1$, the friction equals the stiction value of 911N given in Table 3.1. It is understandable that Figs. 5.2(a), 5.3(a) and (c) show the cylinder to remain stationary till then. Eq. 4.25a accordingly calculates the speed-proportional SMC voltage as zero. So up to $A_1$, the effective control voltage remains constant.

The first inset of Fig. 5.3(d) shows that from $A_1$ onward the net driving force $F_n$ exceeds the stiction limit $F_f$ for some time. Hence, the piston starts accelerating and the partially lubricated Strubeck friction [Sarkar et al. (2013)] starts decreasing with increase in the
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velocity of the extending piston. This continues till $B_1$, after which the friction increases with velocity as a result of the change over to the fully lubricated viscous friction regime [Sarkar et al. (2013)]. Correspondingly, the acceleration reaches a maximum at $t = 0.0764s$ due to the large force gap corresponding to $B_1$ and $C_1$. Of course, the piston acceleration then starts decreasing.

Fig. 5.2: Effect of PI and negative bias voltages to 1-SMC on (a) system response and (b) control voltage.

Once the piston crosses a certain velocity, the negative velocity error, starts dominating over the positive position error. Consequently, $S_1$ in Eq. 4.5 undergoes a sign reversal. This happens at $t = 0.0766s$ marked by $C_1$ in Fig. 5.3(d). Eq. 4.9 results in a corresponding change in $u_v$ from $-\alpha_c$ to $+\alpha_c$. This change causes the force function $F$ in Eq. 4.25b to increase, resulting in sudden drops in both the SMC contribution and the overall voltages. This is seen in the inset of Fig. 5.3(b). The drop in the voltage eventually causes the pressure force to decrease as well with a delay, induced by the oil compressibility. This leads to a sharp reduction of acceleration due to the decreasing net force and the increasing friction. When the net force falls below the friction, the velocity begins to decrease. At this changeover point from an accelerating piston to a decelerating piston, the variation of the friction reveals the
hysteresis effect by a sudden decrease in its value. The consequent reduction in the velocity is reflected as a decrease of $e_{SMC}$ and the overall control voltage at $D_1$ in Fig. 5.3(b).

Between $D_1$ and $E_1$ in Fig. 5.3(b), there is a decelerating phase and correspondingly reducing control voltage. The velocity still remains positive causing the piston to extend. At $E_1$, the piston crosses the set position demand, resulting in a reversal in the sign of $y_e$. This instantly changes the voltage from a positive value to a negative value. In each of the two cases without the PI, a sudden voltage reversal can be inferred from Fig. 5.2(b) as well. Instantly, the bias voltages change from the same positive limit to the respective negative limits. This causes the pressure force and consequently the net force on the piston to decrease below the friction force. The second inset in Fig. 5.3(d) makes this evident. Hence the piston decelerates, causing the velocity to fall sharply. In course of this decreasing phase, the piston velocity falls below the slip value of 0.005m/s and instantly becomes stationary at $F_1$ in Fig. 5.3(c). Fig. 5.3(b) depicts the control voltage to become gradually less negative, as the SMC part tapers of with reduction in the velocity.

For the case without the PI and $-2.0V$ negative bias, the position response in Fig. 5.2(a) subsequent to the voltage reversal exhibits a hold up, a long period of extending overshoot halted by another distinct hold phase and then a retraction to near the regulation demand. This is followed by periodic stick-slip oscillations of smaller amplitude with linked voltage reversals. The interval between these successive stick-slip events about the regulation is the least in case of $-2.5V$ negative bias. For this case, the extended overshoot problem is seen as resolved at the cost of higher-frequency stick-slip oscillations.

Figs. 5.3 to and 5.6 pertain to the case of $-2.0V$ negative bias that is comparable with the deadband. Clearly, the voltage is maintained near the negative threshold beyond $F_1$ in Fig. 5.3(b) and the entire duration corresponding to Fig. 5.4(b). If the voltage variation remains close to the threshold value, the piston dynamics is of course dominated by the flow leakage through the valve. The leakage is seen to give rise to mildly increasing driving pressure force along with a sudden but small drop between $E_1$ and $F_1$ in Fig. 5.3(d). The sudden drop can be explained by the friction characteristics depicted in the second inset in Fig. 5.3(d). Clearly, the friction shows a nonlinear variation with piston velocity and hysteresis with motion undergoing changes between acceleration and deceleration.
Fig. 5.3: Performance of 1-SMC, no PI controller, $e_{s0} = -2V$ – (a) piston position, (b) control voltage, (c) piston velocity and acceleration and (d) forces and the sliding-mode control for $t = 0.025s$ to $0.565s$
Fig. 5.4: Performance of 1-SMC, no PI controller and $e_{ba} = -2V$ in terms of (a) piston position, (b) control voltage, (c) piston velocity and acceleration and (d) forces and $u_t$ for the time interval $t = 0.565$ to 4.32s
Fig. 5.5: Performance of 1-SMC, no PI controller and $e_{in} = -2V$ – (a) piston position, (b) control voltage, (c) piston velocity and acceleration and (d) forces and sliding mode control for $t = 4.32$ to $4.8s$. 

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Fig. 5.6: Performance of 1-SMC, no PI controller and $e_{bn} = -2V$ – (a) piston position; (b) control voltage; (c) piston velocity and acceleration and (d) forces and $u$, for $t = 4.8$ to 6s.

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Fig. 5.4(d) show the friction to rise just above and fall just below the net external force holding a nearly constant value over a significant period. This rise and fall in friction arises due to the hysteresis and the effective force on the piston causes alternate stoppage and resumption of the extension, as captured in Figs. 5.2(a) and 5.4(a) during the early part of the overshooting piston extension. As piston speed falls below the slip value of 0.005m/s, the motion ceases. This switches the friction from a higher value corresponding to an accelerating piston to a lower value of decelerating piston, causing the net force to exceed the friction yet again. Therefore, the piston is seen to pass through phases of intermittent extension and rest. Since the overall velocity beyond the stiction zone is always positive, the piston exhibits increasing displacement interspersed with constant position zones, as seen in Fig. 5.4(a). The velocity oscillations also lead to fluctuations in the SMC contribution of the control voltage. In Fig. 5.4(b), the consequent voltage oscillation is apparent.

During the overshooting extension, the spring force keeps on increasing. At a point, the spring force becomes strong enough to cause reduction in the effective pressure force through an alteration in the leakage flow through the valve. This continues for a long time due to the large deadband of the valve, keeping the piston stationary until the net force overcomes the friction against piston retraction. Fig. 5.5 details the behaviour of this retraction dynamics. This motion begins, when the retracting net force crosses the stiction limit of -600N, as given in Table 3.1. The effective voltage also becomes more negative owing to the negative contribution of the SMC part.

Soon, $S_1$ becomes positive due to the domination of the positive velocity error in Eq. 4.5 and $u_v$ becomes negative. This sudden sign reversal of $u_v$ leads to an increase in the control voltage through an augmentation of the SMC contribution. Correspondingly, the pressure force increases making the net force to surpass the friction. This is manifested as spikes of positive accelerations in Fig. 5.5(c). These accelerations cause the piston velocity to fall within the stiction zone and the piston is intermittently brought to rest. Once this happens, the friction force also jumps to a less negative value due to the hysteresis and the piston retraction is resumed. Hence, the piston approaches the set position through intermittent hold phases. When the position of the piston reaches the demand, the bias voltage becomes +2V. The overall voltage shows a brief positive spike due to reversal of sign of SMC part. Soon as the velocity falls within the slip limit and becomes zero. The control voltage depicted in Fig.5.5 (b) settles to a constant value comprising of +2V and $e_{d0}$ given by Eq. 4.26c.
Fig. 5.6 details a stick-slip cycle around the regulation demand. Fig. 5.2(a) shows more than one such cycle following the long overshoot phase for the –2.0V negative bias without PI. Of course, interaction between the leakage flow through the valve and the nonlinear cylinder friction is responsible for such a cycle. As long as the voltage in Fig. 5.6(b) is maintained close to the threshold level, the stick-slip feature is evident from Fig. 5.6(a) and Figs. 5.6(c) and (d) together make the nonlinear variation of friction with piston velocity quite apparent. In case of the bias of –2.5V, the valve is forced to work beyond its deadband region. As a result, the flow does not show any leakage dominated effect. This explains the elimination of the long overshoot phase and appearance of a high-frequency stick-slip oscillations coupled with voltage reversals observed in Fig. 5.2 for this case.

From Fig. 5.2(b), it is quite evident that the near-regulation PI controller switching through Eqs. 4.29a and b makes the frequency of the voltage reversal by far the lowest. The nature of the voltage variation between two reversals is consistent with the PI controller structure. For instance, at the entry to the near-regulation zone boundary layer with the switched PI control, the voltage shows a large step increase induced by the proportional gain term. During the period of stationary and moving pistons, the integral gain contributes respectively linear and higher-order changes of the voltage with time. Since the frequency of the voltage discontinuity associated with the proportional gain is lower, the corresponding stick-slip events show a significant reduction in frequency. The PI dynamics is now described in greater detail with the help of phase-plane diagram in Fig. 5.7.

Fig. 5.7 presents the PI dynamics in four different time intervals. Fig. 5.7(a) shows the early part beginning from the switching function axis and moving through the fourth quadrant with multiple slope changes. These changes are due to the use of the nonlinear friction model of Eqs. 3.11a and 3.11b under acceleration. Later on, the trajectory shifts to a slow decelerating phase that is evident from the rising trajectory in its approach to the third quadrant and the continuation through it. Upon entering the second quadrant, the trajectory in Fig. 5.7(a) follows a loop towards both the axes. This indicates a phase of decelerated piston extension stopping with a small overshoot above the demand. Fig. 5.7(a) shows a subsequent away-origin shift to a more negative value dominantly along the switching function axis, continuation through the left-half plane and almost reaching the origin by a sloping line. These observations can be correlated with Fig. 5.2 providing better understanding of the control model.
A sudden increase in the overshoot through a slip caused by the fluctuating pressure can be the reason of the away-origin shift. In Fig. 5.2(c), this slip is evident after which the overshoot remains held as almost the same constant for both the cases of with and without PI control. In Fig. 5.2(d), the extending slip is revealed as the voltage becoming more negative at a faster rate. Fig. 5.7(a) captures the slip and the stick respectively as an acceleration causing trajectory migration into the third quadrant and a deceleration causing the trajectory to turn back towards the second quadrant. The outer loop in the second quadrant and the approach towards the origin is linked to the retracting slip evident in Fig. 5.2(a). Figs. 5.7(b) and (c) show the trajectory as cluster of lines. In Fig. 5.7(b), this phase proceeds to very near the origin with reduced amplitude of the clustering lines migrating into the right-half plane. This implies that the piston becomes stationary at a very small undershoot.

While Fig. 5.2(a) shows the long duration of the piston remaining stationary, Fig. 5.2(b) exhibits continuous negative to positive increase of voltage due to the integral contribution. The set of closely packed lines in Fig. 5.7(c) could be attributed to both hysteresis and numerical-error induced bursts of acceleration and deceleration. Of course, the observed loops in Fig. 5.7(d) are associated with the stick-slip event within the time frame of the figure that is apparent in Fig. 5.2(a) corresponding to the scheme with PI switching.
The proposed scheme with PI gains involves discrete voltage jumps. At the time of crossing a boundary layer edge shown in Fig. 5.8, a voltage discontinuity occurs due to the changeover between the bias voltage and the proportional voltage contribution. These arise due to the forms of Eqs. 4.27 and 4.28. Across the demand line, the voltage extraction form contributes another voltage discontinuity that is proportional to the piston velocity. Motion-reversal induced change in the form of friction also contributes to this jump. Besides the voltage discontinuities at the edges of the boundary layer and the demand line, another discontinuity arises due to the trajectory shifting between the positive and negative $S_iS_1$ quadrants. Its role is explained in terms of the performances depicted for different $s_y$ values with 1mm boundary layer thickness in Fig. 5.8 and for different boundary layer thicknesses with $s_y$ equal to $3s^{-1}$ in Fig. 5.9.
For a positive step demand, a higher value of $s_y$ in the switching function defined by Eq. 4.5 extends continuation of the trajectory in the fourth quadrant. As a result, the voltage reduction due to the change of force function from Eq. 4.32a and 4.32b through the sign change of the switching function in Eq. 4.9 gets delayed. Hence with increase in the $s_y$ values, faster piston velocity after the motion onset and higher rise of the voltage are evident in Fig. 5.8. Moreover beyond a particular value of $s_y$, the quadrant-change induced voltage jump takes place in between the jumps across a boundary-layer edge and the demand line. A closer occurrence of all the voltage jumps under the situation makes the underlying pressure fluctuation leading to discrete stick-slip events relatively more pronounced. This explains the high frequency of occurrence of stick-slip events and consequent voltage jumps observed in Fig. 5.8 corresponding to $s_y$ equal to 50s$^{-1}$.

The boundary layer broadening makes the jump decrease in the voltage due to the quadrant change to follow the jump increase at the boundary layer edge induced by the proportional gain. Consequent to this reversal of the sequence, the rate of the piston approaching the
demand line is increased and the voltage to the proportional valve grows faster. These are evident from Figs. 5.9(a) and (b).

The eventual voltage reversal at the demand-line crossing, occurring with a large step change, arrests the motion immediately, thereby invoking a high-amplitude pressure oscillation. This explains the long overshoot phase observed in Fig. 5.9 for the widest boundary layer. For the problem at hand, the thinnest boundary layer used in the figure can be seen as the optimum choice. The observed variations of stationary piston position and constant valve voltage in Figs. 5.9(a) and (b) respectively reveal the achievement of the target demand. Fig. 5.10 shows that higher values of $\alpha_c$ to be a better choice for an early realization of low position error.

![Figure 5.10: Effect of $\alpha_c$ to 1-SMC on (a) system response and (b) control voltage](image)

**5.4 1-SMC Performance with Uncertainties in Friction Model**

Fig. 5.11 compares the performance of the two friction models for the double-rod cylinder given respectively by Eqs. 3.14e and 3.14f. These two models have been differentiated in the figure by positive and negative values of friction coefficients. The predicted response and
voltage variations are very close to each other. In case of the positive friction coefficient, a relatively large overshoot has been predicted. This can be attributed to the use of Eq. 3.14e though for the lower velocity regime including the static piston case Eq. 3.14f is a better model for the nonlinear variation of the friction used for predicting the motion dynamics.

![Graph showing system response and control voltage](image)

**Fig. 5.11:** Effect of friction modelling to 1-SMC on (a) system response and (b) control voltage

Fig. 5.12 shows the predicted performance of the single-rod cylinder. The response does not reveal any significant stick-slip induced hold and motion of the cylinder after the overshoot. However, Fig. 5.11 shows such a feature to be present for the double-rod cylinder. A look at the values of the slip velocities for both the cylinders, listed in Table 3.1, makes the reason for the observed difference quite obvious.

In Fig. 5.13, the switching-function based phase-plots of the two cylinders are shown. As explained earlier in the context of Fig. 5.7(c), the trajectory forming a broad patch in Fig. 5.13(a) about the \( \dot{S} \) axis is due to the near-regulation phase motion dominated by friction hysteresis in case of the double-rod cylinder. Fig. 5.13(b) does not show any prominent hysteresis effect for the single-rod cylinder. This is consistent with the characterization results of these two cylinders discussed by Sarkar et al. (2013).
Fig. 5.12: 1-SMC performance of cylinder S in terms of (a) system response and (b) control voltage

Fig. 5.13: Comparison of 1-SMC performance for Cylinders (a) D with substantial hysteresis and (b) S with low hysteresis
The spiralling trajectory in Fig. 5.13(b) prior to the near-regulation phase motion for the single-rod cylinder is much smoother and wider for the single-rod cylinder. This is the result of its much lower friction, Striebeck and boundary-lubrication parameters evident from Tables 3.1 to 3.3. As a result of the lower friction, the piston responds to the voltage actuation at a faster rate. The consequent higher acceleration and velocity of the piston result in a much wider trajectory.

5.5 Comparison of 1-SMC and 2-SMC Performance

In Figs. 5.14(a) and (b), a comparison of the predicted 1-SMC and 2-SMC performances of the double-rod cylinder has been presented. The SMC parameter $c_\alpha$, the bias settings and the boundary layer thickness in both the cases have been set as $1800 m/s^2$, $\pm 2V$ and $0.001 m$ respectively. In comparison of the PI gain settings of $5500 V/m$ and $3000 V-s/m$ for the 1-SMC case, $5000 V/m$ and $5270 V-s/m$ have been found to yield better performance.

Fig. 5.14: Comparison of 2-SMC and 1-SMC responses – (a) piston position and (b) control voltage.
The improvement is understandable in view of the fact that at the boundary layer edge, a voltage jump arises during the handing over of the control schemes. Of course outside the boundary layer up to the instant of handing over, Fig. 5.10(b) makes it evident that the 1-SMC and 2-SMC voltages remain different. Moreover, the underlying pressure dynamics in both the cases are expected to be different. Of course, the lower proportional gain induces a lower peak voltage in case of the 2-SMC. The prediction of 2-SMC response of piston position in Fig. 5.10(a) is faster but quite similar to that for the 1-SMC. Hence, the final conclusion needs to be drawn from the real-time experiment. The efficacy of the present analysis lies in forming the SMC structure coupled with appropriate PI and bias that would lead to chatter-free performance for a highly nonlinear electrohydraulic system.

5.6 Parametric Study of 2-SMC

The performance of the 2-SMC for $\beta_c$ values of 1 and 10 have been presented in Figs. 5.15(a) to (c). In Fig. 5.15(a), the $S_2 - \dot{S}_2$ response along with $\beta$ – parabolas, as given by Eq. 4.20, for both the cases have been shown. Initially, where the two trajectories are overlapping (Fig.5.15(a)), the velocities of the cylinder are negligible within the stick zone (Fig.5.15(b)) – causing the $e_{SMC}$ to be insignificant, as is seen from Eq. 4.25a. Hence the control voltage is primarily due to the bias voltage. This is evident from the identical voltage values as seen in the region $t \approx 0$ to 0.1s in Fig. 5.15(c). Beyond this region, the $e_{SMC}$ voltages start growing. It is evident from Eq. 4.12a that in the fourth quadrant of the $S_2 - \dot{S}_2$ plot, larger is the value of $\beta_c$, smaller is the value of $u_{i2}$. Consequently, the value of the force function in Eq. 4.25b is also less leading to a higher value of $e_{SMC}$. This is evident from Fig. 5.15(c) in the region $t \approx 0.1$ to 0.2s. Thus in the reaching phase, the controller with $\beta_c$ equal to 10 ensures faster response of the cylinder motion compared to the case with $\beta_c$ equal to 1. This is clearly visible in Fig. 5.15(b).
5.7 Disturbance rejection capability of 2-SMC

Sliding mode controllers are known for their capability of disturbance rejection (Jager 1996, Utkin 1992, Utkin 1977). In the present work the sliding mode controller has been augmented with bias signal to compensate for hard nonlinearities as well as transfer of control authority to a PI controller within a boundary layer near the set point demand. The robustness of the composite control structure has been investigated in Figs. 5.16 and 5.17 for two different cases in the face of different load disturbances, inflicted upon the hydraulic cylinder. In the
first case constant load disturbances of different durations and magnitudes have been applied to the cylinder in the reaching phase i.e., when the cylinder is moving from its initial position towards the set point with some velocity. In the second case, the actuator is residing in its initial position and the robustness of the controller is tested in maintaining this set point in the face of different load disturbances.

Fig. 5.16(a) represents four different load disturbances on the double rod symmetric hydraulic cylinder. The magnitudes and durations of the disturbances are listed in Table 5.2. All the disturbances are injected at \( t = 0.15 \text{s} \). Figs. 5.16(b) and (c) exhibit the corresponding position responses and the control voltages. It can be seen in 5.16(b) that for all the cases, as an effect of the injected disturbance, the actuator deviates from the reaching mode path traced in absence of the disturbance. The deviations, though vary with the magnitude and duration of the impressed disturbance, are only temporary. Both the duration \( (t_{s,d}) \) over which the effect of the disturbance exists and the maximum percentage deviation \( (o_{s,d}) \) from the undisturbed path increases with the amplitude as well as duration of the disturbance. The peak amplitude \( (e_{p,d}) \) of the oscillations sustained by the control voltage to negotiate the disturbance, are entirely dependent on the magnitude of the disturbances, while the duration of control voltage oscillation \( (t_{e,d}) \) due to the disturbance depend on the duration of the disturbance. This study indicates the proposed control structure to be quite robust. Both the duration \( (t_{s,d}) \) over which the effect of the disturbance exists and the maximum percentage deviation \( (o_{s,d}) \) from the undisturbed path increases with the amplitude as well as duration of the disturbance. The peak amplitude \( (e_{p,d}) \) of the oscillations sustained by the control voltage to negotiate the disturbance, are entirely dependent on the magnitude of the disturbances, while the duration of control voltage oscillation \( (t_{e,d}) \) due to the disturbance depend on the duration of the disturbance. These observations are listed in Table 5.2 below. It is seen that the percentage deviation for all the cases are less than ±2.5% indicating low sensitivity to moderate disturbances.

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<tr>
<td>(iii)</td>
<td>+1000</td>
<td>10</td>
<td>+2.2</td>
<td>0.144</td>
<td>4.38</td>
<td>0.072</td>
</tr>
<tr>
<td>(iv)</td>
<td>+600</td>
<td>50</td>
<td>+2.4</td>
<td>0.144</td>
<td>3.76</td>
<td>0.108</td>
</tr>
</tbody>
</table>

Table 5.2: Disturbance and performance parameters
In the second case, the capability of the controller for maintaining a particular set position of cylinder in the face of different disturbances have been checked. The load disturbance amplitudes have been kept above the stick force levels as mentioned in Table 3.1 – below which the system remains insensitive to the disturbances. Constant magnitude loads are applied both for short duration of 10ms as well as a long duration of 0.4s as shown in Fig. 5.17(a). It is seen that even above the stick force levels the net deviation from the set point position is limited to ±0.19% (Fig.5.17(b)) - indicating the invariant nature of the control
structure (Jaeger 1996) to load disturbances. However, this insensitivity in position response is at the cost of a control voltage oscillation of about ±5V as shown in Fig. 5.17(c).

Fig. 5.17: Disturbance rejection by 2-SMC for set point control – (a) load disturbance; (b) piston position and (c) control voltage.
5.8 Performance of 2-SMC for tracking demand

The double rod cylinder is provided with a tracking position demand comprising of a rise phase with a velocity of +0.02m/s for 1s followed by a hold phase at a position of 0.12m for 2s and subsequently a falling phase with a velocity of -0.02m/s for the next 1s. All the control parameters and the same controller structure as used in the set point controller, mentioned in the earlier sections, have been retained and control performance studied. Figs. 5.18 (a) and 5.18 (b) present the position response and the corresponding control voltage. It can be seen that the position response crisscrosses the demand with corresponding oscillations in the control voltage both in the rise and falling phase. In the constant demand phase, the response is consistent with the demand without any voltage oscillation.
5.9 Experimental Results

All the experiments were carried out in the set up illustrated in Fig.2.1. The experiments were carried for a regulation demand of 0.12m piston position from an initial position of 0.1m and initial spring compression of 0.01m. Both the 1-SMC and 2-SMC control structures were used along with the perturbation voltage for the hard nonlinearities and a PI compensation scheme within a boundary layer near the set demand. The controller parameters have been listed in Table 5.3.

<table>
<thead>
<tr>
<th>Description</th>
<th>1-SMC</th>
<th>2-SMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_c$</td>
<td>1800 m/s$^2$</td>
<td>1800 m/s$^2$</td>
</tr>
<tr>
<td>$s_y$</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$e_{hp}$</td>
<td>+2V</td>
<td>+1.9V</td>
</tr>
<tr>
<td>$e_{bm}$</td>
<td>-1.9V</td>
<td>-1.9V</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.001m</td>
<td>0.001m</td>
</tr>
<tr>
<td>$K_P$</td>
<td>2000 V/m</td>
<td>0</td>
</tr>
<tr>
<td>$K_I$</td>
<td>3000 V/m-s</td>
<td>100 V/m-s</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>–</td>
<td>2</td>
</tr>
</tbody>
</table>

A comparison for the 1-SMC case between simulation results of Figs. 5.2(a) and (b) and experimental results of Figs. 5.19 (a) and (b) reveals that the initial hold phase is present for both the cases followed by a rise phase towards the demand. However, the rise time for the experimental result is higher than the simulation result. Also, there is an overshoot in the experimental response. The experimental response shows a small steady state error. The control voltage also exhibits an initial positive spike, followed by a fall and a gradual increase with time for both the cases. However, while in the simulation, the gradual increase of the control voltage is terminated with a sharp negative spike, as the sign of the position error changes, the experimental result shows a much gradual fall. The phase plane plot also indicates that the span of both $S$ and $\dot{S}$ is less in the experimental case. All these discussions are valid in the context of the comparison of the simulation result in Fig. 5.4 and experimental results shown in Figs. 5.20 (a) to (c) for the 2-SMC case as well.
Fig. 5.19: Experimental responses for 1-SMC – (a) piston position; (b) control voltage; (c) $\dot{s}_2 - \dot{s}_1$.

The main cause for the differences in response of the actual hardware with the simulation results lies in the fact that while for simulation, the integration time step has been taken to be $1 \times 10^{-04}$ s, the sampling rate of the actual real time system has been found out to be about $1 \times 10^{-02}$ s. This means the speed at which the controller is fired in the simulation study is 100 times faster than that with which the actual real time system controller acts. This finite delay
in implementation of the control leads to sluggish response and associated response characteristics of the real time system in comparison to the simulation study. However, despite these limitations, it can be seen that design of the controller based on the expertise acquired through detail simulation study has been quite effective in implementation of the real time controller design.

Fig. 5.20: Experimental responses for 2-SMC – (a) piston position; (b) control voltage; (c) $S_S - \dot{S}_S$.
5.10 Summary

In the present work, simulation and experimental investigations on the closed loop performance of the EHAS has been carried out to test the effectiveness of a 1-SMC and a 2-SMC controller along with bias and perturbation voltages – with and without PI switching. The objectives of the study have been to assess the effect the sliding mode controller, the bias voltages and the PI switching on the closed loop performance of the EHAS. Two different values of the negative bias have been tried out with and without PI switching for the 1-SMC controller. Effect of different values of the 1-SMC controller parameters like $s_y$, $a_c$ and 2-SMC controller parameter $\beta_c$, on the performance have been studied. The width of the boundary layer near the set point demand has been varied and its effect studied. To understand the different features of the response, magnified views of different time segments have been presented and discussed. The behavior of the sliding mode controllers have been explained with the phase-plane responses of the system. Robustness of the controller in the face of uncertainties in friction modelling and load disturbances has been explored. Performances of 1-SMC and 2-SMC have been compared. The capability of the 2-SMC for a tracking demand has been presented. Finally, with the expertise obtained through the simulation study, both 1-SMC and 2-SMC were designed and real-time experiments have been conducted.